Irreducible form:

In algebraic expressions, we use the word 'irreducible' in place of 'prime'

$Ex: (i)$ Irreducible' form $of\ 5xy = 5 \times x \times y$

Since $5, x, y$ are cannot be expressed as a product of factors .

(ii) Irreducible form of $3x(x + 2) = 3 \times x \times (x + 2)$

Since $3, x, x + 2$ are cannot be expressed as a product of factors .

(iii) Irreducible form of $10x\ (x\ +\ 2)(y\ +\ 3) = 2\ \times 5\ \times x\ \times (x + 2) \times (y + 3).$

Distributive laws

$(i)\ a \times b + a \times c = a \times (b + c)$

$(ii)\ a \times b - a \times c = a \times (b - c)$

Factorisation of an algebraic expression

Factorisation of an algebraic expression means writing the given expression as a product **of** its factors. These factors can be numbers, variables**,** or an algebraic expression

Method of common factors:

Example 1: Factorise $12a^2b + 15ab^2$

Sol:

$12a^2b$

$= 2 \times 2 \times \underline{3} \times \underline{a} \times a \times \underline{b}$

$= (3 \times a \times b) \times (2 \times 2 \times a)$

$= 3ab \times 4a$

$15ab^2$

$= \underline{3} \times 5 \times \underline{a} \times \underline{b} \times b$

$= (3 \times a \times b) \times (5 \times b)$

$= 3ab \times 5b$

$12a^2b + 15ab^2 = 3ab \times 4a + 3ab \times 5b$

$= 3ab(4a + 5b)$

Example 2: Factorise $10x^2 - 18x^3 + 14x^4$

Sol:

$10x^2$

$= \underline{2} \times 5 \times \underline{x} \times \underline{x}$

$= (2 \times x \times x) \times (5)$

$= 2x^2 \times 5$

$18x^3$

$= \underline{2} \times 3 \times 3 \times \underline{x} \times \underline{x} \times x$

$= (2 \times x \times x) \times (3 \times 3 \times x)$

$= 2x^2 \times 9x$

$14x^4$

$= \underline{2} \times 7 \times \underline{x} \times \underline{x} \times x \times x$

$= (2 \times x \times x) \times (7 \times x \times x)$

$= 2x^2 \times 7x^2$

$10x^2 - 18x^3 + 14x^4 = (2x^2 \times 5) - (2x^2 \times 9x) + (2x^2 \times 7x^2)$

$= 2x^2(5 - 9x + 7x^2)$

$$= 2x^2(7x^2 - 9x + 5)$$

TRY THESE

(i)   **Factorise $12x + 36$**

Sol: $12x + 36$

$$= \left(\underline{2} \times \underline{2} \times \underline{3} \times x\right) + \left(\underline{2} \times \underline{2} \times \underline{3} \times 3\right)$$

$$= 12 \times x + 12 \times 3$$

$$= 12(x + 3)$$

(ii)   **Factorise $22y - 33z$**

Sol : $22y - 33z$

$$= 2 \times \underline{11} \times y - 3 \times \underline{11} \times z$$

$$= 11 \times 2y - 11 \times 3z$$

$$= 11(2y - 3z)$$

(ii)   **Factorise $14pq + 35pqr$**

Sol: $14pq + 35pqr$

$$= \left(2 \times \underline{7} \times \underline{p} \times \underline{q}\right) + \left(5 \times \underline{7} \times \underline{p} \times \underline{q} \times r\right)$$

$$= 7pq \times 2 + 7pq \times 5r$$

$$= 7pq(2 + 5r)$$

Example 3: **Factorise $6xy - 4y + 6 - 9x$.**

Sol: $6xy - 4y + 6 - 9x$

$$= 6xy - 4y - 9x + 6$$

$$= \underline{2y} \times 3x - \underline{2y} \times 2 - \underline{3} \times 3x - (-\underline{3}) \times 2$$

$$= 2y(3x - 2) - 3(3x - 2)$$

$$= (3x - 2)(2y - 3)$$

<div align="center">

EXERCISE 12.1

</div>

1.   **Find the common factors of the given terms.**

(i)   **$12x, 36$**

Sol: $12x = 2 \times 2 \times 3 \times x$

$36 = 2 \times 2 \times 3 \times 3$

Common factors of $12x$,36 are

$2, 2 \times 2, 2 \times 2 \times 3 \Rightarrow 2, 4, 6$

2.   **Factorise the following expressions.**

(i)   **$7x - 42$**

Sol: $7x - 42 = 7 \times x - 7 \times 6 = 7(x - 6)$

(ii) $\quad 6p - 12q$

Sol: $6p - 12q = 6 \times p - 6 \times 2q = 6(p - 2q)$

(iii) $\quad 7a^2 + 14a$

Sol: $7a^2 + 14a$

$\quad = 7a \times a + 7a \times 2$

$\quad = 7a(a + 2)$

(iv) $\quad -16z + 20z^3$

Sol: $-16z + 20z^3$

$\quad = 4z \times (-4) + 4z \times 5z^2$

$\quad = 4z(-4 + 5z^2)$

(v) $\quad 20\,l^2\,m + 30\,a\,l\,m$

Sol: $\quad 20\,l^2\,m + 30\,a\,l\,m$

$\quad = 10lm \times 2l + 10lm \times 3a$

$\quad = 10lm(2l + 3a)$

(vi) $\quad 5\,x^2y - 15\,xy^2$

Sol: $\quad 5\,x^2y - 15\,xy^2$

$\quad = 5xy \times x - 5xy \times 3y$

$\quad = 5xy(x - 3y)$

(vii) $\quad 10\,a^2 - 15\,b^2 + 20\,c^2$

Sol: $\quad 10\,a^2 - 15\,b^2 + 20\,c^2$

$\quad = 5 \times 2a^2 - 5 \times 3b^2 + 5 \times 4c^2$

$\quad = 5(2a^2 - 3b^2 + 4c^2)$

(viii) $\quad -4\,a^2 + 4\,ab - 4\,ca$

Sol: $\quad -4\,a^2 + 4\,ab - 4\,ca$

$\quad = 4a \times (-a) + 4a \times b + 4a \times (-c)$

$\quad = 4a(-a + b - c)$

(ix) $\quad x^2\,y\,z + x\,y^2z + x\,y\,z^2$

Sol: $x^2\,y\,z + x\,y^2z + x\,y\,z^2$

$\quad = xyz \times x + xyz \times y + xyz \times z$

$\quad = xyz(x + y + z)$

(x) $\quad ax^2y + bxy^2 + c\,x\,y\,z$

Sol: $ax^2y + bxy^2 + c\,x\,y\,z$

$= xy \times ax + xy \times by + xy \times cz$

$= xy(ax + by + cz)$

3. Factorise.

(i) $x^2 + x\,y + 8x + 8y$

Sol: $x^2 + x\,y + 8x + 8y$

$= (x \times x + x \times y) + (8 \times x + 8 \times y)$

$= x(x + y) + 8(x + y)$

$= (x + y)(x + 8)$

(ii) $15\,xy - 6x + 5y - 2$

Sol: $15\,xy - 6x + 5y - 2$

$= 3x \times 5y - 3x \times 2 + 1 \times 5y - 1 \times 2$

$= 3x(5y - 2) + 1(5y - 2)$

$= (5y - 2)(3x + 1)$

(iii) $ax + bx - ay - by$

Sol: $ax + bx - ay - by$

$= x \times a + x \times b - y \times a - y \times b$

$= x(a + b) - y(a + b)$

$= (a + b)(x - y)$

(iv) $15\,pq + 15 + 9q + 25p$

Sol: $15\,pq + 15 + 9q + 25p$

$= 15\,pq + 25p + 9q + 15$

$= 5p \times 3q + 5p \times 5 + 3 \times 3q + 3 \times 5$

$= 5p(3q + 5) + 3(3q + 5)$

$= (3q + 5)(5p + 3)$

(v) $z - 7 + 7\,x\,y - x\,y\,z$

Sol: $z - 7 + 7\,x\,y - x\,y\,z$

$= z - 7 - x\,y\,z + 7\,x\,y$

$= 1 \times z - 1 \times 7 - xy \times z + xy \times 7$

$= 1(z - 7) - xy(z - 7)$

$= (z - 7)(1 - xy)$

Factorisation using identities

I. $a^2 + 2ab + b^2 = (a + b)^2 = (a + b)(a + b)$

II. $a^2 - 2ab + b^2 = (a - b)^2 = (a - b)(a - b)$

III. $a^2 - b^2 = (a + b)(a - b)$

IV. $x^2 + (a + b)x + ab = (x + a)(x + b)$

Example 4: Factorise $x^2 + 8x + 16$

Sol: $a^2 + 2ab + b^2 = (a + b)^2$

   $x^2 + 8x + 16$

   $= x^2 + 2 \times x \times 4 + 4^2$

   $= (x + 4)^2$

Example 5: Factorise $4y^2 - 12y + 9$

Sol: $a^2 - 2ab + b^2 = (a - b)^2$

   $4y^2 - 12y + 9$

   $= (2y)^2 - 2 \times 2y \times 3 + (3)^2$

   $= (2y - 3)^2$

Example 6: Factorise $49p^2 - 36$

Sol: $a^2 - b^2 = (a + b)(a - b)$

   $49p^2 - 36 = (7p)^2 - (6)^2$

   $= (7p + 6)(7p - 6)$

Example 7: Factorise $a^2 - 2ab + b^2 - c^2$

Sol: $a^2 - 2ab + b^2 - c^2$

   $= (a^2 - 2ab + b^2) - c^2$

   $= (a - b)^2 - c^2 \ (Identity \ II)$

   $= [(a - b) + c][(a - b) - c] \ (Identity \ III)$

   $= (a - b + c)(a - b - c)$

Example 8: Factorise $m^4 - 256$

Sol: $a^2 - b^2 = (a + b)(a - b)$

   $m^4 - 256 = (m^2)^2 - (16)^2$

   $= (m^2 + 16)(m^2 - 16)$

   $= (m^2 + 16)[m^2 - 4^2]$

   $= (m^2 + 16)(m + 4)(m - 4)$

Example 9: Factorise $x^2 + 5x + 6$

Sol: $x^2 + (a + b)x + ab = (x + a)(x + b)$

$$\boxed{\begin{array}{l} a + b = 2 + 3 = 5 \\ a \times b = 2 \times 3 = 6 \end{array}}$$

$x^2 + 5x + 6$

$= x^2 + (2 + 3)x + 2 \times 3$

$= (x + 2)(x + 3)$

Example 10: Find the factors of $y^2 - 7y + 12$

Sol: $x^2 + (a + b)x + ab = (x + a)(x + b)$

$y^2 - 7y + 12$

$= y^2 + (-3 - 4)y + (-3)(-4)$

$= (y - 3)(y - 4)$

(OR)

$y^2 - 7y + 12 = y^2 - 3y - 4y + 12$

$= y(y - 3) - 4(y - 3)$

$= (y - 3)(y - 4)$

$$\boxed{\begin{aligned} a + b &= -3 - 4 = -7 \\ a \times b &= (-3)(-4) = -12 \end{aligned}}$$

Example 11: Obtain the factors of $z^2 - 4z - 12$

Sol: $x^2 + (a + b)x + ab = (x + a)(x + b)$

$z^2 - 4z - 12 = z^2 + (-6 + 2)z + (-6)(2)$

$= (z - 6)(z + 2)$

Example 12: Find the factors of $3m^2 + 9m + 6$.

Sol: $3m^2 + 9m + 6$

$= 3[m^2 + 3m + 2]$

$= 3[m^2 + (1 + 2)m + 1 \times 2]$

$= 3(m + 1)(m + 2)$

EXERCISE 12.2

1. Factorise the following expressions.

(i) $a^2 + 8a + 16$

Sol: $a^2 + 2ab + b^2 = (a + b)^2$

$a^2 + 8a + 16$

$= a^2 + 2 \times a \times 4 + 4^2$

$= (a + 4)^2$

(ii) $p^2 - 10p + 25$

Sol: $a^2 - 2ab + b^2 = (a - b)^2$

$p^2 - 10p + 25$

$= p^2 - 2 \times p \times 5 + 5^2$

$$= (p - 5)^2$$

(iii) $25m^2 + 30m + 9$

Sol: $a^2 + 2ab + b^2 = (a + b)^2$

$\qquad 25m^2 + 30m + 9$

$\qquad = (5m)^2 + 2 \times 5m \times 3 + (3)^2$

$\qquad = (5m + 3)^2$

(iv) $49y^2 + 84yz + 36z^2$

Sol: $a^2 + 2ab + b^2 = (a + b)^2$

$\qquad 49y^2 + 84yz + 36z^2$

$\qquad = (7y)^2 + 2 \times 7y \times 6z + (6z)^2$

$\qquad = (7y + 6z)^2$

(v) $4x^2 - 8x + 4$

Sol: $a^2 - 2ab + b^2 = (a - b)^2$

$\qquad 4x^2 - 8x + 4$

$\qquad = 4[x^2 - 2x + 1]$

$\qquad = 4[x^2 - 2 \times x \times 1 + 1^2]$

$\qquad = 4(x - 1)^2$

(vi) $121b^2 - 88bc + 16c^2$

Sol: $a^2 - 2ab + b^2 = (a - b)^2$

$\qquad 121b^2 - 88bc + 16c^2$

$\qquad = (11b)^2 - 2 \times 11b \times 4c + (4c)^2$

$\qquad = (11b)^2 - 2 \times 11b \times 4c + (4c)^2$

$\qquad = (11b - 4c)^2$

(vii) $(l + m)^2 - 4lm$

Sol: $(l + m)^2 - 4lm$

$\qquad = (l^2 + 2lm + m^2) - 4lm$

$\qquad = l^2 + 2lm - 4lm + m^2$

$\qquad = l^2 - 2lm + m^2$

$\qquad = (l - m)^2$

(viii)$a^4 + 2a^2 b^2 + b^4$

Sol: $a^4 + 2a^2 b^2 + b^4$

$\qquad = (a^2)^2 + 2 \times a^2 \times b^2 + (b^2)^2$

$$= (a^2 + b^2)^2$$

2. Factorise.

(i)$4p^2 - 9q^2$

Sol: $a^2 - b^2 = (a + b)(a - b)$

$\quad 4p^2 - 9q^2 = (2p)^2 - (3q)^2$

$\quad\quad = (2p + 3q)(2p - 3q)$

(ii) $63a^2 - 112b^2$

Sol: $a^2 - b^2 = (a + b)(a - b)$

$\quad 63a^2 - 112b^2 = 7 \times 9a^2 - 7 \times 16b^2$

$\quad\quad = 7[9a^2 - 16b^2]$

$\quad\quad = 7[(3a)^2 - (4b)^2]$

$\quad\quad = 7(3a + 4b)(3a - 4b)$

(iii) $49x^2 - 36$

Sol: $a^2 - b^2 = (a + b)(a - b)$

$\quad 49x^2 - 36 = (7x)^2 - (6)^2$

$\quad\quad = (7x + 6)(7x - 6)$

(iv) $16x^5 - 144x^3$

Sol: $a^2 - b^2 = (a + b)(a - b)$

$\quad 16x^5 - 144x^3$

$\quad = 16x^3 \times x^2 - 16x^3 \times 9$

$\quad = 16x^3[x^2 - 9]$

$\quad = 16x^3[x^2 - 3^2]$

$\quad = 16x^3(x + 3)(x - 3)$

(v) $(l + m)^2 - (l - m)^2$

Sol: $a^2 - b^2 = (a + b)(a - b)$

$\quad (l + m)^2 - (l - m)^2$

$\quad = [(l + m) + (l - m)][(l + m) - (l - m)]$

$\quad = [l + m + l - m][l + m - l + m]$

$\quad = (2l)(2m)$

$\quad = 4lm$

(vi) $9x^2 y^2 - 16$

Sol: $a^2 - b^2 = (a + b)(a - b)$

$9x^2 y^2 - 16$

$= (3xy)^2 - (4)^2$

$= (3xy + 4)(3xy - 4)$

(vii)$(x^2 - 2xy + y^2) - z^2$

Sol: $(x^2 - 2xy + y^2) - z^2$

$= (x - y)^2 - (z)^2$

$= [(x - y) + z][(x - y) - z]$

$= (x - y + z)(x - y - z)$

(viii)$25a^2 - 4b^2 + 28bc - 49c^2$

Sol: $25a^2 - 4b^2 + 28bc - 49c^2$

$= 25a^2 - (4b^2 - 28bc + 49c^2)$

$= 25a^2 - [(2b)^2 - 2 \times 2b \times 7c + (7c)^2]$

$= (5a)^2 - (2b - 7c)^2$

$= [5a + (2b - 7c)][5a - (2b - 7c)]$

$= (5a + 2b - 7c)(5a - 2b + 7c)$

3. **Factorise the expressions.**

(i)$ax^2 + bx$

Sol: $ax^2 + bx$

$= ax \times x + b \times x$

$= x(ax + b)$

(ii) $7p^2 + 21q^2$

Sol: $7p^2 + 21q^2$

$= 7 \times p^2 + 7 \times 3q^2$

$= 7(p^2 + 3q^2)$

(iii)$2x^3 + 2xy^2 + 2xz^2$

Sol: $2x^3 + 2xy^2 + 2xz^2$

$= 2x \times x^2 + 2x \times y^2 + 2x \times z^2$

$= 2x(x^2 + y^2 + z^2)$

(iv)$am^2 + bm^2 + bn^2 + an^2$

Sol: $am^2 + bm^2 + bn^2 + an^2$

$= m^2(a + b) + n^2(a + b)$

$= (a + b)(m^2 + n^2)$

(v) **(lm + l) + m + 1**

Sol: $(lm + l) + m + 1$

$\qquad = l \times (m + 1) + 1 \times (m + 1)$

$\qquad = (m + 1)(l + 1)$

(vi) **y (y + z) + 9 (y + z)**

Sol: $y (y + z) + 9 (y + z)$

$\qquad = (y + z)(y + 9)$

(vii) **$5y^2 - 20y - 8z + 2yz$**

Sol: $5y^2 - 20y + 2yz - 8z$

$\qquad = (5y \times y - 5y \times 4) + (2z \times y - 2z \times 4)$

$\qquad = 5y(y - 4) + 2z(y - 4)$

$\qquad = (y - 4)(5y + 2z)$

$(viii)$ **10ab + 4a + 5b + 2**

Sol: $10ab + 4a + 5b + 2$

$\qquad = 2a \times 5b + 2a \times 2 + 1 \times 5b + 1 \times 2$

$\qquad = 2a(5b + 2) + 1(5b + 2)$

$\qquad = (5b + 2)(2a + 1)$

(ix) **6xy – 4y + 6 – 9x**

Sol: $6xy - 4y + 6 - 9x$

$\qquad = 6xy - 9x - 4y + 6$

$\qquad = 3x \times 2y - 3x \times 3 - 2 \times 2y + 2 \times 3$

$\qquad = 3x(2y - 3) - 2(2y - 3)$

$\qquad = (2y - 3)(3x - 2)$

4. Factorise.

(i) **$a^4 - b^4$**

Sol: $a^4 - b^4$

$\qquad = (a^2)^2 - (b^2)^2$

$\qquad = (a^2 + b^2)(a^2 - b^2)$

$\qquad = (a^2 + b^2)(a + b)(a - b)$

(ii) **$p^4 - 81$**

Sol: $p^4 - 81$

$\qquad = (p^2)^2 - (9)^2$

$= (p^2 + 9)(p^2 - 9)$

$= (p^2 + 9)(p^2 - 3^2)$

$= (p^2 + 9)(p + 3)(p - 3)$

(iii) $x^4 - (y + z)^4$

Sol: $x^4 - (y + z)^4$

$= (x^2)^2 - [(y + z)^2]^2$

$= [x^2 + (y + z)^2][x^2 - (y + z)^2]$

$= [x^2 + (y + z)^2][x + (y + z)][x - (y + z)]$

$= [x^2 + y^2 + z^2 + 2yz](x + y + z)(x - y - z)$

(v) $a^4 - 2a^2 b^2 + b^4$

Sol: $a^4 - 2a^2 b^2 + b^4$

$= (a^2)^2 + 2 \times a^2 \times b^2 + (b^2)^2$

$= (a^2 - b^2)^2$

$= [(a + b)(a - b)]^2$

$= (a + b)^2 (a - b)^2$

$= (a + b)(a + b)(a - b)(a - b)$

5. *Factorise the following expressions.*

(i) $p^2 + 6p + 8$

Sol: $p^2 + 6p + 8$

$= p^2 + (2 + 4)p + 2 \times 4$

$= (p + 2)(p + 4)$

$$\boxed{\begin{aligned} x^2 + (a + b)x + ab \\ = (x + a)(x + b) \end{aligned}}$$

(ii) $q^2 - 10q + 21$

Sol: $q^2 - 10q + 21$

$= q^2 + (-3 - 7)q + (-3)(-7)$

$= (q - 3)(q - 7)$

$$\boxed{\begin{aligned} x^2 + (a + b)x + ab \\ = (x + a)(x + b) \end{aligned}}$$

(iii) $p^2 + 6p - 16$

Sol: $p^2 + 6p - 16$

$= p^2 + (8 - 2)p + (8)(-2)$

$= (p + 8)(p - 2)$

$$\boxed{\begin{aligned} x^2 + (a + b)x + ab \\ = (x + a)(x + b) \end{aligned}}$$

Division of Algebraic Expressions

Example 13: Do the following divisions.

(i) $-20x^4 \div 10x^2$

Sol: $\dfrac{-20x^4}{10x^2} = \dfrac{-2 \times 2 \times 5 \times x \times x \times x \times x}{2 \times 5 \times x \times x}$

$= -2 \times x \times x = -2x^2$

$(ii) 7x^2 \ y^2 \ z^2 \ \div \ 14xyz$

Sol: $\dfrac{7x^2 \ y^2 \ z^2}{14xyz} = \dfrac{7 \times x \times x \times y \times y \times z \times z}{2 \times 7 \times x \times y \times z}$

$= \dfrac{x \times y \times z}{2} = \dfrac{xyz}{2}$

TRY THESE

(i) Divide $24xy^2 \ z^3$ by $6yz^2$

Sol: $\dfrac{24xy^2 \ z^3}{6yz^2}$

$= \dfrac{2 \times 2 \times 2 \times 3 \times x \times y \times y \times z \times z \times z}{2 \times 3 \times y \times z \times z}$

$= 2 \times 2 \times x \times y \times z$

$= 4xyz$

(ii) Divide $63a^2 \ b^4 \ c^6$ by $7a^2 \ b^2 \ c^3$

Sol: $\dfrac{63a^2 \ b^4 \ c^6}{7a^2 \ b^2 \ c^3}$

$= \dfrac{7 \times 9 \times a^2 \times b^2 \times b^2 \times c^3 \times c^3}{7 \times a^2 \times b^2 \times c^3}$

$= 9 \times b^2 \times c^3$

$= 9b^2c^3$

Example 14: Divide $24(x^2yz + xy^2z + xyz^2)$ by $8xyz$ using both the methods

Sol: $24(x^2yz + xy^2z + xyz^2)$

$= 3 \times 8 \times (xyz \times x + xyz \times y + xyz \times z)$

$= 3 \times 8 \times xyz \times (x + y + z)$

$Now \ \dfrac{24(x^2yz + xy^2z + xyz^2)}{8xyz}$

$= \dfrac{3 \times 8 \times xyz \times (x + y + z)}{8 \times xyz}$

$= 3(x + y + z)$

Method-II

$\dfrac{24(x^2yz + xy^2z + xyz^2)}{8xyz}$

$$= \frac{24x^2yz}{8xyz} + \frac{24xy^2z}{8xyz} + \frac{24xyz^2}{8xyz}$$

$$= 3x + 3y + 3z = 3(x + y + z)$$

Example 15: *Divide* $44(x^4 - 5x^3 - 24x^2)$ *by* $11x(x - 8)$

Sol: $44(x^4 - 5x^3 - 24x^2)$

$$= 44[x^2 \times x^2 - 5x \times x^2 - 24 \times x^2]$$

$$= 44 \times x^2[x^2 - 5x - 24]$$

$$= 44 \times x^2[x^2 + (-8 + 3)x + (-8)(3)]$$

$$= 44 \times x^2[(x - 8)(x + 3)]$$

$$= 4 \times 11 \times x \times x \times (x - 8) \times (x + 3)$$

$$Now \quad \frac{44(x^4 - 5x^3 - 24x^2)}{11x(x - 8)}$$

$$= \frac{4 \times 11 \times x \times x \times (x - 8) \times (x + 3)}{11 \times x \times (x - 8)}$$

$$= 4x(x + 3)$$

Example 16: *Divide* $z(5z^2 - 80)$ *by* $5z(z + 4)$

Sol: $z(5z^2 - 80)$

$$= z \times [5 \times z^2 - 5 \times 16]$$

$$= z \times 5 \times [z^2 - 16]$$

$$= z \times 5 \times [z^2 - 4^2]$$

$$= z \times 5 \times (z + 4) \times (z - 4)$$

$$Now \quad \frac{z(5z^2 - 80)}{5z(z + 4)} = \frac{z \times 5 \times (z + 4) \times (z - 4)}{5 \times z \times (z + 4)}$$

$$= (z - 4)$$

EXERCISE 12.3

1. *Carry out the following divisions.*

(i) $28x^4 \div 56x$

Sol: $\frac{28x^4}{56x} = \frac{28 \times x \times x \times x \times x}{2 \times 28 \times x} = \frac{x \times x}{2} = \frac{x^2}{2}$

(ii) $-36y^3 \div 9y^2$

Sol: $\frac{-36y^3}{9y^2} = \frac{-4 \times 9 \times y \times y \times y}{9 \times y \times y} = -4y$

(iii) $66pq^2r^3 \div 11qr^2$

Sol: $\dfrac{66pq^2\,r^3}{11qr^2} = \dfrac{6 \times 11 \times p \times q \times q \times r \times r \times r}{11 \times q \times r \times r}$

$$= 6 \times p \times q \times r$$

$$= 6pqr$$

$(iv)\mathbf{34x^3y^3\,z^3 \div 51xy^2\,z^3}$

Sol: $\dfrac{34x^3y^3\,z^3}{51xy^2\,z^3}$

$$= \dfrac{2 \times 17 \times x \times x \times x \times y \times y \times y \times z \times z \times}{3 \times 17 \times x \times y \times y \times z \times z \times z}$$

$$= \dfrac{2x^2y}{3}$$

$(v)\mathbf{12a^8\,b^8 \div (-\,6a^6\,b^4)}$

Sol: $\dfrac{12a^8\,b^8}{-\,6a^6\,b^4}$

$$= \dfrac{2 \times 6 \times a^6 \times a^2 \times b^4 \times b^4}{-\,6 \times a^6 \times\,b^4}$$

$$= -2a^2b^4$$

2. *Divide the given polynomial by the given monomial.*

$(i)\mathbf{(5x^2 - 6x) \div 3x}$

Sol: $\dfrac{5x^2 - 6x}{3x}$

$$= \dfrac{5x^2}{3x} - \dfrac{6x}{3x}$$

$$= \dfrac{5x}{3} - 2$$

$(ii)\mathbf{(3y^8 - 4y^6 + 5y^4) \div y^4}$

Sol: $\dfrac{3y^8 - 4y^6 + 5y^4}{y^4}$

$$= \dfrac{3y^8}{y^4} - \dfrac{4y^6}{y^4} + \dfrac{5y^4}{y^4}$$

$$= \dfrac{3y^4 \times y^4}{y^4} - \dfrac{4y^2 \times y^4}{y^4} + \dfrac{5 \times y^4}{y^4}$$

$$= 3y^4 - 4y^2 + 5$$

$(iii)\mathbf{8(x^3y^2\,z^2 + x^2\,y^3\,z^2 + x^2\,y^2\,z^3) \div 4x^2y^2\,z^2}$

Sol: $\dfrac{8(x^3y^2\,z^2 + x^2\,y^3\,z^2 + x^2\,y^2\,z^3)}{4x^2y^2\,z^2}$

$$= \frac{8x^3 y^2 z^2}{4x^2 y^2 z^2} + \frac{8x^2 y^3 z^2}{4x^2 y^2 z^2} + \frac{8x^2 y^2 z^3}{4x^2 y^2 z^2}$$

$$= 2x + 2y + 2z$$

$$= 2(x + y + z)$$

$(iv)$$(x^3 + 2x^2 + 3x) \div 2x$

Sol: $\dfrac{x^3 + 2x^2 + 3x}{2x}$

$$= \frac{x^3}{2x} + \frac{2x^2}{2x} + \frac{3x}{2x}$$

$$= \frac{x^2}{2} + x + \frac{3}{2}$$

(v) $(p^3 q^6 - p^6 q^3) \div p^3 q^3$

Sol: $\dfrac{p^3 q^6 - p^6 q^3}{p^3 q^3}$

$$= \frac{p^3 q^3 \times q^3}{p^3 q^3} - \frac{p^3 q^3 \times p^3}{p^3 q^3}$$

$$= q^3 - p^3$$

3. *Work out the following divisions.*

(i) $(10x - 25) \div 5$

Sol: $\dfrac{10x - 25}{5} = \dfrac{10x}{5} - \dfrac{25}{5} = 2x - 5$

(ii) $(10x - 25) \div (2x - 5)$

Sol: $(10x - 25) = (5 \times 2x - 5 \times 5)$

$$= 5 \times (2x - 5)$$

Now $\dfrac{(10x - 25)}{(2x - 5)}$

$$= \frac{5 \times (2x - 5)}{(2x - 5)}$$

$$= 5$$

(iii) $10y(6y + 21) \div 5(2y + 7)$

Sol: $10y(6y + 21)$

$$= 2 \times 5 \times y \times [3 \times 2y + 3 \times 7]$$

$$= 2 \times 5 \times y \times 3 \times (2y + 7)$$

Now $\dfrac{10y(6y + 21)}{5(2y + 7)}$

$$= \frac{2 \times 5 \times y \times 3 \times (2y + 7)}{5 \times (2y + 7)}$$

$$= 2 \times y \times 3$$

$$= 6y$$

(iv) $9x^2 \; y^2 \; (3z - 24) \div 27xy(z - 8)$

Sol: $9x^2 \; y^2 \; (3z - 24)$

$$= 9x^2 \; y^2 \times [3 \times z - 3 \times 8]$$

$$= 9 \times x \times x \times y \times y \times 3 \times (z - 8)$$

$Now \; \dfrac{9x^2 \; y^2 \; (3z - 24)}{27xy(z - 8)}$

$$= \frac{9 \times x \times x \times y \times y \times 3 \times (z - 8)}{3 \times 9 \times x \times y \times (z - 8)}$$

$$= xy$$

(v) $96abc(3a - 12) \, (5b - 30) \div 144(a - 4) \, (b - 6)$

Sol: $(3a - 12)(5b - 30)$

$$= [3 \times a - 3 \times 4][5 \times b - 5 \times 6]$$

$$= 3 \times (a - 4) \times 5 \times (b - 6)$$

$Now \; \dfrac{96abc(3a - 12)(5b - 30)}{144(a - 4)(b - 6)}$

$$= \frac{2 \times 48 \times abc \times 3 \times (a - 4) \times 5 \times (b - 6)}{3 \times 48 \times (a - 4) \times (b - 6)}$$

$$= 2 \times 5 \times abc$$

$$= 10abc$$

4. *Divide as directed.*

(i) $5(2x + 1)(3x + 5) \div (2x + 1)$

Sol: $\dfrac{5(2x + 1)(3x + 5)}{(2x + 1)} = 5(3x + 5)$

(ii) $26xy(x + 5)(y - 4) \div 13x(y - 4)$

Sol: $\dfrac{26xy(x + 5)(y - 4)}{13x(y - 4)}$

$= \dfrac{2 \times 13 \times x \times y \times (x + 5) \times (y - 4)}{13 \times x \times (y - 4)}$

$= 2 \times y \times (x + 5)$

$= 2y(x + 5)$

(iii) $52pqr\,(p + q)\,(q + r)\,(r + p) \div 104pq(q + r)\,(r + p)$

Sol: $\dfrac{52pqr\,(p + q)(q + r)(r + p)}{104pq(q + r)(r + p)}$

$= \dfrac{52 \times pq \times r \times (p + q)(q + r)(r + p)}{2 \times 52 \times pq \times (q + r)(r + p)}$

$= \dfrac{r(p + q)}{2}$

$= \dfrac{1}{2}r(p + q)$

(iv) $20(y + 4)(y^2 + 5y + 3) \div 5(y + 4)$

Sol: $\dfrac{20(y + 4)(y^2 + 5y + 3)}{5(y + 4)}$

$= \dfrac{4 \times 5 \times (y + 4)(y^2 + 5y + 3)}{5 \times (y + 4)}$

$= 4(y^2 + 5y + 3)$

(v) $x(x + 1)(x + 2)(x + 3) \div x(x + 1)$

Sol: $\dfrac{x(x + 1)(x + 2)(x + 3)}{x(x + 1)}$

$= (x + 2)(x + 3)$

5. *Factorise the expressions and divide them as directed.*

(i) $(y^2 + 7y + 10) \div (y + 5)$

Sol: $y^2 + 7y + 10$

$= y^2 + (2 + 5)y + 2 \times 5$

$= (y + 2)(y + 5)$

Now $\dfrac{(y^2 + 7y + 10)}{(y + 5)}$

$$= \frac{(y + 2)(y + 5)}{(y + 5)}$$

$$= (y + 2)$$

(ii) $(m^2 - 14m - 32) \div (m + 2)$

Sol: $m^2 - 14m - 32$

$$= m^2 + (-16 + 2)m + (-16) \times (2)$$

$$= (m - 16)(m + 2)$$

Now $\dfrac{(m^2 - 14m - 32)}{(m + 2)}$

$$= \frac{(m - 16)(m + 2)}{(m + 2)}$$

$$= (m - 16)$$

(iii) $(5p^2 - 25p + 20) \div (p - 1)$

Sol: $5p^2 - 25p + 20$

$$= 5[p^2 - 5p + 4]$$

$$= 5[p^2 + (-1 - 4)p + (-1)(-4)]$$

$$= 5(p - 1)(p - 4)$$

Now $\dfrac{(5p^2 - 25p + 20)}{(p - 1)}$

$$= \frac{5(p - 1)(p - 4)}{(p - 1)} = 5(p - 4)$$

(iv) $4yz(z^2 + 6z - 16) \div 2y(z + 8)$

Sol: $z^2 + 6z - 16$

$$= z^2 + (8 - 2)z + 8 \times (-2)$$

$$= (z + 8)(z - 2)$$

Now $\dfrac{4yz(z^2 + 6z - 16)}{2y(z + 8)}$

$$= \frac{2y \times 2z \times (z + 8)(z - 2)}{2y \times (z + 8)}$$

$$= 2z(z - 2)$$

(v) $5pq(p^2 - q^2) \div 2p(p + q)$

Sol: $\dfrac{5pq(p^2 - q^2)}{2p(p + q)}$

$$= \frac{5 \times p \times q \times (p + q)(p - q)}{2 \times p \times (p + q)}$$

$$= \frac{5q(p - q)}{2}$$

$$= \frac{5}{2}q(p - q)$$

(vi) $12xy(9x^2 - 16y^2) \div 4xy(3x + 4y)$

Sol: $9x^2 - 16y^2 = (3x)^2 - (4y)^2$

$$= (3x + 4y)(3x - 4y)$$

Now $\dfrac{12xy(9x^2 - 16y^2)}{4xy(3x + 4y)}$

$$= \frac{3 \times 4xy \times (3x + 4y) \times (3x - 4y)}{4xy \times (3x + 4y)}$$

$$= 3(3x - 4y)$$

(vii) $39y^3(50y^2 - 98) \div 26y^2(5y + 7)$

Sol: $50y^2 - 98$

$$= 2 \times 25y^2 - 2 \times 49$$

$$= 2[25y^2 - 49]$$

$$= 2[(5y)^2 - (7)^2]$$

$$= 2(5y + 7)(5y - 7)$$

$Now \ \dfrac{39y^3 \ (50y^2 \ - \ 98)}{26y^2 \ (5y \ + \ 7)}$

$$= \dfrac{3 \times 13 \times y^2 \times y \times 2 \times (5y + 7) \times (5y - 7)}{2 \times 13 \times y^2 \times \ (5y \ + \ 7)}$$

$$= 3y(5y - 7)$$

Please download VI to X class all maths notes from website
https://sureshmathsmaterial.com/