CHAPTER 10

AP VIII CLASS-CBSE (2024-25)
EXPONENTS AND POWERS (Notes)

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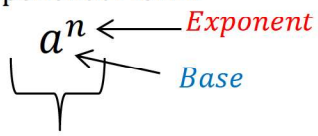
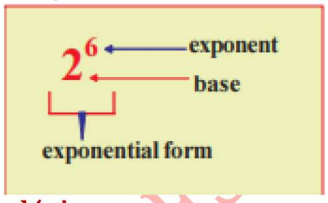
1. $a \times a = a^2$ ('a' squared)
2. $a \times a \times a = a^3$ ('a' cubed)
3. $a \times a \times a \times a = a^4$ ('a' raised to the power of 4)
4. $a \times a \times a \times a \times a = a^5$ ('a' raised to the power of 5)
5. $a \times a \times a \times a \times a \times \ldots .' n' = a^n$, Where 'a' is the base and 'n' is the exponent. This is said to be exponential form.



Exponential form



6. **For any nonzero integer 'a' and integers 'm' and 'n'**

(i) $a^m \times a^n = a^{m+n}$

(ii) $(a^m)^n = a^{mn}$

(iii) $a^m \times b^m = (a \times b)^m$

(iv) $\frac{a^m}{a^n} = a^{m-n}$ *if* $m > n$

(v) $\frac{a^m}{a^n} = \frac{1}{a^{n-m}}$ *if* $n > m$

(vi) $a^0 = 1 \ (a \neq 0)$

(vii) $\frac{a^m}{a^n} = \left(\frac{a}{b}\right)^m$

(viii) $1^{any\ number} = 1$

(ix) $(-1)^{\text{even number}} = 1$

(x) $(-1)^{\text{odd number}} = -1$

7. For any non-zero integer, $\boldsymbol{a^{-m} = \frac{1}{a^m}}$, where m is a positive integer. a^{-m} is the multiplicative inverse of a^m .Also $\boldsymbol{\frac{1}{a^{-m}} = a^m}$. $(a^m \times a^{-m} = 1)$

$(i)\ \frac{1}{a^{-1}} = a\ ; a^{-1} = \frac{1}{a}$ $(ii)\ \left(\frac{a}{b}\right)^{-m} = \left(\frac{b}{a}\right)^m$ $(iii)\ \left(\frac{a}{b}\right)^{-1} = \frac{b}{a}$

8.

$2^2 = 4$	$2^{10} = 1024$	$4^4 = 256$	$7^2 = 49$
$2^3 = 8$	$3^2 = 9$	$4^5 = 1024$	$7^3 = 343$
$2^4 = 16$	$3^3 = 27$	$5^2 = 25$	$8^2 = 64$
$2^5 = 32$	$3^4 = 81$	$5^3 = 125$	$8^3 = 512$
$2^6 = 64$	$3^5 = 243$	$5^4 = 625$	$9^2 = 81$
$2^7 = 128$	$3^6 = 729$	$6^2 = 36$	$9^3 = 729$
$2^8 = 256$	$4^2 = 16$	$6^3 = 216$	$10^2 = 100$
$2^9 = 512$	$4^3 = 64$	$6^4 = 1296$	$10^3 = 1000$

TRY THESE

Find the multiplicative inverse of the following

The multiplicative inverse of $a^{-m} = a^m$

(i) The multiplicative inverse of $2^{-4} = 2^4$

(ii) The multiplicative inverse of $10^{-5} = 10^5$

(iii) The multiplicative inverse of $7^{-2} = 7^2$

(iv) The multiplicative inverse of $5^{-3} = 5^3$

(v) The multiplicative inverse of $10^{-100} = 10^{100}$

Expand the following numbers using exponents.

(i) 1025.63

Sol: $1025.63 = 1000 + 20 + 3 + \frac{6}{10} + \frac{3}{100}$

$= 1 \times 1000 + 2 \times 10 + 3 \times 1 + 6 \times \frac{1}{10} + 3 \times \frac{1}{100}$

$= 1 \times 10^3 + 2 \times 10^1 + 3 \times 10^0 + 6 \times \frac{1}{10^1} + 3 \times \frac{1}{10^2}$

$= 1 \times 10^3 + 2 \times 10^1 + 3 \times 10^0 + 6 \times 10^{-1} + 3 \times 10^{-2}$

$\boxed{\frac{1}{a^m} = a^{-m}}$

(ii) 1256.249

Sol: $1256.249 = 1000 + 200 + 50 + 6 + \frac{2}{10} + \frac{4}{100} + \frac{9}{1000}$

$= 1 \times 1000 + 2 \times 100 + 5 \times 10 + 6 \times 1 + 2 \times \frac{1}{10} + 4 \times \frac{1}{100} + 9 \times \frac{1}{1000}$

$= 1 \times 10^3 + 2 \times 10^2 + 5 \times 10^1 + 6 \times 10^0 + 2 \times \frac{1}{10^1} + 4 \times \frac{1}{10^2} + 9 \times \frac{1}{10^3}$

$= 1 \times 10^3 + 2 \times 10^2 + 5 \times 10^1 + 6 \times 10^0 + 2 \times 10^{-1} + 4 \times 10^{-2} + 9 \times 10^{-3}$

Simplify and write in exponential form.

$(i)\ (-2)^{-3} \times (-2)^{-4} = (-2)^{-3+(-4)} = (-2)^{-7}$

$(ii)\ p^3 \times p^{-10} = p^{3+(-10)} = p^{-7}$

$(iii)\ 3^2 \times 3^{-5} \times 3^6 = 3^{2+(-5)+6} = 3^3$

Example 1: Find the value of

$(i) 2^{-3} = \frac{1}{2^3} = \frac{1}{2 \times 2 \times 2} = \frac{1}{8}$

$(ii) \frac{1}{3^{-2}} = 3^2 = 3 \times 3 = 9$

$$a^{-m} = \frac{1}{a^m}$$

$$\frac{1}{a^{-m}} = a^m$$

Example 2: Simplify

$(i)\ (-4)^5 \times (-4)^{-10}$

Sol: $(-4)^5 \times (-4)^{-10} = (-4)^{5+(-10)}$

(i) $a^m \times a^n = a^{m+n}$

(ii) $\frac{a^m}{a^n} = a^{m-n}$

$= (-4)^{-5}$

$= \frac{1}{(-4)^5}$

$(ii)\ 2^5 \div 2^{-6} = \frac{2^5}{2^{-6}}$

$= 2^{5-(-6)}$

$= 2^{5+6}$

$= 2^{11}$

Example 3: Express 4^{-3} as a power with the base 2.

Sol: $4 = 2 \times 2 = 2^2$

$4^{-3} = (2^2)^{-3} = 2^{2\times(-3)} = 2^{-6}$

$a^m \times b^m = (a \times b)^m$

$(a^m)^n = a^{mn}$

$(-a)^n = a^n$ *if* n *is even*

$(-a)^n = -a^n$ *if* n *is odd*

Example 4: Simplify and write the answer in the exponential form.

$(i)\ (2^5 \div 2^8)^5 \times 2^{-5}$

Sol: $(2^5 \div 2^8)^5 \times 2^{-5}$

$= (2^{5-8})^5 \times 2^{-5}$

$= (2^{-3})^5 \times 2^{-5}$

$= 2^{-15} \times 2^{-5}$

$= 2^{-15-5}$

$= 2^{-20}$

$= \frac{1}{2^{20}}$

$(ii)(-4)^{-3} \times (5)^{-3} \times (-5)^{-3}$

Sol: $(-4)^{-3} \times (5)^{-3} \times (-5)^{-3}$

$= [(-4) \times 5 \times (-5)]^{-3}$

$= 100^{-3}$

$= \frac{1}{100^3}$

$(iii)\ \frac{1}{8} \times (3)^{-3}$

Sol: $\frac{1}{8} \times (3)^{-3}$

$= \frac{1}{2^3} \times (3)^{-3}$

$= (2)^{-3} \times (3)^{-3}$

$= [2 \times 3]^{-3}$

$= 6^{-3} = \frac{1}{6^3}$

$(iv)\ (-3)^4 \times \left(\frac{5}{3}\right)^4$

Sol: $(-3)^4 \times \left(\frac{5}{3}\right)^4$

$= 3^4 \times \frac{5^4}{3^4}$

$= 5^4$

***Example* 5: Find m so that $(-3)^{m+1} \times (-3)^5 = (-3)^7$**

Sol: $(-3)^{m+1} \times (-3)^5 = (-3)^7$

$(-3)^{m+1+5} = (-3)^7$

$(-3)^{m+6} = (-3)^7$

If bases$(\neq 0, \pm 1)$ are equal, then their exponents must be equal.

$m + 6 = 7$

$m = 7 - 6 = 1$

***Example* 6: Find the value of $\left(\frac{2}{3}\right)^{-2}$.**

Sol: $\left(\frac{2}{3}\right)^{-2} = \left(\frac{3}{2}\right)^{2}$

$= \frac{3^2}{2^2}$

$= \frac{9}{4}$

***Example 7*: Simplify (i)** $\left\{\left(\frac{1}{3}\right)^{-2} - \left(\frac{1}{2}\right)^{-3}\right\} \div \left(\frac{1}{4}\right)^{-2}$

Sol: (i) $\left\{\left(\frac{1}{3}\right)^{-2} - \left(\frac{1}{2}\right)^{-3}\right\} \div \left(\frac{1}{4}\right)^{-2}$

$= \left\{\left(\frac{3}{1}\right)^{2} - \left(\frac{2}{1}\right)^{3}\right\} \div \left(\frac{4}{1}\right)^{2}$

$= (3^2 - 2^3) \div 4^2$

$= (9 - 8) \div 16$

$= \frac{1}{16}$

(ii) $\left(\frac{5}{8}\right)^{-7} \times \left(\frac{8}{5}\right)^{-5}$

Sol: $\left(\frac{5}{8}\right)^{-7} \times \left(\frac{8}{5}\right)^{-5}$

$= \left(\frac{8}{5}\right)^{7} \times \left(\frac{5}{8}\right)^{5}$

$= \frac{8^7}{5^7} \times \frac{5^5}{8^5}$

$= \frac{8^{7-5}}{5^{7-5}} = \frac{8^2}{5^2} = \frac{16}{25}$

EXERCISE 10.1

1. Evaluate

(i) $3^{-2} = \frac{1}{3^2} = \frac{1}{9}$

(ii) $(-4)^{-2} = \frac{1}{(-4)^2} = \frac{1}{(-4)(-4)} = \frac{1}{16}$

(iii) $\left(\frac{1}{2}\right)^{-5} = \left(\frac{2}{1}\right)^{5} = 2^5 = 32$

2. Simplify and express the result in power notation with positive exponent.

(i) $(-4)^5 \div (-4)^8 = (-4)^{5-8} = (-4)^{-3} = \frac{1}{(-4)^3}$

(ii) $\left(\frac{1}{2^3}\right)^{2} = \frac{1^2}{(2^3)^2} = \frac{1}{2^6} = \left(\frac{1}{2}\right)^{6}$

(iii) $(-3)^4 \times \left(\frac{5}{3}\right)^{4} = 3^4 \times \frac{5^4}{3^4} = 5^4$

(iv) $(3^{-7} \div 3^{-10}) \times 3^{-5} = \frac{3^{-7}}{3^{-10}} \times 3^{-5}$

$= \frac{3^{10}}{3^7} \times \frac{1}{3^5}$

$$= \frac{3^{10}}{3^{12}}$$

$$= \frac{1}{3^{12-10}}$$

$$= \frac{1}{3^2} = \left(\frac{1}{3}\right)^2$$

(v) $2^{-3} \times (-7)^{-3} = (2 \times -7)^{-3}$

$$= (-14)^{-3}$$

$$= \frac{1}{(-14)^3}$$

$$= \left(\frac{1}{-14}\right)^3$$

3. Find the value of

(i) $(3^0 + 4^{-1}) \times 2^2$

Sol: $(3^0 + 4^{-1}) \times 2^2$

$$= \left(1 + \frac{1}{4}\right) \times 4$$

$$= \frac{5}{4} \times 4 = 5$$

(ii) $(2^{-1} \times 4^{-1}) \div 2^{-2}$

Sol: $(2^{-1} \times 4^{-1}) \div 2^{-2}$

$$= \left(\frac{1}{2} \times \frac{1}{4}\right) \div \frac{1}{2^2}$$

$$= \frac{1}{8} \div \frac{1}{4}$$

$$= \frac{1}{8} \times \frac{4}{1} = \frac{1}{2}$$

(iii) $\left(\frac{1}{2}\right)^{-2} + \left(\frac{1}{3}\right)^{-2} + \left(\frac{1}{4}\right)^{-2}$

Sol: $\left(\frac{1}{2}\right)^{-2} + \left(\frac{1}{3}\right)^{-2} + \left(\frac{1}{4}\right)^{-2}$

$$= 2^2 + 3^2 + 4^2$$

$$= 4 + 9 + 16$$

$$= 29$$

(iv) $(3^{-1} + 4^{-1} + 5^{-1})^0 = 1$

(v) $\left\{\left(\frac{-2}{3}\right)^{-2}\right\}^2 = \left(\frac{-2}{3}\right)^{-2\times 2}$

$$= \left(\frac{-2}{3}\right)^{-4}$$

$$= \left(\frac{-3}{2}\right)^4$$

$$= \frac{(-3)^4}{2^4} = \frac{81}{16}$$

4. Evaluate

(i) $\frac{8^{-1} \times 5^3}{2^{-4}} = \frac{2^4 \times 5^3}{8^1}$

$$= \frac{16 \times 125}{8}$$

$$= 2 \times 125$$

$$= 250$$

(ii) $(5^{-1} \times 2^{-1}) \times 6^{-1}$

$$= \left(\frac{1}{5} \times \frac{1}{2}\right) \times \frac{1}{6}$$

$$= \frac{1}{10} \times \frac{1}{6} = \frac{1}{60}$$

5. Find the value of m for which $5^m \div 5^{-3} = 5^5$

Sol: $5^m \div 5^{-3} = 5^5$

$5^{m-(-3)} = 5^5$

$5^{m+3} = 5^5$

Bases ($\neq$0,+1,-1) same, so their exponents must be equal.

$m+3=5$

$m=5-3$

$m=2$

6. Evaluate

(i) $\left\{\left(\frac{1}{3}\right)^{-1}-\left(\frac{1}{4}\right)^{-1}\right\}^{-1}$

Sol: $\left\{\left(\frac{1}{3}\right)^{-1}-\left(\frac{1}{4}\right)^{-1}\right\}^{-1}$

$=(3-4)^{-1}$

$=(-1)^{-1}$

$=\frac{1}{(-1)}$

$=-1$

(ii) $\left(\frac{5}{8}\right)^{-7}\times\left(\frac{8}{5}\right)^{-4}$

Sol: $\left(\frac{5}{8}\right)^{-7}\times\left(\frac{8}{5}\right)^{-4}$

$=\left(\frac{8}{5}\right)^{7}\times\left(\frac{5}{8}\right)^{4}$

$=\frac{8^7}{5^7}\times\frac{5^4}{8^4}$

$=\frac{8^{7-4}}{5^{7-4}}=\frac{8^3}{5^3}=\frac{512}{125}$

7. Simplify

(i) $\frac{25\times t^{-4}}{5^{-3}\times 10\times t^{-8}}(t\neq 0)$

Sol: $\frac{25\times t^{-4}}{5^{-3}\times 10\times t^{-8}}$

$=\frac{5^1\times t^{-4}}{5^{-3}\times 2\times t^{-8}}$

$=\frac{5^{1+3}\times t^{-4+8}}{2}$

$=\frac{5^4\times t^4}{2}$

$=\frac{625}{2}t^4$

(ii) $\frac{3^{-5}\times 10^{-5}\times 125}{5^{-7}\times 6^{-5}}$

Sol: $\frac{3^{-5}\times 10^{-5}\times 125}{5^{-7}\times 6^{-5}}$

$=\frac{3^{-5}\times(2\times 5)^{-5}\times 5^3}{5^{-7}\times(2\times 3)^{-5}}$

$=\frac{3^{-5}\times(2\times 5)^{-5}\times 5^3}{5^{-7}\times(2\times 3)^{-5}}$

$=\frac{3^{-5}\times 2^{-5}\times 5^{-5}\times 5^3}{5^{-7}\times 2^{-5}\times 3^{-5}}$

$=5^{-5+3+7}=5^5$

Use of Exponents to Express Small Numbers and large Numbers in Standard Form

1. The distance from the Earth to the Sun is **149,600,000,000 m.=1.496×10^{11}m**
2. The speed of light is **300,000,000 m/sec=3×10^8 m/sec.**
3. Thickness of Class VII Mathematics book is 20 mm
4. The average diameter of a Red Blood Cell is **0.000007 mm=7×10^{-6}mm**
5. The thickness of human hair is in the range of **0.005 cm to 0.01 cm=$5\times10^{-3}cm$ to $10^{-2}cm$**
6. The distance of moon from the Earth is **384, 467, 000 m = $3.84467\times10^8\ m$.**
7. The size of a plant cell is **0.00001275 m= $1.275\times10^{-5}m$**
8. Average radius of the Sun is **695000 km= 6.95×10^5km**

9. Mass of propellant in a space shuttle solid rocket booster is **503600 kg**= $\mathbf{5.036 \times 10^5 kg}$
10. Thickness of a piece of paper is **0.0016 cm**= $\mathbf{1.6 \times 10^{-3} cm}$
11. Diameter of a wire on a computer chip is **0.000003 m**= $\mathbf{3 \times 10^{-6} m}$
12. The height of Mount Everest is 8848 m=8.848×10^3m.

TRY THESE

1. Write the following numbers in standard form

$(i)\mathbf{0.000000564} = 5.64 \times 10^{-7}$

$(ii)\mathbf{0.0000021} = 2.1 \times 10^{-6}$

$(iii)\mathbf{21600000} = 2.16 \times 10^{8}$

$(iv)\mathbf{15240000} = 1.524 \times 10^{7}$

Comparing very large and very small numbers

(i) Diameter of the Sun $= 1.4 \times 10^9$ m

(ii) Diameter of the earth $= 1.2756 \times 10^7$ m

(iii) Size of Red Blood cell $= 0.000007$ m $= 7 \times 10^{-6}$ m

(iv) Size of plant cell $= 0.00001275 = 1.275 \times 10^{-5}$ m

(v) Mass of earth is 5.97×10^{24} kg

(vi) Mass of moon is 7.35×10^{22} kg

(vii) Total mass of earth and moon

$= 5.97 \times 10^{24}$ kg $+ 7.35 \times 10^{22}$ kg.

$= 5.97 \times 10^2 \times 10^{22}\ kg + 7.35 \times 10^{22}\ kg.$

$= 5.97 \times 100 \times 10^{22}$ kg $+ 7.35 \times 10^{22}$ kg.

$= 597 \times 10^{22}$ kg $+ 7.35 \times 10^{22}$ kg.

$= (597 + 7.35) \times 10^{22}$ kg.

$= 604.35 \times 10^{22} kg.$

$(viii)$ Distance between Sun and Earth $= 1.496 \times 10^{11}$m

(ix) Distance between Earth and Moon $= 3.84 \times 10^8$m

(x) Distance between Sun and Moon

$= 1.496 \times 10^{11}$m $- 3.84 \times 10^8$m

$= 1.496 \times 10^3 \times 10^8 m - 3.84 \times 10^8 m$

$= 1.496 \times 1000 \times 10^8 m - 3.84 \times 10^8$m

$= 1496 \times 10^8 m - 3.84 \times 10^8 m$

$= (1496 - 3.84) \times 10^8 m$

$= 1492.16 \times 10^8 m$

Example 8: Express the following numbers in standard form.

$(i)\mathbf{0.000035} = 3.5 \times 10^{-5}$

$(ii)\mathbf{4050000} = 4.05 \times 10^6$

Example 9: Express the following numbers in usual form

$(i)\mathbf{3.52 \times 10^5} = 352000$

$(ii)\mathbf{7.54 \times 10^{-4}} = 0.000754$

$(iii)\ \mathbf{3 \times 10^{-5}} = 0.00003$

EXERCISE 10.2

1. Express the following numbers in standard form

$(i)\quad \mathbf{0.0000000000085} = 8.5 \times 10^{-12}$

$(ii)\quad \mathbf{0.00000000000942} = 9.42 \times 10^{-12}$

$(iii)\quad \mathbf{6020000000000000} = 6.02 \times 10^{15}$

$(iv)\quad \mathbf{0.00000000837} = 8.37 \times 10^{-9}$

$(v)\quad \mathbf{31860000000} = 3.186 \times 10^{10}$

2. Express the following numbers in usual form.

$(i)\quad \mathbf{3.02 \times 10^{-6}} = 0.00000302$

$(ii)\quad \mathbf{4.5 \times 10^4} = 45000$

$(iii)\quad \mathbf{3 \times 10^{-8}} = 0.00000003$

$(iv)\quad \mathbf{1.0001 \times 10^9} = 1000100000$

$(v)\quad \mathbf{5.8 \times 10^{12}} = 5800000000000$

$(vi)\mathbf{3.61492 \times 10^6} = 3614920$

3. Express the number appearing in the following statements in standard form.

(i) 1 micron is equal to $\frac{1}{1000000}\ \text{m} = \frac{1}{10^6} = 1 \times 10^{-6} m$

(ii) Charge of an electron is 0.000,000,000,000,000,000,16 coulomb $= 1.6 \times 10^{-19}$ coulomb

(iii) Size of a bacteria is 0.0000005 m $= 5 \times 10^{-7} m$

(iv) Size of a plant cell is 0.00001275 m $= 1.275 \times 10^{-5} m$

(v) Thickness of a thick paper is 0.07 mm $= 7 \times 10^{-2} mm$

4. In a stack there are 5 books each of thickness 20mm and 5 paper sheets each of thickness 0.016 mm. What is the total thickness of the stack?

Sol: Thickness of book=20mm

Thickness of paper sheet=0.016 mm

Number of books=5

and Number of paper sheets=5

The total thickness of the stack

=5×20+5×0.016

=100+0.08

$=100.08$

$=1.0008\times10^{2}$ mm