CHAPTER 10

VII-MATHEMATICS-NCERT(2023-24)

10. Algebraic Expressions (Notes)

https://sureshmathsmaterial.com

1. **Variable**: A variable can take various values. Its value is not fixed.
2. We use letters $x, y, l, m, ..$ etc. to denote variables.
3. A constant has a fixed value. Ex: 4, 100, –17, etc
4. **Algebraic expression**: Algebraic expressions are formed from variables and constants.
5. Expressions are made up of terms. Terms are added to make an expression
6. The addition of the terms 4xy and 7 gives the expression 4xy + 7.
7. $x \times x = x^2$
8. $x \times x \times x = x^3$
9. $2 \times y \times y = 2y^2$
10. $4 \times x \times y = 4xy$
11. $2^2 = 2 \times 2 = 4;\ 3^2 = 3 \times 3 = 9;\ 4^2 = 4 \times 4 = 16;\ 5^2 = 5 \times 5 = 25$
12. $(-2)^2 = (-2) \times (-2) = 4;$
13. $(-3)^2 = (-3) \times (-3) = 9;$
14. $(-4)^2 = (-4) \times (-4) = 16$
15. $2^3 = 2 \times 2 \times 2 = 8;\ 3^3 = 3 \times 3 \times 3 = 27;\ 4^3 = 4 \times 4 \times 4 = 64$
16. $(-2)^3 = (-2) \times (-2) \times (-2) = -8;\ (-3)^3 = (-3) \times (-3) \times (-3) = -27$

TRY THESE

1. **Describe how the following expressions are obtained:**

(i) $7xy + 5$

Sol: We first obtain xy, multiply it by 7 to get $7xy$ and add 5 to get the expression.

(ii) x^2y

Sol: We first obtain x^2 , and multiply it by y to get x^2y.

(iii) $4x^2 - 5x$

Sol: We first obtain x^2 , and multiply it by 4 to get $4x^2$ and subtract $5x$ to get the expression.

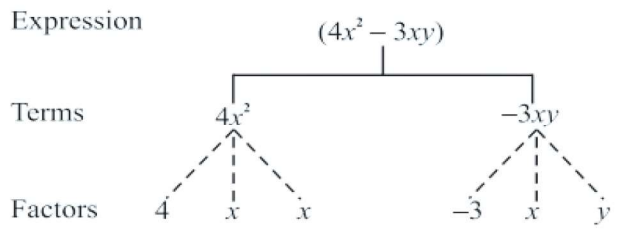
Factors of a term

(i) A term is a product of its factors. The term $-3xy$ is a product of the factors -3, x and y.

(ii) The numerical factor is said to be the numerical coefficient or simply the **coefficient** of the term.

(iii) Factors containing variables are said to be algebraic factors.

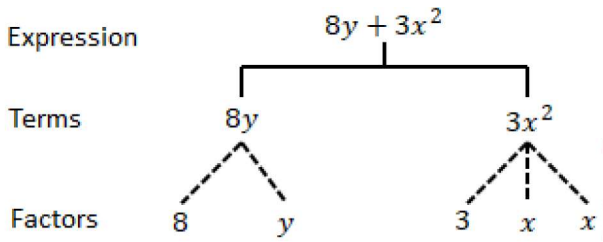
Tree diagram for expression:



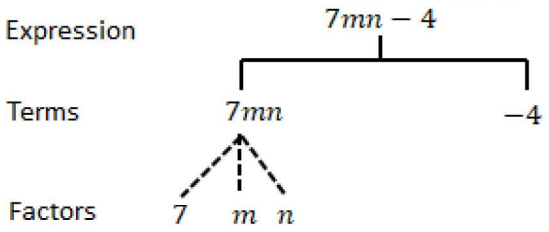
TRY THESE

1. **What are the terms in the following expressions? Show how the terms are formed. Draw a tree diagram for each expression:**

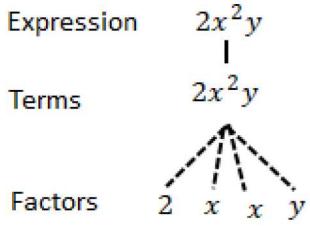
(i) $8y + 3x^2$



(ii) $7mn - 4$



(iii) $2x^2y$



2. **Write three expression each having 4 terms**

Sol: $(i)x^2 + y^2 + xy + 5$

$(ii)\ 2xy + yz - 3xz + 10$

$(iii) -3x^2 + 4xy + 3x - 7y$

3. Identify the coefficients of the terms of following expressions:

Expression	Terms	Coefficient
$4x - 3y$	$4x$	4
	$-3y$	-3
$a + b + 5$	a	1
	b	1
	5	
$2y + 5$	$2y$	2
	5	
$2xy$	$2xy$	2

Example 1: Identify, in the following expressions, terms which are not constants. Give their numerical coefficients: $xy + 4, 13 - y^2, 13 - y + 5y^2, 4p^2q - 3pq^2 + 5$

Sol:

S. No.	Expression	Term (which is not a Constant)	Numerical Coefficient
(i)	$xy + 4$	xy	1
(ii)	$13 - y^2$	$-y^2$	-1
(iii)	$13 - y + 5y^2$	$-y$ $5y^2$	-1 5
(iv)	$4p^2q - 3pq^2 + 5$	$4p^2q$ $-3pq^2$	4 -3

Example 2: (a) What are the coefficients of x in the following expressions?

$4x - 3y, 8 - x + y, y^2x - y, 2z - 5xz$

S. No.	Expression	Term with Factor x	Coefficient of x
(i)	$4x - 3y$	$4x$	4
(ii)	$8 - x + y$	$-x$	-1
(iii)	$y^2x - y$	y^2x	y^2
(iv)	$2z - 5xz$	$-5xz$	$-5z$

(b) What are the coefficients of y in the following expressions?

$4x - 3y, 8 + yz, yz^2 + 5, my + m$

S. No.	Expression	Term with factor y	Coefficient of y
(i)	$4x - 3y$	$-3y$	-3
(ii)	$8 + yz$	yz	z
(iii)	$yz^2 + 5$	yz^2	z^2
(iv)	$my + m$	my	m

Like and Unlike Terms

When terms have the same algebraic factors, they are like terms. When terms have different algebraic factors, they are unlike terms.

Try These

Group the like terms together from the following:

$12x, 12, -25x, -25, -25y, 1, x, 12y, y$

Sol: Group (i): $12x, -25x, \ x$

Group (ii): $-25y, \ 12y, \ y$

Group(iii): 12,1

Types of polynomial:

(1) Monomial: An expression with only one term is called a monomial

Ex: $5xy, \ 7m, \ 2n^2, \ 7x^2y$

(2) Binomial: An expression which contains two unlike terms is called a binomial.

Ex: $7x + 5y, \quad 2m^2 + 5, \ a^2 - b^2$

(3) Trinomial: An expression which contains three terms is called a trinomial

Ex: $x + y + 5, \quad ab + a + b, \ 3x^2 - 5x + 2, \quad m + n + 10$

In general, an expression with one or more terms is called **a polynomial**. Thus a monomial, a binomial and a trinomial are all polynomials.

Example 3: State with reasons, which of the following pairs of terms are of like terms and which are of unlike terms: $(i)\ 7x, 12y\ (ii)\ 15x, -21x\ (iii)\ -4ab, 7ba\ (iv)\ 3xy, 3x$

$(v)\ 6xy^2, 9x^2y\ (vi)\ pq^2, -4pq^2\ (vii)\ mn^2, 10mn$

S. No.	Pair	Factors	Algebraic factors same or different	Like/ Unlike terms	Remarks
(i)	$7x$ $12y$	$7, x$ $12, y$	Different	Unlike	The variables in the terms are different.
(ii)	$15x$ $-21x$	$15, x$ $-21, x$	Same	Like	
(iii)	$-4ab$ $7\,ba$	$-4, a, b$ $7, a, b$	Same	Like	Remember $ab = ba$
(iv)	$3xy$ $3x$	$3, x, y$ $3, x$	Different	Unlike	The variable y is only in one term.
(v)	$6xy^2$ $9x^2y$	$6, x, y, y$ $9, x, x, y$	Different	Unlike	The variables in the two terms match, but their powers do not match.
(vi)	pq^2 $-4pq^2$	$1, p, q, q$ $-4, p, q, q$	Same	Like	Note, numerical factor 1 is not shown

EXERCISE 10.1

1. Get the algebraic expressions in the following cases using variables, constants and arithmetic operations.

(i) Subtraction of z from y.

Sol: $y - z$

(ii) One-half of the sum of numbers x and y.

Sol: $\frac{1}{2}(x + y)$

(iii) The number z multiplied by itself.

Sol: $z \times z = z^2$

(iv) One-fourth of the product of numbers p and q.

Sol: $\frac{1}{4}pq$

(v) Numbers x and y both squared and added.

Sol: $x^2 + y^2$

(vi) Number 5 added to three times the product of numbers m and n.

Sol: $3mn + 5$

(vii) Product of numbers y and z subtracted from 10.

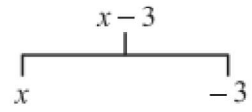
Sol: $10 - yz$

(viii) Sum of numbers a and b subtracted from their product.

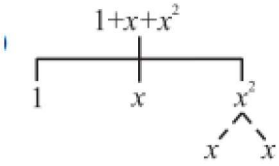
Sol: $ab - (a + b)$

2. **(i) Identify the terms and their factors in the following expressions Show the terms and factors by tree diagrams.**

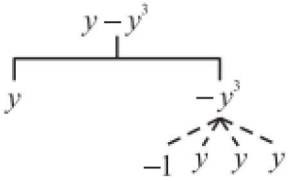
$(a) x - 3$



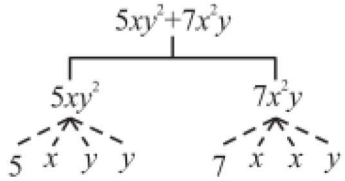
$(b) 1 + x + x^2$



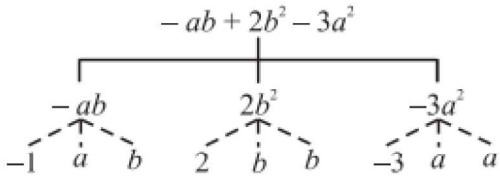
$(c) y - y^3$



$(d) 5xy^2 + 7x^2y$



$(e) - ab + 2b^2 - 3a^2$



(ii) Identify terms and factors in the expressions given below:

$(a) - 4x + 5$ $(b) - 4x + 5y$ $(c) 5y + 3y^2$ $(d) xy + 2x^2y^2$ $(e) pq + q$

$(f) 1.2\,ab - 2.4\,b + 3.6\,a$ $(g) \frac{3}{4}x + \frac{1}{4}$ $(h) 0.1\,p^2 + 0.2\,q^2$

Sol:

	Expression	Terms	Factors
(a)	$-4x + 5$	$-4x$	$-4, x$
		5	5
(b)	$-4x + 5y$	$-4x$	$-4, x$
		$5y$	$5, y$
(c)	$5y + 3y^2$	$5y$	$5, y$

		$3y^2$	$3, y, y$
(d)	$xy + 2x^2y^2$	xy	x, y
		$2x^2y^2$	$2, x, x, y, y$
(e)	$pq + q$	pq	p, q
		q	q
(f)	$1.2ab - 2.4b + 3.6a$	$1.2ab$	$1.2, a, b$
		$-2.4b$	$-2.4, b$
		$3.6a$	$3.6, a$
(g)	$\frac{3}{4}x + \frac{1}{4}$	$\frac{3}{4}x$	$\frac{3}{4}, x$
		$\frac{1}{4}$	
(h)	$0.1p^2 + 0.2q^2$	$0.1p^2$	$0.1, p, p$
		$0.2q^2$	$0.2, q, q$

3. Identify the numerical coefficients of terms (other than constants) in the following expressions:

$(i) 5 - 3t^2$ $(ii) 1 + t + t^2 + t^3$ $(iii) x + 2xy + 3y$ $(iv) 100m + 1000n$

$(v) - p^2q^2 + 7pq$ $(vi) 1.2a + 0.8b$ $(vii) 3.14r^2$ $(viii) 2(l + b)$ $(ix) 0.1y + 0.01y^2$

Sol:

	Expression	Terms	Coefficients
(i)	$5 - 3t^2$	$-3t^2$	-3
(ii)	$1 + t + t^2 + t^3$	t	1
		t^2	1
		t^3	1
(iii)	$x + 2xy + 3y$	x	1
		$2xy$	2
		$3y$	3
(iv)	$100m + 1000n$	$100m$	100
		$1000n$	1000
(v)	$-p^2q^2 + 7pq$	$-p^2q^2$	1
		$7pq$	1
(vi)	$1.2a + 0.8b$	$1.2a$	1.2
		$0.8b$	0.8
(vii)	$3.14r^2$	$3.14r^2$	3.14
(viii)	$2(l + b) = 2l + 2b$	$2l$	2
		$2b$	2

(ix)	$0.1y + 0.01y^2$	$0.1y$	0.1
		$0.01y^2$	0.01

4. (a) Identify terms which contain x and give the coefficient of x.

$(i)y^2x + y$ $(ii)13y^2 - 8yx$ $(iii)x + y + 2$ $(iv)5 + z + zx$ $(v)1 + x + xy$

$(vi)\ 12xy^2 + 25$ $(vii)\ 7x + xy^2$

Sol:

	Expression	Terms with x	Coefficient of x
(i)	$y^2x + y$	y^2x	y^2
(ii)	$13y^2 - 8yx$	$-8yx$	$-8y$
(iii)	$x + y + 2$	x	1
(iv)	$5 + z + zx$	zx	z
(v)	$1 + x + xy$	x and xy	1 and y
(vi)	$12xy^2 + 25$	$12xy^2$	$12y^2$
(vii)	$7 + xy^2$	xy^2	y^2

(b) Identify terms which contain y^2 and give the coefficient of y^2

$(i)\ 8 - xy^2$ $(ii)\ 5y^2 + 7x$ $(iii)\ 2x^2y - 15xy^2 + 7y^2$

	Expression	Terms with y^2	Coefficient of y^2
(i)	$8 - xy^2$	$-xy^2$	$-x$
(ii)	$5y^2 + 7x$	$5y^2$	5
(iii)	$2x^2y - 15xy^2 + 7y^2$	$-15xy^2$ and $7y^2$	$-15x$ and 7

5. Classify into monomials, binomials and trinomials.

$(i)4y - 7z$ → Binomial

$(ii)y^2$ → Monomial

$(iii)x + y - xy$ → Trinomial

$(iv)100$ → Monomial

$(v)ab - a - b$ → Trinomial

$(vi)5 - 3t$ → Binomial

$(vii)4p^2q - 4pq^2$ → Trinomial

$(viii)7mn$ → Monomial

$(ix)z^2 - 3z + 8$ → Trinomial

$(x)a^2 + b^2$ → Binomial

$(xi)z^2 + z$ → Binomial

$(xii)1 + x + x^2$ → Trinomial

6. State whether a given pair of terms is of like or unlike terms

$(i)1, 100$ → Like terms

$(ii)-7x, \frac{5}{2}x$ → Like terms

$(iii)-29x, -29y$ → Unlike terms

$(iv)14xy, 42yx$ → Like terms

$(v)4m^2p, 4mp^2$ → Unlike terms

$(vi)\ 12xz, 12x^2z^2$ → Unlike terms

7. Identify like terms in the following:

(a) $-xy^2, -4yx^2, 8x^2, 2xy^2, 7y, -11x^2, -100x, -11yx, 20x^2y, -6x^2, y, 2xy, 3x$

Sol: $(i) -xy^2, 2xy^2$ $(ii) -4yx^2, 20x^2y$ $(iii) 8x^2, -11x^2, -6x^2$ $(iv) 7y, y$

$(v) -100x, 3x$ $(vi) -11yx, 2xy$

(b) $10pq, 7p, 8q, -p^2q^2, -7qp, -100q, -23, 12p^2q^2, -5p^2, 41, 2405p, 78qp, 13p^2q, qp^2, 701p^2$

$Sol: (i) 10pq, -7qp, 78qp$

$(ii) 7p, 2405p$ $(iii) 8q, -100q$

$(iv) -p^2q^2, 12p^2q^2$ $(v) -23, 41$

$(vi) -5p^2, 701p^2$ $(vii) 13p^2q, qp^2$

FINDING THE VALUE OF AN EXPRESSION

The value of the expression depends on the value of the variable from which the expression is formed.

Example 4:Find the values of the following expressions for x = 2

(i) $x + 4 = 2 + 4 = 6$

(ii) $4x - 3 = (4 \times 2) - 3$

$= 8 - 3 = 5$

(iii) $19 - 5x^2 = 19 - (5 \times 2^2)$

$= 19 - (5 \times 4)$

$= 19 - 20 = -1$

(iv) $100 - 10x^3 = 100 - (10 \times 2^3)$

$= 100 - (10 \times 8)$

$= 100 - 80 = 20$

$2^2 = 2 \times 2 = 4$

$2^3 = 2 \times 2 \times 2 = 8$

Example 5: Find the value of the following expressions when n = – 2.

(i) $5n - 2 = [5 \times (-2)] - 2$

$= -10 - 2 = -12$

$(ii) 5n^2 = [5 \times (-2)^2] = (5 \times 4) = 20$

$5n^2 + 5n - 2 = 20 - 12 = 8$

(iii) $n^3 = (-2)^3 = (-2) \times (-2) \times (-2) = -8$

$n^3 + 5n^2 + 5n - 2 = -8 + 8 = 0$

Example 6:Find the value of the following expressions for a = 3, b = 2.

(i) $a + b = 3 + 2 = 5$

(ii) $7a - 4b = 7 \times 3 - 4 \times 2$

$= 21 - 8 = 13$

(iii) $a^2 + 2ab + b^2$

$= 3^2 + 2 \times 3 \times 2 + 2^2$

$= 9 + 12 + 4 = 25$

(iv) $a^3 - b^3 = 3^3 - 2^3$

$= 3 \times 3 \times 3 - 2 \times 2 \times 2$

$= 9 \times 3 - 4 \times 2$

$= 27 - 8 = 19$

EXERCISE 10.2

1. If m = 2, find the value of

(i) $m - 2 = 2 - 2 = 0$

(ii) $3m - 5 = 3 \times 2 - 5$

$= 6 - 5 = 1$

(iii) $9 - 5m = 9 - 5 \times 2$

$= 9 - 10 = -1$

(iv) $3m^2 - 2m - 7$

$= 3 \times 2^2 - 2 \times 2 - 7$

$= 3 \times 4 - 4 - 7$

$= 12 - 11 = 1$

(v) $\frac{5m}{2} - 4 = \frac{5 \times 2}{2} - 4$

$= \frac{10}{2} - 4$

$= 5 - 4$

$= 1$

2. If p = – 2, find the value of:

(i) $4p + 7 = 4 \times (-2) + 7$

$= -8 + 7 = -1$

(ii) $-3p^2 = (-3) \times (-2)^2$

$= (-3) \times 4 = -12$

$-3p^2 + 4p + 7 = -12 - 1 = -13$

(iii) $-2p^3 = -2 \times (-2)^3$

$= -2 \times (-8) = +16$

$-3p^2 + 4p + 7 = -13$

$-2p^3 - 3p^2 + 4p + 7 = 16 - 13 = 3$

3. Find the value of the following expressions, when $x = -1$:

(i) $2x - 7 = 2 \times (-1) - 7$

$= -2 - 7 = -9$

(ii) $-x + 2 = -(-1) + 2$

$= 1 + 2 = 3$

(iii) $x^2 + 2x + 1 = (-1)^2 + 2(-1) + 1$

$= 1 - 2 + 1 = 2 - 2 = 0$

(iv) $2x^2 - x - 2 = 2 \times (-1)^2 - (-1) - 2$

$= 2 \times 1 + 1 - 2$

$= 2 + 1 - 2 = 3 - 2 = 1$

$(-2)^2 = (-2) \times (-2) = 4$

4. If a = 2, b = – 2, find the value of:

(i) $a^2 + b^2 = (2)^2 + (-2)^2$

$= 4 + 4 = 8$

(ii) $a^2 + ab + b^2 = (2)^2 + 2 \times (-2) + (-2)^2$

$= 4 - 4 + 4 = 4$

(iii) $a^2 - b^2 = (2)^2 - (-2)^2$

$= 4 - 4 = 0$

5. When a = 0, b = – 1, find the value of the given expressions:

(i) $2a + 2b = 2 \times 0 + 2 \times (-1)$

$= 0 - 2 = -2$

$(-1)^2 = (-1) \times (-1) = 1$

(ii) $2a^2 + b^2 + 1 = 2 \times 0^2 + (-1)^2 + 1$

$= 0 + 1 + 1 = 2$

(ii) $2a^2b + 2ab^2 + ab = [2 \times 0^2 \times (-1)] + [2 \times 0 \times (-1)^2] + 0 \times (-1)$

$= 0 + 0 + 0 = 0$

(iii) $a^2 + ab + 2 = 0^2 + 0 \times (-1) + 2$

$= 0 - 0 + 2 = 2$

6. Simplify the expressions and find the value if x = 2

We use distributive property in this simplification

$(i)a(b + c) = a \times b + a \times c$ *and* $(ii)a(b - c) = a \times b - a \times c$

(i) $x + 7 + 4(x - 5)$

$= x + 7 + (4 \times x) - (4 \times 5)$

$= x + 7 + 4x - 20$

$= x + 4x + 7 - 2$

$= 5x - 13$

If $x = 2$

$5x - 13 = (5 \times 2) - 13$

$= 10 - 13 = -3$

(ii) $3(x + 2) + 5x - 7$

$= 3 \times x + 3 \times 2 + 5x - 7$

$= 3x + 6 + 5x - 7$

$= 3x + 5x + 6 - 7$

$= 8x - 1$

If $x = 2$

$8x - 1 = (8 \times 2) - 1$

$= 16 - 1 = 15$

(iii) $6x + 5(x - 2)$

$= 6x + 5 \times x - 5 \times 2$

$= 6x + 5x - 10$

$= 11x - 10$

If $x = 2$

$11x - 10 = (11 \times 2) - 10$

$= 22 - 10 = 12$

(iv) $4(2x - 1) + 3x + 11$

$= (4 \times 2x) - (4 \times 1) + 3x + 11$

$= 8x - 4 + 3x + 11$

$= 8x + 3x - 4 + 11$

$= 11x + 7$

If $x = 2$

$11x + 7 = (11 \times 2) + 7$

$= 22 + 7 = 29$

7. Simplify these expressions and find their values if x = 3, a = – 1, b = – 2.

(i) $\mathbf{3x - 5 - x + 9}$

$= 3x - x - 5 + 9$

$= 2x + 4$

$If\ x = 3\ ;$

$2x + 4 = (2 \times 3) + 4$

$= 6 + 4 = 10$

(ii) $\mathbf{2 - 8x + 4x + 4}$

$= -8x + 4x + 2 + 4$

$= -4x + 6$

$If\ x = 3\ ;$

$-4x + 6 = (-4 \times 3) + 6$

$= -12 + 6 = -6$

(iii) $\mathbf{3a + 5 - 8a + 1}$

$= 3a - 8a + 5 + 1$

$= -5a + 6$

$If\ a = -1$

$-5a + 6 = (-5 \times -1) + 6$

$= 5 + 6 = 11$

(iv) $\mathbf{10 - 3b - 4 - 5b}$

$= -3b - 5b + 10 - 4$

$= -8b + 6$

$If\ b = -2$

$-8b + 6 = (-8 \times -2) + 6$

$= 16 + 6 = 22$

(v) $\mathbf{2a - 2b - 4 - 5 + a}$

$= 2a + a - 2b - 4 - 5$

$= 3a - 2b - 9$

$If\ a = -1\ and\ b = -2$

$3a - 2b - 9$

$= (3 \times -1) - (2 \times -2) - 9$

$= -3 - (-4) - 9$

$= -3 + 4 - 9$

$= -11 + 4 = -8$

8. **(i)** $\mathbf{If\ z = 10, find\ the\ value\ of\ z^3 - 3(z - 10).}$

Sol: $z^3 - 3(z - 10) = z^3 - 3 \times z - 3 \times (-10)$

$= z^3 - 3z + 30$

$If\ z = 10$

$z^3 - 3z + 30 = 10^3 - 3 \times 10 + 30$

$= 1000 - 30 + 30 = 1000$

(ii) $\mathbf{If\ p = -10, find\ the\ value\ of\ p^2 - 2p - 100}$

Sol: $If\ p\ = -\ 10$

$$p^2 - 2p - 100 = (-10)^2 - 2 \times (-10) - 100$$

$$= 100 + 20 - 100 = 20$$

9. What should be the value of a if the value of $2x^2 + x - a$ equals to 5, when x = 0?

Sol: Given: $2x^2 + x - a = 5$

when $x = 0$

$$2 \times 0^2 + 0 - a = 5$$

$$0 - a = 5$$

$$-a = 5$$

$$a = -5$$

10. Simplify the expression and find its value when a = 5 and b = – 3. $2(a^2 + ab) + 3 - ab$

Sol: $2(a^2 + ab) + 3 - ab = (2 \times a^2) + (2 \times ab) + 3 - ab$

$$= 2a^2 + 2ab + 3 - ab$$

$$= 2a^2 + ab + 3$$

$When\ a = 5\ and\ b = -3$

$$2a^2 + ab + 3 = (2 \times 5^2) + 5 \times (-3) + 3$$

$$= (2 \times 25) - 15 + 3$$

$$= 50 - 12 = 38$$