CHAPTER 8

VIII CLASS-NCERT (2023-24)

ALGEBRAIC EXPRESSIONS AND IDENTITIES (Notes)

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1. **Variable**: A variable can take various values and its value cannot be fixed. a, b, x, y, z etc.
2. A constant has a fixed value. For example 6, 8, –10 etc., are some constants
3. Terms are formed as a product of constants and one or more variables.

   Ex: $2x, -3xy, -5x^2, \ldots$.
4. Terms are added or subtracted to form an expression.

(OR)

Expression: An expression is a constant or a variable or combination of these two, using the mathematical operations (+, –, ×, ÷) i.e., terms are added to form expressions.

Examples of expressions are: $2x - 5, -4y + 2, 5x^2, -2xy + 2x + 3y + 7$ *etc*

5. If an expression has at least one algebraic term, then that expression is **Algebraic expression.**
6. The sum of all exponents of the variables in a monomial is the **degree** of the monomial
7. The highest degree among the degrees of the different terms of an algebraic expression is called the degree of that algebraic expression.
8. **Monomial**: Expression that contains only one term is called a monomial.

   Examples: $4x^2, 5xy, -8z, 5xy^2, 10y, \ldots$
9. **Binomial**: Expression that contains two terms is called a binomial.

   Examples: $x + y, a + b, 4l + 5m, 5 - 3xy, ..$
10. **Trinomial**: An expression containing three terms is a trinomial.

    Exp: $a + b + c, 2x + 3y - 5, xy + x^2 + y^2, \ldots$
11. **Polynomial**: An expression containing, one or more terms with non-zero coefficient (with variables having non-negative integers [whole numbers]as exponents) is called a polynomial

    Exp: $a + b + c + d, 3xy, 2x + 5y, \ldots$
12. **Like and Unlike Terms**:

    The terms have same variable with same exponents (powers) are called like terms.

    Example: $(i)\ 2x, 5x, -7x \quad (ii) - 3x^2y, 7x^2y, \frac{2}{3}x^2y$

    Like terms may not have same numerical coefficients.
13. A monomial multiplied by a monomial always gives a monomial.
14. In multiplication of polynomials with polynomials, we should always look for like terms, if any, and combine them

Addition and Subtraction of Algebraic Expressions.

1. We can only combine like terms by adding or subtracting them with one another.
2. Unlike terms cannot combine by adding or subtracting.
3. Subtraction of a number is the same as addition of its additive inverse.

Example 1: Add: $7xy + 5yz - 3zx, 4yz + 9zx - 4y, -3xz + 5x - 2xy$.

Sol:

$$\begin{array}{rl} & 7xy + 5yz - 3zx, \\ + & \quad 4yz + 9zx - 4y \\ + & -2xy \quad -3xz \quad +5x \\ \hline & 5xy + 9yz + 3zx - 4y + 5x \end{array}$$

Example 2: Subtract $5x^2 - 4y^2 + 6y - 3$ from $7x^2 - 4xy + 8y^2 + 5x - 3y$.

Sol:

$$\begin{array}{rl} & 7x^2 - 4xy + 8y^2 + 5x - 3y. \\ & 5x^2 \quad -4y^2 \quad +6y-3 \\ (-) & \quad (+) \quad (-)\ (+) \\ \hline & 2x^2 - 4xy + 12y^2 + 5x - 9y + 3. \end{array}$$

EXERCISE 8.1

1. Add the following

(i) $ab - bc, bc - ca, ca - ab$

Sol: $(ab - bc) + (bc - ca) + (ca - ab)$

$= ab - ab - bc + bc - ca + ca$

$= 0$

(ii) $a - b + ab, b - c + bc, c - a + ac$

Sol: $a - b + ab + b - c + bc + c - a + ac$

$= a - a - b + b + ab - c + c + bc + ac$

$= ab + bc + ca$

(iii) $2p^2q^2 - 3pq + 4, 5 + 7pq - 3p^2q^2$

Sol: $2p^2q^2 - 3pq + 4 + 5 + 7pq - 3p^2q^2$

$= 2p^2q^2 - 3p^2q^2 - 3pq + 7pq + 4 + 5$

$= -p^2q^2 + 4pq + 9$

(iv) $l^2 + m^2, m^2 + n^2, n^2 + l^2, 2lm + 2mn + 2nl$

Sol: $l^2 + m^2 + m^2 + n^2 + n^2 + l^2 + 2lm + 2mn + 2nl$

$= 2l^2 + 2m^2 + 2n^2 + 2lm + 2mn + 2nl$

$= 2(l^2 + m^2 + n^2 + lm + mn + nl)$

2. (a) Subtract $4a - 7ab + 3b + 12$ from $12a - 9ab + 5b - 3$

Sol: $(12a - 9ab + 5b - 3) - (4a - 7ab + 3b + 12)$

$= 12a - 9ab + 5b - 3 - 4a - 12$

$-(+) = -$
$-(-) = +$

$= 12a - 4a - 9ab + 7ab + 5b - 3b - 3 - 12$

$= 8a - 2ab + 2b - 15$

(or)

$$\begin{array}{r} 12a - 9ab + 5b - 3 \\ 4a - 7ab + 3b + 12 \\ (-)\quad (+)\quad (-)\quad (-) \\ \hline 8a - 2ab + 2b - 15 \end{array}$$

(b) Subtract 3xy + 5yz – 7zx from 5xy – 2yz – 2zx + 10xyz

Sol: $(5xy - 2yz - 2zx + 10xyz) - (3xy + 5yz - 7zx)$

$= 5xy - 2yz - 2zx + 10xyz - 3xy - 5yz + 7zx$

$= 5xy - 3xy - 2yz - 5yz - 2zx + 7zx + 10xyz$

$= 2xy - 7yz + 5zx + 10xyz$

(c) Subtract $4p^2q - 3pq + 5pq^2 - 8p + 7q - 10$ from $18 - 3p - 11q + 5pq - 2pq^2 + 5p^2q$

Sol: $(18 - 3p - 11q + 5pq - 2pq^2 + 5p^2q) - (4p^2q - 3pq + 5pq^2 - 8p + 7q - 10)$

$= 18 - 3p - 11q + 5pq - 2pq^2 + 5p^2q - 4p^2q + 3pq - 5pq^2 + 8p - 7q + 10$

$= 18 + 10 - 3p + 8p - 11q - 7q + 5pq + 3pq - 2pq^2 - 5pq^2 + 5p^2q - 4p^2q$

$= 28 + 5p - 18q + 8pq - 7pq^2 + p^2q$

(or)

$$\begin{array}{r} 18 - 3p - 11q + 5pq - 2pq^2 + 5p^2q \\ -10 - 8p - 7q - 3pq + 5pq^2 + 4p^2q \\ (+)\ (+)\quad (+)\quad (+)\quad (-)\quad (-) \\ \hline 28 + 5p - 18q + 8pq - 7pq^2 + p^2q \end{array}$$

8.2 Multiplication of Algebraic Expressions: Introduction:

Multiplying two monomials:

(i) $x \times x = x^2$; $x \times x^2 = x^3$; $x^2 \times x^3 = x^5$

(ii) $4 \times 3x = 4 \times 3 \times x = 12x$

(iii) $5x \times 3y = 5 \times 3 \times x \times y = 15xy$

(iv) $(-7x) \times 5y = (-7) \times 5 \times x \times y = -35xy$

(v) $5x \times 4x^2 = 5 \times 4 \times x \times x^2 = 20x^3$

(vi) $6x \times (-7xyz) = 6 \times (-7) \times x \times xyz = -42x^2yz$

Multiplying three or more monomials:

(i) $2x \times 5y \times 7z = 2 \times 5 \times 7 \times x \times y \times z = 70xyz$

(ii) $4xy \times 5x^2y^2 \times 6x^3y^3 = 4 \times 5 \times 6 \times x \times x^2 \times x^3 \times y \times y^2 \times y^3 = 120x^6y^6$

TRY THESE

Find 4x × 5y × 7z First find 4x × 5y and multiply it by 7z; or first find 5y × 7z and multiply

it by 4x. Is the result the same? What do you observe? Does the order in which you carry out the multiplication matter?

Sol: $4x \times 5y = 4 \times 5 \times x \times y = 20xy$

$(4x \times 5y) \times 7z = 20xy \times 7z$

$= 20 \times 7 \times xy \times z$

$= 140xyz$

$5y \times 7z = 5 \times 7 \times y \times z = 35yz$

$4x \times (5y \times 7z) = 4x \times 35yz$

$= 4 \times 35 \times x \times yz$

$= 140xyz$

We observe the order in which multiply the monomials do not matter.

Also the multiplication of monomials is associative.

Example 3: Complete the table for area of a rectangle with given length and breadth.

Sol:

length	breadth	Area= length× breadth
$3x$	$5y$	$3x \times 5y = 3 \times 5 \times x \times y = 15xy$
$9y$	$4y^2$	$9y \times 4y^2 = 9 \times 4 \times y \times y^2 = 36y^3$
$4ab$	$5bc$	$4ab \times 5bc = 4 \times 5 \times a \times b \times b \times c = 20ab^2c$
$2l^2m$	$3lm^2$	$2l^2m \times 3lm^2 = 2 \times 3 \times l^2 \times l \times m \times m^2 = 6l^3m^3$

Example 4: Find the volume of each rectangular box with given length, breadth and height.

	length	breadth	height
(i)	$2ax$	$3by$	$5cz$
(ii)	m^2n	n^2p	p^2m
(iii)	$2q$	$4q^2$	$8q^3$

Sol: Volume = length × breadth × height

(i) Volume $= (2ax) \times (3by) \times (5cz)$

$= 2 \times 3 \times 5 \times a \times b \times c \times x \times y \times z$

$= 30abcxyz$

(ii) Volume $= (m^2n) \times (n^2p) \times (p^2m)$

$= m^2 \times m \times n \times n^2 \times p \times p^2$

$= m^3n^3p^3$

(iii) Volume $= 2q \times 4q^2 \times 8q^3$

$= 2 \times 4 \times 8 \times q \times q^2 \times q^3$

$= 64q^6$

EXERCISE 8.2

1. **Find the product of the following pairs of monomials.**

(i) 4, 7p

Sol: $4 \times 7p = 28p$

(ii) – 4p, 7p

Sol: $-4p \times 7p$

$= (-4 \times 7) \times p \times p$

$= -28p^2$

(iii) – 4p, 7pq

Sol: $-4p \times 7pq$

$= (-4 \times 7) \times p \times p \times q$

$= -28p^2q$

(iv) $4p^3$, – 3p

Sol: $4p^3 \times (-3p)$

$= (4 \times -3) \times p^3 \times p$

$= -12p^4$

(v) 4p, 0

Sol: $4p \times 0 = 0$

2. **Find the areas of rectangles with the following pairs of monomials as their lengths and breadths respectively.**

(i) (p,q) Length(l) = p and breadth(b) = q

Sol: Area of rectangle = l × b

= p × q

= pq squnits

(ii) (10m,5n) Length(l) = 10m and breadth(b) = 5n

Sol: Area of rectangle = $l \times b$

= 10m × 5n

= 50mn squnits

(iii) **($20x^2, 5y^2$) Length(l) = $20x^2$ and breadth(b) = $5y^2$**

Sol: Area of rectangle = $l \times b$

$= 20x^2 \times 5y^2$

$= 100x^2y^2$ squnits

(iv) ($4x, 3x^2$) Length(l) = 4x and breadth(b) = $3x^2$

Sol: Area of rectangle = $l \times b$

$= 4x \times 3x^2$

$= 12x^3$ squnits

(v) (3mn, 4np) Length(l) = 3mn and breadth(b) = 4np

Sol: Area of rectangle = $l \times b$

$= 3mn \times 4np$

$= 12mn^2p$ squnits

3. **Complete the table of products.**

First monomial→ / Second monomial↓	$2x$	$-5y$	$3x^2$	$-4xy$	$7x^2y$	$-9x^2y^2$
$2x$	$4x^2$	$-10xy$	$6x^3$	$-8x^2y$	$14x^3y$	$-18x^3y^2$
$-5y$	$-10xy$	$25y^2$	$-15x^2y$	$20xy^2$	$-35x^2y^2$	$45x^2y^3$
$3x^2$	$6x^3$	$-15x^2y$	$9x^4$	$-12x^3y$	$21x^4y$	$-27x^4y^2$
$-4xy$	$-8x^2y$	$20xy^2$	$-12x^3y$	$16x^2y^2$	$-28x^3y^2$	$36x^3y^3$
$7x^2y$	$14x^3y$	$-35x^2y^2$	$21x^4y$	$-28x^3y^2$	$49x^4y^2$	$-36x^4y^3$
$-9x^2y^2$	$-18x^3y^2$	$45x^2y^3$	$-27x^4y^2$	$36x^3y^3$	$-36x^4y^3$	$81x^4y^4$

4. **Obtain the volume of rectangular boxes with the following length, breadth and height respectively**

(i) $5a, 3a^2, 7a^4$

Sol: $Volume\ of\ rectangular\ box(cuboid) = l \times b \times h$

$= 5a \times 3a^2 \times 7a^4$

$= 5 \times 3 \times 7 \times a \times a^2 \times a^4$

$= 105a^7\ cubic\ units$

(ii) $2p, 4q, 8r$

Sol: $Volume\ of\ rectangular\ box(cuboid) = l \times b \times h$

$= 2p \times 4q \times 8r$

$= 2 \times 4 \times 8 \times p \times q \times r$

$= 64pqr\ \ cubic\ units$

(iii) $xy, 2x^2y, 2xy^2$

Sol: $Volume\ of\ rectangular\ box(cuboid) = l \times b \times h$

$= xy \times 2x^2y \times 2xy^2$

$= 2 \times 2 \times x \times x^2 \times x \times y \times y \times y^2 cubic\ units$

$= 4x^4y^4\ cubic\ units$

(iv) $a, 2b, 3c$

Sol: $Volume\ of\ rectangular\ box(cuboid) = l \times b \times h$

$= a \times\ 2b \times 3c$

$= 2 \times 3 \times a \times b \times c$

$= 6abc\ \ cubic\ units$

5. **Obtain the product of**

(i) xy, yz, zx

Sol: Product $= xy \times\ yz \times zx = x^2y^2z^2$

(ii) $a, -a^2, a^3$

Sol: Product $= a \times (-a^2) \times a^3 = -a^6$

(iii) $2, 4y, 8y^2, 16y^3$

Sol: Product $= 2 \times 4y \times 8y^2 \times 16y^3$

$= 2 \times 4 \times 8 \times 16 \times y \times y^2 \times y^3$

$= 1024y^6$

(iv) $a, 2b, 3c, 6abc$

Sol: Product $= a \times 2b \times 3c \times 6abc$

$= 2 \times 3 \times 6 \times a \times b \times c \times abc$

$= 36a^2b^2c^2$

(v) $m, -mn, mnp$

Sol: Product $= m \times (-mn) \times mnp$

$= -m^3n^2p$

Multiplying a monomial by a binomial, a trinomial:

Distributive law

$a(b + c) = a \times b + a \times c$

$a(b + c + d) = a \times b + a \times c + a \times d$

$a(b - c) = a \times b - a \times c$

Commutative law

$a \times b = b \times a$

TRY THESE

(i) **Find the product $2x\ (3x + 5xy)$**

Sol: $2x\ (3x + 5xy)$

$= (2x \times 3x) + (2x \times 5xy)$

$= 6x^2 + 10x^2y$

(ii) **Find the product $a^2\ (2ab - 5c)$**

Sol: $a^2\ (2ab - 5c)$

$= (a^2 \times 2ab) - (a^2 \times 5c)$

$= 2a^3b - 5a^2c$

(iii) **Find the product $(4p^2 + 5p + 7) \times 3p$**

Sol: $(4p^2 + 5p + 7) \times 3p$

$= 3p \times (4p^2 + 5p + 7)$

$= (3p \times 4p^2) + (3p \times 5p) + (3p \times 7)$

$= 12p^3 + 15p^2 + 21p$

Example 5: Simplify the expressions and evaluate them as directed:

(i) **$x\ (x - 3) + 2$ for $x = 1$**

Sol: $x\ (x - 3) + 2$

$= x \times x - x \times 3 + 2$

$= x^2 - 3x + 2$

$For\ x\ =\ 1,$

$x^2 - 3x + 2$

$= (1)^2 - 3(1) + 2$

$= 1 - 3 + 2$

$= 3 - 3 = 0$

(ii) 3y (2y – 7)– 3 (y – 4)– 63 for y = –2

Sol: $3y\ (2y - 7) - 3\ (y - 4) - 63$

$= 3y \times 2y - 3y \times 7 - 3 \times y - 3 \times (-4) - 63$

$= 6y^2 - 21y - 3y + 12 - 63$

$= 6y^2 - 24y - 51$

$For\ \ y = -2,$

$6y^2 - 24y - 51$

$= 6(-2)^2 - 24(-2) - 51$

$= 6 \times 4 + 24 \times 2 - 51$

$= 24 + 48 - 51$

$= 72 - 51$

$= 21$

Example 6: Add

(i) 5m (3 – m) and $6m^2$ – 13m

Sol: $5m\ (3 - m) = 5m \times 3 - 5m \times m$

$= 15m - 5m^2$

$5m\ (3 - m) + 6m^2 - 13m$

$= 15m - 5m^2 + 6m^2 - 13m$

$= -5m^2 + 6m^2 + 15m - 13m$

$= m^2 + 2m$

(ii) 4y ($3y^2$ + 5y – 7) and 2 (y^3 – $4y^2$ + 5)

Sol: $4y\ (3y^2 + 5y - 7)$

$= (4y \times 3y^2) + (4y \times 5y) - (4y \times 7)$

$= 12y^3 + 20y^2 - 28y$

$2\ (y^3 - 4y^2 + 5)$

$= (2 \times y^3) - (2 \times 4y^2) + (2 \times 5)$

$= 2y^3 - 8y^2 + 10$

Sum=$12y^3 + 20y^2 - 28y + 2y^3 - 8y^2 + 10$

$= 12y^3 + 2y^3 + 20y^2 - 8y^2 - 28y + 10$

$= 14y^3 + 12y^2 - 28y + 10$

(OR)

$$\begin{array}{rl} & 12y^3 + 20y^2 - 28y \\ (+) & 2y^3 - 8y^2 + 10 \\ \hline & 14y^3 - 12y^2 - 28y + 10 \end{array}$$

Example 7: Subtract 3pq (p – q) from 2pq (p + q).

Sol: $3pq\,(p - q) = 3pq \times p - 3pq \times q = 3p^2q - 3pq^2$

$2pq\,(p + q) = 2pq \times p + 2pq \times q = 2p^2q + 2pq^2$

Subtracting,

$$\begin{array}{rl} & 2p^2q + 2pq^2 \\ & 3p^2q - 3pq^2 \\ (-) \quad & (+) \\ \hline & -p^2q + 5pq^2 \end{array}$$

EXERCISE 8.3

1. Carry out the multiplication of the expressions in each of the following pairs.

(i) 4p, q + r

Sol: $4p \times (q + r)$

$= (4p \times q) \times (4p \times r)$

$= 4pq + 4pr$

(ii) ab, a – b

Sol: $ab \times (a - b)$

$= (ab \times a) - (ab \times b)$

$= a^2b - ab^2$

(iii) a + b, $7a^2b^2$

Sol: $7a^2b^2 \times (a + b)$

$= (7a^2b^2 \times a) + (7a^2b^2 \times b)$

$= 7a^3b^2 + 7a^2b^3$

(iv) $a^2 - 9$, 4a

Sol: $4a(a^2 - 9)$

$= (4a \times a^2) - (4a \times 9)$

$= 4a^3 - 36a$

v) pq + qr + rp, 0

Sol: $0 \times (pq + qr + rp) = 0$

2. Complete the table

(i) $\mathbf{a \times (b + c + d)}$

$= (a \times b) + (a \times c) + (a \times d)$

$= ab + ac + ad$

(ii) $\mathbf{5xy \times (x + y - 5)}$

$= (5xy \times x) + (5xy \times y) - (5xy \times 5)$

$= 5x^2y + 5xy^2 - 25xy$

(iii) $\mathbf{p \times (6p^2 - 7p + 5)}$

$= (p \times 6p^2) - (p \times 7p) + (p \times 5)$

$= 6p^3 - 7pq + 5p$

(iv) $\mathbf{4p^2q^2(p^2 - q^2)}$

$= (4p^2q^2 \times p^2) - (4p^2q^2 \times q^2)$

$= 4p^4q^2 - 4p^2q^4$

(v) $\mathbf{abc \times (a + b + c)}$

$= (abc \times a) + (abc \times b) + (abc \times c)$

$= a^2bc + ab^2c + abc^2$

3. **Find the product**

(i) $\mathbf{(a^2) \times (2a^{22}) \times (4a^{26})}$

Sol: $(a^2) \times (2a^{22}) \times (4a^{26})$

$= (1 \times 2 \times 4) \times (a^2 \times a^{22} \times a^{26})$

$= 8a^{50}$

(ii) $\mathbf{\left(\frac{2}{3}xy\right) \times \left(\frac{-9}{10}x^2y^2\right)}$

Sol: $\left(\frac{2}{3}xy\right) \times \left(\frac{-9}{10}x^2y^2\right)$

$= \left(\frac{2}{3} \times \frac{-9}{10}\right) \times (x \times x^2 \times y \times y^2)$

$= \frac{-3}{5}x^3y^3$

(iii) $\mathbf{\left(-\frac{10}{3}pq^3\right) \times \left(\frac{6}{5}p^3q\right)}$

Sol: $\left(-\frac{10}{3}pq^3\right) \times \left(\frac{6}{5}p^3q\right)$

$= \left(-\frac{10}{3} \times \frac{6}{5}\right) \times (p \times p^3) \times (q^3 \times q)$

$= -4p^4q^4$

(iv) $\mathbf{x \times x^2 \times x^3 \times x^4}$

Sol: $x \times x^2 \times x^3 \times x^4$

$= x^{1+2+3+4}$

$= x^{10}$

4. (a)Simplify 3x (4x – 5) + 3 and find its values for (i)x = 3 (ii)x = $\frac{1}{2}$.

Sol: $3x\,(4x-5)+3$

$=(3x\times 4x)-(3x\times 5)+3$

$=12x^2-15x+3$

(i) *For* $x=3$,

$12x^2-15x+3$

$=12(3)^2-15(3)+3$

$=12\times 9-45+3$

$=108-42$

$=66$

(ii) *For* $x=\frac{1}{2}$

$12x^2-15x+3$

$=12\left(\frac{1}{2}\right)^2-15\left(\frac{1}{2}\right)+3$

$=12\times\frac{1}{4}-15\times\frac{1}{2}+3$

$=3-\frac{15}{2}+3$

$=6-\frac{15}{2}=\frac{12-15}{2}=\frac{-3}{2}$

(b) Simplify a (a^2 + a + 1) + 5 and find its value for (i)a = 0, (ii)a = 1 (iii) *a* = – 1

Sol: $a\,(a^2+a+1)+5$

$=(a\times a^2)+(a\times a)+(a\times 1)+5$

$=a^3+a^2+a+5$

(i) *For* $a=0$

a^3+a^2+a+5

$=0^3+0^2+0+5$

$=0+5$

$=5$

(ii) *For* $a=1$

a^3+a^2+a+5

$=1^3+1^2+1+5$

$=1+1+1+5=8$

(iii) *For* $a=1$

a^3+a^2+a+5

$=(-1)^3+(-1)^2+(-1)+5$

$=-1+1-1+5=4$

5. (a)Add: p (p – q), q (q – r)and r (r – p)

Sol: $p\,(p-q)=p\times p-p\times q=p^2-pq$

$q\,(q-r)=q\times q-q\times r=q^2-qr$

$r\,(r-p)=r\times r-r\times p=r^2-rp$

$Sum=p^2-pq+q^2-qr+r^2-rp$

$=p^2+q^2+r^2-pq-qr-rp$

(*b*) Add: 2x (z – x – y)and 2y (z – y – x)

Sol: $2x\,(z-x-y)+2y\,(z-y-x)$

$=(2x\times z)-(2x\times x)-(2x\times y)+(2y\times z)-(2y\times y)-(2y\times x)$

$=2xz-2x^2-2xy+2yz-2y^2-2xy$

$=-2x^2-2y^2-4xy+2yz+2xz$

(*c*) Subtract: 3l (l – 4 m + 5 n) from 4l (10 n – 3 m + 2 l)

Sol: $4l\,(10n-3m+2l)$

$=(4l\times 10n)-(4l\times 3m)+(4l\times 2l)$

$= 40ln - 12lm + 8l^2$

$3l\,(l - 4m + 5n)$

$= (3l \times l) - (3l \times 4m) + (3l \times 5n)$

$= 3l^2 - 12lm + 15ln$

$Now\ (40ln - 12lm + 8l^2) - (3l^2 - 12lm + 15ln)$

$= 40ln - 12lm + 8l^2 - 3l^2 + 12lm - 15ln$

$= 40ln - 15ln - 12lm + 12lm + 8l^2 - 3l^2$

$= 25ln + 5l^2$

(*d*) Subtract: 3a (a + b + c) – 2 b (a – b + c) from 4c (– a + b + c)

Sol: $4c\,(-a + b + c)$

$= (4c \times -a) + (4c \times b) + (4c \times c)$

$= -4ac + 4bc + 4c^2$

$3a\,(a + b + c) - 2\,b\,(a - b + c)$

$= (3a^2 + 3ab + 3ac) - (2ab - 2b^2 + 2bc)$

$= 3a^2 + 3ab + 3ac - 2ab + 2b^2 - 2bc$

$= 3a^2 + 2b^2 + ab + 3ac - 2bc$

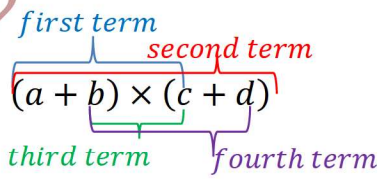
$Now(-4ac + 4bc + 4c^2) - (3a^2 + 2b^2 + ab + 3ac - 2bc)$

$= -4ac + 4bc + 4c^2 - 3a^2 - 2b^2 - ab - 3ac + 2bc$

$= -3a^2 - 2b^2 + 4c^2 - ab + 6bc - 7ac$

Multiplying a binomial by a binomial:

$(a + b) \times (c + d) = a \times (c + d) + b \times (c + d) = a \times c + a \times d + b \times c + b \times d$



(In multiplication of polynomials with polynomials, we should always look for like terms, if any, and combine them).

Example 8: Multiply

(i) (x – 4) and (2x + 3)

Sol: $(x - 4) \times (2x + 3)$

$= x \times (2x + 3) - 4 \times (2x + 3)$

$= (x \times 2x) + (x \times 3) - (4 \times 2x) - (4 \times 3)$

$= 2x^2 + 3x - 8x - 12$

$= 2x^2 - 5x - 12$

(ii) (x – y) and (3x + 5y)

Sol: $(x - y) \times (3x + 5y)$

$= x \times (3x + 5y) - y \times (3x + 5y)$

$= (x \times 3x) + (x \times 5y) - (y \times 3x) - (y \times 5y)$

$= 3x^2 + 5xy - 3xy - 5y^2$

$= 3x^2 + 2xy - 5y^2$

Example 9: Multiply

(i) $(a + 7)$ and $(b - 5)$

Sol: $(a + 7) \times (b - 5)$

$= a \times (b - 5) + 7 \times (b - 5)$

$= a \times b - a \times 5 + 7 \times b - 7 \times 5$

$= ab - 5a + 7b - 35$

(ii) $(a^2 + 2b^2)$ and $(5a - 3b)$

Sol: $(a^2 + 2b^2) \times (5a - 3b)$

$= a^2 \times (5a - 3b) + 2b^2 \times (5a - 3b)$

$= a^2 \times 5a - a^2 \times 3b + 2b^2 \times 5a - 2b^2 \times 3b$

$= 5a^3 - 3a^2b + 10ab^2 - 6b^3$

Multiplying a binomial by a trinomial

$(a + b) \times (p + q + r) = a \times p + a \times q + a \times r + b \times p + b \times q + b \times r$

Ex: $(a + 7) \times (a^2 + 3a + 5)$

$= a \times (a^2 + 3a + 5) + 7 \times (a^2 + 3a + 5)$

$= a \times a^2 + a \times 3a + a \times 5 + 7 \times a^2 + 7 \times 3a + 7 \times 5$

$= a^3 + 3a^2 + 5a + 7a^2 + 21a + 35$

$= a^3 + 3a^2 + 7a^2 + 5a + 21a + 35$

$= a^3 + 10a^2 + 26a + 35$

Example 10: Simplify $(a + b)(2a - 3b + c) - (2a - 3b)c$

Sol: $(a + b)(2a - 3b + c)$

$= (a \times 2a) - (a \times 3b) + (a \times c) + (b \times 2a) - (b \times 3b) + (b \times c)$

$= 2a^2 - 3ab + ac + 2ab - 3b^2 + bc$

$= 2a^2 - ab + ac - 3b^2 + bc$

$(2a - 3b)c = c \times (2a - 3b)$

$= c \times 2a - c \times 3b$

$= 2ac - 3bc$

Now $(a + b)(2a - 3b + c) - (2a - 3b)c$

$= (2a^2 - ab + ac - 3b^2 + bc) - (2ac - 3bc)$

$= 2a^2 - ab + ac - 3b^2 + bc - 2ac + 3bc$

$= 2a^2 - ab + ac - 2ac - 3b^2 + bc + 3bc$

$= 2a^2 - ab - ac - 3b^2 + 4bc$

EXERCISE 8.4

1. Multiply the binomials

(i) (2x + 5)and (4x – 3)

Sol: $(2x + 5) \times (4x - 3)$

$= 2x \times (4x - 3) + 5 \times (4x - 3)$

$= (2x \times 4x) - (2x \times 3) + (5 \times 4x) + (5 \times 3)$

$= 8x^2 - 6x + 20x + 15$

$= 8x^2 + 14x + 15$

(ii) (y – 8)and (3y – 4)

Sol: $(y - 8) \times (3y - 4)$

$= y \times (3y - 4) - 8 \times (3y - 4)$

$= (y \times 3y) - (y \times 4) - (8 \times 3y) - (8 \times -4)$

$= 3y^2 - 4y - 24y + 32$

$= 3y^2 - 28y + 32$

(iii) (2.5l – 0.5m) and (2.5l + 0.5m)

Sol: $(2.5l - 0.5m) \times (2.5l + 0.5m)$

$= 2.5l \times (2.5l + 0.5m) - 0.5m \times (2.5l + 0.5m)$

$= (2.5l \times 2.5l) + (2.5l \times 0.5m) - (0.5m \times 2.5l) - (0.5m \times 0.5m)$

$= 6.25l^2 + 1.25lm - 1.25lm - 0.25m^2$

$= 6.25l^2 - 0.25m^2$

(iv) (a + 3b) and (x + 5)

Sol: $(a + 3b) \times (x + 5)$

$= a \times (x + 5) + 3b \times (x + 5)$

$= (a \times x) + (a \times 5) + (3b \times x) + (3b \times 5)$

$= ax + 5a + 3bx + 15b$

(v)(2pq + 3q²)and (3pq – 2q²)

Sol: $(2pq + 3q^2) \times (3pq - 2q^2)$

$= 2pq \times (3pq - 2q^2) + 3q^2 \times (3pq - 2q^2)$

$= (2pq \times 3pq) - (2pq \times 2q^2) + (3q^2 \times 3pq) - (3q^2 \times 2q^2)$

$= 6p^2q^2 - 4pq^3 + 9pq^3 - 6q^4$

$= 6p^2q^2 + 5pq^3 - 6q^4$

(vi) $\left(\frac{3}{4}a^2 + 3b^2\right)$ **and** $4\left(a^2 - \frac{2}{3}b^2\right)$

Sol: $\left(\frac{3}{4}a^2+3b^2\right)\times 4\left(a^2-\frac{2}{3}b^2\right)$

$=\left(\frac{3}{4}a^2+3b^2\right)\times\left(4a^2-\frac{8}{3}b^2\right)$

$=\frac{3}{4}a^2\times\left(4a^2-\frac{8}{3}b^2\right)+3b^2\times\left(4a^2-\frac{8}{3}b^2\right)$

$=\left(\frac{3}{4}a^2\times 4a^2\right)-\left(\frac{3}{4}a^2\times\frac{8}{3}b^2\right)+(3b^2\times 4a^2)-\left(3b^2\times\frac{8}{3}b^2\right)$

$=3a^4-2a^2b^2+12a^2b^2-8b^4$

$=3a^4+10a^2b^2-8b^4$

2. Find the product.

(i)$(5-2x)(3+x)$

Sol: $(5-2x)(3+x)$

$=5\times(3+x)-2x\times(3+x)$

$=(5\times 3)+(5\times x)-(2x\times 3)-(2x\times x)$

$=15+5x-6x-2x^2$

$=15-x-2x^2$

(ii)$(x+7y)(7x-y)$

Sol: $(x+7y)(7x-y)$

$=x\times(7x-y)+7y\times(7x-y)$

$=(x\times 7x)-(x\times y)+(7y\times 7x)-(7y\times y)$

$=7x^2-xy+49xy-7y^2$

$=7x^2+48xy-7y^2$

(iii)$(a^2+b)(a+b^2)$

Sol: $(a^2+b)(a+b^2)=a^2\times(a+b^2)+b\times(a+b^2)$

$=(a^2\times a)+(a^2\times b^2)+(b\times a)+(b\times b^2)$

$=a^3+a^2b^2+ab+b^3$

(iv) $(p^2-q^2)(2p+q)$

Sol: $(p^2-q^2)(2p+q)$

$=p^2\times(2p+q)-q^2\times(2p+q)$

$=(p^2\times 2p)+(p^2\times q)-(q^2\times 2p)-(q^2\times q)$

$=2p^3+p^2q-2q^2p-q^3$

3. Simplify

(i)$(x^2-5)(x+5)+25$

Sol: $(x^2-5)(x+5)+25$

$=x^2\times(x+5)-5\times(x+5)+25$

$=(x^2\times x)+(x^2\times 5)-(5\times x)-(5\times 5)+25$

$= x^3 + 5x^2 - 5x - 25 + 25$

$= x^3 + 5x^2 - 5x$

(ii) $(a^2 + 5)(b^3 + 3) + 5$

Sol: $(a^2 + 5)(b^3 + 3) + 5$

$= a^2 \times (b^3 + 3) + 5 \times (b^3 + 3) + 5$

$= (a^2 \times b^3) + (a^2 \times 3) + (5 \times b^3) + (5 \times 3) + 5$

$= a^2b^3 + 3a^2 + 5b^3 + 15 + 5$

$= a^2b^3 + 3a^2 + 5b^3 + 20$

(iii) $(t + s^2)(t^2 - s)$

Sol: $(t + s^2)(t^2 - s)$

$= t \times (t^2 - s) + s^2 \times (t^2 - s)$

$= (t \times t^2) - (t \times s) + (s^2 \times t^2) - (s^2 \times s)$

$= t^3 - ts + s^2t^2 - s^3$

(iv) $(a + b)(c - d) + (a - b)(c + d) + 2(ac + bd)$

Sol: $(a + b)(c - d) + (a - b)(c + d) + 2(ac + bd)$

$= a \times (c - d) + b \times (c - d) + a \times (c + d) - b \times (c + d) + 2(ac + bd)$

$= ac - ad + bc - bd + ac + ad - bc - bd + 2ac + 2bd$

$= 2ac - 2bd + 2ac + 2bd$

$= 4ac$

(v) $(x + y)(2x + y) + (x + 2y)(x - y)$

Sol: $(x + y)(2x + y) + (x + 2y)(x - y)$

$= x \times (2x + y) + y \times (2x + y) + x \times (x - y) + 2y \times (x - y)$

$= 2x^2 + xy + 2xy + y^2 + x^2 - xy + 2xy - 2y^2$

$= 3x^2 + 4xy - y^2$

(vi) $(x + y)(x^2 - xy + y^2)$

Sol: $(x + y)(x^2 - xy + y^2)$

$= x \times (x^2 - xy + y^2) + y \times (x^2 - xy + y^2)$

$= (x \times x^2) - (x \times xy) + (x \times y^2) + (y \times x^2) - (y \times xy) + (y \times y^2)$

$= x^3 - x^2y + xy^2 + x^2y - xy^2 + y^3$

$= x^3 - y^3$

(vii) $(1.5x - 4y)(1.5x + 4y + 3) - 4.5x + 12y$

Sol: $(1.5x - 4y)(1.5x + 4y + 3) - 4.5x + 12y$

$= 1.5x \times (1.5x + 4y + 3) - 4y \times (1.5x + 4y + 3) - 4.5x + 12y$

$= 2.25x^2 + 6xy + 4.5x - 6xy - 16y^2 - 12y - 4.5x + 12y$

$= 2.25x^2 - 16y^2$

(viii) $(a + b + c)(a + b - c)$

Sol: $(a + b + c)(a + b - c)$

$= a \times (a + b - c) + b \times (a + b - c) + c \times (a + b - c)$

$= a^2 + ab - ac + ab + b^2 - bc + ac + bc - c^2$

$= a^2 + b^2 - c^2 + 2ab$