CHAPTER 8

VII-MATHEMATICS-NCERT (2024-25)

8. Rational numbers (notes)

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https://sureshmathsmaterial.com

1. **Natural numbers**: The numbers which are used for counting are called Natural numbers and represented with letter N

   N={1,2,3,4,5,.....}

2. **Whole numbers**: If '0' is added to Natural numbers then they are called Whole numbers. And is denoted by 'W'

   W={0,1,2,3,4,5,.....}

3. **Integers**: Combination of positive and negative numbers including 0 are called Integers and represented by 'Z' or 'I'.

   Z={... ... − 4, −3, −2, −1,0,1,2,3,4, ... }

4. **RATIONAL NUMBER**: A number that can be expressed in the form $\frac{p}{q}$, where p and q are integers and q ≠ 0 is called a rational number.

   Ex: $\frac{1}{2}, \frac{-5}{7}, 1\frac{4}{5}, 0.5. 0.33, \ldots..$

5. Integers also rational numbers ($5 = \frac{5}{1}, -7 = \frac{-7}{1}, 0 = \frac{0}{2}$)
6. Rational numbers include integers and fractions.

1. Is the number $\frac{2}{-3}$ rational? Think about it.

Sol: $\frac{2}{-3}$ is rational because it is inthe form $\frac{p}{q}$, $whwre\ p = 2\ and\ q = -3(\neq 0) are\ integers.$

2. List ten rational numbers.

Sol: $\frac{1}{5}, \frac{-3}{4}, \frac{7}{9}, \frac{11}{5}, \frac{5}{-9}, \frac{25}{21}, \frac{37}{145}, 0, -5, 7.$

EQUIVALENT RATIONAL NUMBERS:

By multiplying or dividing the numerator and denominator of a rational number by the same non-zero integer we get equivalent rational number to the given.

Ex: $\frac{10}{-15} = \frac{10 \div (-5)}{-15 \div (-5)} = \frac{-2}{3}$; $\frac{-3}{7} = \frac{-3 \times 2}{7 \times 2} = \frac{-6}{14}$

TRY THESE

Fill in the boxes

(i) $\frac{5}{4}=\frac{5\times 4}{4\times 4}=\frac{5\times 5}{4\times 5}=\frac{5\times(-3)}{4\times(-3)}$

$\frac{5}{4}=\frac{20}{16}=\frac{25}{20}=\frac{-15}{-12}$

(ii) $\frac{-3}{7}=\frac{-3\times 2}{7\times 2}=\frac{-3\times(-3)}{7\times(-3)}=\frac{-3\times 2}{7\times 2}$

$\frac{-3}{7}=\frac{-6}{14}=\frac{9}{-21}=\frac{-6}{14}$

POSITIVE AND NEGATIVE RATIONAL NUMBERS

(i) Both the numerator and denominator of a rational number are positive integers (or negative) is called a positive rational number.

$\frac{2}{3},\frac{5}{7},\frac{11}{45},\frac{-5}{-7},\frac{-4}{-9}\cdots$

(ii) Either numerator or denominator is negative integer is called negative rational number.

$\frac{-2}{3},\frac{5}{-7},\frac{-11}{45},\cdots$

(iii) The number 0 is neither a positive nor a negative rational number.

TRY THESE

Which of these are negative rational numbers?

(i) $\frac{-2}{3}$ (Negative rational number)

(ii) $\frac{5}{7}$ (Positive rational number)

(iii) $\frac{3}{-5}$ (Negative rational number)

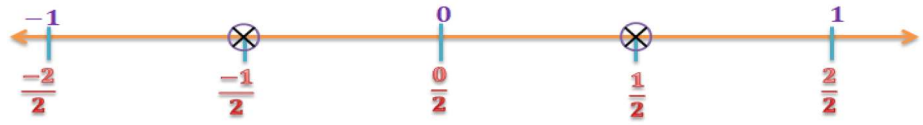
(iv) 0 (Neither positive nor negative)

(v) $\frac{6}{11}$ (Positive rational number)

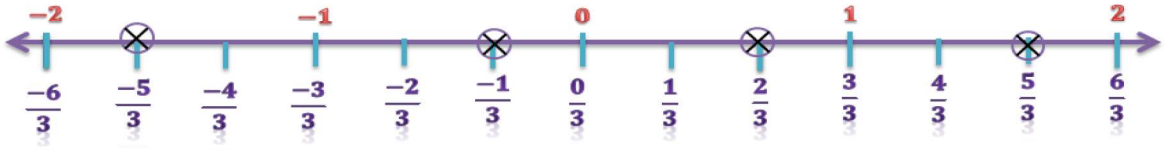
(vi) $\frac{-2}{-9}=\frac{2}{9}$ (Positive rational number)

RATIONAL NUMBERS ON A NUMBER LINE

1. Represent $\frac{1}{2}$ and $\frac{-1}{2}$ on number line.



2. Represent $\frac{-1}{3},\frac{-5}{3},\frac{2}{3}$ and $\frac{5}{3}$ on number line.



RATIONAL NUMBERS IN STANDARD FORM

A rational number is said to be in the standard form if its denominator is a positive integer and the numerator and denominator have no common factor other than 1.

Thus, to reduce the rational number to its standard form, we divide its numerator and denominator by their HCF.

If there is negative sign in the denominator, divide by '– HCF'.

Example1: Reduce $\frac{-45}{30}$ to the standard form.

Sol: $\frac{-45}{30} = \frac{-45 \div 3}{30 \div 3} = \frac{-15 \div 5}{10 \div 5} = \frac{-3}{2}$

Example 2: Reduce to standard form:

(i) $\frac{36}{-24}$

Sol: HCF of $36,24 = 12$

$$\frac{36}{-24} = \frac{36 \div (-12)}{-24 \div (-12)} = \frac{-3}{2}$$

(ii) $\frac{-3}{-15}$

Sol: HCF of $3,15 = 3$

$$\frac{-3}{-15} = \frac{-3 \div (-3)}{-15 \div (-3)} = \frac{1}{5}$$

TRY THESE

Find the standard form of

(i) $\frac{-18}{45}$

Sol: HCF of $18,45 = 9$

$$\frac{-18}{45} = \frac{-18 \div 9}{45 \div 9} = \frac{-2}{5}$$

(ii) $\frac{-12}{18}$

Sol: HCF of $12,18 = 6$

$$\frac{-12}{18} = \frac{-12 \div 6}{18 \div 6} = \frac{-2}{3}$$

COMPARISON OF RATIONAL NUMBERS

(i) To compare rational numbers reduce them to their standard forms and then compare them.

(ii) To compare two negative rational numbers, we compare them ignoring their negative signs and then reverse the order.

(iii) A negative rational number will always be less than a positive rational number.

Exmple 3 : Do $\frac{4}{-9}$ and $\frac{-16}{36}$ represent the same rational number?

Sol: $\frac{4}{-9}=\frac{4\times(-4)}{-9\times(-4)}=\frac{-16}{36}$

$So, \frac{4}{-9}$ and $\frac{-16}{36}$ represent the same rational number.

RATIONAL NUMBERS BETWEEN TWO RATIONAL NUMBERS.

We can find unlimited number of rational numbers between any two rational numbers.

1. Find five rational numbers between $\frac{-5}{7}$ and $\frac{-3}{8}$.

Sol: $\frac{-5}{7}=\frac{-5\times8}{7\times8}=\frac{-40}{56}$ *and* $\frac{-3}{8}=\frac{-3\times7}{8\times7}=\frac{-21}{56}$ $[LCM\ of\ 7,8=56]$

$\frac{-40}{56}<\frac{-21}{56}$

$\frac{-40}{56}<\frac{-39}{56}<\frac{-38}{56}<\frac{-37}{56}<\frac{-36}{56}<\frac{-35}{56}<\frac{-34}{56}<\cdots\ldots\ldots<\frac{-21}{56}$

Five rational numbers between $\frac{-5}{7}$ and $\frac{-3}{8}$ *are* $\frac{-39}{56},\frac{-38}{56},\frac{-37}{56},\frac{-36}{56},\frac{-35}{56}$

Example 4: List three rational numbers between – 2 and – 1.

Sol: $-2<-1$

$\frac{-2}{1}\times\frac{5}{5}<\frac{-1}{1}\times\frac{5}{5}$

$\frac{-10}{5}<\frac{-5}{5}$

$\frac{-10}{5}<\frac{-9}{5}<\frac{-8}{5}<\frac{-7}{5}<\frac{-6}{5}<\frac{-5}{5}$

The three rational numbers between – 2 and – 1 are $\frac{-9}{5},\frac{-8}{5},\frac{-7}{5}$.

Example 5:Write four more numbers in the following pattern:

$\frac{-1}{3},\frac{-2}{6},\frac{-3}{9},\frac{-4}{12},\cdots$

Sol: $\frac{-1}{3},\frac{-1\times2}{3\times2}=\frac{-2}{6},\frac{-1\times3}{3\times3}=\frac{-3}{9},\frac{-1\times4}{3\times4}=\frac{-4}{12}$

The other numbers are

$\frac{-1\times5}{3\times5}=\frac{-5}{15},\quad\frac{-1\times6}{3\times6}=\frac{-6}{18},\quad\frac{-1\times7}{3\times7}=\frac{-7}{21},\quad\frac{-1\times8}{3\times8}=\frac{-8}{24}$

EXERCISE 8.1

1. List five rational numbers between:

(i) -1 and 0

Sol: $-1 < 0$

$$\frac{-1}{1}\times\frac{6}{6} < \frac{0}{1}\times\frac{6}{6}$$

$$\frac{-6}{6} < \frac{0}{6}$$

$$\frac{-6}{6} < \frac{-5}{6} < \frac{-4}{6} < \frac{-3}{6} < \frac{-2}{6} < \frac{-1}{6} < \frac{0}{6}$$

The five rational numbers between – 1 and 0 are $\frac{-5}{6}, \frac{-4}{6}, \frac{-3}{6}, \frac{-2}{6}, \frac{-1}{6}$.

(ii) -2 and -1

Sol: $-2 < -1$

$$\frac{-2}{1}\times\frac{6}{6} < \frac{-1}{1}\times\frac{6}{6}$$

$$\frac{-12}{6} < \frac{-6}{6}$$

$$\frac{-10}{5} < \frac{-9}{6} < \frac{-8}{6} < \frac{-7}{6} < \frac{-6}{6} < \frac{-5}{6} < \frac{-6}{6}$$

The five rational numbers between – 2 and – 1 are $\frac{-9}{6}, \frac{-8}{6}, \frac{-7}{6}, \frac{-6}{6}, \frac{-5}{6}$.

$$\Rightarrow \frac{-3}{2}, \frac{-4}{3}, \frac{-7}{6}, -1, \frac{-5}{6}$$

(iii) $\frac{-4}{5}$ and $\frac{-2}{3}$

Sol: $\frac{-4}{5} < \frac{-2}{3}$

$$\frac{-4\times 9}{5\times 9} < \frac{-2\times 15}{3\times 15}$$

$$\frac{-36}{45} < \frac{-30}{45}$$

$$\frac{-36}{45} < \frac{-35}{45} < \frac{-34}{45} < \frac{-33}{45} < \frac{-32}{45} < \frac{-31}{45} < \frac{-30}{45}$$

The five rational numbers between $\frac{-4}{5}$ *and* $\frac{-2}{3}$ are $\frac{-35}{45}, \frac{-34}{45}, \frac{-33}{45}, \frac{-32}{45}, \frac{-31}{45}$

$$\Rightarrow \frac{-7}{9}, \frac{-34}{45}, \frac{-11}{15}, \frac{-32}{45}, \frac{-31}{45}$$

(iv) $\frac{-1}{2}$ **and** $\frac{2}{3}$

Sol: $\frac{-1}{2} < \frac{2}{3}$

$\frac{-1\times 3}{2\times 3} < \frac{2\times 2}{3\times 2}$

$\frac{-3}{6} < \frac{4}{6}$

$\frac{-3}{6} < \frac{-2}{6} < \frac{-1}{6} < \frac{0}{6} < \frac{1}{6} < \frac{2}{6} < \frac{3}{6} < \frac{4}{6}$

The five rational numbers between $\frac{-1}{2}$ *and* $\frac{2}{3}$ are $\frac{-2}{6}, \frac{-1}{6}, \frac{0}{6}, \frac{1}{6}, \frac{2}{6}$

$\Rightarrow \frac{-1}{3}, \frac{-1}{6}, 0, \frac{1}{6}, \frac{1}{3}$

2. **Write four more rational numbers in each of the following patterns:**

(i) $\frac{-3}{5}, \frac{-6}{10}, \frac{-9}{15}, \frac{-12}{20}, \ldots$

Sol: $\frac{-3}{5}, \quad \frac{-3\times 2}{5\times 2} = \frac{-6}{10}, \quad \frac{-3\times 3}{5\times 3} = \frac{-9}{15}, \quad \frac{-3\times 4}{5\times 4} = \frac{-12}{20}, \ldots$

The other numbers are

$\frac{-3\times 5}{5\times 5}, \quad \frac{-3\times 6}{5\times 6}, \quad \frac{-3\times 7}{5\times 7}, \quad \frac{-3\times 8}{5\times 8}$

$\Rightarrow \frac{-15}{25}, \quad \frac{-18}{30}, \quad \frac{-21}{35}, \quad \frac{-24}{40}$

(ii) $\frac{-1}{4}, \frac{-2}{8}, \frac{-3}{12}, ..$

Sol: $\frac{-1}{4}, \quad \frac{-1\times 2}{4\times 2} = \frac{-2}{8}, \quad \frac{-1\times 3}{4\times 3} = \frac{-3}{12}, ..$

The other numbers are

$\frac{-1\times 4}{4\times 4}, \quad \frac{-1\times 5}{4\times 5}, \quad \frac{-1\times 6}{4\times 6}, \quad \frac{-1\times 7}{4\times 7}$

$\Rightarrow \frac{-4}{16}, \quad \frac{-5}{20}, \quad \frac{-6}{24}, \quad \frac{-7}{28}$

(iii) $\frac{-1}{6}, \frac{2}{-12}, \frac{3}{18}, \frac{4}{-24}, ..$

Sol: $\frac{-1}{6}, \frac{2}{-12}, \frac{3}{-18}, \frac{4}{-24}, \ldots$

$$\frac{-1}{6}, \frac{-1\times(-2)}{6\times(-2)} = \frac{2}{-12}, \frac{-1\times(-3)}{6\times(-3)} = \frac{3}{-18}, \frac{-1\times(-4)}{6\times(-4)} = \frac{4}{-24}, \ldots$$

The other numbers are

$$\frac{-1\times(-5)}{6\times(-5)}, \frac{-1\times(-6)}{6\times(-6)}, \frac{-1\times(-7)}{6\times(-7)}, \frac{-1\times(-8)}{6\times(-8)}$$
$$= \frac{5}{-30}, \frac{6}{-36}, \frac{7}{-42}, \frac{8}{-48},$$

(iv) $\frac{-2}{3}, \frac{2}{-3}, \frac{4}{-6}, \frac{6}{-9}, \ldots$

Sol: $\frac{-2}{3}, \frac{-2\times(-1)}{3\times(-1)} = \frac{2}{-3}, \frac{-2\times(-2)}{3\times(-2)} = \frac{4}{-6}, \frac{-2\times(-3)}{3\times(-3)} = \frac{6}{-9}, \ldots$

The other numbers are

$$\frac{-2\times(-4)}{3\times(-4)}, \frac{-2\times(-5)}{3\times(-5)}, \frac{-2\times(-6)}{3\times(-6)}, \frac{-2\times(-7)}{3\times(-7)}, \ldots$$

$\frac{8}{-12}, \frac{10}{-15}, \frac{12}{-18}, \frac{14}{-21}$

If the numerator and denominator of a rational number are multiplied or divided by a non-zero integer, we get a rational number which is said to be equivalent to the given rational number.

3. Give four rational numbers equivalent to:

(i) $\frac{-2}{7}$

Sol: $\frac{-2}{7} = \frac{-2\times2}{7\times2} = \frac{-2\times3}{7\times3} = \frac{-2\times4}{7\times4} = \frac{-2\times5}{7\times5}$

$$\frac{-2}{7} = \frac{-4}{14} = \frac{-6}{21} = \frac{-8}{28} = \frac{-10}{35}$$

(ii) $\frac{5}{-3}$

Sol: $\frac{5}{-3} = \frac{5\times2}{-3\times2} = \frac{5\times3}{-3\times3} = \frac{5\times4}{-3\times4} = \frac{5\times5}{-3\times5}$

$$\frac{5}{-3} = \frac{10}{-6} = \frac{15}{-9} = \frac{20}{-12} = \frac{25}{-15}$$

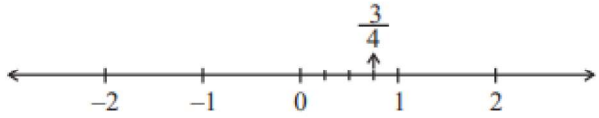
(iii) $\frac{4}{9}$

Sol: $\frac{4}{9} = \frac{4\times2}{9\times2} = \frac{4\times3}{9\times3} = \frac{4\times4}{9\times4} = \frac{4\times5}{9\times5}$

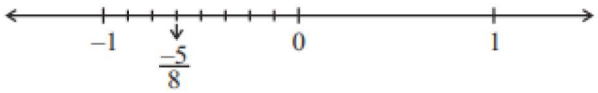
$$\frac{4}{9}=\frac{8}{18}=\frac{12}{27}=\frac{16}{36}=\frac{20}{45}$$

4. Draw the number line and represent the following rational numbers on it:

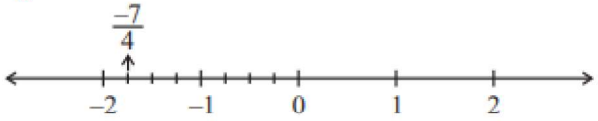
(i) $\frac{3}{4}$



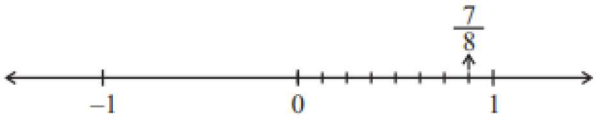
(ii) $\frac{-5}{8}$



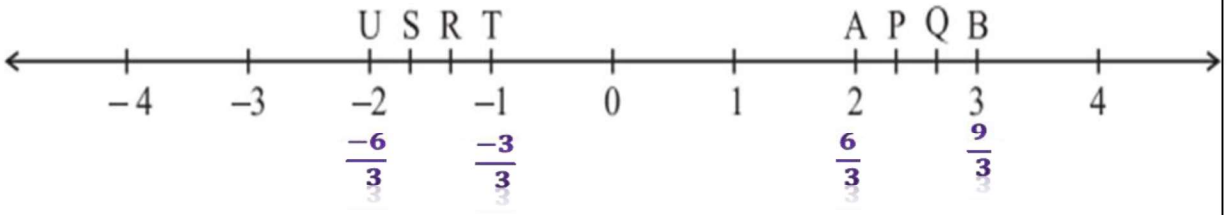
(iii) $\frac{-7}{4}$



(iv) $\frac{7}{8}$



5. The points P, Q, R, S, T, U, A and B on the number line are such that, TR = RS = SU and AP = PQ = QB. Name the rational numbers represented by P, Q, R and S.



Sol: Each one unit divided into three equal parts.

$$-1=\frac{-3}{3},\quad -2=\frac{-6}{3},\quad 2=\frac{6}{3},\quad 3=\frac{9}{3}$$

The rational numbers represented by P, Q, R and S are $\frac{7}{3},\frac{8}{3},\frac{-4}{3},\text{and }\frac{-5}{3}$

6. Which of the following pairs represent the same rational number?

(i) $\frac{-7}{21}$ **and** $\frac{3}{9}$

Sol: $\frac{-7}{21}$ is negative and $\frac{3}{9}$ is positive .

So, $\frac{-7}{21}$ *and* $\frac{3}{9}$ does not represent the same rational number.

(ii) $\frac{-16}{20}$ **and** $\frac{20}{-25}$

Sol: $\frac{-16}{20} = \frac{-16 \div 4}{20 \div 4} = \frac{-4}{5}$

$\frac{20}{-25} = \frac{20 \div (-5)}{-25 \div (-5)} = \frac{-4}{5}$

So, $\frac{-16}{20}$ and $\frac{20}{-25}$ represent the same rational number.

$\frac{-16}{20}$ *cm* $\frac{20}{-25}$

$(-16) \times (-25) = 400$

$(20) \times (20) = 400$

$\frac{-16}{20} = \frac{20}{-25}$

(iii) $\frac{-2}{-3}$ **and** $\frac{2}{3}$

Sol: $\frac{-2}{-3} = \frac{-2 \div (-1)}{-3 \div (-1)} = \frac{2}{3}$

So, $\frac{-2}{-3}$ and $\frac{2}{3}$ represent the same rational number.

(iv) $\frac{-3}{5}$ **and** $\frac{-12}{20}$

Sol: $\frac{-12}{20} = \frac{-12 \div 4}{20 \div 4} = \frac{-3}{5}$

So, $\frac{-3}{5}$ and $\frac{-12}{20}$ represent the same rational number.

(v) $\frac{8}{-5}$ **and** $\frac{-24}{15}$

Sol: $\frac{-24}{15} = \frac{-24 \div (-3)}{15 \div (-3)} = \frac{8}{-5}$

So, $\frac{8}{-5}$ and $\frac{-24}{15}$ represent the same rational number.

(vi) $\frac{1}{3}$ **and** $\frac{-1}{9}$

Sol: $\frac{1}{3}$ is positive and $\frac{-1}{9}$ is negative .

So, $\frac{1}{3}$ and $\frac{-1}{9}$ does not represent the same rational number.

(vii) $\frac{-5}{-9}$ **and** $\frac{5}{-9}$

Sol: $\frac{-5}{-9}$ is positive and $\frac{5}{-9}$ is negative .

So, $\frac{-5}{-9}$ and $\frac{5}{-9}$ does not represent the same rational number.

7. Rewrite the following rational numbers in the simplest form:

(i) $\frac{-8}{6} = \frac{-8 \div 2}{6 \div 2} = \frac{-4}{3}$

(ii) $\frac{25}{45} = \frac{25 \div 5}{45 \div 5} = \frac{5}{9}$

(iii) $\frac{-44}{72} = \frac{-44 \div 4}{72 \div 4} = \frac{-11}{18}$

(iv) $\frac{-8}{10} = \frac{-8 \div 2}{10 \div 2} = \frac{-4}{5}$

8. Fill in the boxes with the correct symbol out of >,< and =.

(i) $\frac{-5}{7} \boxed{<} \frac{2}{3}$

Sol: $\frac{-5}{7} \ldots\ldots\ldots \frac{2}{3}$

$\frac{-5 \times 3}{7 \times 3} \ldots\ldots\ldots \frac{2 \times 7}{3 \times 7}$

$\frac{-15}{21} < \frac{14}{21}$

(Negative number always less than positive number)

(ii) $\frac{-4}{5} \boxed{<} \frac{-5}{7}$

Sol: $\frac{-4}{5} \_\_\_\_ \frac{-5}{7}$

$\frac{-4 \times 7}{5 \times 7} \_\_\_\_ \frac{-5 \times 5}{7 \times 5}$

$\frac{-28}{35} < \frac{-25}{35}$

(iii) $\frac{-7}{8} \boxed{=} \frac{14}{-16}$

Sol: $\frac{-7}{8} \_\_\_\_ \frac{14}{-16}$

$\frac{-7 \times (-2)}{8 \times (-2)} \_\_\_\_ \frac{14}{-16}$

$\frac{14}{-16} = \frac{14}{-16}$

(iv) $\frac{-8}{5} \boxed{>} \frac{-7}{4}$

Sol: $\frac{-8}{5} \_\_\_\_ \frac{-7}{4}$

$\frac{-8 \times 4}{5 \times 4} \_\_\_\_ \frac{-7 \times 5}{4 \times 5}$

$\frac{-32}{20} \boxed{>} \frac{-35}{20}$

(v) $\frac{1}{-3} \boxed{<} \frac{-1}{4}$

Sol: $\frac{1}{-3} \_\_\_\_ \frac{-1}{4}$

$\frac{1 \times (-4)}{-3 \times (-4)} \_\_\_\_ \frac{-1 \times 3}{4 \times 3}$

$\frac{-4}{12} < \frac{-3}{12}$

(vi) $\frac{5}{-11} \boxed{=} \frac{-5}{11}$

Sol: $\frac{5}{-11} \_\_\_\_ \frac{-5}{11}$

$\frac{5 \times (-1)}{-11 \times (-1)} \_\_\_\_ \frac{-5}{11}$

$\frac{-5}{11} = \frac{-5}{11}$

(vii) $0 \boxed{>} \frac{-7}{6}$

9. Which is greater in each of the following:

(i) $\frac{2}{3}, \frac{5}{2}$

Sol: LCM of $3,2 = 6$

$\frac{2}{3} = \frac{2 \times 2}{3 \times 2} = \frac{4}{6}$

$\frac{5}{2} = \frac{5 \times 3}{2 \times 3} = \frac{15}{6}$

$\frac{4}{6} < \frac{15}{6} \Rightarrow \frac{2}{3} < \frac{5}{2}$

So, $\frac{5}{2}$ is greater.

(ii) $\frac{-5}{6}, \frac{-4}{3}$

Sol: LCM of $6,3 = 6$

$\frac{-5}{6} = \frac{-5 \times 1}{6 \times 1} = \frac{-5}{6}$

$\frac{-4}{3} = \frac{-4 \times 2}{3 \times 2} = \frac{-8}{6}$

$\frac{-8}{6} < \frac{-5}{6} \Rightarrow \frac{-4}{3} < \frac{-5}{6}$

So, $\frac{-5}{6}$ is greater.

(iii) $\frac{-3}{4}, \frac{2}{-3}$

Sol: LCM of $4,3 = 12$

$\frac{-3}{4} = \frac{-3 \times 3}{4 \times 3} = \frac{-9}{12}$

Sol: 0 always greater than negative.

$\frac{2}{-3} = \frac{2 \times (-4)}{-3 \times (-4)} = \frac{-8}{12}$

$\frac{-9}{12} < \frac{-8}{12} \Rightarrow \frac{-3}{4} < \frac{2}{-3}$

So, $\frac{2}{-3}$ is greater.

(iv) $\frac{-1}{4}, \frac{1}{4}$

Sol: Every positive is greater than negative.

So, $\frac{1}{4}$ is greater.

(v) $-3\frac{2}{7}, -3\frac{4}{5}$

Sol: LCM of $7,5 = 35$

$-3\frac{2}{7} = -\frac{23}{7} = -\frac{23 \times 5}{7 \times 5} = \frac{-115}{35}$

$-3\frac{4}{5} = \frac{-19}{5} = \frac{-19 \times 7}{5 \times 7} = \frac{-133}{35}$

$\frac{-133}{35} < \frac{-115}{35} \Rightarrow -3\frac{4}{5} < -3\frac{2}{7}$

So, $-3\frac{2}{7}$ is greater.

10. Write the following rational numbers in ascending order:

If we write rational numbers in ascending order then we convert all into like rational numbers.

(i) $\frac{-3}{5}, \frac{-2}{5}, \frac{-1}{5}$

Sol: Ascending order is, $\frac{-3}{5} < \frac{-2}{5} < \frac{-1}{5}$

(ii) $\frac{-1}{3}, \frac{-2}{9}, \frac{-4}{3}$

Sol: LCM of $3,9 = 9$

$$\frac{-1}{3} = \frac{-1 \times 3}{3 \times 3} = \frac{-3}{9}$$

$$\frac{-2}{9} = \frac{-2 \times 1}{9 \times 1} = \frac{-2}{9}$$

$$\frac{-4}{3} = \frac{-4 \times 3}{3 \times 3} = \frac{-12}{9}$$

Ascending order

$$\frac{-12}{9} < \frac{-3}{9} < \frac{-2}{9}$$

$$\therefore \frac{-4}{3} < \frac{-1}{3} < \frac{-2}{9}$$

(iii) $\frac{-3}{7}, \frac{-3}{2}, \frac{-3}{4}$

Sol: LCM of $7,2,4 = 28$

$$\frac{-3}{7} = \frac{-3 \times 4}{7 \times 4} = \frac{-12}{28}$$

$$\frac{-3}{2} = \frac{-3 \times 14}{2 \times 14} = \frac{-42}{28}$$

$$\frac{-3}{4} = \frac{-3 \times 7}{4 \times 7} = \frac{-21}{28}$$

Ascending order

$$\frac{-42}{28} < \frac{-21}{28} < \frac{-12}{28}$$

$$\therefore \frac{-3}{2} < \frac{-3}{4} < \frac{-3}{7}$$

OPERATIONS ON RATIONAL NUMBERS:

Addition

Adding rational numbers with same denominators, we add the numerators keeping the denominators same.

(i) $\frac{7}{3} + \left(\frac{-5}{3}\right) = \frac{7}{3} + \frac{(-5)}{3} = \frac{7 + (-5)}{3} = \frac{2}{3}$

(ii) $\frac{6}{5} + \frac{(-2)}{5} = \frac{6 + (-2)}{5} = \frac{4}{5}$

(iii) $\frac{3}{7} + \frac{(-5)}{7} = \frac{3 + (-5)}{7} = \frac{-2}{7}$

(iv) $\frac{-7}{8} + \frac{5}{8} = \frac{-7 + 5}{8} = \frac{-2}{8} = \frac{-1}{4}$

(v) $\frac{-13}{7} + \frac{6}{7} = \frac{-13 + 6}{7} = \frac{7}{7} = 1$

(vi) $\frac{19}{5} + \left(\frac{-7}{5}\right) = \frac{19 + (-7)}{5} = \frac{12}{5}$

If we add rational numbers with different denominators. Then, we find the equivalent rational numbers of the given rational numbers with their LCM as the denominator and add the rational numbers.

(i) $\frac{-7}{5}+\frac{-2}{3}=\frac{-7\times 3}{5\times 3}+\frac{-2\times 5}{3\times 5}=\frac{-21}{15}+\frac{-10}{15}=\frac{(-21)+(-10)}{15}=\frac{-31}{15}$

(ii) $\frac{-3}{7}+\frac{2}{3}=\frac{-3\times 3}{7\times 3}+\frac{2\times 7}{3\times 7}=\frac{-9}{21}+\frac{14}{21}=\frac{-9+14}{21}=\frac{5}{21}$

(iii) $\frac{-5}{6}+\frac{-3}{11}=\frac{-5\times 11}{6\times 11}+\frac{-3\times 6}{11\times 6}=\frac{-55}{66}+\frac{-18}{66}=\frac{-55+(-18)}{66}=\frac{-73}{66}$

Additive Inverse

$$\frac{a}{b}+\left(\frac{-a}{b}\right)=\frac{a+(-a)}{b}=\frac{0}{b}=0$$

The additive inverse of $\frac{a}{b}=\frac{-a}{b}$

The additive inverse of $\frac{-a}{b}=\frac{a}{b}$

(i) The additive inverse of $\frac{-3}{9}=\frac{3}{9}$

(ii) The additive inverse of $\frac{-9}{11}=\frac{9}{11}$

(iii) The additive inverse of $\frac{5}{7}=\frac{-5}{7}$

Example 6: Satpal walks $\frac{2}{3}$km from a place P, towards east and then from there $1\frac{5}{7}$ km towards west. Where will he be now from P?

Sol: $\frac{2}{3}+\left(-1\frac{5}{7}\right)=\frac{2}{3}+\frac{(-12)}{7}=\frac{2\times 7}{3\times 7}+\frac{(-12)\times 3}{7\times 3}$

$=\frac{14}{21}+\frac{(-36)}{21}=\frac{14+(-36)}{21}=\frac{-22}{21}=-1\frac{1}{21}$

Subtraction

While subtracting two rational numbers, we add the additive inverse of the rational number that is being subtracted, to the other rational number.

$$\frac{a}{b}-\frac{c}{q}=\frac{a\times q-c\times b}{b\times q}$$

(i) $\frac{5}{7}-\frac{3}{8}=\frac{5\times 8-3\times 7}{7\times 8}=\frac{40-21}{56}=\frac{19}{56}$

(ii) $\frac{7}{8}-\frac{5}{9}=\frac{7\times 9-5\times 8}{8\times 9}=\frac{63-40}{72}=\frac{23}{72}$

(iii) $\frac{3}{11}-\frac{8}{7}=\frac{3\times 7-8\times 11}{11\times 7}=\frac{21-88}{77}=\frac{-67}{77}$

(iv) $\frac{7}{9}-\frac{2}{5}=\frac{7\times5-2\times9}{9\times5}=\frac{35-18}{45}=\frac{17}{45}$

(v) $1\frac{2}{3}-2\frac{4}{5}=\frac{5}{3}-\frac{14}{5}=\frac{5\times5-14\times3}{3\times5}=\frac{25-42}{15}=\frac{-17}{15}$

(vi) $\frac{2}{7}-\left(\frac{-5}{6}\right)=\frac{2}{7}+\frac{5}{6}=\frac{2\times6+5\times7}{7\times6}=\frac{12+35}{56}=\frac{47}{56}$

(vii) $2\frac{1}{5}-\frac{(-1)}{3}=\frac{11}{5}+\frac{1}{3}=\frac{11\times3+1\times5}{5\times3}=\frac{33+5}{15}=\frac{38}{15}$

Multiplication

While multiplying a rational number by an integer, we multiply the numerator by that integer, keeping the denominator unchanged.

(i) $\frac{-2}{9}\times(-5)=\frac{-2\times(-5)}{9}=\frac{10}{9}=1\frac{1}{9}$

(ii) $\frac{3}{11}\times(-2)=\frac{3\times(-2)}{11}=\frac{-6}{11}$

(iii) $\frac{-3}{5}\times7=\frac{-3\times7}{5}=\frac{-21}{5}$

(iv) $\frac{-6}{5}\times(-2)=\frac{-6\times(-2)}{5}=\frac{12}{5}$

To multiply two rational numbers, we multiply their numerators and denominators separately, and write theproduct as $\frac{\text{productof numerators}}{\text{product of denominators}}$

(i) $\frac{-3}{8}\times\frac{5}{7}=\frac{-3\times5}{8\times7}=\frac{-15}{56}$

(ii) $\frac{-5}{8}\times\frac{-9}{7}=\frac{-5\times(-9)}{8\times7}=\frac{45}{56}$

(iii) $\frac{-3}{4}\times\frac{1}{7}=\frac{-3\times1}{4\times7}=\frac{-3}{28}$

(iv) $\frac{2}{3}\times\frac{-5}{9}=\frac{2\times(-5)}{3\times9}=\frac{-10}{27}$

Division

Reciprocal :(Multiplicative inverse)

If product of two rational numbers is 1 then they are said to be reciprocals of each other

$\frac{a}{b}\times\frac{b}{a}=1$

Reciprocal of $\frac{a}{b}$ is $\frac{b}{a}$

(i) **The reciprocal of $\frac{2}{7}$** is $\frac{7}{2}$

(ii) The reciprocal of $\frac{-6}{11}$ is $\frac{-11}{6}$

(iii) The reciprocal of $\frac{-8}{5}$ is $\frac{-5}{8}$

To divide one rational number by the other non-zero rational number we multiply the rational number by the reciprocal of the other.

(i) $\frac{4}{9} \div \frac{-5}{7} = \frac{4}{9} \times \frac{-7}{5} = \frac{4 \times (-7)}{9 \times 5} = \frac{-28}{45}$

(ii) $\frac{-6}{5} \div \frac{-2}{3} = \frac{-6}{5} \times \frac{-3}{2} = \frac{(-6) \times (-3)}{5 \times 2} = \frac{18}{10} = \frac{9}{5}$

(iii) $\frac{2}{3} \div \frac{-7}{8} = \frac{2}{3} \times \frac{-8}{7} = \frac{2 \times (-8)}{3 \times 7} = \frac{-16}{21}$

(iv) $\frac{-6}{7} \div \frac{5}{7} = \frac{-6}{7} \times \frac{7}{5} = \frac{(-6) \times \cancel{7}}{\cancel{7} \times 5} = \frac{-6}{5}$

EXERCISE 8.2

1. **Find the sum:**

(i) $\frac{5}{4} + \frac{-11}{4} = \frac{5 + (-11)}{4}$

$= \frac{-6}{4} = \frac{-3}{2}$

(ii) $\frac{5}{3} + \frac{3}{5} = \frac{5 \times 5}{3 \times 5} + \frac{3 \times 3}{5 \times 3}$

$= \frac{25}{35} + \frac{9}{35}$

$= \frac{25 + 9}{35} = \frac{34}{35}$

(iii) $\frac{-9}{10} + \frac{22}{15} = \frac{-9 \times 3}{10 \times 3} + \frac{22 \times 2}{15 \times 2}$

$= \frac{-27}{30} + \frac{44}{30}$

$= \frac{-27 + 44}{30}$

$= \frac{17}{30}$

(iv) $\frac{-3}{-11} + \frac{5}{9} = \frac{-3 \times (-9)}{-11 \times (-9)} + \frac{5 \times 11}{9 \times 11}$

$= \frac{27}{99} + \frac{55}{99}$

$= \frac{27 + 55}{99}$

$= \frac{82}{99}$

(v) $\frac{-8}{19} + \frac{(-2)}{57} = \frac{-8 \times 3}{19 \times 3} + \frac{(-2) \times 1}{57 \times 1}$

$= \frac{-24}{57} + \frac{-2}{57}$

$= \frac{-24 + (-2)}{57}$

$= \frac{-26}{57}$

(vi) $\frac{-2}{3} + 0 = \frac{-2}{3}$

(vii) $-2\frac{1}{3}+4\frac{3}{5}=\frac{-7}{3}+\frac{23}{5}$

$=\frac{-7\times5}{3\times5}+\frac{23\times3}{5\times3}$

$=\frac{-35}{15}+\frac{69}{15}$

$=\frac{-35+69}{15}$

$=\frac{34}{15}$

2. Find

(i) $\frac{7}{24}-\frac{17}{36}=\frac{7\times3}{24\times3}-\frac{17\times2}{36\times2}$

$=\frac{21}{72}-\frac{34}{72}$

$=\frac{21-34}{72}$

$=\frac{-13}{72}$

(ii) $\frac{5}{63}-\left(\frac{-6}{21}\right)=\frac{5}{63}+\frac{6}{21}$

$=\frac{5}{63}+\frac{6\times3}{21\times3}$

$=\frac{5}{63}+\frac{18}{63}$

$=\frac{23}{63}$

(iii) $\frac{-6}{13}-\left(\frac{-7}{15}\right)=\frac{-6}{13}+\frac{7}{15}$

$=\frac{-6\times15}{13\times15}+\frac{7\times13}{15\times13}$

$=\frac{-90}{195}+\frac{91}{195}$

$=\frac{1}{195}$

(iv) $\frac{-3}{8}-\frac{7}{11}=\frac{-3\times11}{8\times11}-\frac{7\times8}{11\times8}$

$=\frac{-33}{88}-\frac{56}{88}$

$=\frac{-33-56}{88}$

$=\frac{-89}{88}$

(v) $-2\frac{1}{9}-6=\frac{-19}{9}-\frac{6}{1}$

$=\frac{-19}{9}-\frac{6\times9}{1\times9}$

$=\frac{-19}{9}-\frac{54}{9}$

$=\frac{-19-54}{9}$

$=\frac{-73}{9}$

3. Find the product:

(i) $\frac{9}{2}\times\left(\frac{-7}{4}\right)=\frac{9\times(-7)}{2\times4}=\frac{-63}{8}$

(ii) $\frac{3}{10}\times(-9)=\frac{3\times(-9)}{10}=\frac{-27}{10}$

(iii) $\frac{-6}{5}\times\frac{9}{11}=\frac{-6\times9}{5\times11}=\frac{-54}{55}$

(iv) $\frac{3}{7}\times\left(\frac{-2}{5}\right)=\frac{3\times(-2)}{7\times5}=\frac{-6}{35}$

(v) $\frac{3}{11}\times\frac{2}{5}=\frac{3\times2}{11\times5}=\frac{6}{55}$

(vi) $\frac{3}{-5}\times\frac{-5}{3}=\frac{3\times(-5)}{-5\times3}=\frac{-15}{-15}=1$

4. Find the value of:

(i) $(-4)\div\frac{2}{3}=\frac{-4}{1}\times\frac{3}{2}=\frac{-4\times3}{1\times2}$

$=\frac{-12}{2}=-6$

(ii) $\frac{-3}{5} \div 2 = \frac{-3}{5} \times \frac{1}{2} = \frac{-3}{10}$

(iii) $\frac{-4}{5} \div (-3) = \frac{-4}{5} \times \frac{1}{-3}$

$= \frac{-4}{-15} = \frac{4}{15}$

(iv) $\frac{-1}{8} \div \frac{3}{4} = \frac{-1}{8} \times \frac{4}{3} = \frac{-4}{24} = \frac{-1}{6}$

(v) $\frac{-2}{13} \div \frac{1}{7} = \frac{-2}{13} \times \frac{7}{1} = \frac{-14}{13}$

(vi) $\frac{-7}{12} \div \left(\frac{-2}{13}\right) = \frac{-7}{12} \times \frac{-13}{2} = \frac{91}{24}$

(vii) $\frac{3}{13} \div \left(\frac{-4}{65}\right) = \frac{3}{13} \times \frac{-65}{4}$

$= \frac{3 \times (-5)}{4} = \frac{-15}{4}$