

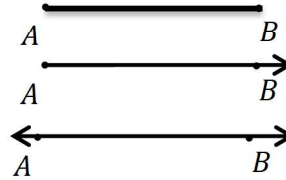
CHAPTER

6

IX-MATHEMATICS-NCERT
6. LINES AND ANGLES (NOTES)

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- Line segment:** A part (or portion) of a line with two end points is called a line-segment
- The line segment AB is denoted by \overline{AB}
- Ray:** A part of a line with one end point is called a ray.
- The ray AB is denoted by \overrightarrow{AB} .
- The line AB is denoted by \overleftrightarrow{AB}
- Sometimes small letters l, m, n, etc. will be used to denote lines.
- If three or more points lie on the same line, they are called collinear points; otherwise they are called non-collinear points.
- An angle is formed when two rays originate from the same end point. The rays making an angle are called the arms of the angle and the end point is called the vertex of the angle
- The angle between 0° and 90° is called acute angle.
- The angle 90° is called right angle
- The angle between 90° and 180° is called obtuse angle.
- The angle 180° is called straight angle.
- The angle between 180° and 360° is called reflex angle.



Name	Acute angle	Right angle	Obtuse angle	Straight angle	Reflex angle	Complete angle
Measure	$0^\circ < x < 90^\circ$	$y = 90^\circ$	$90^\circ < z < 180^\circ$	$s = 180^\circ$	$180^\circ < t < 360^\circ$	$u = 360^\circ$
Illustration						

- Complementary angles:** Two angles whose sum is 90° are called complementary angles.
- Supplementary angles:** Two angles whose sum is 180° are called supplementary angles.
- Adjacent angles:** Two angles are adjacent, if they have a common vertex, a common arm and their non-common arms are on different sides of the common arm.

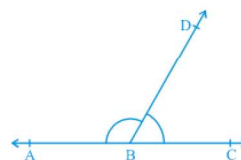


Fig. 6.3 : Linear pair of angles

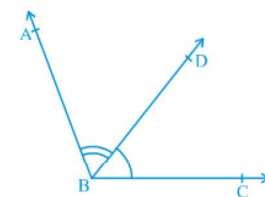


Fig. 6.2 : Adjacent angles

17. Linear pair of angles: the sum of two adjacent angles is 180° , then they are called a linear pair of angles.

18. vertically opposite angles : If \overleftrightarrow{AB} and \overleftrightarrow{CD} intersect at O then

$\angle AOC$ is vertically opposite to $\angle BOD$ and

$\angle AOD$ is vertically opposite to $\angle BOC$

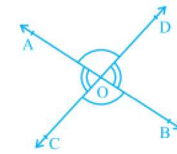
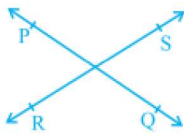


Fig. 6.4 : Vertically opposite angles

19. Intersecting Lines and Non-intersecting Lines:



(i) Intersecting lines



(ii) Non-intersecting (parallel) lines

20. Axiom 6.1 : If a ray stands on a line, then the sum of two adjacent angles so formed is 180°

21. Axiom 6.2 : If the sum of two adjacent angles is 180° , then the non-common arms of the angles form a line.

Theorem 6.1 : If two lines intersect each other, then the vertically opposite angles are equal

Proof: let AB and CD be two lines intersecting at O.

Two pairs of vertically opposite angles are

(i) $\angle AOC$ and $\angle BOD$ (ii) $\angle AOD$ and $\angle BOC$.

Now, ray OA stands on line CD

$\angle AOC + \angle AOD = 180^\circ$ (Linear pair axiom) \rightarrow (1)

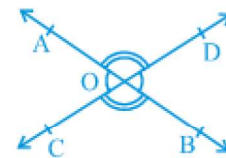
Now, ray OD stands on line AB

$\angle AOD + \angle BOD = 180^\circ$ (Linear pair axiom) \rightarrow (2)

From (1) and (2): $\angle AOC + \angle AOD = \angle AOD + \angle BOD$

$$\angle AOC = \angle BOD$$

Similarly, we can prove $\angle AOD = \angle BOC$.



Example 1 : In Fig. lines PQ and RS intersect each other at point O. If $\angle POR : \angle ROQ = 5 : 7$, find all the angles

Sol: Given $\angle POR : \angle ROQ = 5 : 7$

Let $\angle POR = 5x$ and $\angle ROQ = 7x$

$\angle POR + \angle ROQ = 180^\circ$ (Linear pair of angles)

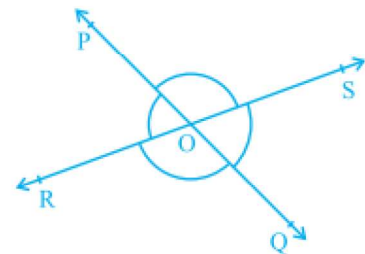
$$5x + 7x = 180^\circ$$

$$12x = 180^\circ$$

$$x = \frac{180^\circ}{12} = 15^\circ$$

$$\angle POR = 5x = 5 \times 15^\circ = 75^\circ$$

$$\angle ROQ = 7x = 7 \times 15^\circ = 105^\circ$$



Now $\angle POS = \angle ROQ = 105^\circ$ (Vertically opposite angles)

$\angle SOQ = \angle POR = 75^\circ$ (Vertically opposite angles)

Example 2 : In Fig. ray OS stands on a line POQ. Ray OR and ray OT are angle bisectors of $\angle POS$ and $\angle SOQ$, respectively. If $\angle POS = x$, find $\angle ROT$

Sol: Given $\angle POS = x$

$\angle POS + \angle SOQ = 180^\circ$ (Linear pair of angles)

$x + \angle SOQ = 180^\circ$

$\angle SOQ = 180^\circ - x$

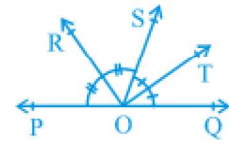
Ray OR and ray OT are angle bisectors of $\angle POS$ and $\angle SOQ$

$\angle ROS = \frac{1}{2} \times \angle POS$ and $\angle SOT = \frac{1}{2} \times \angle SOQ$

$\angle ROS = \frac{1}{2} \times x$ and $\angle SOT = \frac{1}{2} \times (180^\circ - x)$

$\angle ROS = \frac{x}{2}$ and $\angle SOT = 90^\circ - \frac{x}{2}$

$\angle ROT = \angle ROS + \angle SOT = \frac{x}{2} + 90^\circ - \frac{x}{2} = 90^\circ$



Example 3 : In Fig. OP, OQ, OR and OS are four rays. Prove that $\angle POQ + \angle QOR + \angle SOR + \angle POS = 360^\circ$

Sol: Let us produce ray OQ backwards to a point T

TOQ is a line. So, $\angle TOP + \angle POQ = 180^\circ$ (Linear pair axiom) \rightarrow (1)

Similarly $\angle TOS + \angle SOQ = 180^\circ$ (Linear pair axiom) \rightarrow (2)

But $\angle SOQ = \angle SOR + \angle QOR$

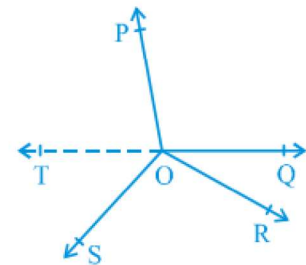
So, (2) becomes $\angle TOS + \angle SOR + \angle QOR = 180^\circ \rightarrow$ (3)

(1)+(3) we get

$\angle TOP + \angle POQ + \angle TOS + \angle SOR + \angle QOR = 180^\circ + 180^\circ = 360^\circ \rightarrow$ (4)

But $\angle TOP + \angle TOS = \angle POS$

Therefore, (4) becomes $\angle POQ + \angle QOR + \angle SOR + \angle POS = 360^\circ$



EXERCISE 6.1

1. In Fig, lines AB and CD intersect at O. If $\angle AOC + \angle BOE = 70^\circ$ and $\angle BOD = 40^\circ$, find $\angle BOE$ and reflex $\angle COE$.

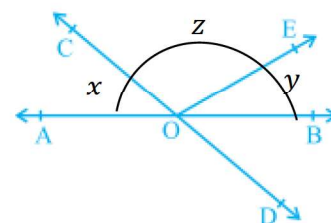
Sol: $\angle AOC = \angle BOD$ (Vertically opposite angles)

$x = 40^\circ$

$\angle AOC + \angle BOE = 70^\circ$

$x + y = 70^\circ$

$40^\circ + y = 70^\circ$



$$y = 70^\circ - 40^\circ$$

$$y = 30^\circ$$

$$x + y + z = 180^\circ \text{ (Linear angles)}$$

$$70^\circ + z = 180^\circ$$

$$z = 180^\circ - 70^\circ$$

$$z = 110^\circ \Rightarrow \angle COE = 110^\circ$$

$$\text{Reflex } \angle COE = 360^\circ - \angle COE = 360^\circ - 110^\circ = 250^\circ$$

2. In Fig, lines XY and MN intersect at O. If $\angle POY = 90^\circ$ and $a : b = 2 : 3$, find c.

Sol: $a : b = 2 : 3$

$$\text{Let } a = 2x \text{ and } b = 3x$$

$$a + b = 90^\circ \Rightarrow 2x + 3x = 90^\circ \Rightarrow 5x = 90^\circ \Rightarrow x = \frac{90^\circ}{5} = 18^\circ$$

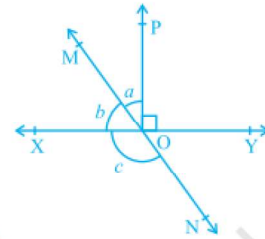
$$a = 2x = 2 \times 18^\circ = 36^\circ \text{ and } b = 3x = 3 \times 18^\circ = 54^\circ$$

$$b + c = 180^\circ \text{ (Linear pair)}$$

$$54^\circ + c = 180^\circ$$

$$c = 180^\circ - 54^\circ$$

$$c = 126^\circ$$



3. In Fig, $\angle PQR = \angle PRQ$, then prove that $\angle PQS = \angle PRT$.

Sol: $\angle PQR = \angle PRQ = x$

$$\angle PQS + \angle PQR = 180^\circ \text{ (Linear pair)}$$

$$\angle PQS + x = 180^\circ \rightarrow (1)$$

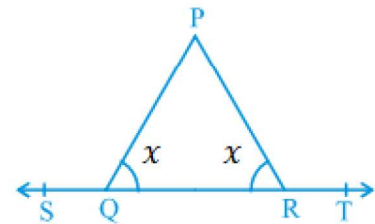
$$\angle PRT + \angle PRQ = 180^\circ \text{ (Linear pair)}$$

$$\angle PRT + x = 180^\circ \rightarrow (2)$$

From (1) and (2)

$$\angle PQS + x = \angle PRT + x$$

$$\angle PQS = \angle PRT$$



4. In Fig, if $x + y = w + z$, then prove that AOB is a line

Sol: $(x + y) + (z + w) = 360^\circ$ (Complete angle)

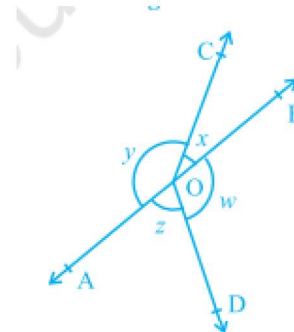
if $x + y = w + z$ then

$$(x + y) + (x + y) = 360^\circ$$

$$2(x + y) = 360^\circ$$

$$(x + y) = \frac{360^\circ}{2} = 180^\circ$$

\therefore AOB is a line



5. In Fig., POQ is a line. Ray OR is perpendicular to line PQ. OS is another ray lying between rays OP and OR. Prove that $\angle ROS = \frac{1}{2}(\angle QOS - \angle POS)$.

Sol: $\angle ROQ + \angle ROP = 180^\circ$ (Linear pair)

$$\text{But } \angle ROQ = 90^\circ$$

$$\text{So, } \angle ROP = 90^\circ$$

$$x + y = 90^\circ$$

$$2(x + y) = 2 \times 90^\circ$$

$$2x + 2y = 180^\circ \rightarrow (1)$$

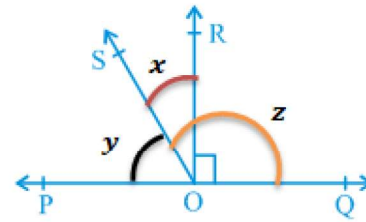
$$y + z = 180^\circ \rightarrow (2) \text{ (Linear pair)}$$

$$2x + 2y = y + z$$

$$2x = y + z - 2y$$

$$2x = z - y \Rightarrow x = \frac{1}{2}(z - y)$$

$$\angle ROS = \frac{1}{2}(\angle QOS - \angle POS)$$



6. It is given that $\angle XYZ = 64^\circ$ and XY is produced to point P. Draw a figure from the given information. If ray YQ bisects $\angle ZYP$, find $\angle XYQ$ and reflex $\angle QYP$.

Sol: $\angle XYZ + \angle ZYP = 180^\circ$ (linear pair)

$$64^\circ + \angle ZYP = 180^\circ$$

$$\angle ZYP = 180^\circ - 64^\circ$$

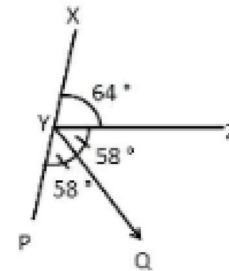
$$\angle ZYP = 116^\circ$$

If ray YQ bisects $\angle ZYP$ then

$$\angle ZYQ = \angle QYP = \frac{1}{2} \times \angle ZYP = \frac{1}{2} \times 116^\circ = 58^\circ$$

$$\angle XYQ = \angle XYZ + \angle ZYQ = 64^\circ + 58^\circ = 122^\circ$$

$$\text{Reflex } \angle QYP = 360^\circ - \angle QYP = 360^\circ - 58^\circ = 302^\circ$$



Lines Parallel to the Same Line

Lines which are parallel to the same line are parallel to each other.

If $l \parallel m$ and $l \parallel n$ then $m \parallel n$

If two parallel lines l and m are cut by a transversal then

- (i) Each pair of corresponding angles are equal in measure.

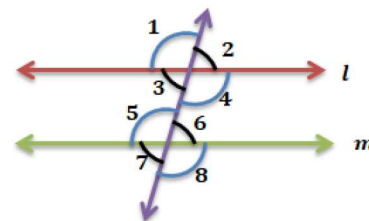
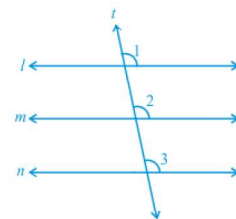
$$\angle 1 = \angle 5; \angle 2 = \angle 6; \angle 3 = \angle 7; \angle 4 = \angle 8$$

- (ii) Each pair of alternate interior angles are equal.

$$\angle 3 = \angle 6; \angle 4 = \angle 5$$

- (iii) Each pair of interior angles on the same side of the transversal are supplementary.

$$\angle 3 + \angle 5 = 180^\circ; \angle 4 + \angle 6 = 180^\circ$$



(iv) Each pair of exterior angles on the same side of the transversal are supplementary

$$\angle 1 + \angle 7 = 180^\circ; \angle 2 + \angle 8 = 180^\circ$$

Example 4 : In Fig. , if $PQ \parallel RS$, $\angle MXQ = 135^\circ$ and $\angle MYR = 40^\circ$, find $\angle XMY$

Sol: Draw a line AB parallel to line PQ

$PQ \parallel AB$ and XM is transversal

$$x + 135^\circ = 180^\circ \text{ (Co-interior angles are supplementary)}$$

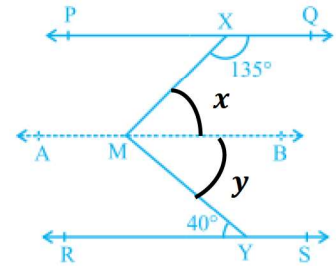
$$x = 180^\circ - 135^\circ$$

$$x = 45^\circ$$

$AB \parallel RS$ and MY is transversal

$$y = 40^\circ \text{ (Alternate interior angles)}$$

$$\angle XMY = x + y = 45^\circ + 40^\circ = 85^\circ$$



Example 5 : If a transversal intersects two lines such that the bisectors of a pair of corresponding angles are parallel, then prove that the two lines are parallel.

Sol: A transversal AD intersects PQ and RS at points B and C respectively.

\overrightarrow{BE} is the bisector of $\angle ABQ$ and \overrightarrow{CG} is the bisector of $\angle BCS$; and

$$\overrightarrow{BE} \parallel \overrightarrow{CG}.$$

$$\angle ABE = \frac{1}{2} \angle ABQ \quad \text{and} \quad \angle BCG = \frac{1}{2} \angle BCS$$

But $BE \parallel CG$ and AD is the transversal.

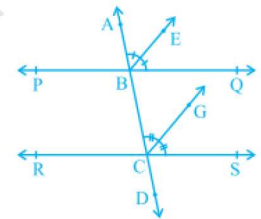
Therefore, $\angle ABE = \angle BCG$ (Corresponding angles axiom)

$$\frac{1}{2} \angle ABQ = \frac{1}{2} \angle BCS$$

$$\angle ABQ = \angle BCS$$

Corresponding angles are equal. From Converse of corresponding angles axiom

$$PQ \parallel RS$$



Example 6 : In Fig. , $AB \parallel CD$ and $CD \parallel EF$. Also $EA \perp AB$. If $\angle BEF = 55^\circ$, find the values of x , y and z .

Sol: $CD \parallel EF$ and DE is transversal

$$y + 55^\circ = 180^\circ \text{ (Co-interior angles are supplementary)}$$

$$y = 180^\circ - 55^\circ = 125^\circ$$

$AB \parallel CD$ and BD is transversal

$$x = y \text{ (corresponding angles)}$$

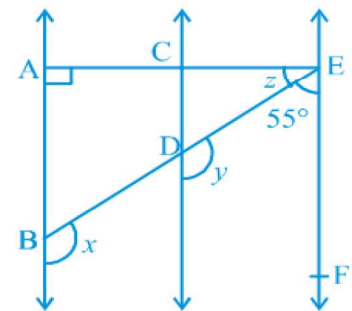
$$x = 125^\circ$$

$AB \parallel EF$ and AE is transversal.

$$\angle EAB + \angle FEA = 180^\circ \text{ (Co-interior angles are supplementary)}$$

$$90^\circ + z + 55^\circ = 180^\circ$$

$$z + 145^\circ = 180^\circ$$



$$z = 180^\circ - 145^\circ$$

$$z = 35^\circ$$

EXERCISE 6.2

1. In Fig, if $AB \parallel CD$, $CD \parallel EF$ and $y : z = 3 : 7$, find x .

Sol: $y : z = 3 : 7$

Let $y = 3a$ and $z = 7a$

But $x = z$ (Alternate interior angles)

$$x = 7a$$

$x + y = 180^\circ$ (co - interior angles are supplementary)

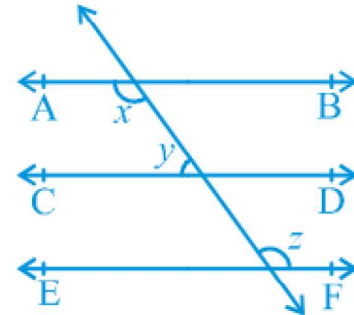
$$7a + 3a = 180^\circ$$

$$10a = 180^\circ$$

$$a = \frac{180^\circ}{10} = 18^\circ$$

$$x = z = 7a = 7 \times 18^\circ = 126^\circ$$

$$y = 3a = 3 \times 18^\circ = 54^\circ$$



2. In Fig, if $AB \parallel CD$, $EF \perp CD$ and $\angle GED = 126^\circ$, find $\angle AGE$, $\angle GEF$ and $\angle FGE$.

Sol: Given $EF \perp CD$

$$\angle FED = \angle FEC = 90^\circ$$

$AB \parallel CD$ and GE transversal

$\angle AGE = \angle GED$ (Alternate interior angles)

But $\angle GED = 126^\circ$

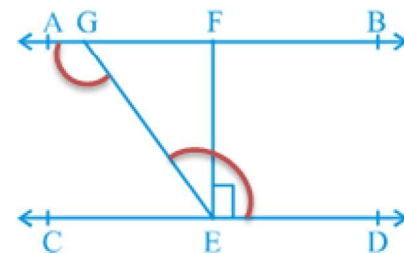
$$\angle AGE = 126^\circ$$

$$\angle GEF = \angle GED - \angle FED = 126^\circ - 90^\circ = 36^\circ$$

$\angle FGE + \angle AGE = 180^\circ$ (Linear pair)

$$\angle FGE + 126^\circ = 180^\circ$$

$$\angle FGE = 180^\circ - 126^\circ = 54^\circ$$



3. In Fig, if $PQ \parallel ST$, $\angle PQR = 110^\circ$ and $\angle RST = 130^\circ$, find $\angle QRS$.

Sol: $PQ \parallel ST$ and QR is transversal

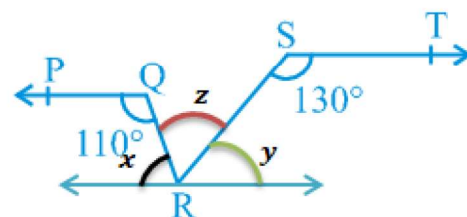
$x + 110^\circ = 180^\circ$ (Co interior angles are supplementary)

$$x = 180^\circ - 110^\circ = 70^\circ$$

similarly $y + 130^\circ = 180^\circ$

$$y + 130^\circ = 180^\circ$$

$$y = 180^\circ - 130^\circ = 50^\circ$$



$$x + y + z = 180^\circ \text{ (Linear angles)}$$

$$70^\circ + 50^\circ + z = 180^\circ$$

$$120^\circ + z = 180^\circ$$

$$z = 180^\circ - 120^\circ = 60^\circ$$

$$\angle QRS = 60^\circ$$

4. In Fig., if $AB \parallel CD$, $\angle APQ = 50^\circ$ and $\angle PRD = 127^\circ$, find x and y .

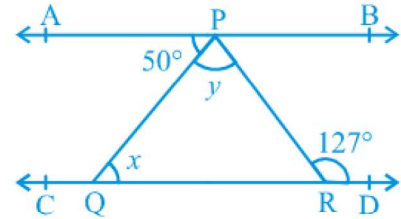
Sol: $AB \parallel CD$ and PQ is transversal

$$x = 50^\circ \text{ (Alternate interior angles)}$$

$AB \parallel CD$ and PR is transversal

$$y + 50^\circ = 127^\circ \text{ (Alternate interior angles)}$$

$$y = 127^\circ - 50^\circ = 77^\circ$$



5. In Fig., PQ and RS are two mirrors placed parallel to each other. An incident ray AB strikes the mirror PQ at B , the reflected ray moves along the path BC and strikes the mirror RS at C and again reflects back along CD . Prove that $AB \parallel CD$.

Sol: Draw BL and CM perpendicular to PQ

$BL \parallel CM$ and BC is transversal

$$\angle 2 = \angle 3 \rightarrow (1) \text{ (Alternate interior angles)}$$

But Angle of incidence = Angle of reflection

$$\angle 1 = \angle 2 \text{ and } \angle 3 = \angle 4 \rightarrow (2)$$

From (1) and (2)

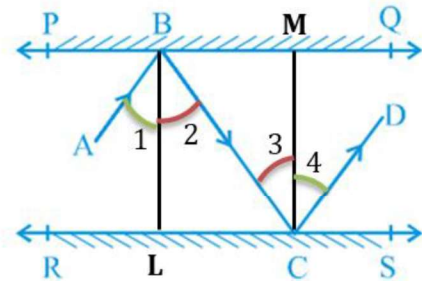
$$\angle 1 = \angle 2 = \angle 3 = \angle 4 \rightarrow (3)$$

$$\Rightarrow \angle 1 + \angle 2 = \angle 3 + \angle 4$$

$$\angle ABC = \angle BCD$$

Pair of alternate interior angles are equal

$$\therefore AB \parallel CD$$



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