CHAPTER 7

X-MATHEMATICS-NCERT-2024-25

COORDINATE GEOMETRY

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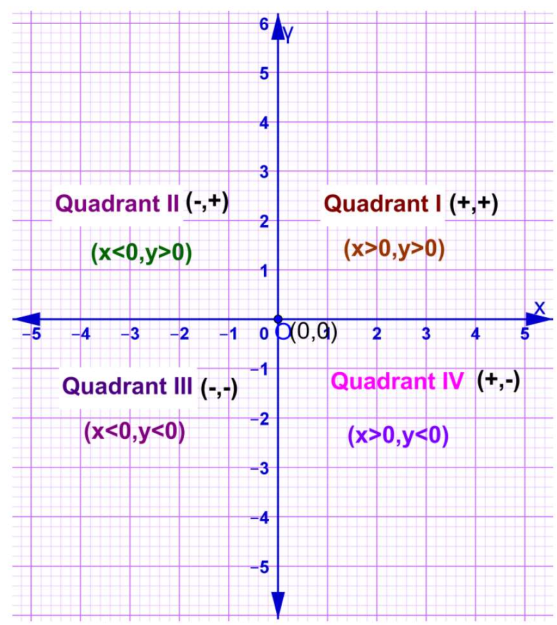
1. The father of coordinate geometry was **René Descartes**.

Rene Descartes (Born March 31,1596 in France-died February 11, 1650 in Sweden)

Rene Descartes was a French mathematician, scientist , and philosopher. He invented analytic geometry, a method of solving geometric problems algebraically and algebraic problems geometrically. The coordinate geometry also introduced by him. He published works on optics, coordinate geometry, physiology, and cosmology, however, he is mostly remembered as the "father of modern philosophy"

2. **Coordinate plane**: A coordinate plane is a two-dimensional plane formed by the intersection of a horizontal line(XOX') and vertical line(YOY').
3. In co-ordinate plane the horizontal number line XOX' is known as X-axis and the vertical number line YOY' is known as Y-axis. The point of intersection of these two axes is called origin denoted by O(0,0)
4. The coordinate plane is divided into four quadrants
5. In point (a,b), 'a' is called X-coordinate(Abscissa) and 'b' is called Y-coordinate (ordinate).



6. $If\ x > 0, y > 0\ then\ (x, y) \in Q_1$
7. $If\ x < 0, y > 0\ then\ (x, y) \in Q_2$
8. $If\ x < 0, y < 0\ then\ (x, y) \in Q_3$
9. $If\ x > 0, y < 0\ then\ (x, y) \in Q_4$
10. $If\ x > 0\ then\ (x, 0) lies\ on\ +ve\ \mathrm{X}-axis$
11. $If\ x < 0\ then\ (x, 0) lies\ on\ -ve\ \mathrm{X}-axis$
12. $If\ y > 0\ then\ (0, y) lies\ on\ +ve\ \mathrm{X}-axis$
13. $If\ y < 0\ then\ (0, y) lies\ on\ -ve\ \mathrm{X}-axis$
14.

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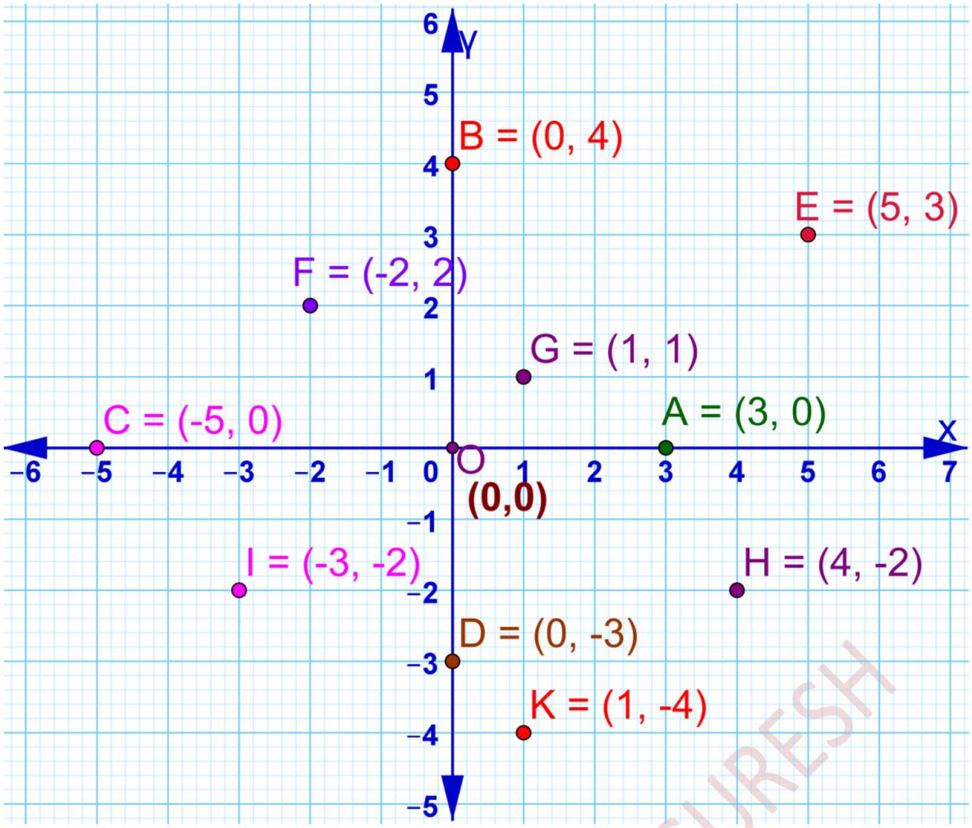
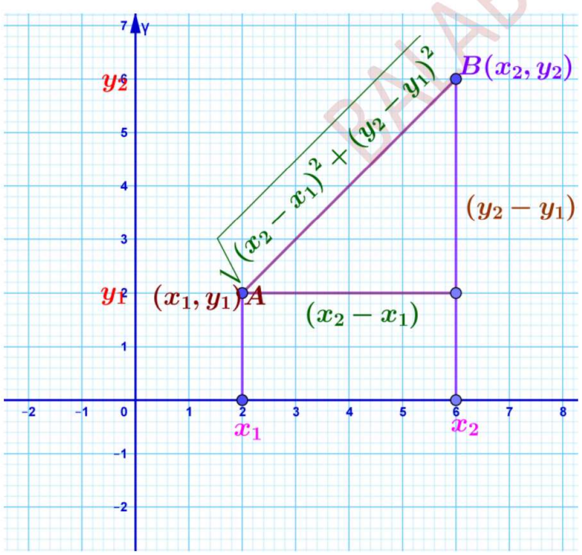
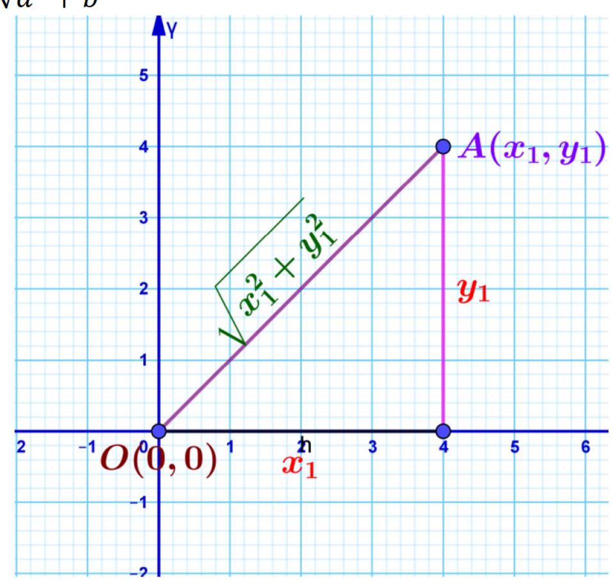


7.2 Distance Formula

1) The distance between two points $P(x_1, y_1)$ and $Q(x_2, y_2)$ is $PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
2) The distance of a point $P(x, y)$ from the origin $(0, 0)$ is $\sqrt{x^2 + y^2}$.
3) The distance between $A(a, 0)$ and $B(0, b)$ is $\sqrt{a^2 + b^2}$





Example-1. **Are the points (3, 2), (–2, –3) and (2, 3) form a triangle?**

Sol: P $(3, 2) = (x_1, y_1)$; Q $(-2, -3) = (x_2, y_2)$

$$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

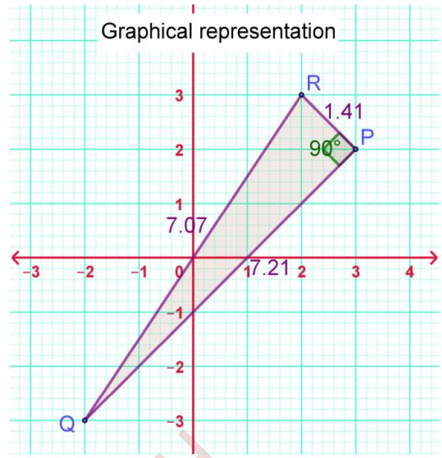
$$= \sqrt{(-2 - 3)^2 + (-3 - 2)^2}$$

$= \sqrt{(-5)^2 + (-5)^2}$

$= \sqrt{25 + 25}$

$= \sqrt{50} = 7.07\ units$

Q (-2, -3) $= (x_1, y_1)$ R (2, 3) $= (x_2, y_2)$



QR $= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(2 + 2)^2 + (3 + 3)^2}$

$= \sqrt{(4)^2 + (6)^2}$

$= \sqrt{16 + 36}$

$= \sqrt{52} = 7.21\ units$

P (3, 2) $= (x_1, y_1)$, R (2, 3) $= (x_2, y_2)$

PR $= \sqrt{(2 - 3)^2 + (3 - 2)^2}$

$= \sqrt{(-1)^2 + (1)^2} = \sqrt{1 + 1} = \sqrt{2} = 1.41\ units$

Since the sum of any two of these distances is greater than the third distance, therefore, the points P,Q and R form a triangle.

$PQ^2 + PR^2 = 50 + 2 = 52 = QR^2$

By the converse of Pythagoras theorem, we have $\angle P = 90°$. Therefore, PQR is a right triangle.

Example-2. Show that the points (1, 7), (4, 2), (–1, –1) and (–4, 4) are the vertices of a square.

Sol: A(1, 7), B(4, 2), C(–1, –1) and D(–4, 4)

A(1, 7)$=(x_1, y_1)$, B(4, 2) $=(x_2, y_2)$,

AB $= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(4 - 1)^2 + (2 - 7)^2}$

$= \sqrt{(3)^2 + (-5)^2}$

$= \sqrt{9 + 25} = \sqrt{34}\ units$

B (4, 2) $=(x_1, y_1)$ C (-1, -1) $=(x_2, y_2)$

BC $= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(-1 - 4)^2 + (-1 - 2)^2}$

$= \sqrt{(-5)^2 + (-3)^2}$

$= \sqrt{25 + 9} = \sqrt{34}\ units$

C (-1, -1) $=(x_1, y_1)$ D (-4, 4) $=(x_2, y_2)$

CD $= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(-4 + 1)^2 + (4 + 1)^2}$

$= \sqrt{(-3)^2 + (5)^2}$

$= \sqrt{9 + 25} = \sqrt{34}\ units$

D (-4, 4) $= (x_1, y_1)$ A(1, 7) $= (x_2, y_2)$

DA $= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(1 + 4)^2 + (7 - 4)^2}$

$= \sqrt{(5)^2 + (3)^2}$

$= \sqrt{25 + 9} = \sqrt{34}\ units$

A (1, 7) $= (x_1, y_1)$ C (-1, -1) $= (x_2, y_2)$

AC $= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(-1 - 1)^2 + (-1 - 7)^2}$

$= \sqrt{(-2)^2 + (-8)^2}$

$= \sqrt{4 + 64} = \sqrt{68}\ units$

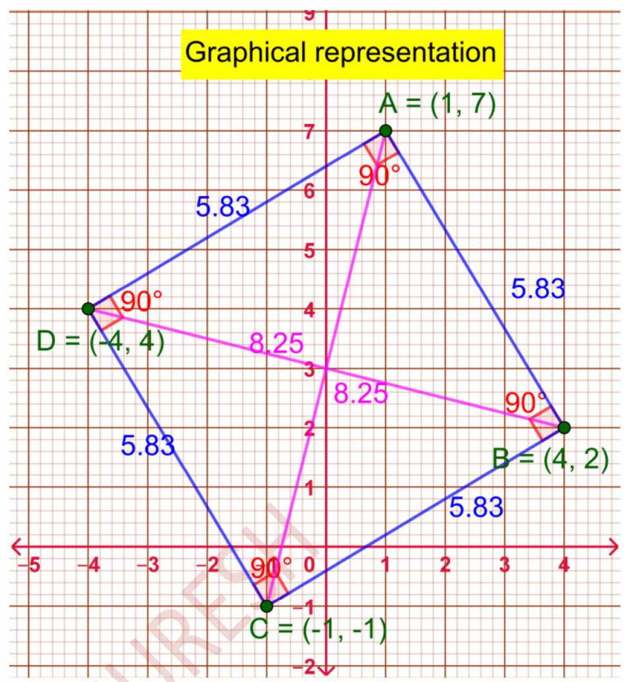
B (4, 2) $= (x_1, y_1)$ D (-4, 4) $= (x_2, y_2)$

BD $= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(-4 - 4)^2 + (4 - 2)^2}$

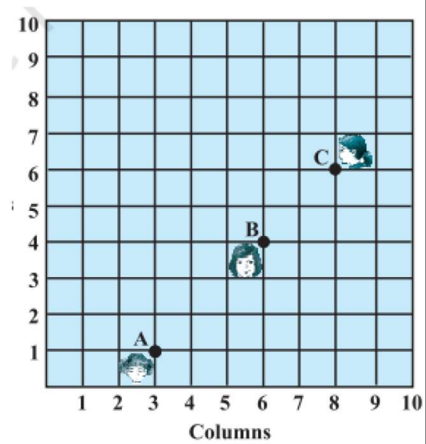
$= \sqrt{(-8)^2 + (2)^2}$

$= \sqrt{64 + 4} = \sqrt{68}\ units$



Since AB = BC = CD = DA and AC = BD. So all the four sides of the quadrilateral ABCD are equal and its diagonals AC and BD are also equal. Therefore, ABCD is a square.

Example 3 : Fig. 7.6 shows the arrangement of desks in a classroom. Ashima, Bharti and Camella are seated at A(3, 1), B(6, 4) and C(8, 6) respectively. Do you think they are seated in a line? Give reasons for your answer.



Sol: A(3, 1) $= (x_1, y_1)$, B(6, 4) $= (x_2, y_2)$

AB $= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(6 - 3)^2 + (4 - 1)^2}$

$= \sqrt{(3)^2 + (3)^2}$

$= \sqrt{9 + 9}$

$= \sqrt{18} = 3\sqrt{2}\ units$

B(6, 4) $= (x_1, y_1)$, C(8, 6) $= (x_2, y_2)$

BC $= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(8-6)^2 + (6-4)^2}$

$= \sqrt{(2)^2 + (2)^2}$

$= \sqrt{4+4}$

$= \sqrt{8} = 2\sqrt{2}\, units$

A(3, 1) $=(x_1, y_1)$, C(8, 6) $=(x_2, y_2)$

AC $= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(8-3)^2 + (6-1)^2}$

$= \sqrt{(5)^2 + (5)^2}$

$= \sqrt{25+25}$

$= \sqrt{50} = 5\sqrt{2}\, units$

Since AB+BC=$3\sqrt{2} + 2\sqrt{2} = 5\sqrt{2} = AC$

We can say that the points A, B and C are collinear. Therefore, they are seated in a line.

Example-4 : Find a relation between x and y such that the point (x , y) is equidistant from the points (7, 1) and (3, 5).

Solution : Let P(x, y) be equidistant from the points A(7, 1) and B(3, 5).

A(7, 1); P(x, y)

AP $= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(x-7)^2 + (y-1)^2}$

$= \sqrt{x^2 - 14x + 49 + y^2 - 2y + 1}$

$= \sqrt{x^2 + y^2 - 14x - 2y + 50}$

B(3, 5). P(x, y)

BP $= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(x-3)^2 + (y-5)^2}$

$= \sqrt{x^2 - 6x + 9 + y^2 - 10y + 25}$

$= \sqrt{x^2 + y^2 - 6x - 10y + 34}$

Given that AP = BP. So, $AP^2 = BP^2$

$x^2 + y^2 - 14x - 2y + 50 = x^2 + y^2 - 6x - 10y + 34$

$x^2 + y^2 - 14x - 2y - x^2 - y^2 + 6x + 10y = 34 - 50$

$-8x + 8y = -16 \Rightarrow x - y = 2$ it is the required relation.

Example-5. Find a point on the y-axis which is equidistant from both the points A(6, 5) and B(– 4, 3).

Sol: let the point P(0, y) on the y-axis be equidistant from A and B.

A(6, 5), P(0, y)

$$AP = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{(0-6)^2 + (y-5)^2}$$

$$= \sqrt{36 + y^2 - 10y + 25}$$

$$= \sqrt{y^2 - 10y + 61}$$

B(– 4, 3), P(0, y)

$$BP = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{(0+4)^2 + (y-3)^2}$$

$$= \sqrt{16 + y^2 - 6y + 9}$$

$$= \sqrt{y^2 - 6y + 25}$$

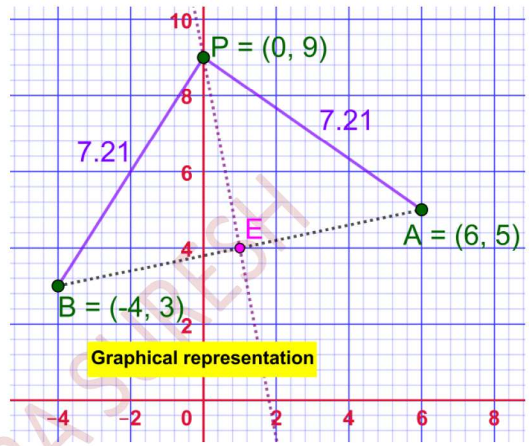
Since $AP = BP \Rightarrow AP^2 = BP^2$

$$y^2 - 10y + 61 = y^2 - 6y + 25$$

$$y^2 - 10y - y^2 + 6y = 25 - 61$$

$$-4y = -36 \Rightarrow y = 9$$

So, the required point is (0, 9).



Let us check our solution:

$$AP = \sqrt{(6-0)^2 + (5-9)^2} = \sqrt{36+16} = \sqrt{52}$$

$$BP = \sqrt{(-4-0)^2 + (3-9)^2} = \sqrt{16+36} = \sqrt{52}$$

EXERCISE 7.1

1. Find the distance between the following pairs of points :

(i) (2, 3) and (4, 1)

(x_1, y_1) (x_2, y_2)

Sol: $Distance = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$$= \sqrt{(4-2)^2 + (1-3)^2}$$

$$= \sqrt{(2)^2 + (-2)^2}$$

$$= \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}\ units$$

(ii) (–5, 7) and (–1, 3)

(x_1, y_1) (x_2, y_2)

Sol: $Distance = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(-1+5)^2 + (3-7)^2}$

$= \sqrt{(4)^2 + (-4)^2}$

$= \sqrt{16+16} = \sqrt{32} = 4\sqrt{2}\, units$

(iii) (–2, –3) and (3, 2)
$(x_1, y_1) \quad (x_2, y_2)$

Sol: $Distance = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(3+2)^2 + (2+3)^2}$

$= \sqrt{(5)^2 + (5)^2}$

$= \sqrt{25+25} = \sqrt{50} = 5\sqrt{2}\, units$

2. Find the distance between the points (0, 0) and (36, 15). Can you now find the distance between the two towns A and B discussed in Section 7.2.

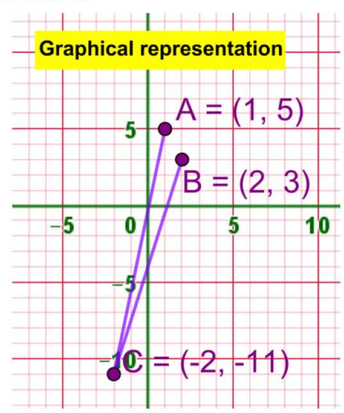
Sol: The distance between $(0, 0)$ and $(36, 15) = \sqrt{36^2 + 15^2} = \sqrt{1296 + 225} = \sqrt{1521} = 39$

Yes, we can find the distance between the two towns A and B.

The distance between town A and B will be 39 km.

3. Determine if the points (1, 5), (2, 3) and (– 2, – 11) are collinear.

Sol: Given points are $A\ (1,5)$, $B\ (2,3)$, $C\ (-2,-1)$



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$A\ (1,5) = (x_1, y_1) \quad B\ (2,3) = (x_2, y_2)$

$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(2-1)^2 + (3-5)^2}$

$= \sqrt{(1)^2 + (-2)^2}$

$= \sqrt{1+4} = \sqrt{5}\, units$

$B\ (2,3) = (x_1, y_1) \quad C\ (-2,-1) = (x_2, y_2)$

$BC = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(-2-2)^2 + (-1-3)^2}$

$= \sqrt{(-4)^2 + (-4)^2}$

$= \sqrt{16+16}$

$= \sqrt{32} = \sqrt{16 \times 2} = 4\sqrt{2}\ units$

$A\ (1,5) = (x_1, y_1) \quad C\ (-2,-1) = (x_2, y_2)$

$AC = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(-2-1)^2 + (-1-5)^2}$

$= \sqrt{(-3)^2 + (-6)^2}$

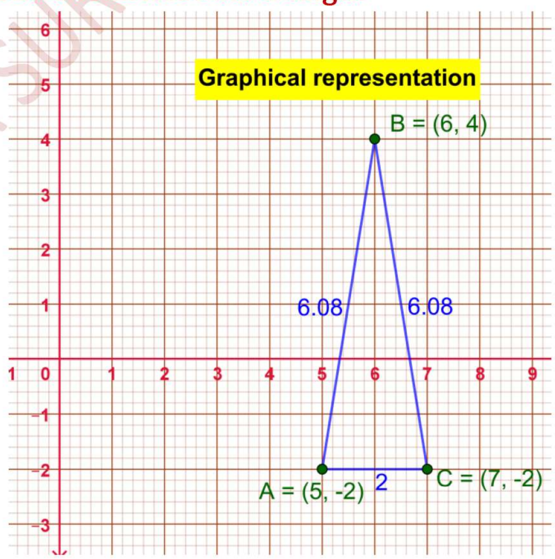
$= \sqrt{9+36}$

$= \sqrt{45} \quad = \sqrt{9 \times 5} = 3\sqrt{5}\ units$

The sum of no two sides is equal to third side.

∴ The given points are not collinear.

4. Check whether (5, – 2), (6, 4) and (7, – 2) are the vertices of an isosceles triangle.



Sol: Given vertices A (5, –2), B(6, 4) and C (7, –2)

A (5, –2), B(6, 4)

$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(6-5)^2 + (4+2)^2}$

$= \sqrt{(1)^2 + (6)^2}$

$= \sqrt{1+36} \ = \sqrt{37}\ units$

B(6, 4), C(7, -2)

$BC = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(7-6)^2 + (-2-4)^2}$

$= \sqrt{(1)^2 + (-6)^2}$

$= \sqrt{1+36} = \sqrt{37}\ units$

A(5, -2), C(7, -2)

$AC = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

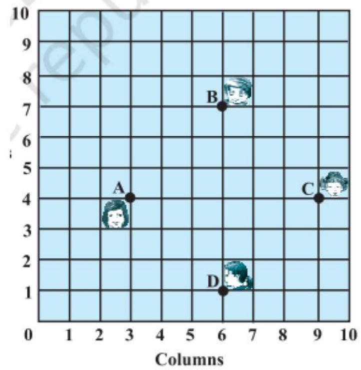
$= \sqrt{(7-5)^2 + (-2+2)^2}$

$= \sqrt{(2)^2 + (0)^2} = \sqrt{4+0} = \sqrt{4} = 2\ units$

Now AB=BC

∴ ΔABC is an isosceles triangle.

5. In a classroom, 4 friends are seated at the points A, B, C and D as shown in Fig. 7.8. Champa and Chameli walk into the class and after observing for a few minutes Champa asks Chameli, "Don't you think ABCD is a square?" Chameli disagrees. Using distance formula, find which of them is correct.



Sol: A=(3,4), B=(6,7), C=(9,4), D=(6,1)

$Distance\ formula = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$AB = \sqrt{(6-3)^2 + (7-4)^2} = \sqrt{3^2 + 3^2} = \sqrt{9+9} = \sqrt{18}$

$BC = \sqrt{(9-6)^2 + (4-7)^2} = \sqrt{3^2 + (-3)^2} = \sqrt{9+9} = \sqrt{18}$

$CD = \sqrt{(6-9)^2 + (1-4)^2} = \sqrt{(-3)^2 + (-3)^2} = \sqrt{9+9} = \sqrt{18}$

$AD = \sqrt{(6-3)^2 + (1-4)^2} = \sqrt{3^2 + (-3)^2} = \sqrt{9+9} = \sqrt{18}$

$AC = \sqrt{(9-3)^2 + (4-4)^2} = \sqrt{6^2 + 0^2} = \sqrt{36+0} = \sqrt{36} = 6$

$BD = \sqrt{(6-6)^2 + (1-7)^2} = \sqrt{0^2 + (-6)^2} = \sqrt{0+36} = \sqrt{36} = 6$

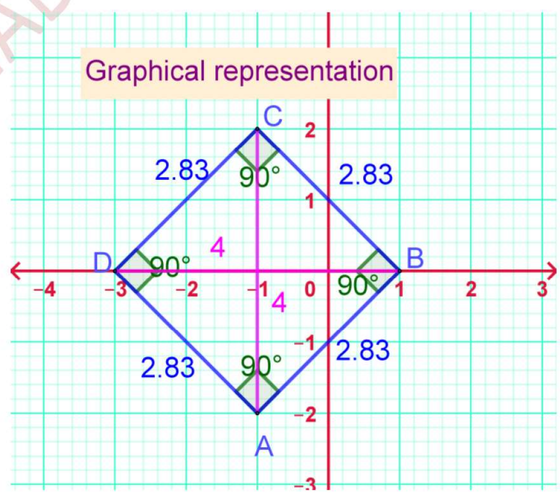
Since AB = BC = CD = DA and AC = BD. So all the four sides are equal and diagonals AC and BD are of equal length. Therefore, ABCD is a square and hence, Champa was correct

6. Name the type of quadrilateral formed, if any, by the following points, and give reasons for your answer:

(i) (– 1, – 2), (1, 0), (– 1, 2), (– 3, 0)

Sol: Given points A(–1, –2), B (1, 0), C (–1, 2), D(–3, 0).

A(–1, –2), B (1, 0)



$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(1+1)^2 + (0+2)^2}$

$= \sqrt{(2)^2 + (2)^2}$

$= \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}\ units$

B (1, 0), C (–1, 2)

$BC = \sqrt{(-1-1)^2 + (2-0)^2}$

$= \sqrt{(-2)^2 + (2)^2}$

$= \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}\ units$

C (–1, 2), D(–3, 0).

$CD = \sqrt{(-3+1)^2 + (0-2)^2}$

$= \sqrt{(-2)^2 + (2)^2}$

$= \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}\ units$

D(–3, 0), A(–1, –2)

$DA = \sqrt{(-1+3)^2 + (-2-0)^2}$

$= \sqrt{(2)^2 + (-2)^2}$

$= \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}\,units$

A(–1, –2), C (-1, 2)

AC $= \sqrt{(-1+1)^2 + (2+2)^2}$

$= \sqrt{(0)^2 + (4)^2}$

$= \sqrt{16} = 4\,units$

B (1, 0), D (–3, 0)

$BC = \sqrt{(-3-1)^2 + (0-0)^2}$

$= \sqrt{(-4)^2 + (0)^2}$

$= \sqrt{16} \; = 4\,units$

Since AB = BC = CD = DA and AC = BD. So all the four sides of the quadrilateral ABCD are equal and its diagonals AC and BD are also equal. Therefore, ABCD is square

(ii) (–3, 5), (3, 1), (0, 3), (–1, – 4)

Sol: Given points A(–3, 5), B(3, 1), C(0, 3), D(–1, –4)

A(–3, 5), B (3, 1)

$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(3+3)^2 + (1-5)^2}$

$= \sqrt{(6)^2 + (-4)^2}$

$= \sqrt{36+16} = \sqrt{52}\,units$

B(3, 1), C(0, 3)

$BC = \sqrt{(0-3)^2 + (3-1)^2}$

$= \sqrt{(-3)^2 + (2)^2}$

$= \sqrt{9+4} = \sqrt{13}\,units$

C(0, 3), D(–1, –4)

$CD = \sqrt{(-1-0)^2 + (-4-3)^2}$

$= \sqrt{(-1)^2 + (-7)^2}$

$= \sqrt{1+49} = \sqrt{50}units$

$D(-1,-4), A(-3,5)$

$DA = \sqrt{(-3+1)^2 + (5+4)^2}$

$= \sqrt{(-2)^2 + (9)^2}$

$= \sqrt{4+81} = \sqrt{85}\ units$

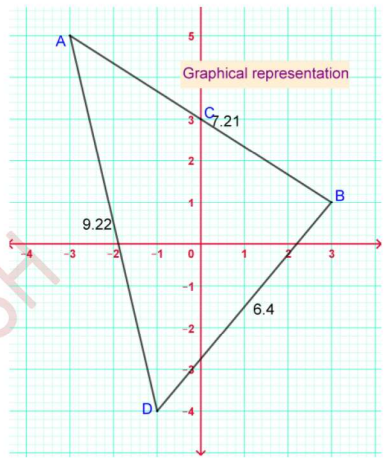
Now AB $\neq$CD$\neq$BC $\neq$DA

All sides are of different length.

Therefore given points form a quadrilateral only.

Higher thinking:

From graphical representation A,C, B are collinear . So, A,B,C and D represents only triangle.



(iii) (4, 5), (7, 6), (4, 3), (1, 2)

Sol: Given points A(4, 5), B(7, 6), C(4, 3), D(1, 2)

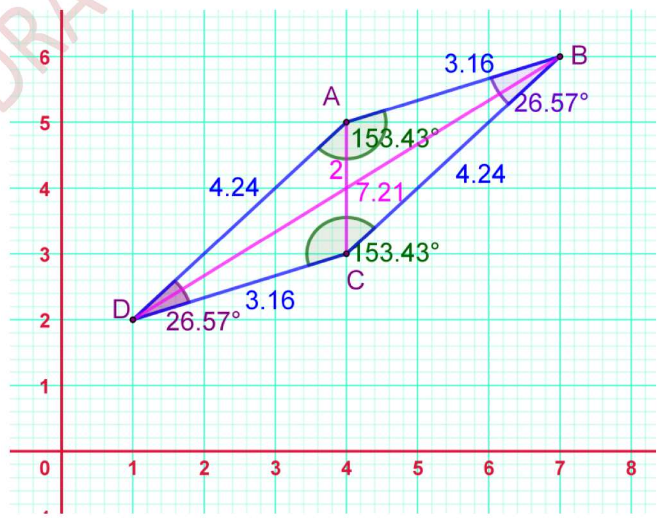
A(4, 5), B(7, 6),

$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(7-4)^2 + (6-5)^2}$

$= \sqrt{(3)^2 + (1)^2}$

$= \sqrt{9+1} = \sqrt{10}\ units$



B(7, 6), C(4, 3),

$BC = \sqrt{(4-7)^2 + (3-6)^2}$

$= \sqrt{(-3)^2 + (-3)^2}$

$= \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}\ units$

C(4, 3), D(1, 2)

$CD = \sqrt{(1-4)^2 + (2-3)^2}$

$= \sqrt{(-3)^2 + (-1)^2}$

$= \sqrt{9+1} = \sqrt{10} units$

$D(1, 2), A(4, 5)$

$DA = \sqrt{(4-1)^2 + (5-2)^2}$

$= \sqrt{(3)^2 + (3)^2}$

$= \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}\, units$

A(4, 5), C (4, 3)

$AC = \sqrt{(4-4)^2 + (3-5)^2}$

$= \sqrt{(0)^2 + (-2)^2}$

$= \sqrt{4} = 2\, units$

$B\ (7,6), D\ (1,2)$

$BD = \sqrt{(1-7)^2 + (2-6)^2}$

$= \sqrt{(-6)^2 + (-4)^2}$

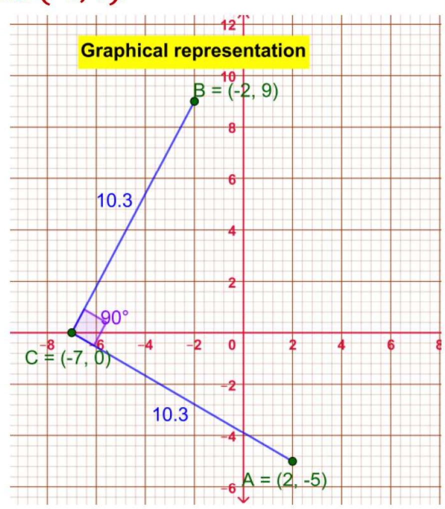
$= \sqrt{36+16} = \sqrt{52}\, units$

Now AB =CD, BC =DA and AC ≠ BD.

Opposite sides are equal and diagonals are not equal.

Therefore given points form a parallelogram.

7. Find the point on the x-axis which is equidistant from (2, –5) and (–2, 9).



Sol: Given points A(2, –5) and B(–2, 9).

Let $P(x, o)$ the point on the X-axis which is equidistant from A and B

A(2, –5), $P(x, o)$

$AP = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(x-2)^2 + (0+5)^2}$

$= \sqrt{x^2 - 4x + 4 + 25}$

$= \sqrt{x^2 - 4x + 29}$

B(–2, 9), $P(x, o)$

$BP = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(x+2)^2 + (0-9)^2}$

$= \sqrt{x^2 + 4x + 4 + 81}$

$= \sqrt{x^2 + 4x + 85}$

Now AP=BP⇒ $AP^2 = BP^2$

$x^2 - 4x + 29 = x^2 + 4x + 85$

$x^2 - 4x - x^2 - 4x = 85 - 29$

$-8x = 56 \Rightarrow x = \frac{56}{-8} = -7$

So, the required point is $(-7, 0)$.

8. Find the values of y for which the distance between the points P(2, – 3) and Q(10, y) is 10 units.

Sol: Given points $P = (2, -3) = (x_1, y_1) Q = (10, y) = (x_2, y_2)$

The distance between given two points = 10

$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = 10$

$\sqrt{(10 - 2)^2 + (y + 3)^2} = 10$

Squaring on both sides we get

$(8)^2 + (y + 3)^2 = 10^2$

$(y + 3)^2 = 10^2 - (8)^2 = 100 - 64 = 36$

$y + 3 = \sqrt{36} = \pm 6$

$y + 3 = 6 \quad or \quad y + 3 = -6$

$y = 6 - 3 \quad or \quad y = -6 - 3$

$y = 3 \; or - 9$

9. If Q(0, 1) is equidistant from P(5, –3) and R(x, 6), find the values of x. Also find the distances QR and PR.

Sol: $Q \text{ is equidistance from } P \text{ and } R$

$PQ = RQ \Rightarrow PQ^2 = RQ^2$

$(5 - 0)^2 + (-3 - 1)^2 = (x - 0)^2 + (6 - 1)^2$

$25 + 16 = x^2 + 25$

$x^2 = 16 \Rightarrow x = \pm 4$

Therefore, point R is (4, 6) or (–4, 6).

$If\ R = (4,6) then\ QR = \sqrt{(4 - 0)^2 + (6 - 1)^2} = \sqrt{16 + 25} = \sqrt{41}$

$PR = \sqrt{(4 - 5)^2 + (6 + 3)^2} = \sqrt{1 + 81} = \sqrt{82}$

$If\ R = (-4,6) then\ QR = \sqrt{(-4-0)^2 + (6-1)^2} = \sqrt{16+25} = \sqrt{41}$

$PR = \sqrt{(-4-5)^2 + (6+3)^2} = \sqrt{81+81} = \sqrt{162} = 9\sqrt{2}$

10. Find a relation between x and y such that the point (x, y) is equidistant from the point (3, 6) and (– 3, 4).

Sol: Let A (–2, 8) and B(–3, –5)

Let P(x,y) is equidistant from the points from A and B.

$AP = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(x+2)^2 + (y-8)^2}$

$= \sqrt{x^2 + 4x + 4 + y^2 - 16y + 64}$

$= \sqrt{x^2 + y^2 + 4x - 16y + 68}$

$BP = \sqrt{(x+3)^2 + (y+5)^2}$

$= \sqrt{x^2 + 6x + 9 + y^2 + 10y + 25}$

$= \sqrt{x^2 + y^2 + 6x + 10y + 34}$

Now AP=BP⇒ $AP^2 = BP^2$

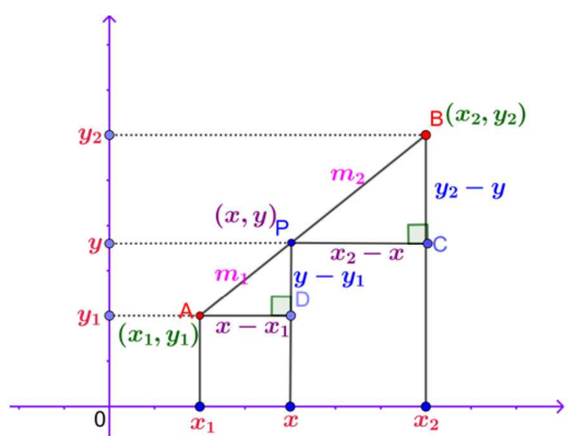
$x^2 + y^2 + 4x - 16y + 68 = x^2 + y^2 + 6x + 10y + 34$

$x^2 + y^2 + 4x - 16y - x^2 - y^2 - 6x - 10y = 34 - 68$

$-2x - 26y = -34$

$\Rightarrow x + 13 = 17\ is\ the\ required\ condition$

7.3 Section Formula



$A(x_1, y_1)$ and $B(x_2, y_2)$ if P(x, y) divides AB internally in the ratio $m_1 : m_2$ then

$P(x, y) = \left(\frac{m_1x_2 + m_2x_1}{m_1 + m_2}, \frac{m_1y_2 + m_2y_1}{m_1 + m_2}\right)$

If $A(x_1, y_1)$ and $B(x_2, y_2)$ then the mid-point of a line segment AB=$\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

If the ratio in which P divides AB is k : 1, then the coordinates of the point P will be

$= \left(\frac{kx_2 + kx_1}{k+1}, \frac{ky_2 + y_1}{k+1}\right)$

The ratio **X –axis** divides line segment joining $A(x_1, y_1)$ and $B(x_2, y_2)$ is $\boldsymbol{-y_1 : y_2}$

The ratio **Y –axis** divides line segment joining $A(x_1, y_1)$ and $B(x_2, y_2)$ is $\boldsymbol{-x_1 : x_2}$

The ratio $P(x, y)$ divides line joining $A(x_1, y_1)$ and $B(x_2, y_2)$ internally $= \frac{x - x_1}{x_2 - x}$ (or) $\frac{y - y_1}{y_2 - y}$

Example-6. Find the coordinates of the point which divides the line segment joining the points (4, –3) and (8, 5) in the ratio 3 : 1 internally.

Sol: Given points $A\,(4, -3)$, $B\,(8,5)$ ratio $= 3 : 1$

(x_1, y_1) (x_2, y_2) $m_1 : m_2$

$$P(x, y) = \left(\frac{m_1x_2 + m_2x_1}{m_1 + m_2}, \frac{m_1y_2 + m_2y_1}{m_1 + m_2}\right)$$

$$= \left(\frac{3 \times 8 + 1 \times (4)}{3 + 1}, \frac{3 \times (5) + 1 \times (-3)}{3 + 1}\right)$$

$$= \left(\frac{24 + 4}{4}, \frac{15 - 3}{4}\right)$$

$$= \left(\frac{28}{4}, \frac{12}{4}\right) = (7,3)$$

The required point $= (7,3)$

Example-7. In what ratio does the point (– 4, 6) divide the line segment joining the points A(– 6, 10) and B(3, – 8)?

Sol: A(– 6, 10) and B(3, – 8) P (– 4, 6)

(x_1, y_1) (x_2, y_2) (x, y)

Let P divides AB in the ratio $m_1 : m_2$

$P = (-4,6)$

$$\left(\frac{m_1x_2 + m_2x_1}{m_1 + m_2}, \frac{m_1y_2 + m_2y_1}{m_1 + m_2}\right) = (-4,6)$$

$$\left(\frac{m_1(3) + m_2(-6)}{m_1 + m_2}, \frac{m_1(-8) + m_2(10)}{m_1 + m_2}\right) = (-4,6)$$

$$\frac{3m_1 - 6m_2}{m_1 + m_2} = -4$$

$$3m_1 - 6m_2 = -4(m_1 + m_2)$$

$$3m_1 - 6m_2 = -4m_1 - 4m_2$$

$$3m_1 + 4m_1 = -4m_2 + 6m_2$$

$$7m_1 = 2m_2$$

$$\frac{m_1}{m_2}=\frac{2}{7}\Rightarrow m_1:m_2=2:7$$

Therefore, the point (– 4, 6) divides the line segment joining the points A(– 6, 10) and B(3, – 8) in the ratio 2 : 7.

Alternatively : Let (– 4, 6) divide AB internally in the ratio k : 1.

$$\left(\frac{3k-6}{k+1},\frac{-8k+10}{k+1}\right)=(-4,6)$$

$$\frac{3k-6}{k+1}=-4$$

$$3k-6=-4k-4$$

$$3k+4k=-4+6$$

$$7k=2$$

$$k:1=2:7$$

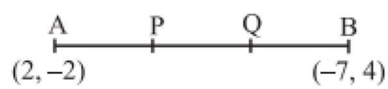
So, the point (– 4, 6) divides the line segment joining the points A(– 6, 10) and B(3, – 8) in the ratio 2 : 7.

TRISECTIONAL POINTS OF A LINE:

1) The points which divide a line segment into 3 equal parts are said to be the Trisectional points.

2) The points divides line segment either 1:2 or 2:1 are called trisectional points.

Example-14. Find the coordinates of the points of trisection of the line segment joining the points A(2,–2) and B(–7, 4).



Sol: A(2,–2) and B(–7, 4).
(x_1, y_1) (x_2, y_2)

Let P divides AB internally in the ratio 1 : 2.= $m_1:m_2$

$$P(x,y)=\left(\frac{m_1x_2+m_2x_1}{m_1+m_2},\frac{m_1y_2+m_2y_1}{m_1+m_2}\right)$$

$$=\left(\frac{1(-7)+2(2)}{1+2},\frac{1(4)+2(-2)}{1+2}\right)$$

$$=\left(\frac{-7+4}{3},\frac{4-4}{3}\right)=\left(\frac{-3}{3},\frac{0}{3}\right)=(-1,0)$$

Let Q divides AB internally in the ratio 2:1=$m_1:m_2$

$$Q(x,y)=\left(\frac{m_1x_2+m_2x_1}{m_1+m_2},\frac{m_1y_2+m_2y_1}{m_1+m_2}\right)$$

$$= \left(\frac{2(-7)+1(2)}{2+1}, \frac{2(4)+1(-2)}{2+1}\right)$$

$$= \left(\frac{-14+2}{3}, \frac{8-2}{3}\right) = \left(\frac{-12}{3}, \frac{6}{3}\right) = (-4,2)$$

Required trisectional points are P(-1, 0) and Q(-4, 2).

Example 9 : Find the ratio in which the y-axis divides the line segment joining the points (5, – 6) and (–1, – 4). Also find the point of intersection.

Sol: A(5, –6)= (x_1, y_1) and B (–1, –4).= (x_2, y_2)

Let Y-Axis divides AB in the ratio K:1.= $m_1 : m_2$

On the y-axis X-coordinate (abscissa) =0

$$\frac{m_1x_2 + m_2x_1}{m_1 + m_2} = 0$$

$$\frac{k \times (-1) + 1 \times (5)}{k+1} = 0 \Rightarrow -k + 5 = 0 \Rightarrow k = 5$$

So, the ratio=5:1=$m_1 : m_2$

$$\text{Now y} - \text{coordinate of } P = \frac{m_1y_2 + m_2y_1}{m_1 + m_2}$$

$$= \frac{5(-4)+1(-6)}{5+1} = \frac{-20-6}{6} = \frac{-26}{6} = \frac{-13}{3}$$

$$\text{The point of intersection} = \left(0, \frac{-13}{3}\right)$$

Example-15. Find the ratio in which the y-axis divides the line segment joining the points (5, –6) and (–1, –4). Also find the point of intersection.

Sol: $A(5, -6) = (x_1, y_1) and B\ (-1, -4) = (x_2, y_2)$.

Let Y-Axis divides AB in the ratio K:1.= $m_1 : m_2$

On the y-axis X-coordinate(abscissa) =0

$$\frac{kx_2 + x_1}{k+1} = 0$$

$$\frac{k \times (-1) + 1 \times (5)}{k+1} = 0 \Rightarrow -k + 5 = 0 \Rightarrow k = 5$$

So, the ratio k:1=5:1

Now y − coordinate of $P = \frac{ky_2 + y_1}{k+1}$

$$= \frac{5(-4)+(-6)}{5+1} = \frac{-20-6}{6} = \frac{-26}{6} = \frac{-13}{3}$$

The point of intersection $= \left(0, \frac{-13}{3}\right)$

Example 10 : If the points A(6, 1), B(8, 2), C(9, 4) and D(p, 3) are the vertices of a parallelogram, taken in order, find the value of p.

Sol: We know that diagonals of parallelogram bisect each other.

Midpoint of AC= midpoint of BD.

$$\left(\frac{6+9}{2}, \frac{1+4}{2}\right) = \left(\frac{8+p}{2}, \frac{2+3}{2}\right)$$

$$\left(\frac{15}{2}, \frac{5}{2}\right) = \left(\frac{8+p}{2}, \frac{5}{2}\right)$$

$$\Rightarrow 8 + P = 15$$

$$\Rightarrow p = 15 - 8 = 7$$

EXERCISE 7.2

1. Find the coordinates of the point which divides the join of (–1, 7) and (4, –3) in the ratio 2 : 3.

Sol: *Given points* $A(-1, 7) = (x_1, y_1)$, $B(4, -3) = (x_2, y_2)$

ratio $= 2 : 3 = m_1 : m_2$

$$P(x, y) = \left(\frac{m_1x_2 + m_2x_1}{m_1 + m_2}, \frac{m_1y_2 + m_2y_1}{m_1 + m_2}\right)$$

$$= \left(\frac{2\times 4 + 3\times(-1)}{2+3}, \frac{2\times(-3) + 3\times 7}{2+3}\right)$$

$$= \left(\frac{8-3}{5}, \frac{-6+21}{5}\right)$$

$$= \left(\frac{5}{5}, \frac{15}{5}\right) = (1,3)$$

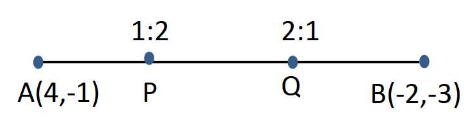
The required point = (1,3)

2. Find the coordinates of the points of trisection of the line segment joining (4, –1) and (–2, –3).

Sol: Given points A(4, –1) and B(–2, –3)

(x_1, y_1) (x_2, y_2)

Let P and Q be the points of trisection of AB



P divides AB in the ratio $1:2 = m_1 : m_2$

$P(x,y) = \left(\frac{m_1x_2 + m_2x_1}{m_1 + m_2}, \frac{m_1y_2 + m_2y_1}{m_1 + m_2}\right)$

$= \left(\frac{1(-2) + 2(4)}{1 + 2}, \frac{1(-3) + 2(-1)}{1 + 2}\right)$

$= \left(\frac{-2 + 8}{3}, \frac{-3 - 2}{3}\right) = \left(\frac{6}{3}, \frac{-5}{3}\right) = \left(2, \frac{-5}{3}\right)$

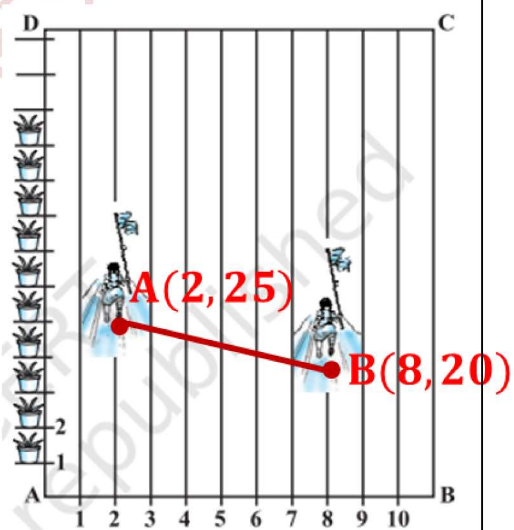
Q divides AB in the ratio $2:1 = m_1:m_2$

$Q(x,y) = \left(\frac{2(-2) + 1(4)}{2 + 1}, \frac{2(-3) + 1(-1)}{2 + 1}\right)$

$= \left(\frac{-4 + 4}{3}, \frac{-6 - 1}{3}\right) = \left(\frac{0}{3}, \frac{-7}{3}\right) = \left(0, \frac{-7}{3}\right)$

The trisection points are $\left(2, \frac{-5}{3}\right)$ and $\left(0, \frac{-7}{3}\right)$

3. To conduct Sports Day activities, in your rectangular shaped school ground ABCD, lines have been drawn with chalk powder at a distance of 1m each. 100 flower pots have been placed at a distance of 1m from each other along AD, as shown in Fig. 7.12. Niharika runs $\frac{1}{4}$th the distance AD on the 2nd line and posts a green flag. Preet runs $\frac{1}{5}$th the distance AD on the eighth line and posts a red flag. What is the distance between both the flags? If Rashmi has to post a blue flag exactly halfway between the line segment joining the two flags, where should she post her flag?



Sol: $AD = 100m$

$Niharika\ posts\ green\ falg\ at\ point\ A = \left(2, \frac{1}{4} \times 100\right) = (2,25)$

$Preet\ posts\ red\ falg\ at\ point\ B = \left(8, \frac{1}{5} \times 100\right) = (8,20)$

The distance between both the flags AB $= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(8-2)^2 + (20-25)^2} = \sqrt{36 + 25} = \sqrt{61}\ m$

Mid point of $AB = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{2 + 8}{2}, \frac{25 + 20}{2}\right) = \left(5, \frac{45}{2}\right)$

Therefore, Rashmi should post her blue flag at 22.5 m on 5th line

4. Find the ratio in which the line segment joining the points (– 3, 10) and (6, – 8) is divided by (– 1, 6).

Sol: $A(-3,10) = (x_1, y_1) and\ B(6,-8) = (x_2, y_2)\ \ P(-1,6) = (x,y).$

Let P divides AB in the ratio $m_1 : m_2$

$$\frac{m_1x_2 + m_2x_1}{m_1 + m_2} = -1$$

$$m_1(6) + m_2(-3) = -1(m_1 + m_2)$$

$$6m_1 - 3m_2 = -m_1 - m_2$$

$$6m_1 + m_1 = -m_2 + 3m_2$$

$$7m_1 = 2m_2 \Rightarrow \frac{m_1}{m_2} = \frac{2}{7} \Rightarrow m_1 : m_2 = 2:7$$

5. Find the ratio in which the line segment joining A (1, – 5) and B(– 4, 5) is divided by the x-axis. Also find the coordinates of the point of division.

Sol: $A\ (1,-5) = (x_1, y_1) and B\ (-4,5) = (x_2, y_2).$

Let X-Axis divides AB in the ratio k:1.= $m_1 : m_2$

On the X-axis y-coordinate=0

$$\frac{m_1y_2 + m_2y_1}{m_1 + m_2} = 0$$

$$\frac{k \times (5) + 1 \times (-5)}{k+1} = 0 \Rightarrow 5k - 5 = 0 \Rightarrow k = 1$$

So, the ratio=1:1=$m_1 : m_2$

$$Now\ x - coordinate\ of\ P = \frac{m_1x_2 + m_2x_1}{m_1 + m_2}$$

$$= \frac{1(-4) + 1(1)}{1+1} = \frac{-4+1}{2} = \frac{-3}{2}$$

The point of division = $\left(\frac{-3}{2}, 0\right)$

6. If (1, 2), (4, y), (x, 6) and (3, 5) are the vertices of a parallelogram taken in order, find x and y.

Sol: The vertices of a parallelogram A(1, 2), B(4, y), C(x, 6) and D(3, 5)

We know that diagonals of parallelogram bisect each other.

midpoint of AC= midpoint of BD.

$$\left(\frac{1+x}{2}, \frac{2+6}{2}\right) = \left(\frac{4+3}{2}, \frac{y+5}{2}\right)$$

$$\left(\frac{1+x}{2}, \frac{8}{2}\right) = \left(\frac{7}{2}, \frac{y+5}{2}\right)$$

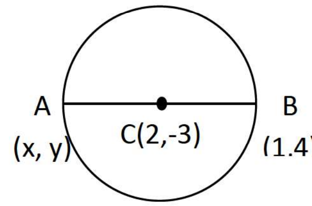
$1 + x = 7 \quad and \quad y + 5 = 8$

$x = 7 - 1 \quad and \quad y = 8 - 5$

$x = 6 \quad and \quad y = 3$

7. Find the coordinates of a point A, where AB is the diameter of a circle whose centre is (2, – 3) and B is (1, 4).

Sol: We know that midpoint of diameter=centre



Midpoint of AB=C

$\left(\frac{x+1}{2}, \frac{y+4}{2}\right) = (2, -3)$

$\frac{x+1}{2} = 2 \quad and \quad \frac{y+4}{2} = -3$

$x + 1 = 2 \times 2 \quad and \quad y + 4 = -3 \times 2$

$x + 1 = 4 \quad and \quad y + 4 = -6$

$x = 4 - 1 \quad and \quad y = -6 - 4$

$x = 3 \quad and \quad y = -10$

$\therefore A = (3, -10)$

8. If A and B are (– 2, – 2) and (2, – 4), respectively, find the coordinates of P such that AP = $\frac{3}{7}$AB and P lies on the line segment AB

Sol: Given points $A\,(-2, -2) = (x_1, y_1)$, $B\,(2, -4) = (x_2, y_2)$

$\text{AP} = \frac{3}{7}\,\text{AB} \Rightarrow \frac{\text{AP}}{\text{AB}} = \frac{3}{7}$

$\Rightarrow \frac{\text{AP}}{\text{AB} - \text{AP}} = \frac{3}{7-3} \Rightarrow \frac{\text{AP}}{\text{PB}} = \frac{3}{4}$

$\therefore$ P divides AB in the ratio =3:4= $m_1 : m_2$

$P(x, y) = \left(\frac{m_1x_2 + m_2x_1}{m_1 + m_2}, \frac{m_1y_2 + m_2y_1}{m_1 + m_2}\right)$

$= \left(\frac{3 \times 2 + 4(-2)}{3+4}, \frac{3 \times (-4) + 4 \times (-2)}{3+4}\right)$

$= \left(\frac{6-8}{7}, \frac{-12-8}{7}\right) = \left(\frac{-2}{7}, \frac{-20}{7}\right)$

The required point P $= \left(\frac{-2}{7}, \frac{-20}{7}\right)$

9. Find the coordinates of the points which divide the line segment joining A(– 2, 2) and B(2, 8) into four equal parts.

Sol: A(−2, 2) and B(2, 8)

Let P,Q and R be the points which divide AB into four equal parts.

$$Q = Midpoint\ of\ AB = \left(\frac{-2+2}{2}, \frac{2+8}{2}\right) = \left(\frac{0}{2}, \frac{10}{2}\right) = (0,5)$$

$$P = Midpoint\ of\ AQ = \left(\frac{-2+0}{2}, \frac{2+5}{2}\right) = \left(\frac{-2}{2}, \frac{7}{2}\right) = \left(-1, \frac{7}{2}\right)$$

$$R = Midpoint\ of\ QB = \left(\frac{0+2}{2}, \frac{5+8}{2}\right) = \left(\frac{2}{2}, \frac{13}{2}\right) = \left(1, \frac{13}{2}\right)$$

The coordinates of the points which divide the line segment joining AB into four equal parts are

$(0,5), \left(-1, \frac{7}{2}\right)\ and\ \left(1, \frac{13}{2}\right)$

10. Find the area of a rhombus if its vertices are (3, 0), (4, 5), (– 1, 4) and (– 2, – 1) taken in order. [Hint : Area of a rhombus = $\frac{1}{2}$(product of its diagonals)]

Sol: Given points A (3,0) , B (4,5) , C (−1,4) , D (−2, −1)

$A\ (3,0) = (x_1, y_1), C\ (-1,4) = (x_2, y_2)$

$$d_1 = AC = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{(-1-3)^2 + (4-0)^2}$$

$$= \sqrt{(-4)^2 + (4)^2}$$

$$= \sqrt{16+16}$$

$$= \sqrt{32}$$

$$= \sqrt{16 \times 2}$$

$$= 4\sqrt{2} units$$

$B(4,5) = (x_1, y_1), D(-2,-1) = (x_2, y_2)$

$$d_2 = BD = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{(-2-4)^2 + (-1-5)^2}$$

$$= \sqrt{(-6)^2 + (-6)^2}$$

$$= \sqrt{36+36}$$

$$= \sqrt{72}$$

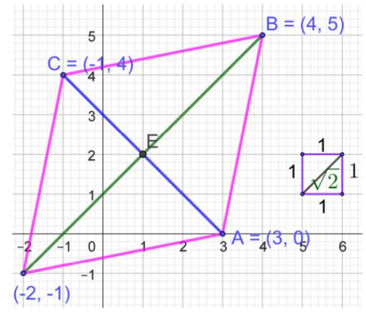
$$= \sqrt{36 \times 2}$$

$$= 6\sqrt{2} units$$

Area of rhombus = $\frac{1}{2} \times d_1 \times d_2 = \frac{1}{2} \times AC \times BD$

$$= \frac{1}{2} \times 4\sqrt{2} \times 6\sqrt{2}$$

$$= \frac{1}{2} \times 24 \times 2 = 24\ \text{sq. units}$$



Some more problems

1. If the mid-point of the line segment joining the points A (3, 4) and B (k, 6) is P (x, y) and x + y – 10 = 0, find the value of k.
2. Name the type of triangle PQR formed by the points $P\ (\sqrt{2},\sqrt{2}), Q\ (-\sqrt{2},-\sqrt{2})\ and\ R\ (-\sqrt{6},\sqrt{6})$.
3. Find the points on the x–axis which are at a distance of $2\sqrt{5}$ from the point (7, –4). How many such points are there?
4. Find the value of a, if the distance between the points A (–3, –14) and B (a, –5) is 9 units.
5. In what ratio does the x–axis divide the line segment joining the points (– 4, – 6) and (–1, 7)? Find the coordinates of the point of division.
6. If P (9a – 2, –b) divides line segment joining A (3a + 1, –3) and B (8a, 5) in the ratio 3 : 1, find the values of a and b.
7. The centre of a circle is (2a, a – 7). Find the values of a if the circle passes through the point (11, –9) and has diameter $10\sqrt{2}$ units
8. The line segment joining the points A(4,-5) and B(4,5) is divided by the point P such that AP;AB=2:5. Find the coordinates of P.
9. Points A(-1,y) and B(5,7) lie on a circle with centre O(2,-3y). Find the value of y .
10. The line segment joining the points P(-3,2) and Q(5,7) is divided by the y-axis in the ratio is.
11. If the mid-point of the line segment joining the points A(3,4) and B(k,6) is P(x,y) and x+y-10=0, find the value of k.
12. Find the area of triangle ABC with A(1,-4) and the mid-points of sides through A being (2,-1) and (0,-1).
13. Find distance between (a cos θ+b sin θ,0) and (0,a sin θ-b cos θ).
14. Find a relation between x and y if the points A(x,y), B(-4,6) and C(-2,3)are collinear.

Answers:

1. k=7
2. An equilateral triangle.
3. (9, 0), (5, 0), 2 points
4. a=-3
5. 6: 7; $(\frac{-34}{13},0)$
6. a = 1 b = –3
7. a=5,3
8. (4,-1)
9. 7 and -1
10. 3:5
11. k=7
12. 12 sq units
13. $\sqrt{a^2+b^2}$
14. $3x=-2y$

MCQ

1. **If the distance between the points (2, –2) and (–1, x) is 5, one of the values of x is**
   (A) –2 (B) 2 (C) –1 (D) 1

2. The mid-point of the line segment joining the points A (–2, 8) and B (– 6, – 4) is
(A) (– 4, – 6) (B) (2, 6) (C) (– 4, 2) (D) (4, 2)
3. The distance of the point P (2, 3) from the x-axis is
(A) 2 (B) 3 (C) 1 (D) 5
4. The distance of the point P (–6, 8) from the origin is
(A) 8 (B) $2\sqrt{7}$ (C) 10 (D) 6
5. AOBC is a rectangle whose three vertices are vertices A (0, 3), O (0, 0) and B (5, 0). The length of its diagonal is
(A) 5 (B) 3 (C) $\sqrt{34}$ (D) 4
6. The perimeter of a triangle with vertices (0, 4), (0, 0) and (3, 0) is
(A) 5 (B) 12 (C) 11 (D) $7+\sqrt{5}$
7. The points (–4, 0), (4, 0), (0, 3) are the vertices of a
(A) right triangle (B) isosceles triangle (C) equilateral triangle (D) scalene triangle
8. The point which divides the line segment joining the points (7, –6) and (3, 4) in ratio 1 : 2 internally lies in the
(A) I quadrant (B) II quadrant (C) III quadrant (D) IV quadrant
9. The fourth vertex D of a parallelogram ABCD whose three vertices are A (–2, 3), B (6, 7) and C (8, 3) is
(A) (0, 1) (B) (0, –1) (C) (–1, 0) (D) (1, 0)
10. If the point P (2, 1) lies on the line segment joining points A (4, 2) and B (8, 4), then
(A) $AP = \frac{1}{3}AB$ (B) $AP = PB$ (C) $PB = \frac{1}{3}AB$ (A) $AP = \frac{1}{2}AB$

11.

1)B	2)C	3)B	4)C	5)C	6)B	7)C	8)B	9)D	10)A

Assertion and reason

1. **Assertion (A):** The value of y is 6, for which the distance between the points P(2, –3) and Q(10, y) is 10

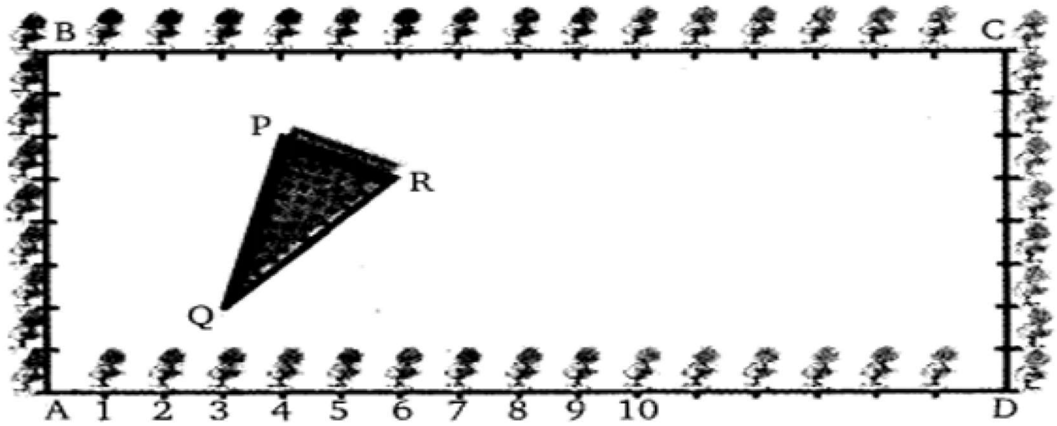
Reason (R): Distance between two given points A (x_1, y_1) and B (x_2, y_2) is given by $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

2.
3.
4.
5.

1)D	2)	3)	4)	5)	6)	7)	8)	9)	10)

CASE STUTDY BASED QUESTIONS

1) **The class X students school in krishnagar have been allotted a rectangular plot of land for their gardening activity. Saplings of Gulmohar are planted on the boundary at a distance of 1 m from each other. There is triangular grassy lawn in the plot as shown in the figure. The students are to sow seeds of flowering plants on the remaining area of the plot**



1. Taking A as origin, find the coordinates of P

a) (4,6) b) (6,4) c) (0,6) d) (4,0)

2. What will be the coordinates of R, if C is the origin?

a) (8,6) b) (3,10) c) (10,3) d) (0,6)

3. What will be the coordinates of Q, if C is the origin?

a) (6,13) b) (-6,13) c) (-13,6) d) (13,6)

4. Calculate the area of the triangles if A is the origin

a) 4.5 b) 6 c) 8 plot. d) 6.25

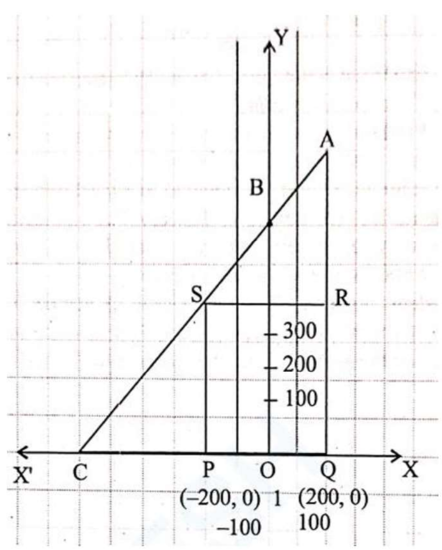
5. Calculate the area of the triangles if C is the origin

a) 8 b) 5 c) 6.25 d) 4.5

Solution:

1. a) (4,6) 2. c) (10,3) 3. d) (13,6) 4. a) 4.5 5. d) 4.5

2) **Jagdish has a field which is in the shape of a right angle triangle AQC he wants to leave a space in the form of square PQRS inside the field for growing wheat and the remaining for growing vegetables (as shown in the figure). In the field there is a pole marked as O.**



Based on the above information, answer the following

questions:

(i) Taking O as origin coordinates of P are (- 200,0) and Q are (200,0). PQRS being a square , what are the coordinates of R and S.

(ii) (a) what is the area of square PQRS ? OR

(b) what is the length of diagonal PR in the square PQRS ?

(iii) If S divides CA in the ratio k:1, what is the value of K, where point A is (200,800)?[CBSE-2023]

Sol: $(i)\ P = (-200,0)\ and\ Q = (200,0)$

$PQ = |x_2 - x_1| = |200 + 200| = 400$

Given PQRS is a square. So, it's all sides are equal

PQ=QR=RS=PS=400

∴S=(-200,400) and R=(200,400)

(ii) (a) The area of square PQRS=(side)^2=400^2=160000 sq units

(b) The length of diagonal PR in the square PQRS=$\sqrt{2} \times side = \sqrt{2} \times 400 = 400\sqrt{2}\ units$

(iii)C=(-600,0)= (x_1, y_1)andA=(200,800)=(x_2, y_2) S=(-200,400)=(x,y)

$$\frac{kx_2 + x_1}{k+1} = x$$

$$\frac{k \times 200 + (-600)}{k+1} = -200$$

$$200k - 600 = -200k - 200$$

$$400k = 400$$

$$k = 1$$

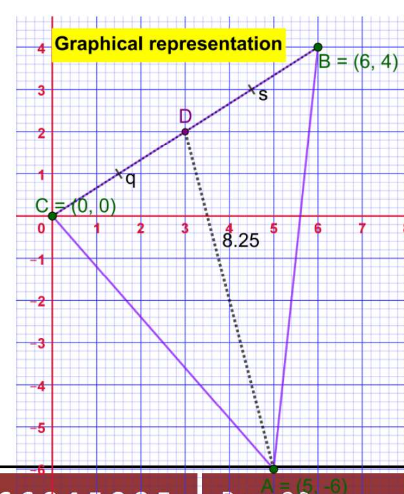
Alternate method:

$$Ratio = \frac{x - x_1}{x_2 - x} = \frac{-200 - (-600)}{200 - (-200)} = \frac{-22 + 600}{200 + 200} = \frac{400}{400} = 1:1 = k:1$$

∴k=1

Previous year problems: [CBSE-2024]

1. **AD is a median of △ABC with vertices A(5,-6), B(6,4) and C(0,0) find length of AD?[**

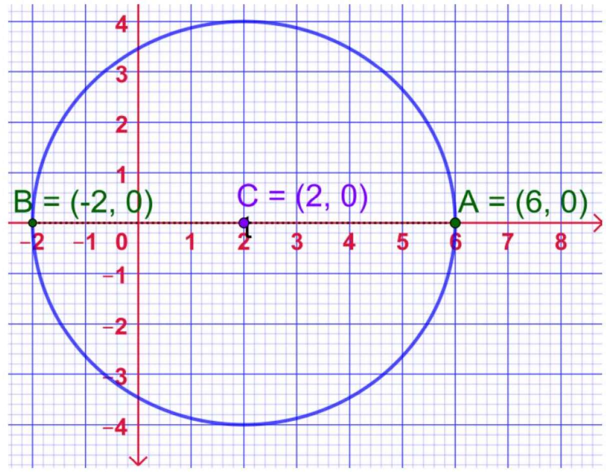


Sol: $D = mid\ point\ of\ BC = \left(\frac{6+0}{2}, \frac{4+0}{2}\right) = (3,2)$

$$AD = \sqrt{(3-5)^2 + (2+6)^2} = \sqrt{4+64} = \sqrt{68}$$

2. The centre of a circle is at (2,0). If one end of a diameter is at (6,0), then the other end is at__

Sol: Centre C=(2,0), one end of a diameter is A= (6,0), other end of a diameter is B= (x,y)



Midpoint of AB=C

$$\left(\frac{6+x}{2}, \frac{0+y}{2}\right) = (2,0)$$

$$\frac{6+x}{2} = 2 \ and \ \frac{y}{2} = 0$$

$$6 + x = 4 \ and \ y = 0$$

$$x = -2 \ and \ y = 0$$

Required other end point=(-2,0)

Shortcut:

$$Other\ end\ point\ of\ diameter = \left(\frac{2-6}{2}, 0\right) = (-2,0)$$

3. Find the ratio in which the point $\left(\frac{8}{5}, y\right)$ divides the line segment joining the points (1, 2) and (2, 3). Also, find the value of y.

Sol: A(1, 2) and B(2, 3) $P\left(\frac{8}{5}, y\right)$

(x_1, y_1) (x_2, y_2) (x, y)

Let P divides AB in the ratio $k : 1$

$$P = \left(\frac{8}{5}, y\right)$$

$$\frac{2k+1}{k+1} = \frac{8}{5}$$

$$10k + 5 = 8k + 8$$

$$2k = 3$$

$$k = \frac{3}{2}$$

$$Required\ ratio = 3:2$$

$$y = \frac{3 \times 3 + 2 \times 2}{3 + 2} = \frac{13}{5}$$

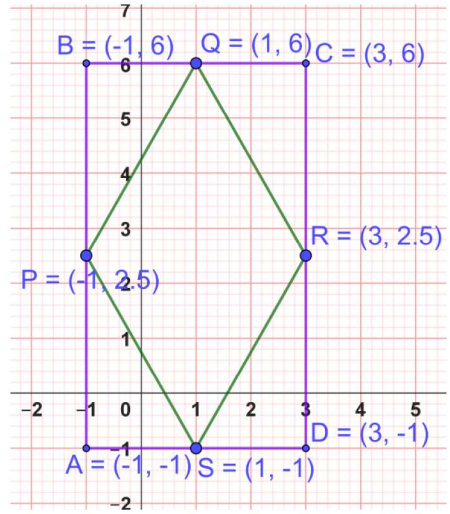
4. ABCD is a rectangle formed by the points A (-1, -1), B (-1, 6), C (3, 6) and D (3, 1). P, Q, R and S are mid-points of sides AB, BC, CD and DA respectively. Show that diagonals of the quadrilateral PQRS bisect each other.

Sol: $P = Midpoint\ of\ AB = \left(\frac{-1-1}{2}, \frac{-1+6}{2}\right) = \left(\frac{-2}{2}, \frac{5}{2}\right) = \left(-1, \frac{5}{2}\right)$

$Q = Midpoint\ of\ BC = \left(\frac{-1+3}{2}, \frac{6+6}{2}\right) = \left(\frac{2}{2}, \frac{12}{2}\right) = (1,6)$

$R = Midpoint\ of\ CD = \left(\frac{3+3}{2}, \frac{6-1}{2}\right) = \left(\frac{6}{2}, \frac{5}{2}\right) = \left(3, \frac{5}{2}\right)$

$S = Midpoint\ of\ AD = \left(\frac{-1+3}{2}, \frac{-1-1}{2}\right) = \left(\frac{2}{2}, \frac{-2}{2}\right)$
$= (1, -1)$



$Midpoint\ of\ diagonal\ PR = \left(\frac{-1+3}{2}, \frac{\frac{5}{2}+\frac{5}{2}}{2}\right) = \left(\frac{2}{2}, \frac{5}{2}\right)$
$= \left(1, \frac{5}{2}\right)$

$Midpoint\ of\ diagonal\ QS = \left(\frac{1+1}{2}, \frac{6-1}{2}\right) = \left(\frac{2}{2}, \frac{5}{2}\right) = \left(1, \frac{5}{2}\right)$

Since coordinates of mid point of QS= coordinates of mid point of PR

Therefore, diagonals PR and QS bisect each other.

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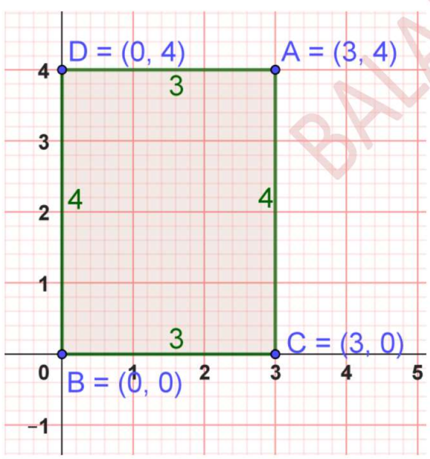
5. The coordinates of the vertex A of a rectangle ABCD whose three vertices are given as B(0,0),C(3,0) and D(0,4) are:

Sol: (3,4)



6. Show that the points (-2, 3), (8, 3) and (6, 7) are the vertices of a right-angled triangle.

Sol: A(2, 3), B(8, 3) and C(6, 7)

$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

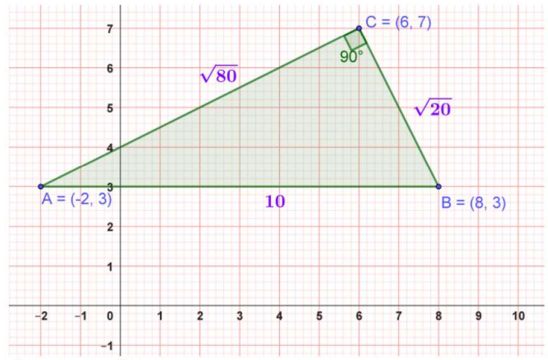
$= \sqrt{(8+2)^2 + (3-3)^2} = 10$

$BC = \sqrt{(6-8)^2 + (7-3)^2}$

$= \sqrt{(-2)^2 + (4)^2} = \sqrt{4+16} = \sqrt{20}$

$AC = \sqrt{(6+2)^2 + (7-3)^2}$

$= \sqrt{64+16} = \sqrt{80}$



$AB^2 = 100$

$and\ BC^2 + AC^2 = 20 + 80 = 100$

$AB^2 = BC^2 + AC^2$

$\therefore$ *The given points are the vertices of a right angled triangle.*

7. If Q(0, 1) is equidistant from P(5.3) and R(x, 6), find the values of x

Sol: $PQ = QR$

$PQ^2 = QR^2$

$(5-0)^2 + (-3-1)^2 = (x-0)^2 + (6-1)^2$

$\Rightarrow 25 + 16 = x^2 + 25$

$\Rightarrow x^2 = 16$

$x = 4, -4$