**CHAPTER 8**

**X-MATHEMATICS-NCERT-2024-25**

# 8. Introduction to Trigonometry

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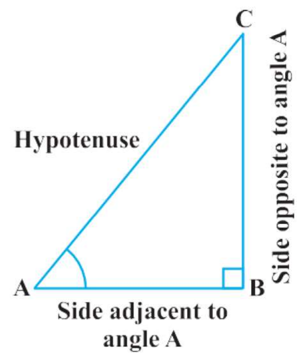
1. The word 'trigonometry' is derived from the Greek words 'tri' (meaning three), 'gon' (meaning sides) and 'metron' (meaning measure).
2. Ancient Greek mathematician Hipparchus is known as the father of trigonometry.
3. **Trigonometric Ratios**

$$(i)\, sine\ of\ \angle A = Sin A = \frac{side\ opposite\ to\ \angle A}{hypotenuse} = \frac{BC}{AC}$$

$$\text{(ii)cosine of } \angle A = \cos A = \frac{\text{side adjacent to } \angle A}{\text{hypotenuse}} = \frac{AB}{AC}$$




$$(iii)\text{tangent of } \angle A = \tan A = \frac{\text{side opposite to } \angle A}{\text{side adjacent to } \angle A} = \frac{BC}{AB}$$

$$(iv)\text{cosecant of } \angle A = \text{cosec } A = \frac{1}{SinA} = \frac{\text{hypotenuse}}{\text{side opposite to } \angle A} = \frac{AC}{BC}$$

$$(v)\text{secant of } \angle A = \sec A = \frac{1}{\cos A} = \frac{\text{hypotenuse}}{\text{side adjacent to } \angle A} = \frac{AC}{AB}$$

$$(vi)\text{cotangent of } \angle A = \cot A = \frac{1}{\tan A} = \frac{\text{side adjacent to } \angle A}{\text{side opposite to } \angle A} = \frac{AB}{BC}$$

$$\tan A = \frac{\text{SinA}}{\cos A} \quad \text{and} \quad \cot A = \frac{\cos A}{\text{SinA}}$$

4. **Relationship between Trigonometric Ratios**

$$(i)\ \sin A = \frac{1}{cosec\ A}\ ; cosec\ A = \frac{1}{\sin A} \implies \sin A \times cosec\ A = 1$$

$$(ii)\ \cos A = \frac{1}{\sec A}\ ;\ \sec A = \frac{1}{\cos A} \implies \cos A \times \sec A = 1$$

$$(iii)\ \tan A = \frac{1}{\cot A};\ \cot A = \frac{1}{\tan A} \implies \tan A \times \cot A = 1$$

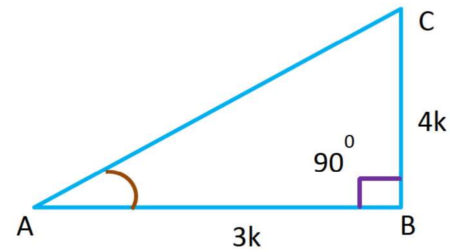
$$\text{(iv) } \tan A = \frac{\sin A}{\cos A} \quad \text{and} \quad \cot A = \frac{\cos A}{\sin A}$$

5. The values of the trigonometric ratios of an angle do not vary with the lengths of the sides of the triangle, if the angle remains the same.

**Example-1. If $\tan A = \frac{4}{3}$, then find the other trigonometric ratio of angle A**

**Sol**: $\tan A = \frac{4}{3} = \frac{BC}{AB}$

BC=4k, AB=3k




In $\Delta ABC$, $\angle B = 90^0$

$AC^2 = AB^2 + BC^2$ (From Pythagoras theorem)

$AC^2 = (3k)^2 + (4k)^2$

$AC^2 = 9k^2 + 16k^2 = 25k^2$

$AC = \sqrt{25k^2} \Rightarrow AC = 5k$

$Sin\ A = \frac{BC}{AC} = \frac{4k}{5k} = \frac{4}{5},$

$cos\ A = \frac{AB}{AC} = \frac{3k}{5k} = \frac{3}{5}$

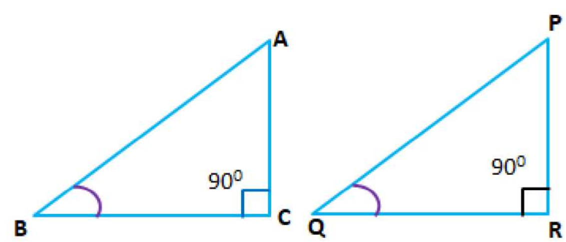
$cosec\ A = \frac{1}{Sin\ A} = \frac{5}{4}$

$Sec\ A = \frac{1}{cos\ A} = \frac{5}{3}$

$cot\ A = \frac{1}{\tan A} = \frac{3}{4}$

**Example 2 : If $\angle B$ and $\angle Q$ are acute angles such that sin B = sin Q, then prove that $\angle B = \angle Q$.**

**Sol**: we have $\sin B = \frac{AC}{AB}$ and $\sin Q = \frac{PR}{PQ}$




Given $\sin B = \sin Q$

$\frac{AC}{AB} = \frac{PR}{PQ}$

$\frac{AC}{PR} = \frac{AB}{PQ} = k, say$

$AC = k.PR$ and $AB = k.PQ$

By using Pythagoras theorem

$$\frac{BC}{QR} = \frac{\sqrt{AB^2 - AC^2}}{\sqrt{PQ^2 - PR^2}} = \frac{\sqrt{k^2PQ^2 - k^2PR^2}}{\sqrt{PQ^2 - PR^2}} = \frac{\sqrt{k^2(PQ^2 - PR^2)}}{\sqrt{PQ^2 - PR^2}} = \frac{k\left(\sqrt{PQ^2 - PR^2}\right)}{\sqrt{PQ^2 - PR^2}} = k$$

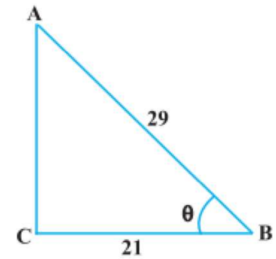
$Hence, \frac{AC}{PR} = \frac{AB}{PQ} = \frac{BC}{QR}$

Then $\Delta ABC \sim \Delta PQR$ (S.S.S similarity)

Therefore, $\angle B = \angle Q$ (In similar triangles corresponding angles are equal)

**Example 3 : Consider Δ ACB, right-angled at C, inwhich AB = 29 units, BC = 21 units and ∠ABC = θ. Determine the values of**

**(i) $\cos^2\theta + \sin^2\theta$ and (ii) $\cos^2\theta - \sin^2\theta$.**




Sol: In $\Delta ABC, \angle C = 90^0$

$AC^2 + BC^2 = AB^2$ (From Pythagoras theorem)

$AC^2 = AB^2 - BC^2$

$AC^2 = 29^2 - 21^2$

$AC^2 = 841 - 441 = 400 \Rightarrow AC = \sqrt{400} \Rightarrow AC = 20$

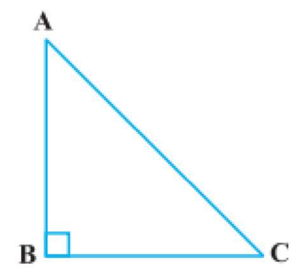
$$\sin\theta = \frac{AC}{AB} = \frac{20}{29}$$

$$\cos\theta = \frac{BC}{AB} = \frac{21}{29}$$

$$(i)\ \cos^2\theta + \sin^2\theta = \left(\frac{21}{29}\right)^2 + \left(\frac{20}{29}\right)^2 = \frac{441}{841} + \frac{400}{841} = \frac{441+400}{841} = \frac{841}{841} = 1$$

$$(ii)\ \cos^2\theta - \sin^2\theta = \left(\frac{21}{29}\right)^2 - \left(\frac{20}{29}\right)^2 = \frac{441}{841} - \frac{400}{841} = \frac{441-400}{841} = \frac{41}{841}$$

**Example 4 : In a right triangle ABC, right-angled at B, if tan A = 1, then verify that 2 sin A cos A = 1**




Sol: $\tan A = 1 = \frac{BC}{AB}$

$BC = AB = k, say$

$$AC = \sqrt{AB^2 + BC^2} = \sqrt{k^2 + k^2} = \sqrt{2k^2} = k\sqrt{2}$$

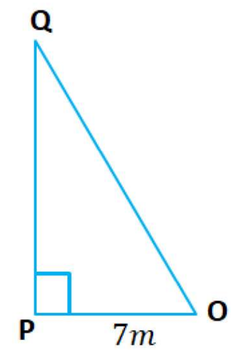
$$\sin A = \frac{BC}{AC} = \frac{k}{k\sqrt{2}} = \frac{1}{\sqrt{2}}$$

$$\cos A = \frac{AB}{AC} = \frac{k}{k\sqrt{2}} = \frac{1}{\sqrt{2}}$$

$$2\sin A\cos A = 2 \times \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}} = 2 \times \frac{1}{2} = 1$$

**Example 5 : In Δ OPQ, right-angled at P, OP = 7 cm and OQ – PQ = 1 cm Determine the values of sin Q and cos Q.**

Solution : In Δ OPQ




$OQ^2 = OP^2 + PQ^2$

$(1 + PQ)^2 = OP^2 + PQ^2$

$1 + PQ^2 + 2PQ = OP^2 + PQ^2$

$1 + 2PQ = OP^2 = 7^2 = 49$

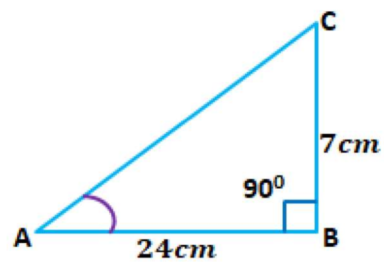
$2PQ = 48 \Rightarrow PQ = 24\ cm$

$OQ = 1 + PQ = 1 + 24 = 25\ cm$

$\sin Q = \frac{OP}{OQ} = \frac{7}{25}\ and\ \cos Q = \frac{PQ}{OQ} = \frac{24}{25}$

## EXERCISE 8.1

**1. In Δ ABC, right-angled at B, AB = 24 cm, BC = 7 cm. Determine :**
**(i) sin A, cos A (ii) sin C, cos C**




Sol: In ΔABC, $\angle B = 90^0$

$AC^2 = AB^2 + BC^2$ (From Pythagoras theorem)

$AC^2 = 24^2 + 7^2$

$AC^2 = 576 + 49$

$AC^2 = 625 \Rightarrow AC = \sqrt{625} \Rightarrow AC = 25\ cm$

(i)

$Sin\ A = \frac{BC}{AC} = \frac{7}{25},$

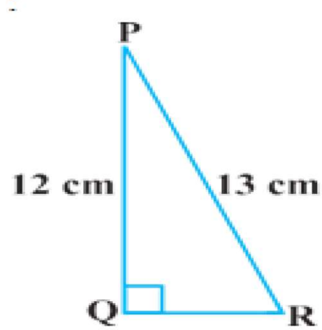
$cos\ A = \frac{AB}{AC} = \frac{24}{25}$

(ii)

$cos\ C = \frac{AB}{AC} = \frac{24}{25}$

$Sin\ C = \frac{BC}{AC} = \frac{7}{25}$

**2. In Fig. 8.13, find tan P – cot R.**




Sol: In ΔPQR, $\angle Q = 90^0$

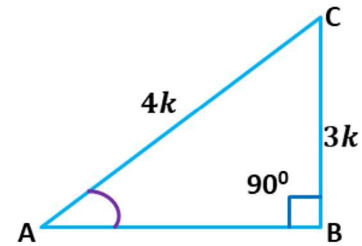
$PR^2 = PQ^2 + QR^2$ (From Pythagoras theorem)

$QR^2 = PR^2 - PQ^2 = 13^2 - 12^2 = 169 - 144 = 25$

$QR = 5\ cm$

$$\tan P - \cot R = \frac{QR}{PQ} - \frac{QR}{PQ} = 0$$

**3. If $\sin A = \frac{3}{4}$ calculate cos A and tan A.**




**Sol:** $\sin A = \frac{3}{4} = \frac{BC}{AC}$

$BC = 3k, AC = 4k$

$AC^2 = AB^2 + BC^2$ (Pythagoras theorem)

$(4k)^2 = AB^2 + (3k)^2$

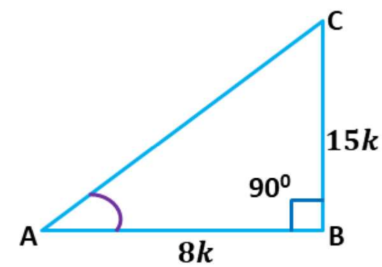
$AB^2 = 16k^2 - 9k^2 = 7k^2$

$AB = \sqrt{7k^2} = \sqrt{7}k$

$\cos A = \frac{AB}{AC} = \frac{\sqrt{7}k}{4k} = \frac{\sqrt{7}}{4}$

$\tan A = \frac{BC}{AB} = \frac{3k}{\sqrt{7}k} = \frac{3}{\sqrt{7}}$

**4. Given 15 cot A = 8, find sin A and sec A**




Sol: $\cot A = \frac{8}{15} = \frac{AB}{BC}$

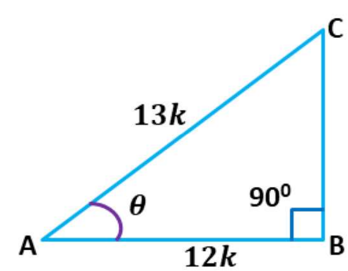
$AB = 8k; BC = 15k$

$AC = \sqrt{AB^2 + BC^2} = \sqrt{64k^2 + 225k^2} = \sqrt{289k^2} = 17k$

$\sin A = \frac{BC}{AC} = \frac{15k}{17k} = \frac{15}{17}$

$\sec A = \frac{AC}{AB} = \frac{17k}{8k} = \frac{17}{8}$

**5. Given $\sec\theta = \frac{13}{12}$ calculate all other trigonometric ratios.**




**Sol:** $\sec\theta = \frac{13}{12} = \frac{AC}{AB}$

$AC = 13k; AB = 12k$

$BC = \sqrt{AC^2 - AB^2} = \sqrt{169k^2 - 144k^2} = \sqrt{25k^2} = 5k$

$\sin\theta = \frac{BC}{AC} = \frac{5k}{13k} = \frac{5}{13}$

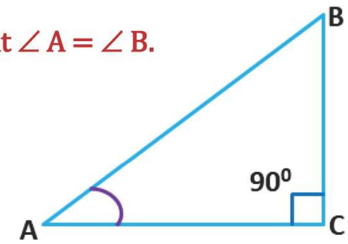
$\cos\theta = \frac{AB}{AC} = \frac{12k}{13k} = \frac{12}{13}$

$\tan\theta = \frac{BC}{AB} = \frac{5k}{12k} = \frac{5}{12}$

$cosec\,\theta = \frac{1}{\sin\theta} = \frac{13}{5}$

$\cot\theta = \frac{1}{\tan\theta} = \frac{12}{5}$

**6. If ∠ A and ∠ B are acute angles such that cos A = cos B, then show that ∠ A = ∠ B.**




**Sol**: $\cos A = \cos B \Rightarrow \frac{AC}{AB} = \frac{BC}{AB}$

$AC = BC$

$\angle A = \angle B$ (angles opposite to equal sides of triangle are equal.)

**7. Given $\cot\theta = \frac{7}{8}$ , then evaluate (i) $\frac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)}$ (ii) $\frac{(1+\sin\theta)}{\cos\theta}$**

Sol: $\cot\theta = \frac{7}{8} = \frac{AB}{BC}$

$AC^2 = AB^2 + BC^2 = (7k)^2 + (8k)^2 = 49k^2 + 64k^2 = 113k^2$

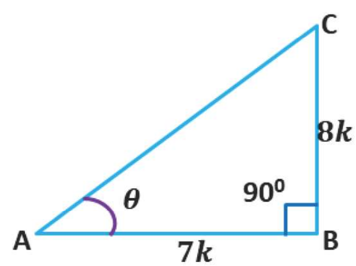
$AC = \sqrt{113}k$

$\sin\theta = \frac{BC}{AC} = \frac{8k}{\sqrt{113}k} = \frac{8}{\sqrt{113}}$, $\cos\theta = \frac{AB}{AC} = \frac{7k}{\sqrt{113}k} = \frac{7}{\sqrt{113}}$

(i) $\frac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)}$

$$= \frac{\left(1+\frac{8}{\sqrt{113}}\right)\left(1-\frac{8}{\sqrt{113}}\right)}{\left(1+\frac{7}{\sqrt{113}}\right)\left(1-\frac{7}{\sqrt{113}}\right)}$$

$$= \frac{(\sqrt{113}+8)(\sqrt{113}-8)}{(\sqrt{113}+7)(\sqrt{113}-7)} = \frac{113-64}{113-49} = \frac{49}{64}$$



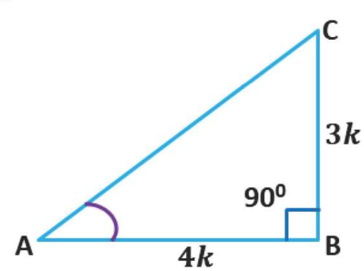

(ii) $\frac{1+\sin\theta}{\cos\theta} = \frac{1+\frac{8}{\sqrt{113}}}{\frac{7}{\sqrt{113}}} = \frac{\sqrt{113}+8}{7}$

**8. If 3 cot A = 4, check whether $\frac{1-\tan^2 A}{1+\tan^2 A} = \cos^2 A - \sin^2 A$ or not.**

**Sol**: $\cot A = \frac{4}{3} = \frac{AB}{BC}$

$AB = 4k; BC = 3k$

$AC^2 = AB^2 + BC^2 = (4k)^2 + (3k)^2 = 16k^2 + 9k^2 = 25k^2$



$AC = 5k$

$\tan A = \frac{1}{\cot A} = \frac{3}{4}$

$\sin A = \frac{3k}{5k} = \frac{3}{5}; \cos A = \frac{4k}{5k} = \frac{4}{5}$

$\frac{1 - tan^2 A}{1 + tan^2 A} = \frac{1 - \left(\frac{3}{4}\right)^2}{1 + \left(\frac{3}{4}\right)^2} = \frac{1 - \frac{9}{16}}{1 + \frac{9}{16}} = \frac{16 - 9}{16 + 9} = \frac{7}{25}$

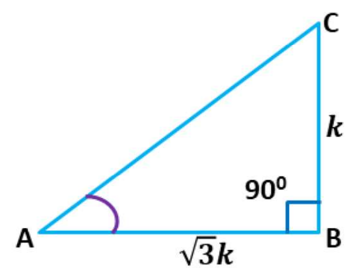
$\cos^2 A - \sin^2 A = \left(\frac{4}{5}\right)^2 - \left(\frac{3}{5}\right)^2 = \frac{16}{25} - \frac{9}{25} = \frac{16 - 9}{25} = \frac{7}{25}$

$\frac{1 - tan^2 A}{1 + tan^2 A} = \cos^2 A - \sin^2 A = \frac{7}{25}$

**9. In triangle ABC, right − angled at B, if $\tan A = \frac{1}{\sqrt{3}}$ find the value of:**
**(i) sin A cos C + cos A sin C (ii) cos A cos C – sin A sin C**

**Sol:** $\tan A = \frac{1}{\sqrt{3}} = \frac{BC}{AB}$

$BC = k; AB = \sqrt{3}k$




$AC^2 = AB^2 + BC^2 = \left(\sqrt{3}k\right)^2 + (k)^2 = 3k^2 + k^2 = 4k^2$

$AC = 2k$

$\sin A = \frac{k}{2k} = \frac{1}{2}; \cos A = \frac{\sqrt{3}k}{2k} = \frac{\sqrt{3}}{2}$

$\sin C = \frac{\sqrt{3}k}{2k} = \frac{\sqrt{3}}{2}; \cos C = \frac{k}{2k} = \frac{1}{2}$

$(i) \sin A \cos C + \cos A \sin C = \frac{1}{2} \times \frac{1}{2} + \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2}$

$= \frac{1}{4} + \frac{3}{4} = \frac{4}{4} = 1$

$(ii) \cos A \cos C - \sin A \sin C = \frac{\sqrt{3}}{2} \times \frac{1}{2} - \frac{1}{2} \times \frac{\sqrt{3}}{2}$

$= \frac{\sqrt{3}}{4} - \frac{\sqrt{3}}{4} = 0$

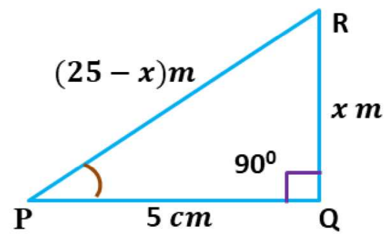
**10. In Δ PQR, right-angled at Q, PR + QR = 25 cm and PQ = 5 cm. Determine the values of sin P, cos P and tan P**

**Sol:** PQ=5 cm, PR + QR = 25 cm.

$Let\ QR = x\ cm$

$PR + x = 25$

$PR = (25 - x)cm$




$PR^2 = QR^2 + PQ^2$(Pythagoras theorem)

$(25 - x)^2 = x^2 + 5^2$

$625 + x^2 - 50x = x^2 + 25$

$50x = 625 - 25 = 600$

$x = 12$

$QR = 12\ cm, PR = 25 - 12 = 13\ cm$

$\sin P = \frac{QR}{PR} = \frac{12}{13};$

$\cos P = \frac{PQ}{PR} = \frac{5}{13};$

$\tan P = \frac{QR}{PQ} = \frac{12}{5}$

**11. State whether the following are true or false. Justify your answer.**

(i) The value of tan A is always less than 1

Sol: False

If opposite side and adjacent side are equal then tan A=1

(ii) $\sec A = \frac{12}{5}$ for some value of angle A.

Sol: True.

$\sec A = \frac{hypotenuse}{side\ adjacent\ to\ angle\ A} > 1 \Rightarrow \sec A > 1$

(iii) cos A is the abbreviation used for the cosecant of angle A

Sol: False.

cos A is the abbreviation used for "cosine of angle A"

(iv) cot A is the product of cot and A.

Sol: False.

cot A means "cotangent of ∠A"

(v) $\sin \theta = \frac{4}{3}$ for some angle θ.

Sol: False.

$\sin \theta = \frac{side\ opposite\ to\ angle\ \theta}{hypotenuse} < 1\ (Since\ side\ is\ less\ than\ hypotenuse)$

$\sin \theta = \frac{4}{3} > 1\ is\ not\ possible\ for\ some\ angle.$

## Trigonometric Ratios of Some Specific Angles

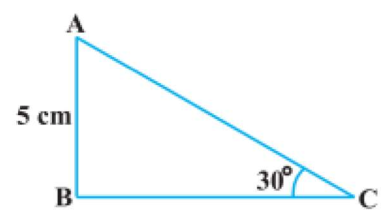
| | $\angle A$ | $0^o$ | $30^o$ | $45^o$ | $60^o$ | $90^o$ |
|---|---|---|---|---|---|---|
| | | $\sqrt{\frac{0}{4}}=0$ | $\sqrt{\frac{1}{4}}=\frac{1}{2}$ | $\sqrt{\frac{2}{4}}=\sqrt{\frac{1}{2}}=\frac{1}{\sqrt{2}}$ | $\sqrt{\frac{3}{4}}=\frac{\sqrt{3}}{2}$ | $\sqrt{\frac{4}{4}}=1$ |
| | **sin A** | 0 | $\frac{1}{2}$ | $\frac{1}{\sqrt{2}}$ | $\frac{\sqrt{3}}{2}$ | 1 |
| | **cos A** | 1 | $\frac{\sqrt{3}}{2}$ | $\frac{1}{\sqrt{2}}$ | $\frac{1}{2}$ | 0 |
| $\frac{\sin A}{\cos A}$ | ***tan A*** | 0 | $\frac{1}{\sqrt{3}}$ | 1 | $\sqrt{3}$ | not defined |
| $\frac{1}{\tan A}$ | ***cot A*** | not defined | $\sqrt{3}$ | 1 | $\frac{1}{\sqrt{3}}$ | 0 |
| $\frac{1}{\cos A}$ | ***sec A*** | 1 | $\frac{2}{\sqrt{3}}$ | $\sqrt{2}$ | 2 | not defined |
| $\frac{1}{\sin A}$ | ***cosec A*** | not defined | 2 | $\sqrt{2}$ | $\frac{2}{\sqrt{3}}$ | 1 |

**Example 6 : In Δ ABC, right-angled at B, AB = 5 cm and ∠ ACB = 30°. Determine the lengths of the sides BC and AC.**

**Sol:** $\tan C = \frac{AB}{BC}$

$\tan 30^0 = \frac{5}{BC}$

$\frac{1}{\sqrt{3}} = \frac{5}{BC} \Rightarrow BC = 5\sqrt{3}cm$



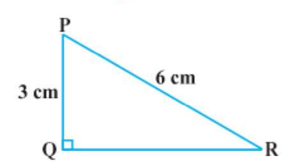

$\sin 30^0 = \frac{5}{AC}$

$\frac{1}{2} = \frac{5}{AC} \Rightarrow AC = 10cm$

**Example 7 : In Δ PQR, right-angled at Q, PQ = 3 cm and PR = 6 cm. Determine ∠ QPR and ∠ PRQ.**

**Sol:** $\sin R = \frac{PQ}{PR} = \frac{3}{6} = \frac{1}{2} = \sin 30^0$

$\angle PRQ = 30^0$




$\angle QPR = 90^0 - 30^0 = 60^0$

**Example 8 :** ***If*** $\boldsymbol{\sin(A-B) = \frac{1}{2}, \cos(A+B) = \frac{1}{2}, 0^0 < A+B \le 90^0, A > B}$, ***find A and B***

***Sol:*** $\sin(A-B) = \frac{1}{2} = \sin 30^0$

$\Rightarrow A - B = 30^0$----------(1)

$\cos(A+B) = \frac{1}{2} = \cos 60^0$

$\Rightarrow A + B = 60^0$----------(2)

(1)+(2)$\Rightarrow A - B + A + B = 30^0 + 60^0$

$\Rightarrow 2A = 90^0 \Rightarrow A = \frac{90^0}{2} = 45^0$

Substitute $A = 45^0$ value in (2)

$45^0 + B = 60^0$

$\Rightarrow B = 60^0 - 45^0 = 15^0$

$\therefore A = 45^0 \quad and \quad B = 15^0$

## EXERCISE 8.2

**1. Evaluate the following :**

**(i) $\sin 60^0 \cos 30^0 + \sin 30^0 \cos 60^0$**

**Sol**: $\sin 60^0 \cos 30^0 + \sin 30^0 \cos 60^0 = \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} + \frac{1}{2} \times \frac{1}{2} = \frac{3}{4} + \frac{1}{4} = \frac{3+1}{4} = \frac{4}{4} = 1$

**(ii) $2\tan^2 45^0 + \cos^2 30^0 - \sin^2 60^2$.**

**Sol**: $2\tan^2 45^0 + \cos^2 30^0 - \sin^2 60^2$.

$$= 2 \times (1)^2 + \left(\frac{\sqrt{3}}{2}\right)^2 - \left(\frac{\sqrt{3}}{2}\right)^2 = 2 \times 1 = 2$$

**(ii) $\frac{\cos 45^0}{\sec 30^0 + cosec\, 60^0}$**

**Sol**: $$\frac{\cos 45^0}{\sec 30^0 + cosec\, 30^0} == \frac{\frac{1}{\sqrt{2}}}{\frac{2}{\sqrt{3}} + 2} = \frac{\frac{1}{\sqrt{2}}}{\frac{2+2\sqrt{3}}{\sqrt{3}}}$$

$$= \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2+2\sqrt{3}} = \frac{\sqrt{3}}{\sqrt{2}(2+2\sqrt{3})}$$

$$= \frac{\sqrt{3}}{2\sqrt{2}(\sqrt{3}+1)} \times \frac{\sqrt{2}(\sqrt{3}-1)}{\sqrt{2}(\sqrt{3}-1)} = \frac{\sqrt{3} \times \sqrt{2}(\sqrt{3}-1)}{4(3-1)} = \frac{3\sqrt{2}-\sqrt{6}}{8}$$

**(iii) $\frac{\sin 30^0 + \tan 45^0 - cosec\, 60^0}{\sec 30^0 + \cos 60^0 + \cot 45^0}$**

**Sol**: $$\frac{\sin 30^0 + \tan 45^0 - cosec\, 60^0}{\sec 30^0 + \cos 60^0 + \cot 45^0} = \frac{\frac{1}{2} + 1 - \frac{2}{\sqrt{3}}}{\frac{2}{\sqrt{3}} + \frac{1}{2} + 1}$$

$$= \frac{\frac{\sqrt{3}+2\sqrt{3}-4}{2\sqrt{3}}}{\frac{4+\sqrt{3}+2\sqrt{3}}{2\sqrt{3}}} = \frac{\sqrt{3}+2\sqrt{3}-4}{4+\sqrt{3}+2\sqrt{3}} = \frac{3\sqrt{3}-4}{3\sqrt{3}+4}$$

$$= \frac{3\sqrt{3}-4}{3\sqrt{3}+4} \times \frac{3\sqrt{3}-4}{3\sqrt{3}-4} = \frac{27-24\sqrt{3}+16}{27-16} = \frac{43-24\sqrt{3}}{11}$$

**(v) $\frac{5cos^2 60^0 + 4\sec^2 30^0 - tan^2 45^0}{sin^2 30^0 + cos^2 30^0}$**

**Sol**: $$\frac{5cos^2 60^0 + 4\sec^2 30^0 - tan^2 45^0}{sin^2 30^0 + cos^2 30^0}$$

$$= \frac{5 \times \left(\frac{1}{2}\right)^2 + 4 \times \left(\frac{2}{\sqrt{3}}\right)^2 - (1)^2}{\left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2}$$

$$= \frac{\frac{5}{4} + \frac{16}{3} - 1}{\frac{1}{4} + \frac{3}{4}}$$

$$= \frac{\frac{15 + 64 - 12}{12}}{\frac{1+3}{4}} = \frac{67}{12}$$

**2. Choose the correct option and justify your choice :**

(i) $\dfrac{2\tan 30^0}{1 + tan^2 30^0}$ (A) $\sin 60^0$ (B) $\cos 60^0$ (C) $\tan 60^0$ (D) $\sin 30^0$ **[A]**

**Sol**: $\dfrac{2\,tan\,30^0}{1 + tan^2 30^0} = \dfrac{2 \times \frac{1}{\sqrt{3}}}{1 + \left(\frac{1}{\sqrt{3}}\right)^2} = \dfrac{\frac{2}{\sqrt{3}}}{1 + \frac{1}{3}} = \dfrac{\frac{2}{\sqrt{3}}}{\frac{4}{3}} = \dfrac{2 \times 3}{4 \times \sqrt{3}} = \dfrac{\sqrt{3}}{2} = sin\,60^0$

(ii) $\dfrac{1 - tan^2 45^0}{1 + tan^2 45^0} =$ (A) $\tan 90^0$ (B)1 (C) $\sin 45^0$ (D) 0 **[D]**

**Sol**: $\dfrac{1 - tan^2 45^0}{1 + tan^2 45^0} = \dfrac{1-1}{1+1} = \dfrac{0}{2} = 0$

0

(iii) $\dfrac{2\,tan\,30^0}{1 - tan^2 30^0} =$ (A) $\cos 60^0$ (B) $\sin 60^0$ (C) $\tan 60^0$ (D) $\sin 30^0$ **[C]**

**Sol**: $\dfrac{2\,tan\,30^0}{1 - tan^2 30^0} = \dfrac{2 \times \frac{1}{\sqrt{3}}}{1 - \left(\frac{1}{\sqrt{3}}\right)^2} = \dfrac{\frac{2}{\sqrt{3}}}{1 - \frac{1}{3}} = \dfrac{\frac{2}{\sqrt{3}}}{\frac{2}{3}} = \dfrac{2}{\sqrt{3}} \times \dfrac{3}{2} = \dfrac{3}{\sqrt{3}} = \dfrac{\sqrt{3} \times \sqrt{3}}{\sqrt{3}} = \sqrt{3} = \tan 60^0$

**3. If $\tan(A + B) = \sqrt{3}$ and $\tan(A - B) = \frac{1}{\sqrt{3}}$ ; $0° < A + B \leq 90°$; $A > B$, find A and B.**

***Sol***: $\tan(A + B) = \sqrt{3} = \tan 60^0$

$A + B = 60^0 \rightarrow (1)$

$\tan(A - B) = \dfrac{1}{\sqrt{3}} = \tan 30^0$

$A - B = 30^0 \rightarrow (2)$

$(1) + (2) \Rightarrow A + B + A - B = 60^0 + 30^0$

$2A = 90^0 \Rightarrow A = 45^0$

$From\ (1); 45^0 + B = 60^0 \Rightarrow B = 15^0$

**4. State whether the following are true or false. Justify your answer**

**(i) sin (A + B) = sin A + sin B.**

Sol: False.

Let $A = 30^0$ $and\ B = 60^0$

$\sin(A + B) = \sin(30^0 + 60^0) = \sin 90^0 = 1$

$sin\ A + sin\ B = \sin 30^0 + sin\ 60^0 = \frac{1}{2} + \frac{\sqrt{3}}{2} = \frac{1+\sqrt{3}}{2}$

$\sin(A + B) \neq sin\ A + sin\ B$

**(ii) The value of sin θ increases as θ increases.**

Sol: True.

$$\sin 0^0 = 0; \sin 30^0 = \frac{1}{2}; \sin 45^0 = \frac{1}{\sqrt{2}}; \sin 60^0 = \frac{\sqrt{3}}{2}; \sin 90^0 = 1$$

**(iii) The value of cos θ increases as θ increases.**

Sol: False.

$$\cos 0^0 = 1; \cos 30^0 = \frac{\sqrt{3}}{2}; \cos 45^0 = \frac{1}{\sqrt{2}}; \cos 60^0 = \frac{1}{2}; \cos 90^0 = 0$$

**(iv) sin θ = cos θ for all values of θ.**

Sol: False.

$\sin\theta = \cos\theta$ is true for only $45^0$

**(v) cot A is not defined for A = 0°.**

Sol: True.

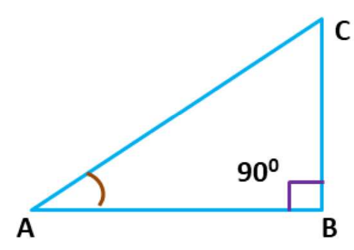
$$\cot 0^0 = \frac{\cos 0^0}{\sin 0^0} = \frac{1}{0} \text{ is not defined.}$$

## 8.1 Trigonometric Identities

An equation is called an identity when it is true for all values of the variables involved. Similarly, an equation involving trigonometric ratios of an angle is called a trigonometric identity, if it is true for all values of the angle(s) involved.

**Identity I: $sin^2A + cos^2A = 1$**

$In\ \Delta ABC, \angle B = 90^0$




$AB^2 + BC^2 = AC^2$(Pythagoras theorem)

Dividing each term by $AC^2$, we get

$$\frac{AB^2}{AC^2} + \frac{BC^2}{AC^2} = \frac{AC^2}{AC^2}$$

$$\left(\frac{AB}{AC}\right)^2 + \left(\frac{BC}{AC}\right)^2 = 1$$

$sin^2A + cos^2A = 1$

**Identity II: $\sec^2 A - tan^2 A = 1 (0° \leq A < 90°)$**

$\sin^2 A + \cos^2 A = 1$ (from Identity I)

Dividing each term by $cos^2A$, we get

$$\frac{sin^2A}{cos^2A}+\frac{cos^2A}{cos^2A}=\frac{1}{cos^2A}$$

$tan^2A+1=sec^2A$

$\sec^2 A-tan^2 A=1$

***Identity III***: $\mathbf{cosec^2 A-cot^2 A=1(0^\circ < A\le 90^\circ)}$

$sin^2A+cos^2A=1$ (*from Identity I*)

Dividing each term by $sin^2A$, we get

$$\frac{sin^2A}{sin^2A}+\frac{cos^2A}{sin^2A}=\frac{1}{sin^2A}$$

$1+cot^2 A=cosec^2 A$

$cosec^2 A-cot^2 A=1$

| | | |
|---|---|---|
| $sin^2A+cos^2A=1$ | $\sec^2 A-\tan^2 A=1$ | $cosec^2A-cot^2A=1$ |
| $\sin^2 A=1-\cos^2 A$ | $\sec^2 A=1+\tan^2 A$ | $cosec^2A=1+cot^2A$ |
| $\sin A=\sqrt{1-\cos^2 A}$ | $\tan^2 A=\sec^2 A-1$ | $cot^2A=cosec^2A-1$ |
| $cos^2A=1-sin^2A$ | $\sec A+\tan A=\frac{1}{\sec A-\tan A}$ | $cosec\,A+cot\,A=\frac{1}{cosecA-cotA}$ |
| $\cos A=\sqrt{1-sin^2A}$ | $\sec A-\tan A=\frac{1}{\sec A+\tan A}$ | $cosec\,A-cot\,A=\frac{1}{cosecA+cotA}$ |

**Example 9 : Express the ratios cos A, tan A and sec A in terms of sin A.**

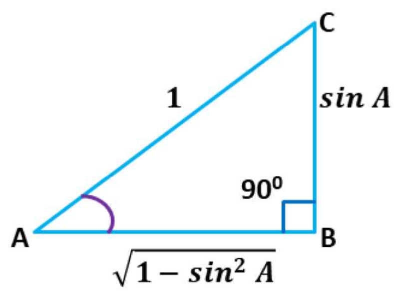
***Solution***: $sin^2A+cos^2A=1$

$cos^2A=1-sin^2A$

$\cos A=\sqrt{1-sin^2A}$

$$\tan A=\frac{\sin A}{\cos A}=\frac{\sin A}{\sqrt{1-sin^2A}}$$

$$\sec A=\frac{1}{\cos A}=\frac{1}{\sqrt{1-sin^2A}}$$




**Example 10 : Prove that sec A (1 – sin A)(sec A + tan A) = 1.**

***Sol***: $secA\,(1-sinA)\,(secA+tanA)$

$$=\frac{1}{\cos A}\times(1-sinA)\left(\frac{1}{\cos A}+\frac{\sin A}{\cos A}\right)$$

$$=\frac{(1-sinA)}{\cos A}\times\frac{(1+sinA)}{cos\,A}$$

$$=\frac{1-sin^2A}{cos^2A}$$

$$=\frac{cos^2A}{cos^2A}=1$$

**Example 11 : Prove that** $\mathbf{\frac{\cot A-\cos A}{\cot A+\cos A}=\frac{cosec\,A-1}{cosec\,A+1}}$

**Sol:** $\dfrac{\cot A - \cos A}{\cot A + \cos A} = \dfrac{\frac{\cos A}{\sin A} - \cos A}{\frac{\cos A}{\sin A} + \cos A} = \dfrac{\cos A\left(\frac{1}{\sin A} - 1\right)}{\cos A\left(\frac{1}{\sin A} + 1\right)} = \dfrac{\left(\frac{1}{\sin A} - 1\right)}{\left(\frac{1}{\sin A} + 1\right)} = \dfrac{cosec\ A - 1}{cosec\ A + 1}$

**Example 12 :** **Prove that $\dfrac{\sin\theta - \cos\theta + 1}{\sin\theta + \cos\theta - 1} = \dfrac{1}{\sec\theta - \tan\theta}$ using the identity $\sec^2\theta = 1 + \tan^2\theta$**

**Sol:** LHS $= \dfrac{\sin\theta - \cos\theta + 1}{\sin\theta + \cos\theta - 1}$

Dividing numerator and denominator by cos θ.

$$= \frac{\frac{\sin\theta - \cos\theta + 1}{\cos\theta}}{\frac{\sin\theta + \cos\theta - 1}{\cos\theta}} = \frac{\tan\theta - 1 + \sec\theta}{\tan\theta + 1 - \sec\theta} = \frac{(\tan\theta + \sec\theta) - 1}{(\tan\theta - \sec\theta) + 1}$$

$$= \frac{(\tan\theta + \sec\theta) - 1}{(\tan\theta - \sec\theta) + 1} \times \frac{(\tan\theta - \sec\theta)}{(\tan\theta - \sec\theta)}$$

$$= \frac{(\tan^2\theta - \sec^2\theta) - (\tan\theta - \sec\theta)}{[(\tan\theta - \sec\theta) + 1] \times (\tan\theta - \sec\theta)}$$

$$= \frac{-1 - \tan\theta + \sec\theta}{[\tan\theta - \sec\theta + 1] \times (\tan\theta - \sec\theta)}$$

$$= \frac{-1[\tan\theta - \sec\theta + 1]}{[\tan\theta - \sec\theta + 1] \times (\tan\theta - \sec\theta)}$$

$$= \frac{-1}{\tan\theta - \sec\theta} = \frac{1}{\sec\theta - \tan\theta}$$

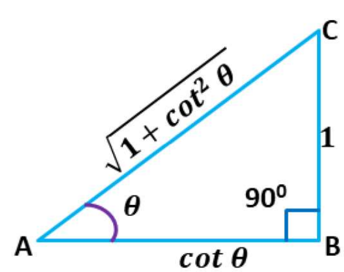


## EXERCISE 8.3

**1. Express the trigonometric ratios sin A, sec A and tan A in terms of cot A**

***Sol:*** $cosec^2 A = 1 + cot^2 A$

$cosec\ \theta = \sqrt{1 + cot^2 A}$



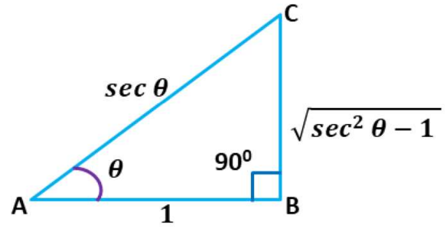

$\text{Sin}\ \theta = \dfrac{1}{cosec\ \theta} = \dfrac{1}{\sqrt{1 + cot^2 A}}$

$\sec\theta = \sqrt{1 + tan^2\theta} = \sqrt{1 + \dfrac{1}{cot^2\theta}} = \sqrt{\dfrac{cot^2\theta + 1}{cot^2\theta}} = \dfrac{\sqrt{1 + cot^2\theta}}{\cot\theta}$

$\cos\theta = \dfrac{1}{\sec\theta} = \dfrac{\cot\theta}{\sqrt{1 + cot^2\theta}}$

$\tan\theta = \dfrac{1}{\cot\theta}$

**2. Write all the other trigonometric ratios of ∠ A in terms of sec A.**




***Sol***: $\cos A = \frac{1}{\sec A}$

$$\sin A = \sqrt{1 - cos^2 A} = \sqrt{1 - \frac{1}{sec^2 A}} = \sqrt{\frac{sec^2 A - 1}{sec^2 A}} = \frac{\sqrt{sec^2 A - 1}}{\sec A}$$

$$cesec\ A = \frac{1}{\sin A} = \frac{\sec A}{\sqrt{sec^2 A - 1}}$$

$$\tan A = \sqrt{sec^2 A - 1}$$

$$\cot A = \frac{1}{\tan A} = \frac{1}{\sqrt{sec^2 A - 1}}$$

**3. Choose the correct option. Justify your choice.**

**(*i*) $9\ sec^2 A - 9\ tan^2 A$** [B]

(A) 1 (B) 9 (C) 8 (D) 0

Sol: $9\ sec^2 A - 9\ tan^2 A = 9(sec^2 A - tan^2 A) = 9 \times 1 = 9$

**(ii) (1 + tan θ + sec θ) (1 + cot θ – cosec θ) =** [C]

(A) 0 (B) 1 (C) 2 (D) –1

***Sol***: (1 + tan θ + sec θ)(1 + cot θ – cosec θ)

$$= \left(1 + \frac{\sin\theta}{\cos\theta} + \frac{1}{\cos\theta}\right)\left(1 + \frac{\cos\theta}{\sin\theta} - \frac{1}{\sin\theta}\right)$$

$$= \left(\frac{\cos\theta + \sin\theta + 1}{\cos\theta}\right)\left(\frac{\sin\theta + \cos\theta - 1}{\sin\theta}\right)$$

$$= \left[\frac{(\sin\theta + \cos\theta)^2 - 1^2}{\sin\theta\cos\theta}\right]$$

$$= \left(\frac{sin^2\theta + cos^2\theta + 2\sin\theta\cos\theta - 1}{\sin\theta\cos\theta}\right)$$

$$= \frac{1 + 2\sin\theta\cos\theta - 1}{\sin\theta\cos\theta}$$

$$= \frac{2\sin\theta\cos\theta}{\sin\theta\cos\theta} = 2$$



**(iii) (sec A + tan A) (1 – sin A) =** [ ]

(A) sec A (B) sin A (C) cosec A (D) cos A

***Sol***: $(\sec A + \tan A)(1 - \sin A) = \left(\frac{1}{\cos A} + \frac{\sin A}{\cos A}\right)(1 - \sin A)$

$$= \frac{(1 + \sin A)}{\cos A}(1 - \sin A) = \frac{1 - \sin^2 A}{\cos A} = \frac{\cos^2 A}{\cos A} = \cos A$$

**(*iv*) $\frac{1 + tan^2 A}{1 + cot^2 A} =$** [D]

(A) $sec^2$ A (B) −1 (C) $cot^2$ A (D) $tan^2$ A

***Sol***: $\frac{1 + tan^2 A}{1 + cot^2 A} = \frac{1 + tan^2 A}{1 + \frac{1}{tan^2 A}} = \frac{1 + tan^2 A}{\frac{1 + tan^2 A}{tan^2 A}} = tan^2 A$

**4. Prove the following identities, where the angles involved are acute angles for which the expressions are defined.**

$(i)\ (cosec\ \theta - cot\ \theta)^2 = \dfrac{1-\cos\theta}{1+\cos\theta}$

**Sol**: $(cosec\ \theta - cot\ \theta)^2$

$= \left(\dfrac{1}{\sin\theta} - \dfrac{\cos\theta}{\sin\theta}\right)^2$

$= \left(\dfrac{1-\cos\theta}{\sin\theta}\right)^2$

$= \dfrac{(1-\cos\theta)^2}{\sin^2\theta}$

$= \dfrac{(1-\cos\theta)^2}{1-\cos^2\theta}$

$= \dfrac{(1-\cos\theta)(1-\cos\theta)}{(1+\cos\theta)(1-\cos\theta)}$

$= \dfrac{1-\cos\theta}{1+\cos\theta}$

$(ii)\ \dfrac{\cos A}{1+\sin A} + \dfrac{1+\sin A}{\cos A} = 2\sec A$

**Sol**: $\dfrac{\cos A}{1+\sin A} + \dfrac{1+\sin A}{\cos A} = \dfrac{\cos^2 A + (1+\sin A)^2}{\cos A\,(1+\sin A)} = \dfrac{\cos^2 A + \sin^2 A + 1 + 2.\sin A}{\cos A\,(1+\sin A)}$

$= \dfrac{1+1+2.\sin A}{\cos A\,(1+\sin A)} = \dfrac{2+2.\sin A}{\cos A\,(1+\sin A)} = \dfrac{2(1+\sin A)}{\cos A\,(1+\sin A)} = \dfrac{2}{\cos A} = 2\sec A$

$(iii)\ \dfrac{\tan\theta}{1-\cot\theta} + \dfrac{\cot\theta}{1-\tan\theta} = 1 + \sec\theta\ cosec\ \theta$

**Sol**: $\dfrac{\tan\theta}{1-\cot\theta} + \dfrac{\cot\theta}{1-\tan\theta} = \dfrac{\frac{\sin\theta}{\cos\theta}}{1-\frac{\cos\theta}{\sin\theta}} + \dfrac{\frac{\cos\theta}{\sin\theta}}{1-\frac{\sin\theta}{\cos\theta}} = \dfrac{\frac{\sin\theta}{\cos\theta}}{\frac{\sin\theta-\cos\theta}{\sin\theta}} + \dfrac{\frac{\cos\theta}{\sin\theta}}{\frac{\cos\theta-\sin\theta}{\cos\theta}}$

$= \dfrac{\sin^2\theta}{\cos\theta\,(\sin\theta-\cos\theta)} - \dfrac{\cos^2\theta}{\sin\theta\,(\sin\theta-\cos\theta)} = \dfrac{\sin^3\theta-\cos^3\theta}{\sin\theta\cos\theta\,(\sin\theta-\cos\theta)}$

$= \dfrac{(\sin\theta-\cos\theta)(\sin^2\theta+\cos^2\theta+\sin\theta\cos\theta)}{\sin\theta\cos\theta\,(\sin\theta-\cos\theta)} = \dfrac{1+2\sin\theta\cos\theta}{\sin\theta\cos\theta} = \dfrac{1}{\sin\theta\cos\theta} + \dfrac{\sin\theta\cos\theta}{\sin\theta\cos\theta}$

$= \sec\theta\ cosec\ \theta + 1$

$(iv)\ \dfrac{1+\sec A}{\sec A} = \dfrac{\sin^2 A}{1-\cos A}$

**Sol**: RHS $= \dfrac{\sin^2 A}{1-\cos A} = \dfrac{1-\cos^2 A}{1-\cos A} = \dfrac{(1+\cos A)(1-\cos A)}{1-\cos A}$

$= 1+\cos A = 1 + \dfrac{1}{\sec A} = \dfrac{\sec A + 1}{\sec A} = LHS$

$(v)\ \dfrac{\cos A - \sin A + 1}{\cos A + \sin A - 1} = cosec\ A + \cot A$, *using the identity* $cosec^2 A = 1 + cot^2 A$

**Sol:** $\dfrac{\cos A - \sin A + 1}{\cos A + \sin A - 1} = \dfrac{\dfrac{\cos A}{\sin A} - \dfrac{\sin A}{\sin A} + \dfrac{1}{\sin A}}{\dfrac{\cos A}{\sin A} + \dfrac{\sin A}{\sin A} - \dfrac{1}{\sin A}} = \dfrac{\cot A - 1 + cosec\ A}{\cot A + 1 - cosec\ A}$

$= \dfrac{\cot A - (1 - cosec\ A)}{\cot A + (1 - cosec\ A)} \times \dfrac{\cot A - (1 - cosec\ A)}{\cot A - (1 - cosec\ A)}$

$= \dfrac{\cot^2 A + (1 - cosec\ A)^2 - 2\cot A(1 - cosec\ A)}{\cot^2 A - (1 - cesec\ A)^2}$

$= \dfrac{\cot^2 A + 1 + cosec^2\ A - 2cosec\ A - 2\cot A + 2\cot A\, cesec\ A}{\cot^2 A - (1 + cosec^2\ A - 2cosec\ A)}$

$= \dfrac{cosec^2\ A + cosec^2\ A + 2\cot A\, cosec\ A - 2cosec\ A - 2\cot A}{\cot^2 A - 1 - cosec^2\ A + 2cosec\ A}$

$= \dfrac{2cosec^2\ A + 2\cot A\, cosec\ A - 2cosec\ A - 2\cot A}{\cot^2 A - 1 - cosec^2\ A + 2cosec\ A}$

$= \dfrac{2cosec\ A(cosec\ A + \cot A) - 2(cosec\ A + \cot A)}{\cot^2 A - cosec^2\ A - 1 + 2cosec\ A}$

$= \dfrac{(cosec\ A + \cot A)(2cosec\ A - 2)}{-1 - 1 + 2cosec\ A}$

$= \dfrac{(cosec\ A + \cot A)(2cosec\ A - 2)}{2cosec\ A - 2} = (cosec\ A + \cot A)$

(*vi*) $\sqrt{\dfrac{1 + \sin A}{1 - \sin A}} = \sec A + \tan A$

**sol:** $\sqrt{\dfrac{1 + sin\ A}{1 - sin\ A}}$

$= \sqrt{\dfrac{(1 + sin\ A)(1 + sin\ A)}{(1 - sin\ A)(1 + sin\ A)}}$

$= \sqrt{\dfrac{(1 + sin\ A)^2}{1 - sin^2 A}}$

$= \dfrac{\sqrt{(1 + sin\ A)^2}}{\sqrt{cos^2 A}}$

$= \dfrac{1 + sinA}{\cos A}$

$= \dfrac{1}{\cos A} + \dfrac{\sin A}{\cos A}$

$= \sec A + \tan A$

(*vii*) $\dfrac{\sin\theta - 2sin^3\theta}{2cos^3\theta - \cos\theta} = \tan\theta$

**Sol:** $\dfrac{\sin\theta - 2sin^3\theta}{2cos^3\theta - \cos\theta} = \dfrac{\sin\theta(1 - 2sin^2\theta)}{\cos\theta\,(2cos^2\theta - 1)} = \dfrac{\sin\theta(1 - 2sin^2\theta)}{\cos\theta\,(2(1 - sin^2\theta) - 1)} = \dfrac{\sin\theta(1 - 2sin^2\theta)}{\cos\theta\,(2 - 2sin^2\theta - 1)}$

$$= \frac{\sin\theta(1-2sin^2\theta)}{\cos\theta(1-2sin^2\theta)} = \frac{\sin\theta}{\cos\theta} = \tan\theta$$

**(viii)** $(\sin A + cosecA)^2 + (\cos A + \sec A)^2 = 7 + tan^2A + cot^2A$

**Sol:** $L.H.S = (sin\,A + cosecA)^2 + (cos\,A + sec\,A)^2$

$$= sin^2A + cosec^2A + 2\sin A.cosec\,A + cos^2A + sec^2A + 2.cos\,A.\sec A$$
$$= sin^2A + cos^2A + cosec^2A + sec^2A + 2\sin A \times \frac{1}{\sin A} + 2cos\,A \times \frac{1}{\cos A}$$
$$= 1 + cot^2A + 1 + tan^2A + 1 + 2 + 2$$
$$= 7 + tan^2A + cot^2A$$
$$= R.H.S$$

**(ix)** $(\text{cosec A} - \sin \text{A})(\sec \text{A} - \cos \text{A}) = \frac{1}{\tan \text{A} + \cot \text{A}}$

**Sol:** $LHS = (\text{cosec A} - \sin\text{A})(\sec\text{A} - \cos\text{A})$

$$= \left(\frac{1}{\sin A} - \sin A\right)\left(\frac{1}{\cos A} - \cos A\right) = \frac{1-sin^2A}{\sin A} \times \frac{1-cos^2A}{\cos A}$$
$$= \frac{cos^2A}{\sin A} \times \frac{sin^2A}{\cos A} = \cos A.\sin A$$

$$\text{RHS} = \frac{1}{\tan\text{A} + \cot\text{A}} = \frac{1}{\frac{\sin A}{\cos A} + \frac{\cos A}{\sin A}} = \frac{1}{\frac{sin^2A + cos^2A}{\cos A.\sin A}} = \frac{\cos A.\sin A}{1} = \cos A.\sin A$$

**(x)** $\frac{1+tan^2A}{1+cot^2A} = \left(\frac{1-\tan A}{1-\cot A}\right)^2 = tan^2A$

**Sol:** $\frac{1+tan^2A}{1+cot^2A} = \frac{\sec^2 A}{cosec^2A} = \frac{1}{cos^2A} \div \frac{1}{sin^2A} = \frac{1}{cos^2A} \times \frac{sin^2A}{1} = \frac{sin^2A}{cos^2A} = tan^2A$

$$\left(\frac{1-\tan A}{1-\cot A}\right)^2 = \left(\frac{1-\frac{\sin A}{\cos A}}{1-\frac{\cos A}{\sin A}}\right)^2 = \left(\frac{\frac{\cos A - \sin A}{\cos A}}{\frac{\sin A - \cos A}{\sin A}}\right)^2$$
$$= \left(\frac{(\cos A - \sin A)^2}{\cos^2 A} \times \frac{sin^2A}{(\sin A - \cos A)^2}\right) = \frac{sin^2A}{cos^2A} = tan^2A$$

## Some more problems

1. Show that $\sqrt{(1-cos^2\theta)sec^2\theta} = \tan\theta$
2. Prove that $sin^6\theta + cos^6\theta + 3sin^2\theta cos^2\theta = 1$
3. If $\sin\theta + \cos\theta = \sqrt{3}$, then prove that $\tan\theta + \cot\theta = 1$
4. *Prove* $\frac{\sin\theta}{1+\cos\theta} + \frac{1+\cos\theta}{\sin\theta} = 2\,\text{cosec}\,\theta$
5. $\frac{\tan A}{1+\sec A} - \frac{\tan A}{1-\sec A} = 2\,\text{cosec}\,\theta$
6. Simplify $(1 + \tan^2\theta)(1 - \sin\theta)(1 + \sin\theta)$
7. If $2\sin^2\theta - \cos^2\theta = 2$, then find the value of $\theta$
8. *Show that* $tan^2\theta + tan^4\theta = sec^4\theta - sec^2\theta$
9. If $1 + \sin^2\theta = 3\sin\theta\cos\theta$, then prove that $\tan\theta = 1$ or $1/2$
10. $a\cos\theta + b\sin\theta = m$ and $a\sin\theta - b\cos\theta = n$, then prove that $a^2+b^2=m^2+n^2$
11. *Prove th* $\sqrt{\frac{\sec A-1}{\sec A+1}} + \sqrt{\frac{\sec A+1}{\sec A-1}} = 2cosec\,A$
12. *Prove that* $\frac{\tan\theta}{1-\cot\theta} + \frac{\cot\theta}{1-\tan\theta} = 1 + \sec\theta\ cosec\,\theta$

13. The value of $\left(sin^2\theta + \frac{1}{1+\tan^2\theta}\right)$
14. If $\sec\theta = \sqrt{2}$ then the value of $\frac{1+\tan\theta}{\sin\theta}$.
15. If $\sec\theta + \tan\theta = p$, then $\tan\theta$.
16. Prove that: $\frac{\sin\theta}{\cot\theta + cosec\,\theta} = 2 + \frac{\sin\theta}{\cot\theta - cosec\,\theta}$
17. Prove that $\frac{\tan\theta}{1-\cot\theta} + \frac{\cot\theta}{1-\tan\theta} = 1 \tan\theta + \cot\theta = \sec\theta\, cosec\,\theta + 1$
18. $\sin A + sin^2 A = 1$, then find the value of the expression $(cos^2 A + cos^4 A)$
19. If $\sin\theta + \cos\theta = \sqrt{2}$, prove that $\tan\theta + \cot\theta = 2$

Answers: 13)1  14) $2\sqrt{2}$  15) $\frac{p^2-1}{2p}$  18)1

**MCQ**

1. **The value of (sin30° + cos30°) – (sin60° + cos60° ) is**
   (A) – 1  (B) 0  (C) 1  (D) 2
2. **The value of (sin 45° + cos 45°) is**
   (A) $\frac{1}{\sqrt{2}}$  (B) $\sqrt{2}$  (C) $\frac{\sqrt{3}}{2}$  (D) 1
3. **If $\cos A = \frac{4}{5}$, then the value of tan A is**
   (A) $\frac{3}{5}$  (B) $\frac{3}{4}$  (C) $\frac{4}{3}$  (D) $\frac{5}{3}$
4. **Given that $\sin\theta = \frac{a}{b}$, then cosθ is equal to**
   (A) $\frac{b}{\sqrt{b^2-a^2}}$  (B) $\frac{b}{a}$  (C) $\frac{\sqrt{b^2-a^2}}{b}$  (D) $\frac{a}{\sqrt{b^2-a^2}}$
5. **If ΔABC is right angled at C, then the value of cos (A+B) is**
   (A) 0  (B) 1  (C) 1 2  (D) $\sqrt{3}/2$
6. **If sin A + $sin^2$ A = 1, then the value of the expression ($cos^2$ A + $cos^4$ A) is**
   (A) 1  (B) 1/ 2  (C) 2  (D) 3
7. **If $4\tan\theta = 3$, then $\frac{4\sin\theta - \cos\theta}{4\sin\theta + \cos\theta}$ is equal to**
   (A) 2 /3  (B) 1/ 3  (C) 1/ 2  (D) 3/ 4
8. **If sin θ – cos θ = 0, then the value of ($sin^4$ θ + $cos^4$ θ) is**
   (A) 1  (B) 3/ 4  (C) 1/ 2  (D) 1 /4

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 1)B | 2)B | 3)B | 4)C | 5)A | 6)A | 7)C | 8)C |

1. **Assertion**: The value of sin60°cos30°+sin30°cos60° is 1.
   **Reason**: sin90° = 1 and cos90° = 0.
2. **Assertion:** The value of $2\tan^2 45° + \cos^2 30° - \sin^2 60°$ is 2.
   **Reason**: The value of tan45° = 1, cos30° = $\sqrt{3}/2$, and sin60° = $\sqrt{3}/2$
3. **Assertion:** If $x = 2\sin^2\theta$ and $y = 2\cos^2\theta + 1$, then the value of $x + y = 3$.
   **Reason**: For any value of θ, $\sin^2\theta + \cos^2\theta = 1$.
4. **Assertion:** sin A is the product of sin A.

Reason: The value of sin θ increases as θ increases.

5. Assertion: In a right ΔABC, right angled at B, if tan A = 1, then 2sinA.cosA = 1.

   Reason: cosec A is the abbreviation used for cosecant of angle A.

6. Assertion: In a right ΔABC, right angled at B, if tan A = 12/5, then sec A = 13/5.

   Reason: cotA is the product of cot and A.

7. Assertion: If $x\sin^3\theta + y\cos^3\theta = \sin\theta\cos\theta$ and $x\sin\theta = y\cos\theta$, then $x^2 + y^2 = 1$.

   Reason: For any value of θ, $\sin^2\theta + \cos^2\theta = 1$.

8. Assertion: $(\cos^4 A - \sin^4 A)$ is equal to $2\cos^2 A - 1$.

   Reason: The value of cos θ decreases as θ increases.

9. Assertion: If $\cos A + \cos^2 A = 1$, then $\sin^2 A + \sin^4 A = 1$.

   Reason: $\sin^2 A + \cos^2 A = 1$, for any value of A.

10. Assertion: sin(A + B) = sinA + sinB.

    Reason: For any value of θ, $1 + \tan^2\theta = \sec^2\theta$.

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1)b | 2)a | 3)a | 4)d | 5)b | 6)c | 7)a | 8)b | 9)a | 10)d |

## PREVIOUS YEAR PROBLEMS:

### CBSE-2024

1. ***If*** $\sec\theta - \tan\theta = m$, ***then the value of*** $\sec\theta - \tan\theta = \frac{1}{m}$

2. ***If*** $\cos(\alpha+\beta) = 0$, ***then value of*** $\cos\left(\frac{\alpha+\beta}{2}\right) = \frac{1}{\sqrt{2}}$

3. ***Evaluate*** : $2\sqrt{2}\cos 45^0 \sin 30^0 + 2\sqrt{3}\cos 30^0$

*Sol*: $2\sqrt{2} \times \frac{1}{\sqrt{2}} \times \frac{1}{2} + 2\sqrt{3} \times \frac{\sqrt{3}}{2} = 1 + 3 = 4$

4. ***If*** $A = 60^0$ ***and*** $B = 30^0$, ***verify that*** : $\sin(A+B) = \sin A\cos B + cas\, A \sin B$

*Sol*: $LHS = \sin(A+B) = sin(60^0 + 30^0) = \sin 90^0 = 1$

$RHS = \sin A\cos B + \cos A\sin B = sin\, 60^0\, cos\, 30^0 + sin\, 30^0\, cos\, 60^0$

$= \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} + \frac{1}{2} \times \frac{1}{2} = \frac{3}{4} + \frac{1}{4} = \frac{3+1}{4} = \frac{4}{4} = 1$

∴LHS=RHS

### CBSE-2023

5. ***If*** $4\cot^2 45^0 - \sec^2 60^0 + \sin^2 60^0 + p = \frac{3}{4}$, ***then find the value of p.***

*Sol*: $4\cot^2 45^0 - \sec^2 60^0 + \sin^2 60^0 + p = \frac{3}{4}$

$$4 \times (1)^2 - (2)^2 + \left(\frac{\sqrt{3}}{2}\right)^2 + p = \frac{3}{4}$$

$$4 - 4 + \frac{3}{4} + p = \frac{3}{4}$$

$$p = 0$$

**6.** ***If*** $\cos A + \cos^2 A = 1$, ***then find the value of*** $\sin^2 A + \sin^4 A$.

**Sol**: *Given* : $\cos A + \cos^2 A = 1$

| |
|---|
| $\sin^2 A + \cos^2 A = 1$ |
| $\sin^2 A = 1 - \cos^2 A$ |
| $\cos^2 A = 1 - \sin^2 A$ |

$$\cos A = 1 - \cos^2 A$$

$$\cos A = \sin^2 A$$

Squaring on both sides

$$\cos^2 A = \sin^4 A$$

$$1 - \sin^2 A = \sin^4 A$$

$$\sin^2 A + \sin^4 A = 1$$

**7.** ***Prove that***: $\left(\frac{1}{\cos\theta} - \cos\theta\right)\left(\frac{1}{\sin\theta} - \sin\theta\right) = \frac{1}{\tan\theta + \cot\theta}$

**Sol**: LHS $= \left(\frac{1}{\cos\theta} - \cos\theta\right)\left(\frac{1}{\sin\theta} - \sin\theta\right) = \left(\frac{1 - \cos^2\theta}{\cos\theta}\right)\left(\frac{1 - \sin^2\theta}{\sin\theta}\right)$

$$= \frac{\sin^2\theta}{\cos\theta} \times \frac{\cos^2\theta}{\sin\theta} = \sin\theta . \cos\theta$$

RHS $= \frac{1}{\tan\theta + \cot\theta} = \frac{1}{\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}}$

$$= \frac{\sin\theta . \cos\theta}{\sin^2\theta + \cos^2\theta} = \sin\theta . \cos\theta$$

∴LHS=RHS