CHAPTER 6

X-MATHEMATICS-NCERT-2024-25

Triangles

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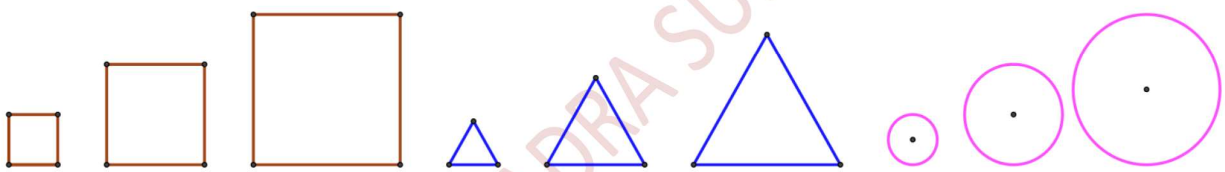
https://sureshmathsmaterial.com

1. **congruent figures**: The figures that have the same shape and size are called congruent figures
2. The two triangles are congruent if the **sides** and **angles** of one triangle are **equal** to the **corresponding** sides and angles of the other triangle.
3. If Δ PQR is congruent to Δ ABC, we write Δ PQR ≅ Δ ABC.
4. Congruent triangles corresponding parts are equal and we write in short 'CPCT' for corresponding parts of congruent triangles.
5. **Similar figures:**

   Two figures having the same shape but not necessarily the same size are called **similar figures**.

   Examples: 1) All squares are similar 2) All equilateral triangles are similar 3) All circles are similar



6. All the congruent figures are similar but the similar figures need not be congruent.
7. Two polygons of the same number of sides are similar, if (i) their corresponding angles are equal and (ii) their corresponding sides are in the same ratio (or proportion).

EXERCISE 6.1

1. Fill in the blanks with similar / not similar.

(i) All squares are <u>similar</u>

(ii) All equilateral triangles are <u>similar.</u>

(iii) All isosceles triangles are <u>not similar</u>.

(iv) Two polygons with same number of sides are similar. If their corresponding **angles are equal** and **corresponding sides** are equal.

(v) Reduced and Enlarged photographs of an object are <u>similar</u>.

(vi) Rhombus and squares are <u>not similar</u> to each other.

2. Give two different examples of pair of

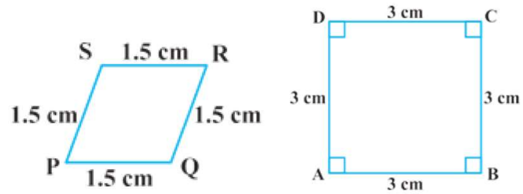
(i) Similar figures:

Example: 1. All squares 2. All circles. 3. All equilateral triangles.

(ii) Non-similar figures:

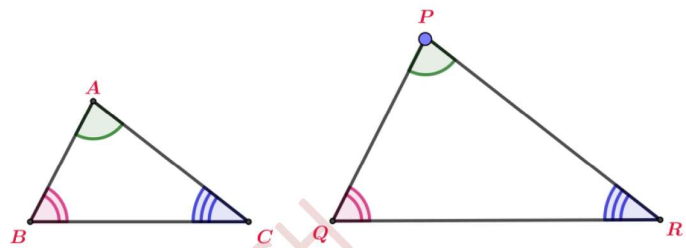
Examples: 1. Square, Rectangle 2. Rectangle, Rhombus

3. State whether the following quadrilaterals are similar or not:



Sol: Corresponding angles are not equal. So, the quadrilateral are not similar.

6.3 Similarity of Triangles



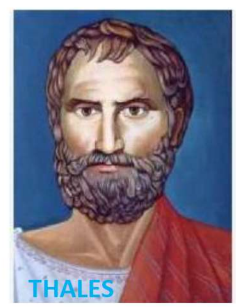
1. Two triangles are similar, if
(i) their corresponding angles are equal and
(ii) their corresponding sides are in the same ratio (or proportion).

2. In ΔABC and ΔPQR

(i) $\angle A = \angle P, \angle B = \angle Q, \angle C = \angle R$ (ii) $\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR}$

Then ΔABC is similar to ΔPQR. It is denoted by ΔABC ~ ΔPQR .

(Symbol '~' is read as "Is similar to")

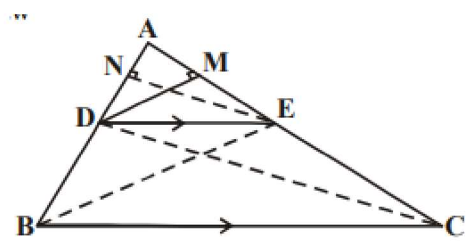


Basic proportionality theorem (Thales theorem)

If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, then the other two sides are divided in the same ratio.

Given: In ΔABC, DE ∥ BC which intersects sides AB and A C at D and E respectively

RTP: $\frac{AD}{DB} = \frac{AE}{EC}$



Construction: Join B, E and C, D and then draw DM ⊥ AC and EN ⊥ AB.

$$Proof: Area\ of\ \Delta ADE = \frac{1}{2} \times AD \times EN$$

$$Area\ of\ \Delta BDE = \frac{1}{2} \times BD \times EN$$

$$So, \frac{ar(\Delta ADE)}{ar(\Delta BDE)} = \frac{\frac{1}{2} \times AD \times EN}{\frac{1}{2} \times BD \times EN} = \frac{AD}{DB} \rightarrow (1)$$

$$Area\ of\ \Delta ADE = \frac{1}{2} \times AE \times DM$$

$$Area\ of\ \Delta CDE = \frac{1}{2} \times EC \times DM$$

Area of triangle
$= \frac{1}{2} \times Base \times Height$

$$\frac{ar(\Delta\ ADE)}{ar(\Delta CDE)} = \frac{\frac{1}{2} \times AE\ \times DM}{\frac{1}{2} \times EC\ \times DM} = \frac{AE}{EC} \rightarrow (2)$$

But ΔBDE and ΔCDE are on the same base DE and between same parallels BC and DE.

So $ar(\Delta BDE) = ar(\Delta CDE) \rightarrow (3)$

From (1) (2) and (3), we have

$$\frac{ar(\ \Delta ADE)}{ar(\ \Delta BDE)} = \frac{ar(\ \Delta ADE)}{ar(\Delta CDE)}$$

$$\Rightarrow \frac{AD}{DB} = \frac{AE}{EC}$$

Hence proved

Theorem-6.2 :(Converse of basic proportionality theorem)

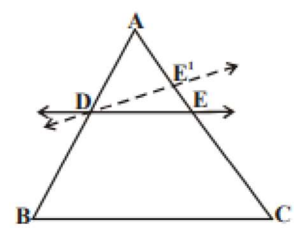
If a line divides two sides of a triangle in the same ratio, then the line is parallel to the third side.

Given : In ΔABC, a line DE is drawn such that $\frac{AD}{DB} = \frac{AE}{EC}$

RTP : DE || BC

Proof: Assume that DE is not parallel to BC then draw the line DE^{I} ∥ BC

In ΔABC ; DE^{I} ∥ BC



$So, \frac{AD}{DB} = \frac{AE'}{E'C}$ (*From Basic proportionality therem*)

$But\ \frac{AD}{DB} = \frac{AE}{EC}$ (*Given*)

$$\therefore \frac{AE}{EC} = \frac{AE'}{E'C} \Rightarrow \frac{AE}{EC} + 1 = \frac{AE'}{E'C} + 1$$

$$\frac{AE + EC}{EC} = \frac{AE' + E'C}{E'C}$$

$$\frac{AC}{EC} = \frac{AC}{E'C}$$

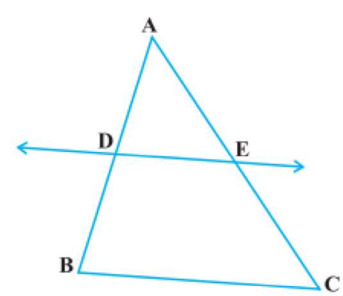
$$\Rightarrow EC = E'C$$

E and E′ must coincide

∴ DE ∥ BC

Example 1 : If a line intersects sides AB and AC of a Δ ABC at D and E respectively and is parallel to BC, prove that

$$\boldsymbol{\frac{AD}{AB} = \frac{AE}{AC}}$$



Solution: In ΔABC, DE ||BC

$\frac{AD}{DB} = \frac{AE}{EC}$ (*from basic propertionality theorem*)

$\Rightarrow \frac{DB}{AD} = \frac{EC}{AE}$

$\Rightarrow 1 + \frac{DB}{AD} = 1 + \frac{EC}{AE}$

$\Rightarrow \frac{AD + DB}{AD} = \frac{AE + EC}{AE}$

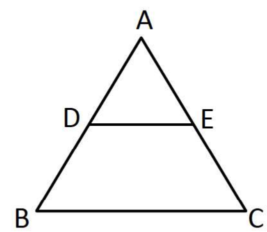
$\Rightarrow \frac{AB}{AD} = \frac{AC}{AE}$

In ΔABC, DE $\parallel$ BC then

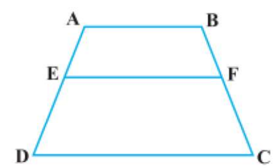
(i) $\frac{AD}{DB} = \frac{AE}{EC}$; $\frac{BD}{DA} = \frac{CE}{EA}$

(ii) $\frac{AB}{AD} = \frac{AC}{AE}$; $\frac{AD}{AB} = \frac{AE}{AC}$

(iii) $\frac{AB}{DB} = \frac{AC}{EC}$; $\frac{BD}{AB} = \frac{EC}{AC}$



Example 2 : ABCD is a trapezium with AB || DC. E and F are points on non-parallel sides AD and BC respectively such that EF is parallel to AB (see Fig. 6.14). Show that $\frac{AE}{ED} = \frac{BF}{FC}$



Solution : Let us join AC to intersect EF at G.

AB || DC and EF || AB (given)

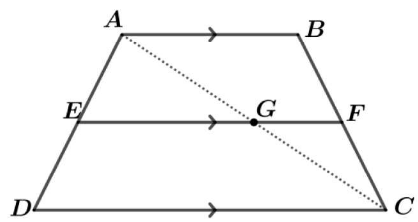
$\Rightarrow$ EF || DC (Lines parallel to the same line are parallel to each other)

In ΔADC, EG || DC

$\Rightarrow \frac{AE}{ED} = \frac{AG}{GC}$ (by BPT) $\rightarrow$ (1)

Similarly, In ΔCAB, GF || AB

$\Rightarrow \frac{AG}{GC} = \frac{BF}{FC}$ (by BPT) $\rightarrow$ (2)



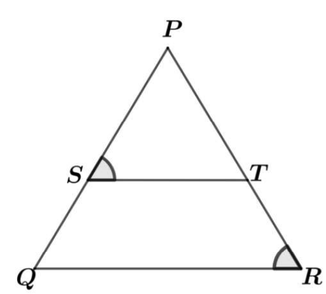
From (1) & (2)

$\frac{AE}{ED} = \frac{BF}{FC}$

Example3: In ΔPQR, ST is a line such that $\frac{PS}{SQ} = \frac{PT}{TR}$ and also $\angle PST = \angle PRQ$. Prove that ΔPQR is an isosceles triangle.

Sol: In ΔPQR, ST is a line such that

$\frac{PS}{SQ} = \frac{PT}{TR}$



$\Rightarrow \mathrm{ST} \parallel \mathrm{QR}$ (*by converse of* BPT)

$\angle PST = \angle PQR$ (*corresponding angles*) $\rightarrow (1)$

But $\angle PST = \angle PRQ$ (*given*) $\rightarrow (2)$

From (1) and (2)

$\angle \mathrm{PQR} = \angle \mathrm{PRQ}$

$\Rightarrow \mathrm{PR} = \mathrm{PQ}$ (*Sides opposite to equal angles are equal*)

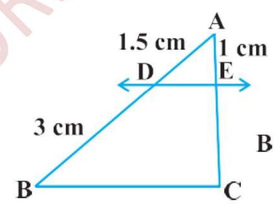
Now in $\Delta\mathrm{PQR}$, $\mathrm{PR} = \mathrm{PQ}$

$\therefore \Delta\mathrm{PQR}$ is an isosceles triangle.

EXERCISE 6.2

1. In Fig. 6.17, (i) and (ii), DE || BC. Find EC in (i) and AD in (ii)

Sol: In $\Delta\mathrm{ABC}$, $\mathrm{DE} \parallel \mathrm{BC}$



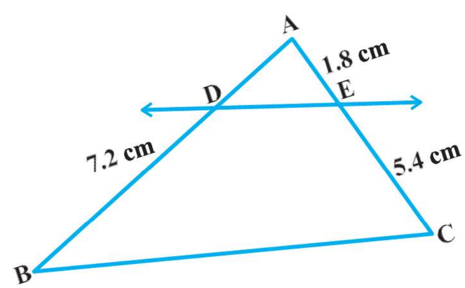
From basic property theorem

$$\frac{\mathrm{AD}}{\mathrm{DB}} = \frac{\mathrm{AE}}{\mathrm{EC}} \Rightarrow \frac{1.5}{3} = \frac{1}{EC}$$

$$\Rightarrow \mathrm{EC} = \frac{1\times3}{1.5} = \frac{3}{1.5} = \frac{30}{15} = 2$$

$\therefore EC = 2\ cm$

(*ii*)



Sol: *In* $\Delta\mathrm{ABC}$, $\mathrm{DE} \parallel \mathrm{BC}$

From basic property theorem

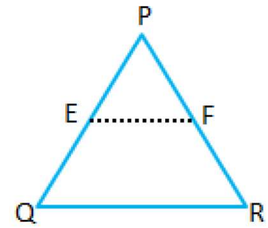
$$\frac{\mathrm{AD}}{\mathrm{DB}} = \frac{\mathrm{AE}}{\mathrm{EC}} \Rightarrow \frac{AD}{7.2} = \frac{1.8}{5.4} = \frac{18}{54} = \frac{1}{3}$$

$$\Rightarrow AD = \frac{7.2}{3} = 2.4$$

$\therefore AD = 2.4\ cm$

2. E and F are points on the sides PQ and PR respectively of a Δ PQR. For each of the following cases, state whether EF || QR :

(i) PE = 3.9 cm, EQ = 3 cm, PF = 3.6 cm and FR = 2.4 cm



Sol: $\frac{PE}{EQ} = \frac{3.9}{3} = 1.3$; $\frac{PF}{FR} = \frac{3.6}{2.4} = \frac{3}{2} = 1.5$

$\frac{PE}{EQ} \neq \frac{PF}{FR} \Rightarrow EF \nparallel QR$

(ii) PE = 4 cm, QE = 4.5 cm, PF = 8 cm and RF = 9 cm

Sol: $\frac{PE}{EQ} = \frac{4}{4.5} = \frac{40}{45} = \frac{8}{9}$; $\frac{PF}{FR} = \frac{8}{9}$

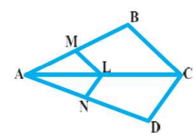
$\frac{PE}{EQ} = \frac{PF}{FR} \Rightarrow EF \parallel QR$

(iii) PQ = 1.28 cm PR = 2.56 cm PE = 1.8 cm and PF = 3.6 cm

Sol: $\frac{PQ}{PE} = \frac{1.28}{1.8} = \frac{128}{180} = \frac{32}{45}$; $\frac{PR}{PF} = \frac{2.56}{3.6} = \frac{256}{360} = \frac{64}{90} = \frac{32}{45}$

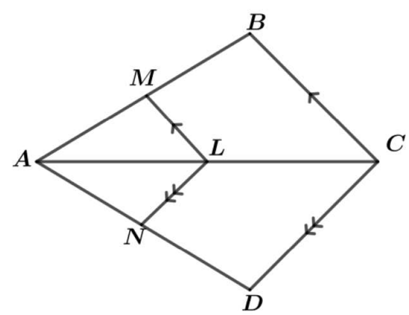
$\frac{PQ}{PE} = \frac{PR}{PF} \Rightarrow EF \parallel QR$

3. In Fig. 6.18, if LM || CB and LN || CD, prove that $\frac{AM}{AB} = \frac{AN}{AD}$



Sol: In ΔACB, LM || CB

$\Rightarrow \frac{AL}{LC} = \frac{AM}{MB}$ (*by* BPT) $\rightarrow (1)$



In ΔACD, LN || CD

$\Rightarrow \frac{AL}{LC} = \frac{AN}{ND}$ (*by* BPT) $\rightarrow (2)$

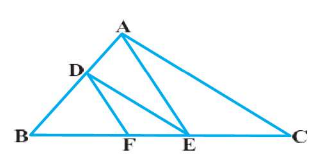
From (1) and (2)

$\frac{AM}{MB} = \frac{AN}{ND}$

$\Rightarrow \frac{AM}{AM + MB} = \frac{AN}{AN + ND}$

$\Rightarrow \frac{AM}{AB} = \frac{AN}{AD}$

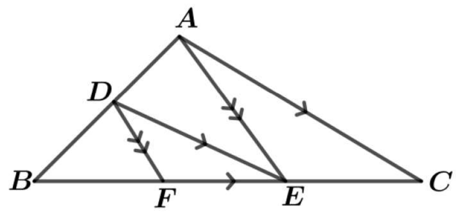
4. In Fig. 6.19, DE || AC and DF || AE. Prove that $\frac{BF}{FE} = \frac{BE}{EC}$



Sol: *In* ΔABC , DE $\parallel$ AC

$\Rightarrow \frac{BD}{DA} = \frac{BE}{EC} \;(\,by\ BPT) \rightarrow (1)$

$In\ \Delta AEB$, DF ∥ AE

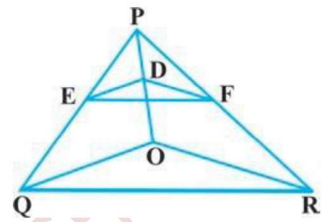


$$\Rightarrow \frac{BD}{DA} = \frac{BF}{FE} \;(\,by\ BPT) \rightarrow (2)$$

From (1) and (2)

$$\frac{BF}{FE} = \frac{BE}{EC}$$

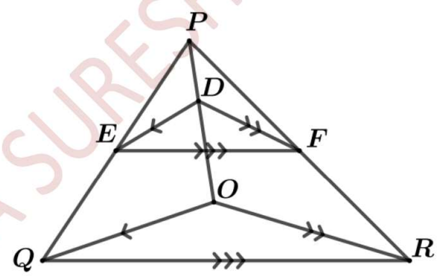
5. In Fig. 6.20, DE || OQ and DF || OR. Show that EF || QR.



Solution: $In\ \Delta PQO$, ED ∥ QO

$$\Rightarrow \frac{PD}{DO} = \frac{PE}{EQ} \;(\,by\ BPT) \rightarrow (1)$$

$In\ \Delta POR$, DF ∥ OR



$$\Rightarrow \frac{PD}{DO} = \frac{PF}{FR} \;(\,by\ BPT) \rightarrow (2)$$

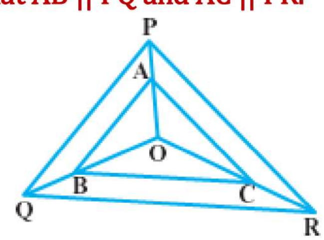
From (1) and (2)

$$\frac{PE}{EQ} = \frac{PF}{FR}$$

$\Rightarrow$ EF ∥ QR $(by\ converse\ of\ BPT)$

Hence proved

6. In Fig. 6.21, A, B and C are points on OP, OQ and OR respectively such that AB || PQ and AC || PR. Show that BC || QR.

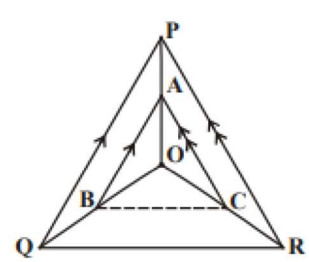


Sol: $In\ \Delta PQO$, $AB \parallel PQ$

$$\Rightarrow \frac{OA}{AP} = \frac{OB}{BQ} \;(\,by\ BPT) \rightarrow (1)$$

$In\ \Delta PRO$, $AC \parallel PR$

$$\Rightarrow \frac{OA}{AP} = \frac{OC}{CR} \;(\,by\ BPT) \rightarrow (2)$$



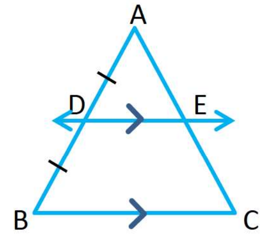
From (1) and (2)

$$\frac{OB}{BQ} = \frac{OC}{CR}$$

$\Rightarrow$ BC ∥ QR $(by\ converse\ of\ BPT)$

7. Using Theorem 6.1, prove that a line drawn through the mid-point of one side of a triangle parallel to another side bisects the third side. (Recall that you have proved it in Class IX).

Solution: Let $in\ \Delta ABC\ , D\ is\ midpoint\ of\ AB\ and\ DE \parallel BC$



$\Rightarrow \frac{AD}{DB} = \frac{AE}{EC}$ (*by BPT*)

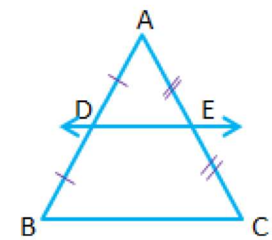
$\Rightarrow 1 = \frac{AE}{EC}$ (*Since D is midpoint of AB ie,* $AD = DB$)

$\Rightarrow AE = EC$

$\Rightarrow E\ is\ mid\ point\ of\ AC$

$\Rightarrow \overleftrightarrow{DE}\ bisects\ AC$

8. **Using Theorem 6.2, prove that the line joining the mid-points of any two sides of a triangle is parallel to the third side. (Recall that you have done it in Class IX).**



Sol: *Given*: $In\ \Delta ABC\ , D\ is\ midpoint\ of\ AB\ and\ E\ is\ midpoint\ of\ AC$

RTP: $DE \parallel BC$

Proof:

$AD = DB\ \ and\ \ AE = EC$

($D\ is\ midpoint\ of\ AB\ and\ E\ is\ midpoint\ of\ AC$)

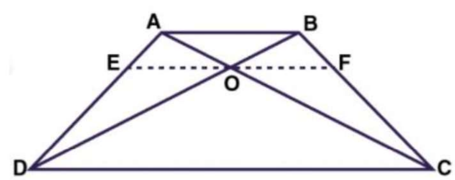
$\Rightarrow \frac{AD}{DB} = 1$ and $\frac{AE}{EC} = 1$

$\Rightarrow \frac{AD}{DB} = \frac{AE}{EC}$

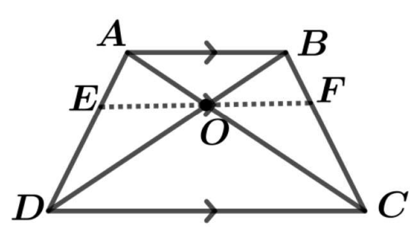
$\Rightarrow DE \parallel BC$ (*by converse of BPT*)

9. **ABCD is a trapezium in which AB || DC and its diagonals intersect each other at the point O. Show that**

$$\frac{AO}{BO} = \frac{CO}{DO}$$



Given: ABCD is a trapezium in which AB||DC and its diagonals intersect each other at point 'O'



RTP: $\frac{AO}{BO} = \frac{CO}{DO}$

Construction: Through 'O' draw a line $EF \parallel AB \parallel DC$.

Proof: $In\ \Delta ADC, EO \parallel DC$

$\frac{AE}{ED} = \frac{AO}{OC} \rightarrow (1)$

$In\ \Delta ADB, EO \parallel AB$

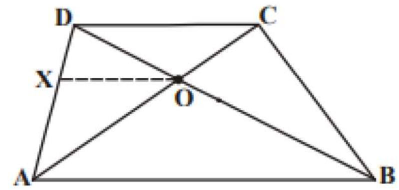
$\frac{AE}{ED} = \frac{BO}{OD} \rightarrow (2)$

From (1) and (2)

$$\frac{AO}{OC}=\frac{BO}{OD}\Rightarrow\frac{AO}{BO}=\frac{OC}{OD}\Rightarrow\frac{AO}{BO}=\frac{CO}{DO}$$

Hence proved.

10. The diagonals of a quadrilateral ABCD intersect each other at the point O such that $\frac{AO}{BO}=\frac{CO}{DO}$ Show that ABCD is a trapezium.



Given: In quadrilateral ABCD, $\frac{AO}{BO}=\frac{CO}{DO}$

RTP : ABCD is a trapezium.

Construction: Through 'O' draw a line parallel to AB which meets DA at X.

Proof : In ΔDAB, XO || AB (by construction)

$$\Rightarrow\frac{AX}{XD}=\frac{BO}{OD}\quad \text{(by B. P. T)}\rightarrow(1)$$

But $\frac{AO}{BO}=\frac{CO}{DO}$ (given)

$$\Rightarrow\frac{AO}{CO}=\frac{BO}{DO}\rightarrow(2)$$

From (1) and (2)

$$\frac{AX}{XD}=\frac{AO}{CO}$$

In ΔADC, XO is a line such that $\frac{AX}{XD}=\frac{AO}{OC}$

⇒ XO ∥ DC (From converse of BPT)

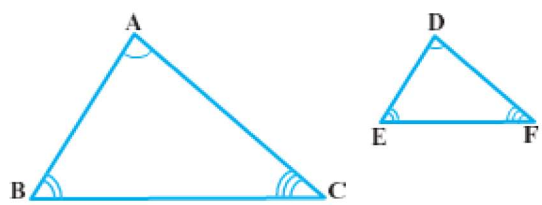
⇒ AB ∥ DC

In quadrilateral ABCD, AB || DC

⇒ ABCD is a trapezium

6.4 Criteria for Similarity of Triangles

Two triangles are similar, if (i) their correspondingangles are equal and (ii) their corresponding sides are in the same ratio (or proportion).



In Δ ABC andΔDEF, if

(i) ∠A = ∠D, ∠B = ∠E, ∠C =∠F and

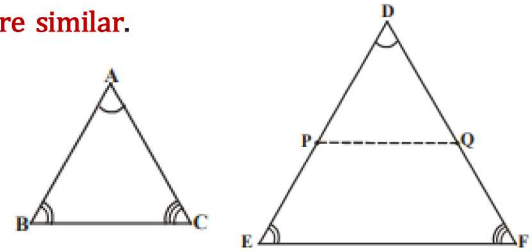
(ii) $\frac{AB}{DE}=\frac{BC}{EF}=\frac{CA}{FD}$, then the two triangles are similar .

We write the similarity of these two triangles as'ΔABC ~ Δ DEF' and read it as 'triangle ABC is similar to triangle DEF'.

The symbol '~' stands for 'is similar to'

Theorem 6.3 (AAA criterion for similarity of triangles)

If in two triangles, corresponding angles are equal, then their corresponding sides are in the same ratio (or proportion) and hence the two triangles are similar.



Given: : In triangles ABC and DEF,

$\angle A = \angle D$, $\angle B = \angle E$ and $\angle C = \angle F$

RTP : $\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$

Construction: Locate points P and Q on DE and DF respectively, such that AB = DP and AC = DQ. Join PQ.

Proof: *In* ΔABC , ΔDPQ

$AB = DP$ (Construction)

$AC = DQ$ (Construction)

$\angle A = \angle D$ (Given)

$\Delta ABC \cong \Delta DPQ$ (SAS congruency)

$\Rightarrow \angle B = \angle P$ (CPCT)

But $\angle B = \angle E$ (given)

$\therefore \angle P = \angle E \Rightarrow PQ \parallel EF$

$\Rightarrow \frac{DP}{PE} = \frac{DQ}{QF}$ (by BPT)

$\Rightarrow \frac{DP}{DP + PE} = \frac{DQ}{DQ + QF}$

$\Rightarrow \frac{DP}{DE} = \frac{DQ}{DF}$

$\Rightarrow \frac{AB}{DE} = \frac{AC}{DF}$

Similarly $\frac{AB}{DE} = \frac{BC}{EF}$

So $\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$

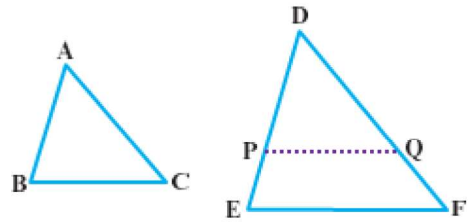
Hence proved.

AA similarity criterion for two triangles:

If two angles of one triangle are respectively equal to two angles of another triangle, then the two triangles are similar.

SSS (Side–Side–Side) similarity criterion for two triangles.

Theorem 6.4 : If in two triangles, sides of one triangle are proportional to (i.e., in the same ratio of) the sides of the other triangle, then their corresponding angles are equal and hence the two triangles are similar.



Given: ΔABC and ΔDEF are such that

$$\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF} (< 1)$$

RTP : $\angle A = \angle D$, $\angle B = \angle E$, $\angle C = \angle F$

Construction : Locate points P and Q on DE and DF respectively such that AB = DP and AC = DQ. Join PQ.

Proof: $\frac{AB}{DE} = \frac{AC}{DF}$ (Given)

$\Rightarrow \frac{DP}{DE} = \frac{DQ}{DF}$ (*from construction*)

$\Rightarrow PQ \parallel EF$ (*By converse of BPT*)

So $\angle P = \angle E$ and $\angle Q = \angle F$ (corresponding angles)

In $\Delta DPQ, \Delta DEF$ corresponding angles are equal

$\Delta DPQ \sim \Delta DEF$

$\Rightarrow \frac{DP}{DE} = \frac{DQ}{DF} = \frac{PQ}{EF}$

$\Rightarrow \frac{AB}{DE} = \frac{AC}{DF} = \frac{PQ}{EF}$

$\Rightarrow \frac{BC}{EF} = \frac{PQ}{EF} \Rightarrow BC = PQ$

In $\Delta ABC, \Delta DPQ$

AB = DP and AC = DQ (by construction)

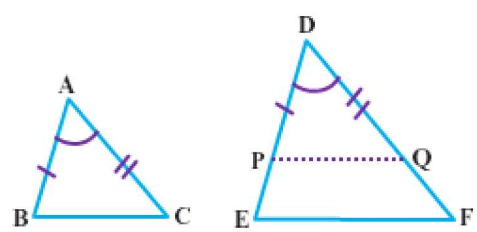
BC = PQ (proved)

$\Rightarrow \Delta ABC \cong \Delta DPQ$

So $\angle A = \angle D$, $\angle B = \angle P = \angle E$ and $\angle C = \angle Q = \angle F$ (CPCT)

SAS (Side–Angle–Side) similarity criterion for two triangles.

Theorem 6.5 : If one angle of a triangle is equal to one angle of the other triangle and the sides including these angles are proportional, then the two triangles are similar.



Given : In ΔABC and ΔDEF

$$\frac{AB}{DE} = \frac{AC}{DF} (< 1) \quad and \quad \angle A = \angle D$$

RTP : $\Delta ABC \sim \Delta DEF$

Construction: Locate points P and Q on DE and DF respectively such that AB = DP and AC = DQ. Join PQ.

Proof: $\frac{AB}{DE} = \frac{AC}{DF}$ (Given)

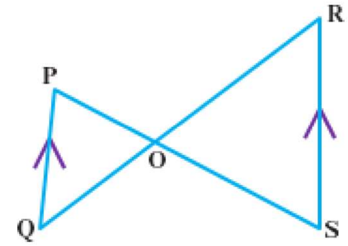
$\Rightarrow \frac{DP}{DE} = \frac{DQ}{DF}$ (*from construction*)

$\Rightarrow PQ \parallel EF$ and $\Delta ABC \cong \Delta DPQ$

So $\angle A = \angle D$, $\angle B = \angle P$, $\angle C = \angle Q$

$\Rightarrow$ So $\angle A = \angle D$, $\angle B = \angle E$, $\angle C = \angle F$

$\Rightarrow \Delta ABC \sim \Delta DEF$ (AAA similarity)



Example 4 : In Fig. 6.29, if PQ || RS, prove that Δ POQ ~ Δ SOR.

Solution : PQ || RS (Given)

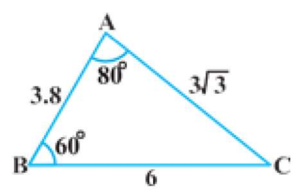
So, ∠ P = ∠ S (Alternate angles)

and ∠ Q = ∠ R (Alternate angles)

∠ POQ = ∠ SOR (Vertically opposite angles)

∴ Δ POQ ~ Δ SOR (AAA similarity criterion)

Example 5 : Observe adjacent figure and then find ∠ P



Solution : In ΔABC and ΔPQR,

$\frac{AB}{RQ} = \frac{3.8}{7.6} = \frac{1}{2}$; $\frac{BC}{PQ} = \frac{6}{12} = \frac{1}{2}$; $\frac{CA}{PR} = \frac{3\sqrt{3}}{6\sqrt{3}} = \frac{1}{2}$

$\frac{AB}{RQ} = \frac{BC}{PQ} = \frac{CA}{PR}$

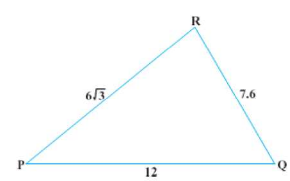
Δ ABC ~ ΔRQP (By SSS similarity)

∠C =∠P (Corresponding angles of similar triangles CAST)

∠ C = 180° – ∠ A – ∠B(Angle sum property)

= 180° – 80° – 60° = 40°

∠P = 40°



Example 6 : In Fig. 6.31, OA . OB = OC . OD. Show that ∠ A = ∠ C and ∠ B = ∠ D.

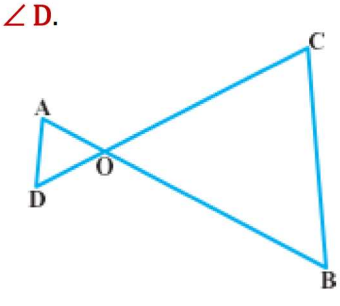
Solution : OA .OB = OC .OD (Given)

$\frac{OA}{OC} = \frac{OD}{OB}$

∠ AOD = ∠ COB (Vertically opposite angles)

Δ AOD ~ Δ COB (SAS similarity criterion)

∠ A = ∠ C and ∠ D = ∠ B (Corresponding angles of similar triangles)



Example 7 : A girl of height 90 cm is walking away from the base of a lamp-post at a speed of 1.2 m/s. If the lamp is 3.6 m above the ground, find the length of her shadow after 4 seconds.

Solution :

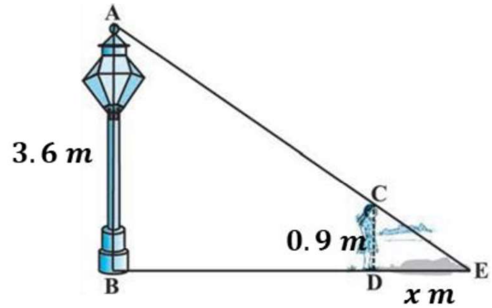
Lamp post (AB)=3.6 m

Height of girl (CD)=90cm=0.9 m

Length of shadow=DE=$x\ m$

Distance from pole to girl(BD)=$speed \times time$

$= 1.2 \times 4 = 4.8\ m$



In ΔABE and ΔCDE

$\angle E = \angle E$ (commom)

$\angle D = \angle B = 90^0$(lamp − post as well as the girl are standing vertical to the ground)

$\Delta ABE \sim \Delta CDE(\ AA\ similarity)$

$\frac{BE}{DE} = \frac{AB}{CD}$

$\frac{BD + DE}{DE} = \frac{AB}{CD}$

$\frac{4.8 + x}{x} = \frac{3.6}{0.9} = 4$

$4x = 4.8 + x$

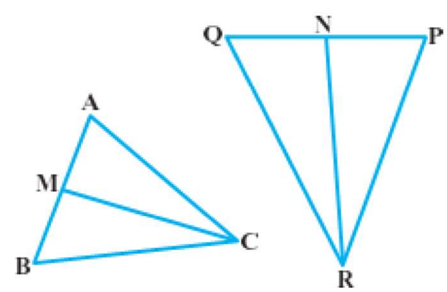
$3x = 4.8$

$x = \frac{4.8}{3} = 1.6$

The length of shadow of girl after 4 seconds=1.6 m

Example 8 : In Fig. 6.33, CM and RN are respectively the medians of Δ ABC and Δ PQR. If Δ ABC ~ Δ PQR, prove that:

(*i*) ΔCMB ~ ΔRNQ (ii) $\frac{CM}{RN} = \frac{AB}{PQ}$



Sol: $\Delta ABC \sim \Delta PQR$

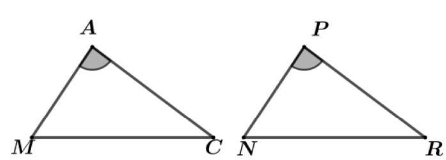
$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR} \rightarrow (1)$

CM and RN are medians of similar triangles ΔABC and ΔPQR

$AM = MB = \frac{AB}{2} \Rightarrow AB = 2AM = 2MB \rightarrow (2)$

$PN = NQ = \frac{PQ}{2} \Rightarrow PQ = 2PN = 2NQ \rightarrow (3)$

From (1),(2) and (3)



$\frac{2\text{AM}}{2\text{PN}} = \frac{\text{AC}}{\text{PR}} \Rightarrow \frac{\text{AM}}{\text{PN}} = \frac{\text{AC}}{\text{PR}} \rightarrow (4)$

In ΔAMC , ΔPNR

$\angle\text{A} = \angle\text{P}$ (CAST)

$\frac{\text{AM}}{\text{PN}} = \frac{\text{AC}}{\text{PR}} \ \left(from\ (4)\right)$

$\therefore \Delta\text{AMC} \sim \Delta\text{PNR}$ (SAS similarity)-→(i)

$\frac{\text{CM}}{\text{RN}} = \frac{\text{AC}}{\text{PR}}$

$But\ \frac{\text{AC}}{\text{PR}} = \frac{\text{AB}}{\text{PQ}}\ (from\ (1))$

$\frac{\text{CM}}{\text{RN}} = \frac{\text{AB}}{\text{PQ}} \rightarrow (ii)$

$A\ gain\ \frac{\text{AB}}{\text{PQ}} = \frac{\text{BC}}{\text{QR}}\left(from\ (1)\right)$

$\frac{\text{CM}}{\text{RN}} = \frac{\text{BC}}{\text{QR}}$

$Also\ \frac{\text{CM}}{\text{RN}} = \frac{\text{AB}}{\text{PQ}} = \frac{2BM}{2QN}$

$i.e.,\frac{\text{CM}}{\text{RN}} = \frac{BM}{QN}$

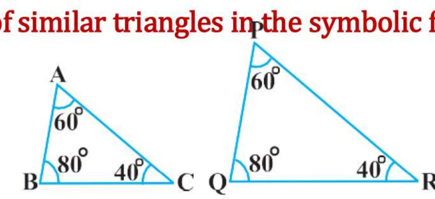
$\frac{\text{CM}}{\text{RN}} = \frac{\text{BC}}{\text{QR}} = \frac{BM}{QN}$

$\Delta\text{CMB} \sim \Delta\text{RNQ}$ (SSS similarity)

EXERCISE 6.3

1. **State which pairs of triangles in Fig. 6.34 are similar. Write the similarity criterion used by you for answering the question and also write the pairs of similar triangles in the symbolic form :**

(i)



(i)

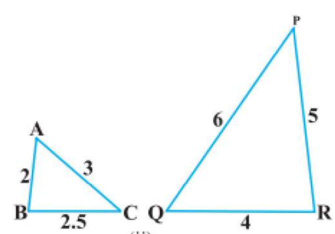
Sol: $\angle A = \angle P = 60^0$

$\angle B = \angle Q = 80^0$

$\angle C = \angle R = 40^0$

$\Delta ABC \sim \Delta PQR(\ AAA\ similarity)$

$(\boldsymbol{ii})$



(ii)

Sol: $\frac{AB}{QR} = \frac{2}{4} = \frac{1}{2}$

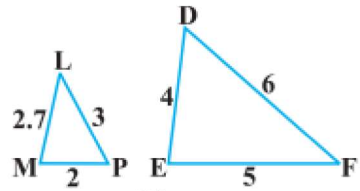
$\frac{BC}{RP} = \frac{2.5}{5} = \frac{1}{2}$

$\frac{CA}{PR} = \frac{3}{6} = \frac{1}{2}$

$\frac{AB}{QR} = \frac{BC}{RP} = \frac{CA}{PR}$

$\Delta ABC \sim \Delta QRP(\,SSS\ similarity)$

(*iii*)



(iii)

Sol: $\frac{MP}{ED} = \frac{2}{4} = \frac{1}{2}$

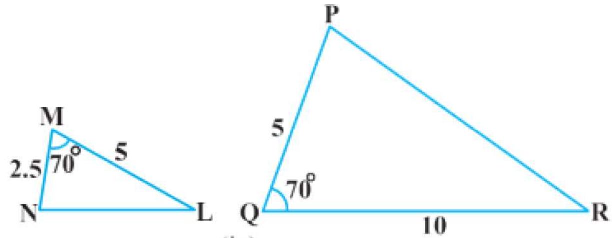
$\frac{PL}{DF} = \frac{3}{6} = \frac{1}{2}$

$\frac{LM}{FE} = \frac{2.7}{5} = \frac{27}{50}$

$\frac{MP}{ED} = \frac{PL}{DF} \neq \frac{LM}{FE}$

$\Delta MPL, \Delta EDF\ are\ not\ similar.$

(iv)



(iv)

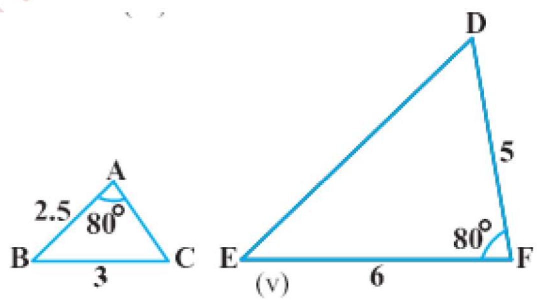
Sol: $\frac{MN}{QP} = \frac{2.5}{5} = \frac{1}{2}$

$\frac{ML}{QR} = \frac{5}{10} = \frac{1}{2}$

$\angle M = \angle Q = 70^0$

$\Delta MNL \sim \Delta QPR(\,SAS\ similarity)$

(v)



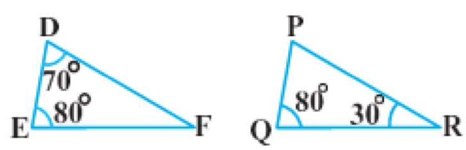
(v)

Don't say they are similar or not.

If AC=3 and DE=6 then

ΔABC~ΔFDE (SSS or SAS similarity)

(vi)



(vi)

Sol: $\angle D + \angle E + \angle F = 180^0$(angle sum property)

$70^0 + 80^0 + \angle F = 180^0$

$\angle F = 180^0 - 150^0 = 30^0$

$\angle E = \angle Q = 80^0$

$\angle F = \angle R = 30^0$

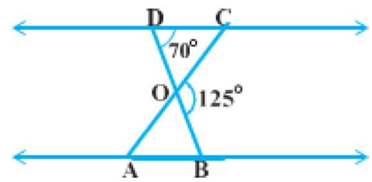
ΔDEF~ΔPQR(AA similarity)

2. In adjacent Fig Δ ODC ~ Δ OBA, ∠ BOC = 125° and ∠ CDO = 70°. Find ∠ DOC, ∠ DCO and ∠ OAB.

Sol: ∠DOC + ∠ COB = 180° (Linear pair)

⇒ ∠DOC = 180° – 125° (Given, ∠ BOC = 125°)

$\Rightarrow \angle DOC = 55°$



$\angle DCO + \angle CDO + \angle DOC = 180°$ (Angle sum property)

$\Rightarrow \angle DCO + 70° + 55° = 180°$

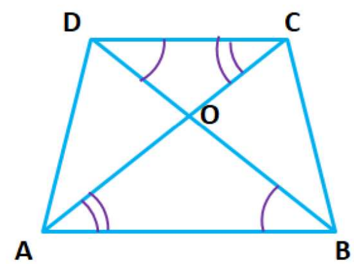
$\Rightarrow \angle DCO = 55°$

It is given that, $\Delta ODC \sim \Delta OBA$

$\angle OAB = \angle OCD$(Corresponding angles of similar triangles are equal)

$\Rightarrow \angle OAB = 55°$

3. Diagonals AC and BD of a trapezium ABCD with AB || DC intersect each other at the point O. Using a similarity criterion for two triangles, show that $\frac{OA}{OC} = \frac{OB}{OD}$



Sol: In ΔAOB and ΔCOD

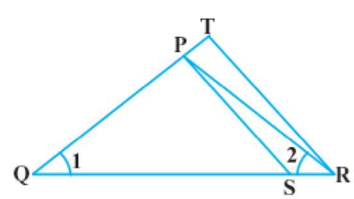
$\angle OAB = \angle OCD$ (AB || DC, alternate interior angles)

$\angle OBA = \angle ODC$ (AB || DC, alternate interior angles)

$\Delta AOB \sim \Delta COD$(AA similarity)

$\frac{OA}{OC} = \frac{OB}{OD}$ (corresponding sides are propertional)

4. In adjacent Fig. , $\frac{QR}{QS} = \frac{QT}{PR}$ = and $\angle 1 = \angle 2$. Show that $\Delta PQS \sim \Delta TQR$.



Sol: In ΔPQR, $\angle PQR = \angle PRQ$

$\Rightarrow PQ = PR$(Sides opposite to equal angles are equal)$\rightarrow$(1)

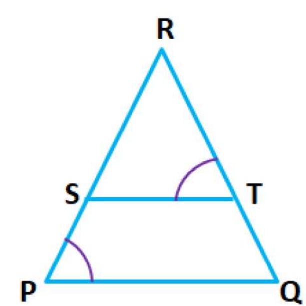
$Given, \frac{QR}{QS} = \frac{QT}{PR} \Rightarrow \frac{QR}{QS} = \frac{QT}{QP}$ (from (1))$\rightarrow$(2)

In ΔPQS and ΔTQR

$\frac{QR}{QS} = \frac{QT}{QP}$ and $\angle Q = \angle Q$

$\therefore \Delta PQS \sim \Delta TQR$ [By SAS similarity criterion]

5. S and T are points on sides PR and QR of Δ PQR such that $\angle P = \angle RTS$. Show that $\Delta RPQ \sim \Delta RTS$.



Sol: In ΔRPQ and ΔRTS

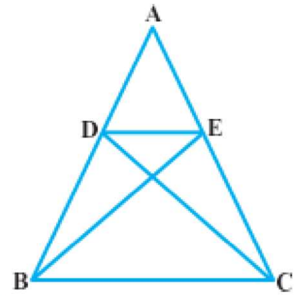
$\angle QPR = \angle RTS$ (Given)

$\angle R = \angle R$ (Common angle)

$\therefore \Delta RPQ \sim \Delta RTS$ (AA similarity criterion)

6. In Fig. 6.37, if Δ ABE ≅ Δ ACD, show that Δ ADE ~ Δ ABC.

Sol: Given, ΔABE ≅ ΔACD.



AB=AC and AE=AD (By CPCT)→(1)

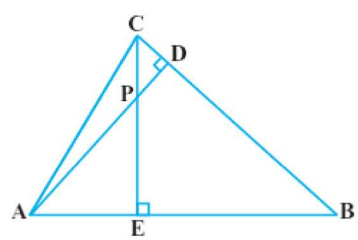
In ΔADE and ΔABC

$\frac{AD}{AB} = \frac{AE}{EC}$ $(from\ (1))$

$\angle A = \angle A$ [Common angle]

$\therefore \Delta ADE \sim \Delta ABC$ [SAS similarity criterion]

7. In adjacent Fig., altitudes AD and CE of Δ ABC intersect each other at the point P. Show that: (i) Δ AEP ~ Δ CDP (ii) Δ ABD ~ Δ CBE (iii) Δ AEP ~ Δ ADB (iv) Δ PDC ~ Δ BEC



Sol:(i) In ΔAEP and ΔCDP,

$\angle AEP = \angle CDP$ (90° each)

$\angle APE = \angle CPD$ (Vertically opposite angles)

$\Delta AEP \sim \Delta CDP$ (by AA similarity criterion)

(ii) In ΔABD and ΔCBE,

$\angle ADB = \angle CEB$ (90° each)

$\angle ABD = \angle CBE$ (Common Angles)

$\therefore \Delta ABD \sim \Delta CBE$(by AA similarity criterion)

(iii) In ΔAEP and ΔADB,

$\angle AEP = \angle ADB$ (90° each)

$\angle PAE = \angle DAB$ (Common Angles)

$\therefore \Delta AEP \sim \Delta ADB$(by AA similarity criterion)

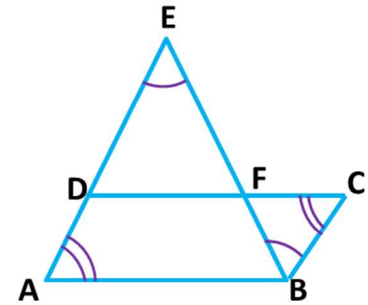
(iv) In ΔPDC and ΔBEC,

$\angle PDC = \angle BEC$ (90° each)

$\angle PCD = \angle BCE$ (Common angles)

$\therefore \Delta PDC \sim \Delta BEC$(by AA similarity criterion)

8. E is a point on the side AD produced of a parallelogram ABCD and BE intersects CD at F. Show that $\Delta ABE \sim \Delta CFB$



Sol: In ΔABE and ΔCFB

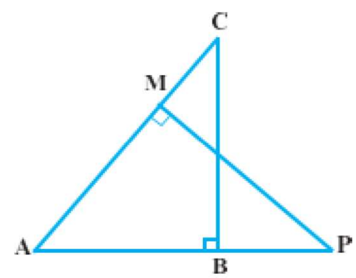
$\angle A = \angle C$ (Opposite angles of a parallelogram)

$\angle AEB = \angle CBF$ (AE || BC ,alternate interior angles as)

$\therefore \Delta ABE \sim \Delta CFB$ (AA similarity criterion)

9. In Fig. 6.39, ABC and AMP are two right triangles, right angled at B and M respectively. Prove that:

$(i) \Delta ABC \sim \Delta AMP (ii) \frac{CA}{PA} = \frac{BC}{MP}$



Sol: (i) In ΔABC and ΔAMP

$\angle BAC = \angle MAP$ (common angles)

$\angle ABC = \angle AMP = 90°$

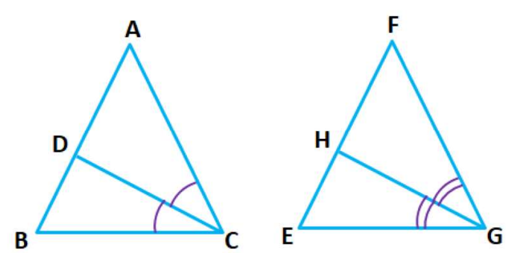
$\therefore \Delta ABC \sim \Delta AMP$ (AA similarity criterion)

(ii) $\Delta ABC \sim \Delta AMP$ (from(i))

$$\frac{CA}{PA} = \frac{BC}{MP} \text{ (Ratio of corresponding sides are equal in similar triangles)}$$

10. CD and GH are respectively the bisectors of ΔACB and ΔEGF such that D and H lie on sides AB and FE of ΔABC and ΔEFG respectively. If $\Delta ABC \sim \Delta FEG$, show that:

$(i) \frac{CD}{GH} = \frac{AC}{FG} (ii) \Delta DCB \sim \Delta HGE (iii) \Delta DCA \sim \Delta HGF$



Sol: (i) Given $\Delta ABC \sim \Delta FEG$.

$\therefore \angle A = \angle F, \angle B = \angle E$, and $\angle ACB = \angle FGE$ (Corresponding angles of similar triangles)$\rightarrow$(1)

$\therefore \angle ACD = \angle FGH$ and $\angle DCB = \angle HGE$ (Angle bisector)$\rightarrow$(2)

In ΔACD and ΔFGH,

$\angle ACD = \angle FGH$ (from(2))

$\angle A = \angle F$ (from(1))

∴ ΔACD ~ ΔFGH (AA similarity criterion)

$\Rightarrow \frac{CD}{GH} = \frac{AC}{FG}$

(ii) In ΔDCB and ΔHGE,

$\angle DCB = \angle HGE$ (from (2))

$\angle B = \angle E$ (from (1))

∴ ΔDCB ~ ΔHGE (AA similarity criterion)

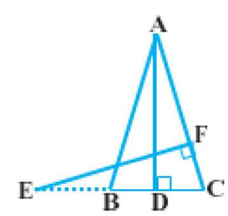
(iii) In ΔDCA and ΔHGF,

$\angle ACD = \angle FGH$ (from (2))

$\angle A = \angle F$ (from (1))

∴ ΔDCA ~ ΔHGF (AA similarity criterion)

11. In adjacent Fig., E is a point on side CB produced of an isosceles triangle ABC with AB = AC. If AD ⊥ BC and EF ⊥ AC, prove that Δ ABD ~ Δ ECF.



Sol: Given, ABC is an isosceles triangle.

∴ AB = AC

⇒ $\angle ABD = \angle ECF$ (Angles opposite to equal sides)→(1)

In ΔABD and ΔECF,

$\angle ADB = \angle EFC = 90°$

$\angle ABD = \angle ECF$(from (1))

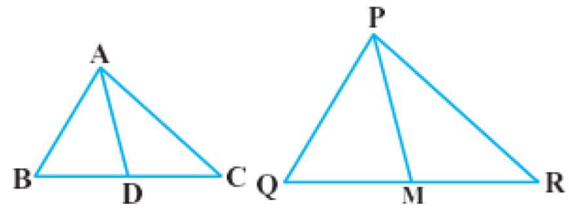
∴ ΔABD ~ ΔECF (By AA similarity)

12. Sides AB and BC and median AD of a triangle ABC are respectively proportional to sides PQ and QR and median PM of ΔPQR (see Fig. 6.41). Show that ΔABC ~ Δ PQR.

Sol: AD and PM are medians of ΔABC and Δ PQR

BC = 2BD = 2DC and QR = 2QM = 2MR→(1)

Given $\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AD}{PM} \rightarrow (2)$



$\Rightarrow \frac{AB}{PQ} = \frac{2BD}{2QM} = \frac{AD}{PM}\ (from(1))$

$\Rightarrow \frac{AB}{PQ} = \frac{BD}{QM} = \frac{AD}{PM}$

$\Rightarrow \Delta ABD \sim \Delta PQM$ [SSS similarity criterion]

$\therefore \angle ABD = \angle PQM$ [Corresponding angles of two similar triangles are equal]

$\Rightarrow \angle ABC = \angle PQR \rightarrow (3)$

In ΔABC and ΔPQR

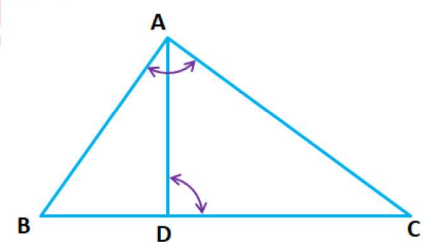
$\frac{AB}{PQ} = \frac{BC}{QR}\ (from(2))$

$\angle ABC = \angle PQR\ (from(3))$

$\Delta ABC \sim \Delta PQR$ [SAS similarity criterion]

13. D is a point on the side BC of a triangleABC such that $\angle ADC = \angle BAC$. Show that $CA^2 = CB.CD$

Sol: In ΔADC and ΔBAC,



$\angle ADC = \angle BAC$ (given)

$\angle ACD = \angle BCA$ (Common angles)

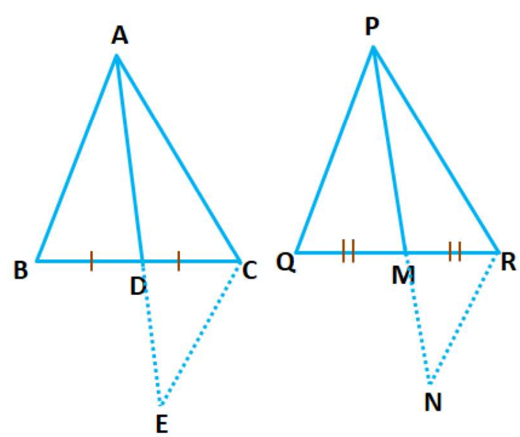
$\therefore \Delta ADC \sim \Delta BAC$ (AA similarity criterion)

$\frac{CA}{CB} = \frac{CD}{CA}$ (corresponding sides of similar triangles are in proportion)

$\Rightarrow CA^2 = CB \times CD$

14. Sides AB and AC and median AD of a triangle ABC are respectively proportional to sides PQ and PR and median PM of another triangle PQR. Show that $\Delta ABC \sim \Delta PQR$.

Sol: Produce AD to E so that AD = DE. Join CE, Similarly produce PM to N such that PM = MN, also Join RN.



In ΔABD and ΔCDE

AD = DE (By Construction.)

BD = DC (AD is the median)

$\angle ADB = \angle CDE$ (Vertically opposite angles)

$\therefore \Delta ABD \cong \Delta CDE$ (SAS criterion of congruence)

$\Rightarrow AB = CE$ (By CPCT)$\rightarrow(1)$

In ΔPQM and ΔMNR,

PM = MN (By Construction)

QM = MR (PM is the median)

$\angle PMQ = \angle NMR$ (Vertically opposite angles)

$\therefore \Delta PQM = \Delta MNR$ (SAS criterion of congruence)

$\Rightarrow PQ = RN$ [By CPCT]$\rightarrow$(2)

$Given\ \frac{AB}{PQ} = \frac{AC}{PR} = \frac{AD}{PM} \rightarrow(3)$

From (1),(2) and (3)

$\Rightarrow \frac{CE}{RN} = \frac{AC}{PR} = \frac{2AD}{2PM}$

$\Rightarrow \frac{CE}{RN} = \frac{AC}{PR} = \frac{AE}{PN}$

$\therefore \Delta ACE \sim \Delta PRN$ (SSS similarity)

$\angle CAE = \angle RPN$ (corresponding angles of similar triangles are equal)

$\Rightarrow \angle CAD = \angle PRM$

Similarly $\angle BAD = \angle QRM$

$\Rightarrow \angle CAD + \angle BAD = \angle PRM + \angle QRM$

$\Rightarrow \angle BAC = \angle QPR \rightarrow(4)$

In ΔABC and ΔPQR

$\frac{AB}{PQ} = \frac{AC}{PR}\ (Given)$

$\angle BAC = \angle QPR$(From(4))

$\therefore \Delta ABC \sim \Delta PQR$ (SAS similarity criterion)

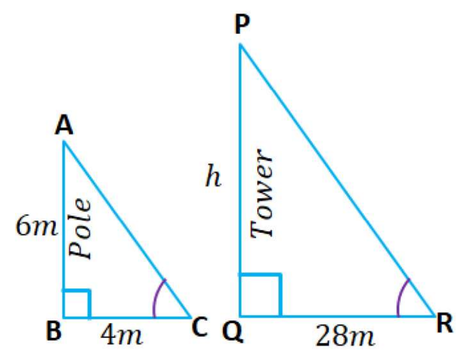
15. A vertical pole of length 6 m casts a shadow 4 m long on the ground and at the same time a tower casts a shadow 28 m long. Find the height of the tower.

Sol: Length of pole (AB)=6 m

Length of shadow of pole (BC)=4m

Let Height of tower (PQ)=h m

Length of shadow of the tower (QR)=28 m



In ΔABC and ΔPQR,

$\angle B = \angle Q = 90°$

$\angle C = \angle R$ (angular elevation of sun)

$\therefore \Delta ABC \sim \Delta PQR$ (AA similarity criterion)

$\frac{AB}{PQ} = \frac{BC}{QR}$(In similar trianglescorresponding sides are proportional)

$\frac{6}{h} = \frac{4}{28} \Rightarrow h = \frac{6 \times 28}{4} = 6 \times 7 = 42$

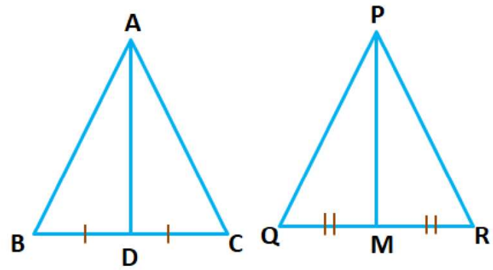
∴ The height of the tower is 42 m.

16. If AD and PM are medians of triangles ABC and PQR, respectively where $\Delta ABC \sim \Delta PQR$, prove that $\frac{AB}{PQ} = \frac{AD}{PM}$

Sol: $\Delta ABC \sim \Delta PQR$

In similar triangles corresponding angles are equal and corresponding sides are proportional

$\Rightarrow \angle A = \angle P, \angle B = \angle Q, \angle C = \angle R$ and



$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR} \rightarrow (1)$

Since AD and PM are medians

BC=2BD and QR=2QM

In ΔABD and ΔPQM,

$\angle B = \angle Q$ (from (1))

$\frac{AB}{PQ} = \frac{BC}{QR}$ (from(1))

$\Rightarrow \frac{AB}{PQ} = \frac{2BD}{2QM}$

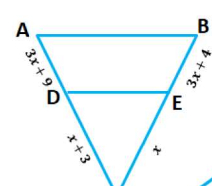
$\Rightarrow \frac{AB}{PQ} = \frac{BD}{QM}$

∴ ΔABD ~ ΔPQM (SAS similarity criterion)

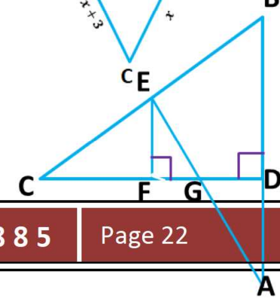
$\Rightarrow \frac{AB}{PQ} = \frac{AD}{PM} (CSST)$

Some problems for student brain boosting

1. Legs (sides other than the hypotenuse) of a right triangle are of lengths 16cm and 8 cm. Find the length of the side of the largest square that can be inscribed in the triangle.



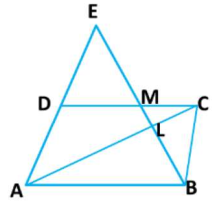
2. Find the value of x for which DE||AB in adjacent Fig.,



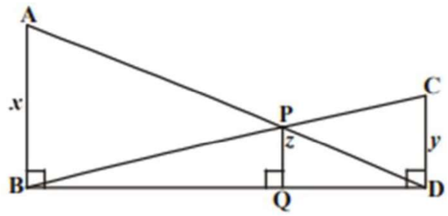
3. If $\Delta ABC \sim \Delta DEF$, AB = 4 cm, DE = 6 cm, EF = 9 cm and FD = 12 cm, find the perimeter of ΔABC

4. In the given figure ,CD is the perpendicular bisector of AB. EF is perpendicular to CD. AE intersects CD at G. Prove that $\frac{CF}{CD} = \frac{FG}{DG}$ (CBSE-2023)

5. In the given figure, ABCD is a parallelogram.BE bisectsCD at M and intersects Ac at L. Prove that EL=2BL



6. In adjacent fig., PA,QB and RC are each perpendicular to AC. If x=8 cm and z=6 cm, then find y.

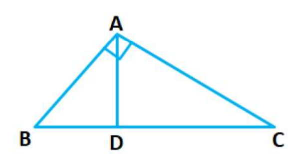


Answers:1)16/3 cm 2)2 3)18 cm 6)24/7 cm

MCQ

1. D and E are respectively the points on the sides AB and AC of a triangle ABC such that AD = 2 cm, BD = 3 cm, BC = 7.5 cm and DE BC. Then, length of DE (in cm) is
   (A) 2.5 (B) 3 (C) 5 (D) 6

2. In Fig. 6.2, ∠BAC = 90° and AD ———————— BC. Then,
   (A) BD.CD = BC² (B) AB.AC = BC² (C) BD.CD = AD² (D) AB.AC = AD²



3. If ΔA B C ~ ΔE D F and ΔA B C is not similar to ΔD E F, then which of the following is not true?
   (A) BC.EF = A C.FD (B) AB.EF = AC.DE (C) BC.DE = AB.EF (D) BC.DE = AB.FD

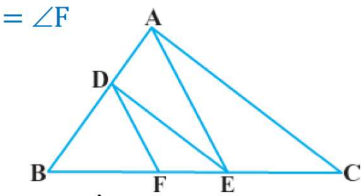
4. If in two triangles ABC and PQR$\frac{AB}{QR} = \frac{BC}{PR} = \frac{CA}{PQ}$ then
   (A) ΔPQR ~ ΔCAB (B) ΔPQR ~ ΔABC(C) ΔCBA ~ ΔPQR (D) ΔBCA ~ ΔPQR

5. If in two triangles DEF and PQR, D = Q and R = E, then which of the following is not true?
   (A) $\frac{EF}{PR} = \frac{DF}{PQ}$ (B) $\frac{DE}{PQ} = \frac{EF}{RP}$ (C) $\frac{DE}{QR} = \frac{DF}{PQ}$ (D) $\frac{EF}{RP} = \frac{DE}{QR}$

6. It is given that ΔABC ~ ΔDFE, ∠A =30°,∠C = 50°, AB = 5 cm, AC = 8 cm and DF= 7.5 cm. Then, the following is true:
   (A) DE = 12 cm, ∠F = 50° (B) DE = 12 cm, ∠F = 100° (C) EF = 12 cm, ∠D = 100° (D) EF = 12 cm, ∠D = 30°

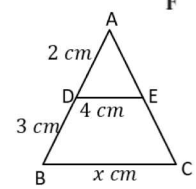
7. If in triangles ABC and DEF, $\frac{AB}{DE} = \frac{BC}{FD}$, then they will be similar, when

(A) ∠B = ∠E (B) ∠A = ∠D (C) ∠B = ∠D (D) ∠A = ∠F



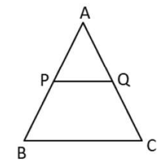
8. In ΔABC and ΔDEF, $\frac{AB}{DE}=\frac{BC}{FD}$. Which of the following makes the two triangles similar?(CBSE-2023)

(A) ∠A=∠D (B) ∠B=∠D (C) ∠B=∠E (D) ∠A=∠F



9. In the given figure, DE||BC. The value of x is

(A) 6 (B) 12.5 (C) 8 (D) 10



10. In ΔABC,PQ||BC. If PB=6 cm, AP=4 cm, AQ=8 cm, find the length of AC.

(A) 12 cm (B) 20 cm (C) 6 cm (D) 14 cm

11. **Assertion (A):** The sides of two similar triangles are in the ratio 2 : 5, then the areas of these triangles are in the

ratio 4 : 25.

Reason (R): The ratio of the areas of two similar triangles is equal to the square of the ratio of their sides.

12. **Assertion (A):** If two sides of a right angle are 7 cm and 8 cm, then its third side will be 9 cm.

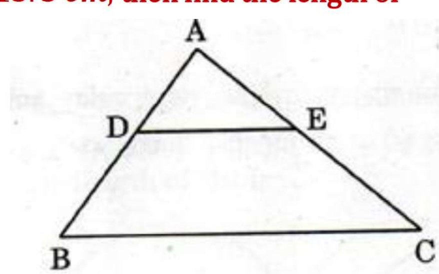
Reason (R): In a right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

13.

1)B	2)C	3)C	4)A	5)D	6)	7)	8)	8)B	9)D	10)B	11)A	12)D	

Previous year problems:

1. **In ΔABC , $DE \parallel BC$. If $AD = 4\ cm$, $AB = 9\ cm$ and $AC = 13.5\ cm$, then find the length of EC?[CBSE-2024]**



Sol: In ΔABC , $DE \parallel BC$

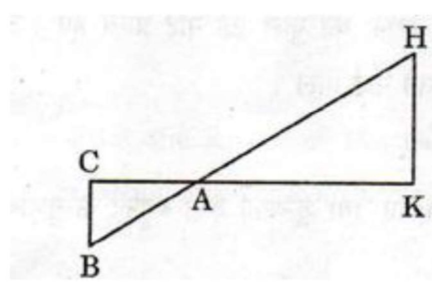
$\frac{AB}{AD}=\frac{AC}{AE}$ (By basic propertionality theorem)

$\frac{9}{4}=\frac{13.5}{AE}$

$AE=\frac{4}{9}\times 13.5 = 4\times 1.5 = 6\ cm$

$EC = AC - AE = 13.5 - 6 = 7.5\ cm$

2. **In the given figure , $\Delta AHK \sim \Delta ABC$. If AK=8 cm,BC=3.2cm and HK=6.4 cm, then find the length of AC.[CBSE-2024]**



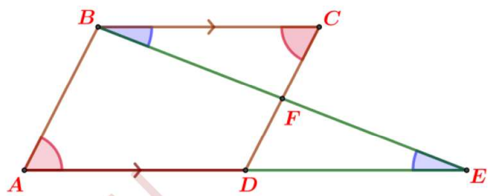
Sol: In the given figure , $\Delta AHK \sim \Delta ABC$

$$\frac{AC}{AK} = \frac{BC}{HK}\ (corresponding\ sides\ are\ proportional)$$

$$\frac{AC}{8} = \frac{3.2}{6.4}$$

$$AC = \frac{32}{64} \times 8 = 4\ cm$$

3. E is a point on the side AD produced of a parallelogram ABCD and BE intersects CD at EF. Show that $\triangle ABE \sim \Delta CFB$ [CBSE-2024]



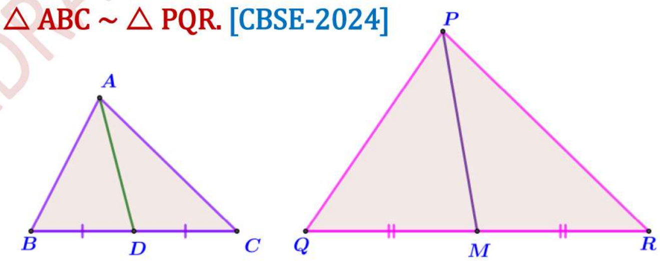
Sol: $In \triangle ABE, \Delta CFB$

$\angle A = \angle C$ (opposite angles of a parallelogram)

$\angle AEB = \angle CBF$ (Alternate interior angles)

$\Delta ABE \sim \Delta CFB$ (AA similarity)

4. Sides AB, BC and the median AD of △ ABC are respectively proportional to sides PQ, QR and the median PM of another △ PQR. Prove that △ ABC ~ △ PQR. [CBSE-2024]



5.

Sol: Given

$$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AD}{PM}$$

$$\frac{AB}{PQ} = \frac{2BD}{2QM} = \frac{AD}{PM}\ (D, M\ are\ midpoints\ of\ Bc\ and\ QR$$

$$\frac{AB}{PQ} = \frac{BD}{QM} = \frac{AD}{PM}$$

$\Delta ABD \sim \Delta PQM$

$\angle ABD = \angle PQM (\ CAST)$

$\angle ABC = \angle PQR \rightarrow (1)$

$In\ \Delta ABC, \Delta PQR$

$$\frac{AB}{PQ} = \frac{BC}{QR}(Given)$$

$$\angle ABC = \angle PQR\big(from(1)\big)$$

$$\Delta ABC \sim \Delta PQR(SAS\ similarity\ criterion)$$