CHAPTER 5

X-MATHEMATICS-NCERT-2024-25

ARITHMETIC PROGRESSIONS

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Arithmetic Progressions

1. An arithmetic progression is a list of numbers in which each term is obtained by adding a fixed number to the preceding term except the first term.
2. This fixed number is called the common difference of the AP it is denoted by d
3. The first term is denoted by a_1, $second\ term = a_2$, $third\ term = a_3, \dots \dots ..$
4. **General form of AP**: $\boldsymbol{a, a+d, a+2d, a+3d} \dots \dots$
5. $\boldsymbol{a_1} = a$; $\boldsymbol{a_2} = a+d$; $\boldsymbol{a_3} = a+2d$; $\boldsymbol{a_7} = a+6d$; $\boldsymbol{a_{13}} = a+12d$; $\boldsymbol{a_{20}} = a+19d$; ...
6. $\boldsymbol{n^{th}}$ **term of AP** : $a_n = a + (n-1)d$
7. $If\ \boldsymbol{a, b, c\ are\ in\ AP}\ then\ b - a = c - b \Rightarrow 2b = a + c \Rightarrow b = \frac{a+c}{2}$
8. In the list of numbers $a_1, a_2, a_3, a_4, ..$ if the differences $a_2 - a_1, a_3 - a_2, a_4 - a_3, \dots .$

give the same value, i.e., if $a_{k+1} - a_k$ is the same for different values of k, then the given list of numbers is an AP.

Example1: **For the AP $\frac{3}{2}, \frac{1}{2}, \frac{-1}{2}, \frac{-3}{2}, \dots \dots$ write the first term a and the common difference d.**

Sol: $a_1 = \frac{3}{2}, a_2 = \frac{1}{2}, a_3 = \frac{-1}{2}, a_4 = \frac{-3}{2}, \dots$

First term $= a = a_1 = \frac{3}{2}$

Common difference $= d = a_2 - a_1 = \frac{1}{2} - \frac{3}{2} = \frac{1-3}{2} = \frac{-2}{2} = -1$

Example 2 : Which of the following list of numbers form an AP? If they form an AP, write the next two terms:

(i) 4, 10, 16, 22, . . .

Sol: $a_1 = 4, a_2 = 10, a_3 = 16, a_4 = 22$

$a_2 - a_1 = 10 - 4 = 6$

$a_3 - a_2 = 16 - 10 = 6$

$a_4 - a_3 = 22 - 16 = 6$

$a_2 - a_1 = a_3 - a_2 = a_4 - a_3 = \cdots$

i.e., $a_{k+1} - a_k$ is same every time

So, the given list of numbers forms an AP with the common difference $d = 6$

The next two terms are: $22 + 6 = 28$ and $28 + 6 = 34$

(ii) 1, – 1, – 3, – 5, . . .

Sol: $a_1 = 1, a_2 = -1, a_3 = -3, a_4 = -5, ..$

$a_2 - a_1 = -1 - 1 = -2$

$a_3 - a_2 = -3 - (-1) = -3 + 1 = -2$

$a_4 - a_3 = -5 - (-3) = -5 + 3 = -2$

$a_2 - a_1 = a_3 - a_2 = a_4 - a_3 = \cdots$

i.e., $a_{k+1} - a_k$ is same every time

So, the given list of numbers forms an AP with the common difference $d = -2$

The next two terms are: $-5 - 2 = -7$ *and* $-7 - 2 = -9$

(iii) – 2, 2, – 2, 2, – 2, . . .

Sol: $a_1 = -2, a_2 = 2, a_3 = -2, a_4 = 2, a_5 = -2$

$a_2 - a_1 = 2 - (-2) = 2 + 2 = 4$

$a_3 - a_2 = -2 - 2 = -4$

$a_2 - a_1 \neq a_3 - a_2$

So, the given list of numbers does not form an AP.

(iv) 1, 1, 1, 2, 2, 2, 3, 3, 3, . . .

Sol: $a_1 = 1, a_2 = 1, a_3 1, = a_4 = 2,$

$a_2 - a_1 = 1 - 1 = 0$

$a_3 - a_2 = 1 - 1 = 0$

$a_4 - a_3 = 2 - 1 = 1$

$a_3 - a_2 \neq a_4 - a_3$

So, the given list of numbers does not form an AP.

EXERCISE 5.1

1. **In which of the following situations, does the list of numbers involved make an arithmetic progression, and why?**

(i) The taxi fare after each km when the fare is ₹ 15 for the first km and ₹ 8 for each additional km.

Sol: Taxi fare for first km= ₹15

Taxi fare for second km= ₹15+₹8=₹23

Taxi fare for third km=₹23+₹8=₹31

Taxi fare for fourth km=₹31+₹8=₹39

∴ The taxi fares are ₹15, ₹23, ₹31, ₹39,……

$a_2 - a_1 = a_3 - a_2 = a_4 - a_3 = \cdots = 8$

It is an arithmetic progression with common difference=8

(ii) The amount of air present in a cylinder when a vacuum pump removes 1/ 4 of the air remaining in the cylinder at a time.

Sol: let the amount of air present in cylinder=x

(If a vacuum pump removes $\frac{1}{4}$ of the air then the remaining air is $\frac{3}{4}$ of it)

When vacuum pump use first time remaining air $= \frac{3}{4} \times x = \frac{3x}{4}$

Vacuum pump use second time remaining air $= \frac{3}{4} \times \frac{3x}{4} = \frac{9x}{16}$

Vacuum pump use third time remaining air $= \frac{3}{4} \times \frac{9x}{16} = \frac{27x}{64}$

List of air present in cylinder is $x, \frac{3x}{4}, \frac{9x}{16}, \frac{27x}{64}, \ldots$

$a_2 - a_1 = \frac{3x}{4} - x = \frac{3x - 4x}{4} = \frac{-x}{4}$

$a_3 - a_2 = \frac{9x}{16} - \frac{3x}{4} = \frac{9x - 12x}{16} = \frac{-3x}{16}$

$a_2 - a_1 \neq a_3 - a_2$

So, the given list of numbers does not form an AP.

(iii) The cost of digging a well after every metre of digging, when it costs ₹ 150 for the first metre and rises by ₹ 50 for each subsequent metre.

Sol: The cost of digging for one metre=₹150

The cost of digging for two metres=₹150+₹50=₹200

The cost of digging for three metres=₹200+₹50=₹250

The cost of digging for four metres=₹250+₹50=₹300

The costs are ₹150, ₹200, ₹250, ₹300,….

$a_2 - a_1 = a_3 - a_2 = a_4 - a_3 = \cdots = ₹50$

It is an arithmetic progression with common difference=₹50

(iv) The amount of money in the account every year, when ₹ 10000 is deposited at compound interest at 8 % per annum.

Sol: $\mathrm{P} = ₹10000$, $R = 8\ \%$,

$$A = P\left(1 + \frac{R}{100}\right)^n$$

$$First\ year\ ammount = 10000\left(1 + \frac{8}{100}\right)^1 = 10000 \times \frac{108}{100} = ₹10800$$

$$Second\ year\ ammount = 10000\left(1 + \frac{8}{100}\right)^2 = 10000 \times \frac{108}{100} \times \frac{108}{100} = ₹11664$$

$$Third\ year\ ammount = 10000\left(1 + \frac{8}{100}\right)^1 = 10000 \times \frac{108}{100} \times \frac{108}{100} \times \frac{108}{100} = ₹12597.12$$

The amounts are ₹10000, ₹10800, ₹11664, ₹12597.12, ...

$a_2 - a_1 = ₹10800 - ₹10000 = ₹800$

$a_3 - a_2 = ₹11664 - ₹10800 = ₹864$

$a_2 - a_1 \neq a_3 - a_2$

The given situations does not form an AP

2. Write first four terms of the AP, when the first term a and the common difference d are given as follows:

(i) $\boldsymbol{a = 10, d = 10}$

Sol: $a_1 = a = 10$

$a_2 = a + d = 10 + 10 = 20$

$a_3 = a + 2d = 10 + 2 \times 10 = 10 + 20 = 30$

$a_4 = a + 3d = 10 + 3 \times 10 = 10 + 30 = 40$

The first four terms of AP are 10,20,30,40

(ii) $\boldsymbol{a = -2, d = 0}$

Sol: $a_1 = a = -2$

$a_2 = a + d = -2 + 0 = -2$

$a_3 = a + 2d = -2 + 2 \times 0 = -2 + 0 = -2$

$a_4 = a + 3d = -2 + 3 \times 0 = -2 + 0 = -2$

The first four terms of AP are $-2, -2, -2, -2, ..$

(iii) **a = 4, d = – 3**

Sol: $a_1 = a = 4$

$a_2 = a + d = 4 + (-3) = 4 - 3 = 1$

$a_3 = a + 2d = 4 + 2 \times (-3) = 4 - 6 = -2$

$a_4 = a + 3d = 4 + 3 \times (-3) = 4 - 9 = -5$

The first four terms of AP are $4, 1, -2, -5$

(iv) $\boldsymbol{a = -1, d = \frac{1}{2}}$

Sol: $a_1 = a = -1$

$a_2 = a + d = -1 + \frac{1}{2} = \frac{-2+1}{2} = \frac{-1}{2}$

$a_3 = a + 2d = -1 + 2 \times \left(\frac{1}{2}\right) = -1 + 1 = 0$

$a_4 = a + 3d = -1 + 3 \times \left(\frac{1}{2}\right) = -1 + \frac{3}{2} = \frac{-2+3}{2} = \frac{1}{2}$

The first four terms of AP are $-1, \frac{-1}{2}, 0, \frac{1}{2}$

(v) $\boldsymbol{a = -1.25, d = -0.25}$

Sol: $a_1 = a = -1.25$

$a_2 = a + d = -1.25 + (-0.25) = -1.25 - 0.25 = -1.5$

$a_3 = a + 2d = -1.25 + 2 \times (-0.25) = -1.25 - 0.50 = -1.75$

$a_4 = a + 3d = -1.25 + 3 \times (-0.25) = -1.25 - 0.75 = -2$

The first four terms of AP are $-1.25, -1.5, -1.75, -2$

3. For the following APs, write the first term and the common difference:

(i) 3, 1, – 1, – 3, . .

Sol: First term=a=3

Common difference=$d = a_2 - a_1 = 1 - 3 = -2$

(ii) – 5, – 1, 3, 7, . . .

Sol: First term=a=-5

Common difference=$d = a_2 - a_1 = -1 + 5 = 4$

(iii) $\frac{1}{3}, \frac{5}{3}, \frac{9}{3}, \frac{13}{3}, \ldots$

Sol: First term=a=$\frac{1}{3}$

Common difference $= d = a_2 - a_1 = \frac{5}{3} - \frac{1}{3} = \frac{5-1}{3} = \frac{4}{3}$

(iv) 0.6, 1.7, 2.8, 3.9, . . .

Sol: First term=a=0.6

Common difference=$d = a_2 - a_1 = 1.7 - 0.6 = 1.1$

4. Which of the following are APs ? If they form an AP, find the common difference d and write three more terms.

(i) 2, 4, 8, 16, . . .

Sol: $a_1 = 2, a_2 = 4, a_3 = 8, a_4 = 16$

$a_2 - a_1 = 4 - 2 = 2$

$a_3 - a_2 = 8 - 4 = 4$

$a_2 - a_1 \neq a_3 - a_2$

So, the given list of numbers does not form an AP

(ii) $\mathbf{2, \frac{5}{2}, 3, \frac{7}{2} \ldots.}$

Sol: $a_1 = 2, a_2 = \frac{5}{2} a_3 = 3, a_4 = \frac{7}{2}$

$$a_2 - a_1 = \frac{5}{2} - 2 = \frac{5-4}{2} = \frac{1}{2}$$

$$a_3 - a_2 = 3 - \frac{5}{2} = \frac{6-5}{2} = \frac{1}{2}$$

$$a_4 - a_3 = \frac{7}{2} - 3 = \frac{7-6}{3} = \frac{1}{2}$$

$$a_2 - a_1 = a_3 - a_2 = a_4 - a_3 = \cdots$$

i.e., $a_{k+1} - a_k$ is same every time

So, the given list of numbers forms an AP with the common difference $d = \frac{1}{2}$

The next three terms are: $\frac{7}{2} + \frac{1}{2} = \frac{8}{2}, \frac{8}{2} + \frac{1}{2} = \frac{9}{2}, \frac{9}{2} + \frac{1}{2} = \frac{10}{2} \Rightarrow 4, \frac{9}{2}, 5$

(iii) $\mathbf{-1.2, -3.2, -5.2, -7.2, \ldots}$

Sol: $a_1 = -1.2, a_2 = -3.2, a_3 = -5.2, a_4 = -7.2$

$$a_2 - a_1 = -3.2 - (-1.2) = -3.2 + 1.2 = -2$$

$$a_3 - a_2 = -5.2 - (-3.2) = -5.2 + 3.2 = -2$$

$$a_4 - a_3 = -7.2 - (-5.2) = -7.2 + 5.2 = -2$$

$$a_2 - a_1 = a_3 - a_2 = a_4 - a_3 = \cdots$$

i.e., $a_{k+1} - a_k$ is same every time

So, the given list of numbers forms an AP with the common difference $d = -2$

The next three terms are: $-7.2 - 2 = -9.2, -9.2 - 2 = -11.2, -11.2 - 2 = -13.2$

$\Rightarrow -9.2, -11.2, -13.2$

(iv) $\mathbf{-10, -6, -2, 2, \ldots}$

Sol: $a_1 = -10, a_2 = -6, a_3 = -2, a_4 = 2$

$$a_2 - a_1 = -6 - (-10) = -6 + 10 = 4$$

$$a_3 - a_2 = -2 - (-6) = -2 + 6 = 4$$

$$a_4 - a_3 = 2 - (-2) = 2 + 2 = 4$$

$$a_2 - a_1 = a_3 - a_2 = a_4 - a_3 = \cdots$$

i.e., $a_{k+1} - a_k$ is same every time

So, the given list of numbers forms an AP with the common difference $d = 4$

The next three terms are: 2+4=6,6+4=10,10+4=14

$\Rightarrow$ 6,10,14

(v) $\mathbf{3, 3+\sqrt{2}, 3+2\sqrt{2}, 3+3\sqrt{2}, \ldots \ldots}$

Sol: $a_1 = 3, a_2 = 3 + \sqrt{2}, a_3 = 3 + 2\sqrt{2}, a_4 = 3 + 3\sqrt{2}$

$$a_2 - a_1 = 3 + \sqrt{2} - 3 = \sqrt{2}$$

$a_3 - a_2 = 3 + 2\sqrt{2} - (3 + \sqrt{2}) = 3 + 2\sqrt{2} - 3 - \sqrt{2} = \sqrt{2}$

$a_4 - a_3 = 3 + 3\sqrt{2} - (3 + 2\sqrt{2}) = 3 + 3\sqrt{2} - 3 + 2\sqrt{2} = \sqrt{2}$

$a_2 - a_1 = a_3 - a_2 = a_4 - a_3 = \cdots$

i.e., $a_{k+1} - a_k$ is same every time

So, the given list of numbers forms an AP with the common difference $d = \sqrt{2}$

The next three terms are:−

$(3 + 3\sqrt{2}) + \sqrt{2} = 3 + 4\sqrt{2}; (3 + 4\sqrt{2}) + \sqrt{2} = 3 + 5\sqrt{2}; (3 + 5\sqrt{2}) + \sqrt{2} = 3 + 6\sqrt{2}$

(vi) 0.2, 0.22, 0.222, 0.2222, . . .

Sol: $a_1 = 0.2, a_2 = 0.22, a_3 = 0.222, a_4 = 0.2222$

$a_2 - a_1 = 0.22 - 0.2 = 0.02$

$a_3 - a_2 = 0.222 - 0.22 = 0.002$

$a_2 - a_1 \neq a_3 - a_2$

So, the given list of numbers does not form an AP

(vii) $0, -4, -8, -12, \ldots.$

Sol: $a_1 = 0, a_2 = -4, a_3 = -8, a_4 = -12$

$a_2 - a_1 = -4 - 0 = -4$

$a_3 - a_2 = -8 - (-4) = -8 + 4 = -4$

$a_4 - a_3 = -12 - (-8) = -12 + 8 = -4$

$a_2 - a_1 = a_3 - a_2 = a_4 - a_3 = \cdots$

i.e., $a_{k+1} - a_k$ is same every time

So, the given list of numbers forms an AP with the common difference $d = -4$

The next three terms are:−

$-12 - 4 = -16; \quad -16 - 4 = -20; \quad -20 - 4 = -24$

(viii) $-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \ldots.$

Sol: $a_2 - a_1 = -\frac{1}{2} - \left(-\frac{1}{2}\right) = -\frac{1}{2} + \frac{1}{2} = 0$

$a_3 - a_2 = -\frac{1}{2} - \left(-\frac{1}{2}\right) = -\frac{1}{2} + \frac{1}{2} = 0$

$a_4 - a_3 = -\frac{1}{2} - \left(-\frac{1}{2}\right) = -\frac{1}{2} + \frac{1}{2} = 0$

$a_2 - a_1 = a_3 - a_2 = a_4 - a_3 = \cdots$

i.e., $a_{k+1} - a_k$ is same every time

So, the given list of numbers forms an AP with the common difference $d = 0$

The next three terms are: $-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}$

(ix) 1,3,9,27,…….

Sol: $a_2 - a_1 = 3 - 1 = 2$

$a_3 - a_2 = 9 - 3 = 6$

$a_2 - a_1 \neq a_3 - a_2$

So, the given list of numbers does not form an AP

(x) $\boldsymbol{a, 2a, 3a, 4a, \ldots \ldots}$

Sol: $a_2 - a_1 = 2a - a = a$

$a_3 - a_2 = 3a - 2a = a$

$a_4 - a_3 = 4a - 3a = a$

$a_2 - a_1 = a_3 - a_2 = a_4 - a_3 = \cdots$

i.e., $a_{k+1} - a_k$ is same every time

So, the given list of numbers forms an AP with the common difference $d = a$

The next three terms are: $5a, 6a, 7a$

(xi) $\boldsymbol{a, a^2, a^3, a^4, \ldots}$

Sol: $a_2 - a_1 = a^2 - a = a(a-1)$

$a_3 - a_2 = a^3 - a^2 = a^2(a-1)$

$a_2 - a_1 \neq a_3 - a_2$

So, the given list of numbers does not form an AP

(xii) $\boldsymbol{\sqrt{2}, \sqrt{8}, \sqrt{18}, \sqrt{32}, \ldots \ldots}$

Sol: $\sqrt{2}, 2\sqrt{2}, 3\sqrt{2}, 4\sqrt{2}, \ldots \ldots$

$\sqrt{8} = \sqrt{4 \times 2} = \sqrt{4} \times \sqrt{2} = 2\sqrt{2}$

$\sqrt{18} = \sqrt{9 \times 2} = \sqrt{9} \times \sqrt{2} = 3\sqrt{2}$

$\sqrt{32} = \sqrt{16 \times 2} = \sqrt{16} \times \sqrt{2} = 4\sqrt{2}$

$a_2 - a_1 = 2\sqrt{2} - \sqrt{2} = \sqrt{2}$

$a_3 - a_2 = 3\sqrt{2} - 2\sqrt{2} = \sqrt{2}$

$a_4 - a_3 = 4\sqrt{2} - 3\sqrt{2} = \sqrt{2}$

$a_2 - a_1 = a_3 - a_2 = a_4 - a_3 = \cdots$

i.e., $a_{k+1} - a_k$ is same every time

So, the given list of numbers forms an AP with the common difference $d = \sqrt{2}$

The next three terms are: $5\sqrt{2}, 6\sqrt{2}, 7\sqrt{2}$

$\Rightarrow \sqrt{50}, \sqrt{72}, \sqrt{98}$

(xiii) $\boldsymbol{\sqrt{3}, \sqrt{6}, \sqrt{9}, \sqrt{12}, \ldots \ldots}$

Sol: $a_2 - a_1 = \sqrt{6} - \sqrt{3}$

$a_3 - a_2 = \sqrt{9} - \sqrt{6} = 3 - \sqrt{6}$

$a_2 - a_1 \neq a_3 - a_2$

So, the given list of numbers does not form an AP

(xiv) $\boldsymbol{1^2, 3^2, 5^2, 7^2, \ldots}$

Sol: $a_2 - a_1 = 3^2 - 1^2 = 9 - 1 = 8$

$a_3 - a_2 = 5^2 - 3^2 = 25 - 9 = 16$

$a_2 - a_1 \neq a_3 - a_2$

So, the given list of numbers does not form an AP

(xv) $1^2, 5^2, 7^2, 73, \ldots$

Sol: $a_2 - a_1 = 5^2 - 1^2 = 25 - 1 = 24$

$a_3 - a_2 = 7^2 - 5^2 = 49 - 25 = 24$

$a_3 - a_2 = 73 - 7^2 = 73 - 49 = 24$

$a_2 - a_1 = a_3 - a_2 = a_4 - a_3 = \cdots$

i.e., $a_{k+1} - a_k$ is same every time

So, the given list of numbers forms an AP with the common difference $d = 24$

The next three terms are: 73+24, 97+24,121+24

$\Rightarrow$ 97,121,145

n[th] Term of an AP(general term of the AP)

The nth term a_n of the AP with first term a and common difference d is given by

$$a_n = a + (n - 1)\,d.$$

If there are **m** terms in the AP, then **m** represents the last term which is sometimes also denoted by l.

n[th] Term of an AP from the end

If d be the common difference and l be the last term of an AP, then n[th] term from the end $= l - (n-1)d$

Example 3 : Find the 10[th] term of the AP : 2, 7, 12, . . .

Sol: Given AP is 2, 7, 12,. . .

$a = 2;\ d = a_2 - a_1 = 7 - 2 = 5$

The 10th term $= a_{10} = a + 9d$

$= 2 + 9 \times (5)$

$= 2 + 45 = 47$

Example 4 : Which term of the AP : 21, 18, 15, . . . is – 81? Also, is any term 0? Give reason for your answer.

Sol: First term=a=21

Common difference=$d = a_2 - a_1 = 18 - 21 = -3$

$Let\ a_n = -81$

$\Rightarrow a + (n-1)d = -81$

$\Rightarrow 21 + (n-1) \times (-3) = -81$

$\Rightarrow (n-1) \times (-3) = -81 - 21 = -102$

$\Rightarrow n - 1 = \dfrac{-102}{-3} = 34$

$\Rightarrow n = 34 + 1 = 35$

$\therefore$ -81 is the 35^{th} term of the given AP.

$Let\ a_n = 0$

$\Rightarrow a + (n-1)d = 0$

$\Rightarrow 21 + (n-1) \times (-3) = 0$

$\Rightarrow (n-1) \times (-3) = -21$

$\Rightarrow n - 1 = \dfrac{-21}{-3} = 7$

$\Rightarrow n = 7 + 1 = 8$

$\therefore$ The 8^{th} term of the given AP is 0.

Example 5 : Determine the AP whose 3rd term is 5 and the 7th term is 9.

Sol: 3^{rd} term of AP=5$\Rightarrow a + 2d = 5 \rightarrow (1)$

7^{th} term of AP=9 $\Rightarrow a + 6d = 9 \rightarrow (2)$

$(2) - (1) \Rightarrow$ $a + 6d = 9$

$$\begin{array}{r} a + 2d = 5 \\ (-)\,(-)\quad(-) \\ \hline 4d = 4 \\ \hline \end{array}$$

$d = 1$

Substitute d=1 value in (1)

$a + 2 \times 1 = 5$

$a = 5 - 2$

$a = 3$

Hence, the required AP is 3,4,5,6,……

Example 6 : Check whether 301 is a term of the list of numbers 5, 11, 17, 23, . . .

Sol: Given list of numbers 5, 11, 17, 23, ….

$a_2 - a_1 = 11 - 5 = 6$

$a_3 - a_2 = 17 - 11 = 6$

$a_4 - a_3 = 23 - 17 = 6$

$a_2 - a_1 = a_3 - a_2 = a_4 - a_3 = \cdots$

i.e., a_{k+1} – a_k is same every time

So, the given list of numbers forms an AP with the common difference $d = 6$ and $a = 5$

$Let\ a_n = 301$

$\Rightarrow a + (n-1)d = 301$

$\Rightarrow 5 + (n-1) \times (6) = 301$

$\Rightarrow (n-1) \times (6) = 301 - 5 = 296$

$\Rightarrow n - 1 = \frac{296}{6} = \frac{148}{3}$

$\Rightarrow n = \frac{148}{3} + 1 = \frac{151}{3}$ it is not a positive integer

So, 301 is not a term of the given list of numbers.

Example 7 : How many two-digit numbers are divisible by 3?

Sol: The list of two-digit numbers divisible by 3 is : 12, 15, 18, . . . , 99

Clearly it is an AP. $a = 12$ $and\ d = 15 - 12 = 3$

$Let\ a_n = 99 \Rightarrow a + (n-1)d = 99$

$12 + (n-1) \times 3 = 99$

$(n-1) \times 3 = 99 - 12 = 87$

$n - 1 = \frac{87}{3} = 29$

$n = 29 + 1 = 30$

So, there are 30 two-digit numbers divisible by 3.

Example 8 : Find the 11th term from the last term (towards the first term) of the AP : 10, 7, 4, . . ., – 62.

Sol: Given AP is 10,7,4,…

$a = 10, d = 7 - 10 = -3$

$Let\ a_n = -62 \Rightarrow a + (n-1)d = -62$

$10 + (n-1) \times (-3) = -62$

$(n-1) \times (-3) = -62 - 10 = -72$

$n - 1 = \frac{-72}{-3} = 24$

$n = 24 + 1 = 25$

So, there are 25 terms in the given AP.

The 11^{th} term from the last $= (25-10)^{th}\ term$

$= 15^{th}\ term = a + 14d$

$= 10 + 14 \times (-3)$

$= 10 - 42 = -32$

The 11^{th} term from the last of the AP is -32.

Alternative Solution 1:

If we write the given AP in the reverse order then

$a = -62 \text{ and } d = 3$

$11\, th\ term = a + 10d = -62 + 10 \times 3 = -62 + 30 = -32$

Alternative Solution 2:

$l = -62\,; d = -3$

n^{th} term from the last of the AP series$= l - (n-1)d$

$11^{th}\ term\ from\ the\ last\ of\ the\ AP\ series = l - 10d$

$= -62 - 10(-3) = -62 + 30 = -32$

Example 9 : A sum of ₹ 1000 is invested at 8% simple interest per year. Calculate the interest at the end of each year. Do these interests form an AP? If so, find the interest at the end of 30 years making use of this fact.

Sol: simple interest$(I) = \dfrac{P \times T \times R}{100}$

Here P=1000, R= 8%

The interest at the end of 1^{st} year $= \dfrac{1000 \times 1 \times 8}{100} =$ ₹ 80

The interest at the end of 2^{nd} year $= \dfrac{1000 \times 2 \times 8}{100} =$ ₹ 160

The interest at the end of 3^{rd} year $= \dfrac{1000 \times 3 \times 8}{100} =$ ₹ 240

The interests are 80,160,240,.....

$a_2 - a_1 = a_3 - a_2 = a_4 - a_3 = \cdots = 80$

The interests form an AP with $a = 80, d = 80$

The interest at the end of 30 years$= a_{30} = a + 29d$

$= 80 + 29 \times 80 =$ ₹2400

Example 10 : In a flower bed, there are 23 rose plants in the first row, 21 in the second, 19 in the third, and so on. There are 5 rose plants in the last row. How many rows are there in the flower bed?

Sol: The number of rose plants in the 1^{st},2^{nd} ,3^{rd} ,....rows are

23, 21, 19,....., 5 Clearly it is an AP

$a = 23, d = 21 - 23 = -2$

$Let\ a_n = 5 \Rightarrow a + (n-1)d = 5$

$23 + (n-1) \times (-2) = 5$

$(n-1) \times (-2) = 5 - 23 = -18$

$n - 1 = \dfrac{-18}{-2} = 9$

$n = 9 + 1 = 10$

So, there are 10 rows in the flower bed.

EXERCISE 5.2

1. Fill in the blanks in the following table, given that a is the first term, d the common difference and an the nth term of the AP:

(i) $\boldsymbol{a = 7, d = 3, n = 8, a_n = ?}$

Sol: $a_n = a + (n-1)d$

$= 7 + (8-1) \times 3$

$= 7 + 7 \times 3$

$= 7 + 21 = 28$

(ii) $\boldsymbol{a = -18, d = ?, n = 10, a_n = 0}$

Sol: $a_n = 0$

$a + (n-1)d = 0$

$-18 + (10-1)d = 0$

$9d = 18$

$d = \frac{18}{9} = 2$

(iii) $\boldsymbol{a = ?, d = -3, n = 18, a_n = -5}$

Sol: $a_n = -5$

$a + (n-1)d = -5$

$a + (18-1) \times (-3) = -5$

$a + 17 \times (-3) = -5$

$a - 51 = -5$

$a = -5 + 51 = 46$

(iv) $\boldsymbol{a = -18.9, d = 2.5, n = ?, a_n = 3.6}$

Sol: $a_n = 3.6$

$a + (n-1)d = 3.6$

$-18.9 + (n-1) \times (2.5) = 3.6$

$(n-1) \times (2.5) = 3.6 + 18.9$

$n - 1 = \frac{22.5}{2.5} = 9$

$n = 9 + 1 = 10$

(v) $\boldsymbol{a = 3.5, d = 0, n = 105, a_n = ?}$

Sol: $a_n = a + (n-1)d$

$= 3.5 + (105-1) \times 0 = 3.5$

2. Choose the correct choice in the following and justify :
(i) 30th term of the AP: 10, 7, 4, . . . , is
(A) 97 (B) 77 (C) –77 (D) – 87 [C]

Sol: Given A.P is 10,7,4,……..

$a = 10, d = 7 - 10 = -3$

30^{th} term of the A.P $= a + 29d$

$= 10 + 29 \times (-3)$

$= 10 - 87 = -77$

(ii) 11th term of the AP: – 3, $\frac{-1}{2}$, 2 …, is

(A) 28 (B) 22 (C) –38 (D) – 48 1/ 2 [B]

Sol: Given A.P is $-3, \frac{-1}{2}, 2, \dots..$

$$a = -3, \quad d = a_2 - a_1 = \frac{-1}{2} - (-3) = \frac{-1}{2} + 3 = \frac{-1+6}{2} = \frac{5}{2}$$

11^{th} term of the A.P$= a + 10d$

$= -3 + 10 \times \left(\frac{5}{2}\right)$

$= -3 + 25 = 22$

3. In the following APs, find the missing terms in the boxes :

(i) 2, ☐ , 26

Sol: $a_1 = a = 2$

$a_3 = a + 2d = 26$

$\Rightarrow 2 + 2d = 26$

$\Rightarrow 2d = 26 - 2$

$\Rightarrow d = \frac{24}{2} = 12$

Now $a_2 = a + d = 2 + 12 = 14$

(ii) ☐ , 13, ☐ , 3

Sol: $a_2 = a + d = 13 \rightarrow (1)$

$a_4 = a + 3d = 3 \rightarrow (2)$

(2)-(1)$\Rightarrow$ $a + 3d = 3$

$a + d = 13$

$(-)\ (-)\ (-)$

$2d = -10$

$d = \frac{-10}{2} = -5$

Substitute $d = -5$ in (1)

$a - 5 = 13$

$a = 13 + 5 = 18$

Now $a_1 = a = 18$

$a_3 = a + 2d = 13 + 2(-5) = 18 - 10 = 8$

(iii) 5, ☐, ☐, $9\frac{1}{2}$

Sol: $a_1 = a = 5$

$a_4 = a + 3d = \frac{19}{2}$

$5 + 3d = \frac{19}{2}$

$3d = \frac{19}{2} - 5 = \frac{19 - 10}{2} = \frac{9}{2}$

$d = \frac{9}{2 \times 3} = \frac{3}{2}$

$a_2 = a + d = 5 + \frac{3}{2} = \frac{13}{2}$

$a_3 = a + 2d = 5 + 2 \times \frac{3}{2} = 5 + 3 = 8$

(iv) − 4, ☐, ☐, ☐, ☐, 6

Sol: $a_1 = -4 \Rightarrow a = -4$

$a_6 = 6 \Rightarrow a + 5d = 6$

$-4 + 5d = 6$

$5d = 6 + 4 = 10$

$d = \frac{10}{5} = 2$

$a_2 = a + d = -4 + 2 = -2$

$a_3 = a + 2d = -4 + 2 \times 2 = -4 + 4 = 0$

$a_4 = a + 3d = -4 + 3 \times 2 = -4 + 6 = 2$

$a_5 = a + 4d = -4 + 4 \times 2 = -4 + 8 = 4$

(v) ☐, 38, ☐, ☐, ☐, −22

Sol: $a_2 = 38 \Rightarrow a + d = 38 \rightarrow (1)$

$a_6 = -22 \Rightarrow a + 5d = -22 \rightarrow (2)$

$(2) - (1) \Rightarrow a + 5d = -22$

$$\begin{array}{r} a + d = 38 \\ (-)\ (-)\quad (-) \\ \hline 4d = -60 \\ \hline \end{array}$$

$d = \frac{-60}{4} = -15$

Substitute $d = -15$ in (1)

$a - 15 = 38$

$a = 38 + 15 = 53$

$a_1 = a = 53$

$a_3 = a + 2d = 53 + 2(-15) = 53 - 30 = 23$

$a_4 = a + 3d = 53 + 3(-15) = 53 - 45 = 8$

$a_5 = a + 4d = 53 + 4(-15) = 53 - 60 = -7$

4. Which term of the AP : 3, 8, 13, 18, . . . ,is 78?

Sol: given A.P: 3, 8, 13, 18, . .

$a = 3; d = 8 - 3 = 5$

$let\ a_n = 78$

$a + (n-1)d = 78$

$3 + (n-1) \times 5 = 78$

$(n-1) \times 5 = 78 - 3 = 75$

$n - 1 = \frac{75}{5} = 15$

$n = 15 + 1 = 16$

∴ 78 is the 16^{th} term of A.P

5. Find the number of terms in each of the following APs :
(i) 7, 13, 19, . . . , 205

Sol: $a = 7, d = 13 - 7 = 6$

$let\ a_n = 205$

$a + (n-1)d = 205$

$7 + (n-1) \times 6 = 205$

$(n-1) \times 6 = 205 - 7 = 198$

$n - 1 = \frac{198}{6} = 33$

$n = 33 + 1 = 34$

The number of terms in given A.P are 34.

(ii) $18, 15\frac{1}{2}, 13, \ldots, -47$

Sol: $a = 18,$

$d = \frac{31}{2} - 18 = \frac{31 - 36}{2} = \frac{-5}{2}$

$let\ a_n = -47$

$a + (n-1)d = -47$

$18 + (n-1) \times \left(\frac{-5}{2}\right) = -47$

$(n-1) \times \left(\frac{-5}{2}\right) = -47 - 18$

$(n-1) \times \left(\frac{-5}{2}\right) = -65$

$n - 1 = -65 \times \frac{-2}{5} = 26$

$n = 26 + 1 = 27$

The number of terms in given A.P are 27.

6. Check whether – 150 is a term of the AP : 11, 8, 5, 2 . . .

Sol: $a = 11, d = 8 - 11 = -3$

$let\ a_n = -150$

$a + (n-1)d = -150$

$11 + (n-1) \times (-3) = -150$

$(n-1) \times (-3) = -150 - 11 = -161$

$n - 1 = \frac{-161}{-3} = \frac{161}{3}$ *it is not a natural number*

∴ −150 is not a term of given AP

7. Find the 31st term of an AP whose 11th term is 38 and the 16th term is 73.

Sol: 11^{th} term is 38⇒ $a + 10d = 38 \rightarrow (1)$

16^{th} term is 73⇒ $a + 15d = 73 \rightarrow (2)$

$(2) - (1) \Rightarrow a + 15d = 73$

$a + 10d = 38$

$(-)\ (-)\ \ (-)$

$5d = 35$

$\Rightarrow d = \frac{35}{5} = 7$

Substitute d=7 in (1)

$a + 10 \times 7 = 38$

$a = 38 - 70 = -32$

31^{st} term $= a + 30d$

$= -32 + 30 \times 7$

$= -32 + 210$

$= 178$

8. An AP consists of 50 terms of which 3rd term is 12 and the last term is 106. Find the 29th term.

Sol: In AP 3rd term = 12

$a + 2d = 12 \rightarrow (1)$

Last term=50th term=106

$a + 49d = 106 \rightarrow (2)$

$(2) - (1) \Rightarrow a + 49d - a - 2d = 106 - 12$

$47d = 94 \Rightarrow d = 2$

Substitute d=1 in (1) we get

$a + 2 \times 2 = 12$

$a = 12 - 4 = 8$

$a_{29} = a + 28d = 8 + 28 \times 2 = 8 + 56 = 64$

9. If the 3rd and the 9th terms of an AP are 4 and – 8 respectively, which term of this AP is zero?

Sol: 3rd term of an A.P=4$\Rightarrow a + 2d = 4 \rightarrow (1)$

9th term of an A.P$= -8 \Rightarrow a + 8d = -8 \rightarrow (2)$

$(2) - (1) \Rightarrow a + 8d = -8$

$$\begin{array}{r} a + 2d = \ \ 4 \\ (-)\ (-)\quad (-) \\ \hline 6d = -12 \\ \hline \end{array}$$

$$d = \frac{-12}{6} = -2$$

Substitute d=-2 in (1) we get

$a + 2 \times (-2) = 4$

$a - 4 = 4$

$a = 4 + 4 = 8$

$let\ a_n = 0$

$a + (n-1)d = 0$

$8 + (n-1) \times (-2) = 0$

$(n-1) \times (-2) = 0 - 8$

$n - 1 = \frac{-8}{-2} = 4$

$n = 4 + 1 = 5$

$\therefore$ The 5th term of A.P is ‘0’

10. The 17th term of an AP exceeds its 10th term by 7. Find the common difference.

Sol: 17th term of an AP=10th term+7

$a + 16d = a + 9d + 7$

$a + 16d - a - 9d = 7$

$7d = 7 \Rightarrow d = 1$

The common difference=1

11. Which term of the AP : 3, 15, 27, 39, . . . will be 132 more than its 54th term?

Sol: $a = 3$; $\text{d} = 15 - 3 = 12$

$Let\ a_n = a_{54} + 132$

$a + (n-1)d = a + 53d + 132$

$(n-1)d = 53d + 132$

$(n-1) \times 12 = 53 \times 12 + 132$

$(n-1) \times 12 = 768$

$n - 1 = \frac{768}{12} = 64$

$n = 65$

Therefore, 65^{th} term will be 132 more than 54^{th} term.

12. Two APs have the same common difference. The difference between their 100th terms is 100, what is the difference between their 1000th terms?

Sol: Let the first A.P is $a, a + d, a + 2d, a + 3d, \ldots..$

The second A.P is $b, b + d, b + 2d, b + 3d, \ldots ....$

The difference between their 100^{th} terms = 100

$a_{100} - b_{100} = 100$

$(a + 99d) - (b + 99d) = 100$

$a + 99d - b - 99d = 100$

$a - b = 100 \rightarrow (1)$

The difference between their 1000th terms $= a_{1000} - b_{1000}$

$= (a + 999d) - (b + 999d)$

$= a + 999d - b - 999d$

$= a - b$

$= 100 \quad (from\ (1))$

The difference between their 1000^{th} terms = 100.

13. How many three-digit numbers are divisible by 7?

Sol: The three-digit numbers are divisible by 7 are

105,112, 119,……., 994

$a = 105, d = 7$

$let\ a_n = 994$

$a + (n-1)d = 994$

$105 + (n-1) \times 7 = 994$

$(n-1) \times 7 = 994 - 105 = 889$

$n - 1 = \frac{889}{7} = 127$

$n = 127 + 1 = 128$

∴ 128 three digit numbers are divisible by 7

14. How many multiples of 4 lie between 10 and 250?

Sol: Multiples of 4 lie between 10 and 250 are

12,16,20,…….,248

$a = 12, d = 4$

$let\ a_n = 248$

$a + (n-1)d = 248$

$12 + (n-1) \times 4 = 248$

$(n-1) \times 4 = 248 - 12 = 236$

$n - 1 = \frac{236}{4} = 59$

$n = 59 + 1 = 60$

∴ 60 multiples of 4 lie between 10 and 250.

15. For what value of n, are the nth terms of two APs: 63, 65, 67, . . . and 3, 10, 17, . . . equal?

Sol: First A.P : 63,65,67,….

$a = 63, d = 2$

$a_n = a + (n-1)d$

$= 63 + (n-1) \times 2$

$= 63 + 2n - 2$

$= 2n + 61$

Second A.P: 3,10,17,…..

$a = 3, d = 7$

$a_n = a + (n-1)d$

$= 3 + (n-1) \times 7$

$= 3 + 7n - 7 = 7n - 4$

If n^{th} terms of two A.Ps are equal then

$7n - 4 = 2n + 61$

$7n - 2n = 61 + 4$

$5n = 65$

$n = \frac{65}{5} = 13$

∴ 13^{th} terms of the two A.Ps are equal.

16. Determine the AP whose third term is 16 and the 7th term exceeds the 5th term by 12.

Sol: Third term of AP=16⇒ $a + 2d = 16 \rightarrow (1)$

7^{th} term=5^{th} term + 12

$a + 6d = a + 4d + 12$

$a + 6d - a - 4d = 12$

$2d = 12$

$d = 6$

Substitute $d = 6$ in (1) we get

$a + 2 \times 6 = 16$

$a = 16 - 12 = 4$

The required AP is $a, a + d, a + 2d, a + 3d, ...$

⇒ 4,10,16,22, … ….

17. Find the 20th term from the last term of the AP : 3, 8, 13, . . ., 253.

Sol: $a = 3, d = 8 - 3 = 5$

$let \ \ a_n = l = 253$

$a + (n - 1)d = 253$

$3 + (n - 1) \times 5 = 253$

$(n - 1) \times 5 = 253 - 3 = 250$

$n - 1 = \frac{250}{5} = 50$

$n = 50 + 1 = 51$

The 20^{th} term from the end of the AP= (51-20)+1=32^{th} term from first

$= a + 31d = 3 + 31 \times 5 = 3 + 155$=158

(OR)

$a = 3, \quad d = 8 - 3 = 5$

$a_n = l = 253$

n^{th}term from the end of the AP $= l - (n - 1)d$

20^{th}term from the end of the AP $= 253 - 19 \times 5 = 253 - 95 = 158$

18. The sum of the 4th and 8th terms of an AP is 24 and the sum of the 6th and 10th terms is 44. Find the first three terms of the AP.

Sol: 4^{th} term +8^{th} term of an AP =24

$\Rightarrow a + 3d + a + 7d = 24$

$\Rightarrow 2a + 10d = 24$

$\Rightarrow a + 5d = 12 \rightarrow (1)$

6^{th} term $+10^{th}$ term of an AP =44

$\Rightarrow a + 5d + a + 9d = 44$

$\Rightarrow 2a + 14d = 44$

$\Rightarrow a + 7d = 22 \rightarrow (2)$

$(2) - (1) \Rightarrow a + 7d = 22$

$$\begin{array}{r} a + 5d = 12 \\ (-)\ (-)\quad (-) \\ \hline 2d = 10 \\ \hline \end{array}$$

$d = 5$

Substitute d=5 in (1) we get

$a + 5 \times 5 = 12$

$a = 12 - 25$

$a = -13$

$\therefore$ The first three terms of AP are $a, a + d, a + 2d$

$\Rightarrow -13, -13 + 5, -13 + 10$

$\Rightarrow -13, -8, -3$

19. Subba Rao started work in 1995 at an annual salary of ₹ 5000 and received an increment of ₹ 200 each year. In which year did his income reach ₹ 7000?

Sol: subbarao salary in 1995=₹5000, Increment =₹200

Salary in 1996=5000+200=₹5200

Salary in 1997=5200+200=₹5400

Salary in 1998=5400+200=₹5600

The salaries are ₹5000, ₹5200, ₹5400, ₹5600,…….forms an AP

$a = 5000, d = 200$

Let $a_n = 7000$

$a + (n - 1) \times 200 = 7000$

$5000 + (n - 1) \times 200 = 7000$

$(n - 1) \times 200 = 7000 - 5000 = 2000$

$n - 1 = \frac{2000}{200} = 10$

$n = 10 + 1$

$n = 11$

$\therefore$ In 11^{th} year subbarao income reached 7000

20. Ramkali saved ₹ 5 in the first week of a year and then increased her weekly savings by ₹ 1.75. If in the nth week, her weekly savings become ₹ 20.75, find n.

Sol: Ramkali savings:

First week=₹ 5

Second week=₹ 5+₹ 1.75=₹ 6.75

Third week=₹ 6.75+₹ 1.75=₹ 8.50

Fourth week=₹ 8.50+₹ 1.75=₹ 10.25

.....

Ramkali's savings in the consecutive weeks are ₹ 5, ₹ 6.75, ₹ 8.50, ₹ 10.25,...

These are in AP with a=5 and d=1.75

The nth week savings = ₹ 20.75

$a + (n - 1)d = 20.75$

$5 + (n - 1) \times 1.75 = 20.75$

$(n - 1) \times 1.75 = 20.75 - 5$

$(n - 1) \times 1.75 = 15.75$

$n - 1 = \frac{15.75}{1.75} = \frac{1575}{175} = 9$

$n = 9 + 1$

$n = 10$

5.4 Sum of First n Terms of an AP

Carl Fredrich Gauss (1777-1855) is a great German Mathematician

i. If first term of an AP is $\boldsymbol{a}$ and common difference is $\boldsymbol{d}$ then

Sum of first n terms $= S_n = \frac{n}{2}[2a + (n - 1)d] = \frac{n}{2}(a + a_n)$

ii. If first term is $\boldsymbol{a}$, last term is $\boldsymbol{l}$ and number of terms is $\boldsymbol{n}$ then

$S_n = \frac{n}{2}(a + l)$

iii. $a_n = S_n - S_{n-1}$

Example 11 : Find the sum of the first 22 terms of the AP : 8, 3, –2, . . .

Solution : Here, a = 8, d = 3 – 8 = –5, n = 22.

$S_n = \frac{n}{2}[2a + (n - 1)d]$

$$S_{22}=\frac{22}{2}[2\times 8+(22-1)(-5)]=11[16-105]=11\times(-89)=-979$$

Example-12. If the sum of the first 14 terms of an AP is 1050 and its first term is 10, find the 20th term

Sol: $a=10, n=14,$

$S_{14}=1050$

$\frac{n}{2}[2a+(n-1)d]=1050$

$\frac{14}{2}[2\times 10+(14-1)d]=1050$

$7[20+13d]=1050$

$20+13d=\frac{1050}{7}=150$

$13d=150-20$

$13d=130$

$d=10$

20th term$=a+19d$

$=10+19\times 10=10+190=200$

Example-13. How many terms of the AP : 24, 21, 18, . . . must be taken so that their sum is 78?

Sol: $a=24, d=21-24=-3$

$Let\ S_n=78$

$\frac{n}{2}[2a+(n-1)d]=78$

$n[2\times 24+(n-1)(-3)]=2\times 78$

$n[48-3n+3]=156$

$n[-3n+51]=156$

$-3n^2+51n-156=0$

$3n^2-51n+156=0$

$n^2-17n+52=0$

$(n-4)(n-13)=0$

$n-4=0\quad or\quad n-13=0$

$n=4\ or\ 13$

So, the number of terms is either 4 or 13.

Remark: Two answers are possible because the sum of the terms from 5th to 13th will be zero.

Example-14. (i) Find the sum of the first 1000 positive integers.

Sol: The first 1000 positive integers are 1,2,3,4,.....,1000

$a=1, d=1, n=1000$

$S_n=\frac{n}{2}[a+l]$

$$S_{1000} = \frac{1000}{2}[1 + 1000] = 500 \times 1001 = 500500$$

(ii) Find the sum of the first n positive integers

Sol: $a = 1, d = 1, n = n$

$$S_n = \frac{n}{2}[a + l] = \frac{n}{2}(1 + n) = \frac{n(n+1)}{2}$$

$$The\ sum\ of\ the\ first\ n\ positive\ integers = \frac{n(n+1)}{2}$$

Example-15. Find the sum of first 24 terms of the list of numbers whose nth term is given by $\boldsymbol{a_n = 3 + 2n}$

Sol: $a_n = 3 + 2n$

$a_1 = 3 + 2 \times 1 = 3 + 2 = 5$

$a_2 = 3 + 2 \times 2 = 3 + 4 = 7$

$a_3 = 3 + 2 \times 3 = 3 + 6 = 9$

List of numbers are 5,7,9,...... clearly it is an AP

$a = 5, d = 7 - 5 = 2, n = 24$

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$S_{24} = \frac{24}{2}[10 + (24 - 1) \times 2]$$

$= 12[10 + 23 \times 2]$

$= 12 \times 56$

$= 672$

Example-16. A manufacturer of TV sets produced 600 sets in the third year and 700 sets in the seventh year. Assuming that the production increases uniformly by a fixed number every year, find : (i) the production in the 1st year (ii) the production in the 10th year (iii) the total production in first 7 years.

Sol: $a_3 = 600, a_7 = 700$

$a_7 = 700 \Rightarrow a + 6d = 700 \rightarrow (1)$

$a_3 = 600 \Rightarrow a + 2d = 600 \rightarrow (2)$

$4d = 100$

$$d = \frac{100}{4} = 25$$

Substitute d=25 in (2)

$a + 2 \times 25 = 600$

$a + 50 = 600$

$a = 600 - 50 = 550$

(i) The production in the 1st year=550

(ii) The production in the 10th year$=a+9d$

$= 550 + 9 \times 25$

$= 550 + 225$

$= 775$

(iii) The total production in first 7 years$=S_7$

$= \frac{7}{2}[2 \times 550 + (7-1) \times 25]$

$= \frac{7}{2}[1100 + 6 \times 25]$

$= \frac{7}{2}[1250] = 7 \times 625 = 4375$

$S_n = \frac{n}{2}[2a + (n-1)d]$

EXERCISE 5.3

1. Find the sum of the following APs:

(i) 2, 7, 12, . . ., to 10 terms.

Sol: $a = 2, d = 7 - 2 = 5, n = 10$

$S_n = \frac{n}{2}[2a + (n-1)d]$

$S_{10} = \frac{10}{2}[2 \times 2 + (10-1) \times 5]$

$= 5[4 + 45]$

$= 5 \times 49$

$= 245$

(ii) –37, –33, –29, . . ., to 12 terms.

Sol: $a = -37, d = -33 + 37 = 4, n = 12$

$S_n = \frac{n}{2}[2a + (n-1)d]$

$S_{12} = \frac{12}{2}[2 \times (-37) + (12-1) \times 4]$

$= 6[-74 + 44]$

$= 6 \times (-30)$

$= -180$

(iii) 0.6, 1.7, 2.8, . . ., to 100 terms

Sol: $a = 0.6, d = 1.7 - 0.6 = 1.1, n = 100$

$S_n = \frac{n}{2}[2a + (n-1)d]$

$S_{100} = \frac{100}{2}[2 \times 0.6 + (100-1) \times 1.1]$

$= 50[1.2 + 99 \times 1.1]$

$= 50[1.2 + 108.9]$

$= 50 \times 110.1$

$= 5505$

(iv) $\frac{1}{15}, \frac{1}{12}, \frac{1}{10}, \ldots\ldots, to\ 11\ terms$

Sol: $a = \frac{1}{15}, d = \frac{1}{12} - \frac{1}{15} = \frac{5-4}{60} = \frac{1}{60}, n = 11$

$S_n = \frac{n}{2}[2n + (n-1)d]$

$S_{11} = \frac{11}{2}\left[2\left(\frac{1}{15}\right) + (11-1)\left(\frac{1}{60}\right)\right]$

$= \frac{11}{2}\left[\frac{2}{15} + 10 \times \frac{1}{60}\right]$

$= \frac{11}{2}\left[\frac{2}{15} + \frac{1}{6}\right]$

$= \frac{11}{2}\left[\frac{4+5}{30}\right]$

$= \frac{11}{2} \times \frac{9}{30} = \frac{11}{2} \times \frac{3}{10}$

$= \frac{33}{20} = 1\frac{13}{20}$

2.Find the sums given below

(i) $7 + 10\frac{1}{2} + 14 + \cdots + 84$

Sol: $a = 7, d = 10\frac{1}{2} - 7 = 3\frac{1}{2} = \frac{7}{2}, l = 84$

$l = a_n = 84$

$a + (n-1)d = 84$

$7 + (n-1)\left(\frac{7}{2}\right) = 84$

$(n-1)\left(\frac{7}{2}\right) = 84 - 7$

$n - 1 = 77 \times \frac{2}{7} = 22$

$n = 22 + 1 = 23$

$S_n = \frac{n}{2}(a + l)$

$S_{23} = \frac{23}{2}(7 + 84)$

$= \frac{23}{2} \times 91$

$= \frac{2093}{2} = 1046\frac{1}{2}$

(ii) 34 + 32 + 30 + . . . + 10

Sol: $a = 34, d = 32 - 34 = -2$

$l = a_n = a + (n-1)d = 10$

$34 + (n-1)(-2) = 10$

$(n-1)(-2) = 10 - 34$

$(n-1)(-2) = -24$

$n - 1 = \frac{-24}{-2} = 12$

$n = 12 + 1 = 13$

$S_n = \frac{n}{2}(a + l)$

$S_{13} = \frac{13}{2}(34 + 10)$

$= \frac{13}{2} \times 44$

$= 13 \times 22$

$= 286$

(iii) –5 + (–8) + (–11) + . . . + (–230)

Sol: $a = -5, d = -8 + 5 = -3$

$l = a_n = a + (n-1)d = -230$

$-5 + (n-1)(-3) = -230$

$(n-1)(-3) = -230 + 5$

$(n-1)(-3) = -225$

$n - 1 = \frac{-225}{-3} = 75$

$n = 75 + 1 = 76$

$S_n = \frac{n}{2}(a + l)$

$S_{76} = \frac{76}{2}[-5 + (-230)]$

$= 38 \times (-235)$

$= -8930$

3. **In an AP:**

(i) $\boldsymbol{Given\ a = 5, d = 3, a_n = 50, find\ n\ and\ S_n}$.

Sol: $a_n = 50$

$a + (n-1)d = 50$

$5 + (n-1) \times 3 = 50$

$(n-1) \times 3 = 50 - 5$

$(n-1)\times 3 = 45$

$n-1 = \frac{45}{3} = 15$

$n = 15+1$

$n = 16$

$S_n = \frac{n}{2}[2a+(n-1)d]$

$S_{16} = \frac{16}{2}[2\times 5+(16-1)\times 3]$

$= 8[10+15\times 3]$

$= 8[10+45]$

$= 8\times 55 = 440$

(ii) Given $a = 7, a_{13} = 35$, find d and S_{13}.

Sol: $a_{13} = 35$

$a+12d = 35$

$7+12d = 35$

$12d = 35-7$

$12d = 28$

$d = \frac{28}{12} = \frac{7}{3}$

$S_n = \frac{n}{2}[2a+(n-1)d]$

$S_{13} = \frac{13}{2}\left[2\times 7+(13-1)\times \frac{7}{3}\right]$

$= \frac{13}{2}\left[14+12\times \frac{7}{3}\right]$

$= \frac{13}{2}[14+28]$

$= \frac{13}{2}\times 42 = 13\times 21 = 273$

(iii) Given $a_{12} = 37, d = 3$, find a and S_{12}.

Sol: $a_{12} = 37$

$a+11d = 37$

$a+11\times 3 = 37$

$a+33 = 37$

$a = 37-33 = 4$

$S_n = \frac{n}{2}[2a+(n-1)d]$

$S_{12} = \frac{12}{2}[2\times 4+(12-1)\times 3]$

$= 6[8 + 33]$

$= 6 \times 41 = 246$

(iv) $\boldsymbol{Given\ a_3 = 15, S_{10} = 125, find\ d\ and\ a_{10}}$

Sol: $a_3 = 15$

$a + 2d = 15 \Rightarrow a = 15 - 2d \rightarrow (1)$

$S_{10} = 125$

$$\frac{10}{2}[2a + (10 - 1)d] = 125$$

$$[2(15 - 2d) + 9d] = \frac{125}{5}$$

$30 - 4d + 9d = 25$

$5d = 25 - 30$

$$d = \frac{-5}{5} = -1$$

Substitute $d = -1$ in (1)

$a = 15 - 2 \times (-1) = 15 + 2 = 17$

$a_n = a + 9d$

$= 17 + 9 \times (-1)$

$= 17 - 9 = 8$

(v)given $\mathbf{d = 5, S_9 = 75}$, find a and $\mathbf{a_9}$.

Sol: $S_9 = 75$

$$\frac{9}{2}[2a + (9 - 1) \times 5] = 75$$

$$\frac{9}{2}[2a + (9 - 1) \times 5] = 75$$

$$\frac{9}{2}[2a + 40] = 75$$

$18a + 360 = 150$

$18a = 150 - 360$

$18a = -210$

$$a = \frac{-210}{18} = \frac{-35}{3}$$

$$a_9 = a + 8d = \frac{-35}{3} + 8 \times 5 = \frac{-35}{3} + 40 = \frac{-35 + 120}{3} = \frac{85}{3}$$

(vi) $\boldsymbol{Given\ a = 2, d = 8, S_n = 90, find\ n\ and\ a_n}$.

Sol: $S_n = 90$

$$\frac{n}{2}[2a + (n - 1)d] = 90$$

$\frac{n}{2}[2 \times 2 + (n-1) \times 8] = 90$

$n[4 + 8n - 8] = 90 \times 2$

$4n + 8n^2 - 8n - 180 = 0$

$8n^2 - 4n - 180 = 0$

$2n^2 - n - 45 = 0$

$2n^2 - 10n + 9n - 45 = 0$

$2n(n-5) + 9(n-5) = 0$

$(n-5)(2n+9) = 0$

$n - 5 = 0 \; or \; 2n + 9 = 0$

$n = 5 \; or \; n = \frac{-9}{2}$

$\therefore n = 5$ (n *is a natural number*)

$a_n = a_5 = a + 4d$

$= 2 + 4 \times 8 = 2 + 32 = 34$

(vii) Given $a = 8, a_n = 62, S_n = 210$, find n and d.

Sol: $S_n = 210$

$\frac{n}{2}(a + a_n) = 210$

$\frac{n}{2}(8 + 62) = 210$

$n = \frac{210 \times 2}{70} = 6$

$a_n = 62$

$a + (n-1)d = 62$

$8 + (6-1)d = 62$

$5d = 62 - 8 = 54$

$d = \frac{54}{5}$

(viii) Given $a_n = 4, d = 2, S_n = -14$, find n and a.

Sol: $a_n = 4$

$a + (n-1)d = 4$

$a + (n-1) \times 2 = 4$

$a + 2n - 2 = 4$

$a = 4 - 2n + 2$

$a = 6 - 2n \rightarrow (1)$

$S_n = -14$

$\frac{n}{2}[a+a_n]=-14$

$n[6-2n+4]=-14\times 2$

$n[10-2n]=-28$

$10n-2n^2+28=0$

$-2n^2+10n+28=0$

$n^2-5n-14=0$

$(n-7)(n+2)=0$

$n-7=0\ or\ n+2=0$

$n=7\ or\ n=-2$

$\therefore n=7$ (n is a natural number)

From (1)

$a=6-2\times 7=6-14=-8$

(ix) given a = 3, n = 8, S = 192, find d

Sol: $S=192$

$\frac{8}{2}[2\times 3+(8-1)d]=192$

$4[6+7d]=192$

$24+28d=192$

$28d=192-24$

$28d=168$

$d=\frac{168}{28}=6$

(x) Given $l=28, S=144$, and there are total 9 terms. Find a.

Sol: $l=a_n=28, S=144, n=9$

$S=144$

$\frac{n}{2}[a+l]=144$

$\frac{9}{2}[a+28]=144$

$a+28=\frac{144\times 2}{9}$

$a=32-28=4$

4. How many terms of the AP : 9, 17, 25, . . . must be taken to give a sum of 636?

Sol: $a=9; d=17-9=8$

$S_n=636$

$\frac{n}{2}[2a+(n-1)d]=636$

$\frac{n}{2}[2 \times 9 + (n-1) \times 8] = 636$

$n[18 + 8n - 8] = 636 \times 2$

$18n + 8n^2 - 8n - 1272 = 0$

$8n^2 + 10n - 1272 = 0$

$4n^2 + 5n - 636 = 0$

$4n^2 + 53n - 48n - 636 = 0$

$n(4n + 53) - 12(4n + 53) = 0$

$(4n + 53)(n - 12) = 0$

$n = \frac{-53}{4}$ $or\ n = 12$

$n = 12(\ n\ is\ natural\ number)$

5. **The first term of an AP is 5, the last term is 45 and the sum is 400. Find the number of terms and the common difference.**

Sol: $a = 5; l = a_n = 45;\ S_n = 400$

$S_n = 400$

$\frac{n}{2}(a + l) = 400$

$\frac{n}{2}(5 + 45) = 400$

$n = \frac{400 \times 2}{50} = 16$

$a_n = 45$

$a + (n-1)d = 45$

$5 + (16 - 1) \times d = 45$

$15d = 45 - 5$

$d = \frac{40}{15} = \frac{8}{3}$

6. **The first and the last terms of an AP are 17 and 350 respectively. If the common difference is 9, how many terms are there and what is their sum?**

Sol: $a = 17\ , d = 9\ and\ \ l = a_n = 350$

$a_n = 350$

$a + (n-1)d = 350$

$17 + (n-1) \times 9 = 350$

$(n-1)\times 9 = 350 - 17$

$n - 1 = \frac{333}{9} = 37$

$n = 37 + 1 = 38$

$S_n = \frac{n}{2}(a + l)$

$= \frac{38}{2}(17 + 350)$

$= 19 \times 367 = 6973$

There are 38 terms and their sum is 6973.

Given A.P. contains 38 terms and the sum of the terms is 6973.

7. Find the sum of first 22 terms of an AP in which d = 7 and 22nd term is 149.

Sol: $a_{22} = 149$

$a + 21d = 149$

$a + 21 \times 7 = 149$

$a + 147 = 149$

$a = 2$

$S_n = \frac{n}{2}[a + l]$

$S_{22} = \frac{22}{2}[2 + 149] = 11 \times 151 = 1661$

8. Find the sum of first 51 terms of an AP whose second and third terms are 14 and 18 respectively.

Sol: $a_2 = 14 \Rightarrow a + d = 14 \rightarrow (1)$

$a_3 = 18 \Rightarrow a + 2d = 18 \rightarrow (2)$

$(2) - (1) \Rightarrow a + 2d = 18$

$a + d = 14$

$(-)\ (-)\ (-)$

d = 4

Substitute d=4 in (1)

$a + 4 = 14$

$a = 14 - 4 = 10$

$S_n = \frac{n}{2}[2a + (n-1)d]$

$S_{51} = \frac{51}{2}[2 \times 10 + (51 - 1) \times 4]$

$= \frac{51}{2}[20 + 50 \times 4]$

$= \frac{51}{2} \times 220$

$= 51 \times 110$

$= 5610$

9. If the sum of first 7 terms of an AP is 49 and that of 17 terms is 289, find the sum of first n terms.

Sol: $S_n = \frac{n}{2}[2a + (n-1)d]$

$S_7 = 49 \Rightarrow \frac{7}{2}[2a + (7-1)d] = 49$

$\Rightarrow [2a + 6d] = \frac{2 \times 49}{7}$

$\Rightarrow 2a + 6d = 14$

$\Rightarrow a + 3d = 7 \rightarrow (1)$

$S_{17} = 289 \Rightarrow \frac{17}{2}[2a + (17-1)d] = 289$

$\Rightarrow [2a + 16d] = \frac{2 \times 289}{17}$

$\Rightarrow 2a + 16d = 34$

$\Rightarrow a + 8d = 17 \rightarrow (2)$

$(2) - (1) \Rightarrow a + 8d = 17$

$a + 3d = 7$

$(-)\ (-)\quad (-)$

$5d = 10$

$d = 2$

Substitute d=2 in (1)

$a + 3 \times 2 = 7 \Rightarrow a + 6 = 7 \Rightarrow a = 1$

$S_n = \frac{n}{2}[2a + (n-1)d]$

$= \frac{n}{2}[2 \times 1 + (n-1)2]$

$= \frac{n}{2}[2 + 2n - 2]$

$= \frac{n}{2} \times 2n = n^2$

10. Show that $a_1, a_2, \ldots, a_n, \ldots$ form an AP where a_n is defined as below. Also find the sum of the first 15 terms in each case

(i) $a_n = 3 + 4n$

Sol: $a_n = 3 + 4n$

$a_1 = 3 + 4 \times 1 = 3 + 4 = 7$

$a_2 = 3 + 4 \times 2 = 3 + 8 = 11$

$a_3 = 3 + 4 \times 3 = 3 + 12 = 15$

$a_4 = 3 + 4 \times 4 = 3 + 16 = 19$

The list of terms are 7,11, 15,19,.....

$a_2 - a_1 = 11 - 7 = 4$

$a_3 - a_2 = 15 - 11 = 4$

$a_4 - a_3 = 19 - 15 = 4$

$a_2 - a_1 = a_3 - a_2 = a_4 - a_3 = \cdots$

i.e., $a_{k+1} - a_k$ is same every time

So, the given list of numbers forms an AP. $a = 7, d = 4$

$S_n = \frac{n}{2}[2a + (n-1)d]$

$S_{15} = \frac{15}{2}[2 \times 7 + (15 - 1) \times 4]$

$= \frac{15}{2}[14 + 56]$

$= \frac{15}{2} \times 70 = 15 \times 35 = 525$

(ii) $a_n = 9 - 5n$

Sol: $a_n = 9 - 5n$

$a_1 = 9 - 5 \times 1 = 9 - 5 = 4$

$a_2 = 9 - 5 \times 2 = 9 - 10 = -1$

$a_3 = 9 - 5 \times 3 = 9 - 15 = -6$

$a_4 = 9 - 5 \times 4 = 9 - 20 = -11$

The list of terms is $4, -1, -6, -11, \ldots \ldots$

$a_2 - a_1 = -1 - 4 = -5$

$a_3 - a_2 = -6 + 1 = -5$

$a_4 - a_3 = -11 + 6 = -5$

$a_2 - a_1 = a_3 - a_2 = a_4 - a_3 = \cdots$

i.e., $a_{k+1} - a_k$ is same every time

So, the given list of numbers forms an AP. $a = 4, d = -5$

$S_n = \frac{n}{2}[2a + (n-1)d]$

$S_{15} = \frac{15}{2}[2 \times 4 + (15 - 1) \times (-5)]$

$= \frac{15}{2}[8 - 70]$

$= \frac{15}{2} \times (-62) = 15 \times (-31) = -465$

11. **If the sum of the first n terms of an AP is $4n - n^2$, what is the first term? What is the sum of first two terms? What is the second term? Similarly, find the 3rd, the 10th and the nth terms**

Sol: $S_n = 4n - n^2$

$S_1 = 4 \times 1 - 1^2 = 4 - 1 = 3$

$S_2 = 4 \times 2 - 2^2 = 8 - 4 = 4$

$S_3 = 4 \times 3 - 3^2 = 12 - 9 = 3$

$S_4 = 4 \times 4 - 4^2 = 16 - 16 = 0$

$a_1 = S_1 = 3$

$a_2 = S_2 - S_1 = 4 - 3 = 1$

$a_3 = S_3 - S_2 = 3 - 4 = -1$

$\therefore a = 3, d = a_2 - a_1 = 1 - 3 = -2$

$a_{10} = a + 9d = 3 + 9 \times (-2) = 3 - 18 = -15$

$a_n = a + (n-1)d = 3 + (n-1) \times (-2) = 3 - 2n + 2 = 5 - 2n$

12. **Find the sum of the first 40 positive integers divisible by 6.**

Sol: The first 40 positive integers divisible by 6 are

$6 \times 1, 6 \times 2, 6 \times 3, \ldots \ldots \ldots, 6 \times 40$

$\Rightarrow 6, 12, 18, \ldots \ldots, 240$

$a = 6, d = 6, n = 40, l = 240$

$S_n = \frac{n}{2}[a + l]$

$S_{40} = \frac{40}{2}[6 + 240]$

$= 20 \times 246 = 4920$

Shortcut:

$S_{40} = 6 + 12 + 18 + \cdots .. + 240$

$= 6(1 + 2 + 3 + 4 + \cdots + 40)$

$= 6 \times \frac{40 \times 41}{2} = 3 \times 1640 = 4920$

13. **Find the sum of the first 15 multiples of 8.**

Sol: The multiples of 8 are 8, 16, 24, 32, ...

These numbers are in an A.P.

$a = 8 \quad , d = 8 \quad , n = 15$

The sum of first n terms $S_n = \frac{n}{2}[2a + (n - 1)d]$

$S_{15} = \frac{15}{2}[2 \times 8 + (15 - 1)8]$

$= \frac{15}{2}[16 + 14 \times 8]$

$= \frac{15}{2}[16 + 112]$

$= \frac{15}{2} \times 128$

$= 15 \times 64 = 960$

14. Find the sum of the odd numbers between 0 and 50.

Sol: The odd numbers lying between 0 and 50 are 1, 3, 5, 7, 9 ... 49

These odd numbers are in an A.P.

$a = 1; \quad d = 2; \quad l = 49$

We know that nth term of AP, $a_n = l = a + (n - 1)d$

49 = 1 + (n - 1) 2

48 = 2(n - 1)

n - 1 = 24

n = 25

$S_n = \frac{n}{2} [a + l]$

$S_{25} = \frac{25}{2} (1 + 49)$

$= \frac{25}{2} \times 50$

$= 25 \times 25$

$= 625$

15. A contract on construction job specifies a penalty for delay of completion beyond a certain date as follows: ₹200 for the first day, ₹ 250 for the second day, ₹ 300 for the third day, etc., the penalty for each succeeding day being ₹ 50 more than for the preceding day. How much money the contractor has to pay as penalty, if he has delayed the work by 30 days?

Sol: Penalty for 1st day = Rs. 200

Penalty for 2nd day = Rs. 250

Penalty for 3rd day = Rs. 300

These penalties are in A.P. a = 200, d = 50, n = 30

$S_n = \frac{n}{2} [2a + (n - 1) d]$

$$S_{30} = \frac{30}{2}[2 \times 200 + (30 - 1)50]$$

$= 15[400 + 1450]$

$= 15 \times 1850$

$= 27750$

Therefore, the contractor has to pay Rs. 27750 as a penalty.

16. **A sum of 700 is to be used to give seven cash prizes to students of a school for their overall academic performance. If each prize is 20 less than its preceding prize, find the value of each of the prizes.**

Sol: Let the prizes be $x, x-20, x-40, x-60, x-80, x-100, x-120$

$a = x, d = -20, l = x - 120$

$S_7 = 700$

$\frac{n}{2}(a+l) = 700$

$\frac{7}{2}[x + x - 120] = 700$

$2x - 120 = \frac{700 \times 2}{7} = 200$

$2x = 200 + 120 = 320$

$x = 160$

The prizes are ₹160, ₹140, ₹120, ₹100, ₹80, ₹60, ₹40.

17. **In a school, students thought of planting trees in and around the school to reduce air pollution. It was decided that the number of trees, that each section of each class will plant, will be the same as the class, in which they are studying, e.g., a section of Class I will plant 1 tree, a section of Class II will plant 2 trees and so on till Class XII. There are three sections of each class. How many trees will be planted by the students?**

Sol: Trees planted by each class are

$3 \times 1, 3 \times 2, 3 \times 3, \ldots\ldots\ldots\ldots, 3 \times 12$

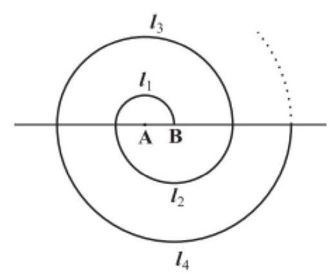
$\Rightarrow 3, 6, 9, \ldots\ldots\ldots, 36$ it is an AP

$a = 3, d = 3, n = 12, l = 36$

$S_n = \frac{n}{2}[a + l]$

$S_{12} = \frac{12}{2}[3 + 36] = 6 \times 39 = 234$

Total plants=234



18. A spiral is made up of successive semicircles, with centres alternately at A and B, starting with centre at A, of radii 0.5 cm, 1.0 cm, 1.5 cm, 2.0 cm, . . . as shown in Fig. 5.4. What is the total length of such a spiral made up of thirteen consecutive semicircles? (Take $\pi = \frac{22}{7}$)

Sol: The radii are 0.5 cm, 1.0 cm, 1.5 cm, 2.0 cm,……. these terms are in AP

$a = 0.5, d = 0.5, n = 13$

$l_1 = \pi \times r = \pi \times 0.5 = \pi \times \frac{1}{2} = \frac{\pi}{2}$

$l_2 = \pi \times 1 = \pi,\qquad l_3 = \pi \times 1.5 = \pi \times \frac{3}{2} = \frac{3\pi}{2}, \dots\dots$

Total length of spiral$= l_1 + l_2 + l_3 + \cdots . + l_{13}$

$= \frac{\pi}{2} + \pi + \frac{3\pi}{2} + \cdots \dots .13\ terms$

$= \frac{\pi}{2}[1 + 2 + 3 + \cdots 13\ terms]$

$$= \frac{\pi}{2}\left[\frac{13(13+1)}{2}\right] = \frac{\pi}{2}\left(\frac{13 \times 14}{2}\right) = \frac{\pi}{2} \times 91 = \frac{22}{7} \times \frac{1}{2} \times 91 = 11 \times 13 = 143\ cm$$

19. 200 logs are stacked in the following manner: 20 logs in the bottom row, 19 in the next row, 18 in the row next to it and so on. In how may rows are the 200 logs placed and how many logs are in the top row?

Sol: The logs in rows are 20,19,18,……is an AP

$a = 20, d = -1$

$S_n = 200$

$\frac{n}{2}[2a + (n-1)d] = 200$

$\frac{n}{2}[2 \times 20 + (n-1) \times (-1)] = 200$

$n[40 - n + 1] = 200 \times 2$

$41n - n^2 - 400 = 0$

$-n^2 + 41n - 400 = 0$

$n^2 - 41n + 400 = 0$

$(n-16)(n-25) = 0$

$n - 16 = 0\ \ or\ \ n - 25 = 0$

$n = 16\ or\ n = 25$

$\therefore n = 16\ (\ n\ cannot\ be\ 25)$

$a_{16} = a + 15d = 20 + 15(-1) = 20 - 15 = 5$

The number of logs in the top row=5.

20. In a potato race, a bucket is placed at the starting point, which is 5 m from the first potato, and the other potatoes are placed 3 m apart in a straight line. There are ten potatoes in the line. A competitor starts from the bucket, picks up the nearest potato, runs back with it, drops it in the bucket, runs back to pick up the next potato, runs to the bucket to drop it in, and she continues in the same way until all the potatoes are in the bucket. What is the total distance the competitor has to run?

Sol: The distance of first ball (from bucket)=5m

The distance of second ball=5+3=8m

The distance of third ball= 8+3=11 m

The distance of fourth ball= 11+3=14 m

……….

The distance of fourth ball= 11+3=14 m

The distance covered the competitor for 1st ,2nd ,3rd, …. Balls are

$2 \times 5m, 2 \times 8m, 2 \times 11m, \dots \dots (10\ terms)$

$10m, 16m, 22m, \dots \dots \dots . (10\ terms)\ \ clearly\ these\ terms\ are\ in\ AP$

$a = 10, d = 6, n = 10$

$S_n = \frac{n}{2}[2a + (n-1)d]$

$= \frac{10}{2}[2 \times 10 + (10-1) \times 6]$

$= 5[20 + 54] = 5 \times 74 = 370m$

CASE STUDY BASED QUESTINS

1) **India is competitive manufacturing location due to the low cost of manpower and strong technical and engineering capabilities contributing to higher quality production runs. The production of TV sets in a factory increases uniformly by a fixed number every year. It produced 16000 sets in 6th year and 22600 in 9th year.**

Based on the above information, answer the following questions:

(i) **Find the production during first year.**

(ii) **Find the production during 8th year.**

(iii) Find the production during first 3 years.

(iv) In which year, the production is Rs 29,200.

(v) Find the difference of the production during 7th year and 4th year

Sol: The production of TV sets in a factory increases uniformly by a fixed number every year.

Number of TV sets produced by the factory in the 6^{th} year=16000 and in the 9^{th} year=22600

$a + 5d = 16000 \rightarrow (1) and\ a + 8d = 22600 \rightarrow (2)$

From $(2) - (1)$: $\mathrm{a + 8d - a - 5d = 22600 - 16000}$

$3d = 6600$

$d = 2200$

$From\ (1): a + 5 \times 2200 = 16000$

$a = 16000 - 1100 = 5000$

(i) The production during the first year=a=₹5000

(ii) The production during the 8^{th} year=a+7d=5000+7×2200=5000+15400=20400

(iii)The production during first 3 years $= \frac{n}{2}[2a + (n-1)d] = \frac{3}{2}[2 \times 5000 + 2 \times 2200]$

$= \frac{3}{2}[10000 + 11000] = \frac{3}{2} \times 21000 = 3 \times 10500 = 31500$

$(iv)\ a_n = 29200 \Rightarrow a + (n-1)d = 29200$

$5000 + (n-1) \times 2200 = 29200$

$(n-1) \times 2200 = 29200 - 5000$

$(n-1) \times 2200 = 24200$

$n - 1 = \frac{24200}{2200} = 11$

$n = 11 + 1 = 12$

In 12^{th} year the production is 29200

(v) The difference of the production during 7th year and 4th year$= a_7 - a_4$

$= (a + 6d) - (a + 3d)$

$= a + 6d - a - 3d = 3d = 3 \times 2200 = 6600$

2) Your friend Veer wants to participate in a 200m race. He can currently run that distance in 51 seconds and with each day of practice it takes him 2 seconds less. He wants to do in 31 seconds .

(i) Which of the following terms are in AP for the given situation

a) 51,53,55…. b) 51, 49, 47…. c) -51, -53, -55…. d) 51, 55, 59…

(ii) What is the minimum number of days he needs to practice till his goal is achieved

a) 10 b) 12 c) 11 d) 9

(iii) Which of the following term is not in the AP of the above given situation

a) 41 b) 30 c) 37 d) 39

(iv) If n^{th} term of an AP is given by $a_n = 2n + 3$ then common difference of an AP is

a) 2 b) 3 c) 5 d) 1

(v) The value of x, for which 2x, x+ 10, 3x + 2 are three consecutive terms of an AP

a) 6 b) -6 c) 18 d) -18

Sol: (i) b

First day performance=51 sec

Second day performance=51-2=49 sec

Third day performance=49-2=47 sec

Performances in each day are 51, 49,47,.....(seconds)

These are in AP with a=51 and d=-2

(ii) c

$a_n = 31$

$a + (n-1)d = 31$

$51 + (n-1)(-2) = 31$

$(n-1)(-2) = 31 - 51 = -20$

$(n-1) = \frac{-20}{-2} = 10$

$n = 11$

∴11 days he needs to practice till his goal to achieve.

(iii) b

All terms are odd numbers.

(iv)a

$a_n = 2n + 3$

$a_1 = 2 \times 1 + 3 = 2 + 3 = 5$

$a_2 = 2 \times 2 + 3 = 4 + 3 = 7$

$d = a_2 - a_1 = 7 - 5 = 2$

(v) a

$a_1 = 2x, a_2 = x + 10, a_3 = 3x + 2$

$a_2 - a_1 = a_3 - a_2$

$x + 10 - 2x = 3x + 2 - x - 10$

$10 - x = 2x - 8$

$3x = 18$

$x = 6$

3) Your elder brother wants to buy a car and plans to take loan from a bank for his car. He repays his total loan of Rs 1,18,000 by paying every month starting with the first instalment of Rs 1000. If he increases the instalment by Rs 100 every month , answer the following:

(i) The amount paid by him in 30th installment is

a) 3900 b) 3500 c) 3700 d) 3600

(ii) The amount paid by him in the 30 installlments is

a) 37000 b) 73500 c) 75300 d) 75000

(iii) What amount does he still have to pay offer 30th installment?

a) 45500 b) 49000 c) 44500 d) 54000

(iv) If total instalments are 40 then amount paid in the last installment?

a) 4900 b) 3900 c) 5900 d) 9400

(v) The ratio of the 1st installment to the last installment is

a) 1:49 b) 10:49 c) 10:39 d) 39:10

Sol:

(i) a) 3900

First installment=₹1000

Second installment=₹1000+₹100=₹1100

Third installment=₹1000+2×₹100=₹1200

The installments : ₹1000,₹1100,₹1200,...are in AP

$a = 1000 \ and \ d = 100$

Amount paid in 30th installment=a+29d=1000+29×100=1000+2900=₹3900

(ii) b) 73500

$$Amount\ paid\ in\ 30\ installments = \frac{n}{2}[2a + (n-1)d]$$

$$= \frac{30}{2}[2 \times 1000 + 29 \times 100] = 1500 \times 4900 = ₹73500$$

(iii) c)44500

Total amount he still have to pay after the 30th instalment=

$$Total\ loan\ amount - \ Amount\ paid\ in\ 30\ installments = 118000 - 73500 = ₹44500$$

(iv) a)4900

Amount paid in last installement $= a_{40} = a + 39d = 1000 + 39 \times 100 = 4900$

(v) b)10:49

1st installment : last installment=1000:4900=10:49

Some more problems from exemplar and previous papers

1. For the AP: –3, –7, –11, ..., can we find directly $a_{30} - a_{20}$ without actually finding a30 and a20? Give reasons for your answer.
2. Is 0 a term of the AP: 31, 28, 25, ...? Justify your answer.
3. Find the value of the middle most term (s) of the AP : –11, –7, –3,..., 49.
4. The sum of the first three terms of an AP is 33. If the product of the first and the third term exceeds the second term by 29, find the AP. (Hint: Let the three terms in AP be a – d, a, a + d)
5. Find a, b and c such that the following numbers are in AP: a, 7, b, 23, c.
6. Find whether 55 is a term of the AP: 7, 10, 13,--- or not. If yes, find which term it is.
7. Determine k so that $k^2 + 4k + 8$, $2k^2 + 3k + 6$, $3k^2 + 4k + 4$ are three consecutive terms of an AP.
8. If the nth terms of the two APs: 9, 7, 5, ... and 24, 21, 18,... are the same, find the value of n. Also find that term.
9. Find the 12th term from the end of the AP: –2, –4, –6,..., –100.
10. Which term of the AP: 53, 48, 43,... is the first negative term?
11. In an A.P., the sum of the first n terms is given by $S_n=6n-n^2$.Find the 30^{th} term(CBSE-2023)

Answers:

1. 40
2. **No**
3. The two middle most terms are 17 and 21
4. 2,11,20
5. $a = -1; b = 15; c = 31$
6. Yes, 17^{th} term
7. k=0
8. 16^{th} term ;-21
9. -78
10. 12^{th} term
11. -53.

MCQ

1. The 10th term of the AP: 5, 8, 11, 14, ... is

   (A) 32 (B) 35 (C) 38 (D) 185
2. In an AP if a = –7.2, d = 3.6, a_n = 7.2, then n is

   (A) 1 (B) 3 (C) 4 (D) 5

3. In an AP, if d = –4, n = 7, an = 4, then a is
(A) 6 (B) 7 (C) 20 (D) 28

4. In an AP, if a = 3.5, d = 0, n = 101, then a_n will be
(A) 0 (B) 3.5 (C) 103.5 (D) 104.5

5. The 21st term of the AP whose first two terms are –3 and 4 is
(A) 17 (B) 137 (C) 143 (D) –143

6. Which term of the AP: 21, 42, 63, 84,... is 210?
(A) 9^{th} (B) 10^{th} (C) 11^{th} (D) 12^{th}

7. If the common difference of an AP is 5, then what is a_{18} -a_{13} ?
(A) 5 (B) 20 (C) 25 (D) 30

8. If 7 times the 7th term of an AP is equal to 11 times its 11th term, then its 18th term will be
(A) 7 (B) 11 (C) 18 (D) 0

9. In an AP if a = 1, a_n = 20 and S_n = 399, then n is
(A) 19 (B) 21 (C) 38 (D) 42

10. If the numbers n – 2, 4n – 1 and 5n + 2 are in AP, find the value of n.
(A) 2 (B) 1 (C) 3 (D) 4

11. The famous mathematician associated with finding the sum of the first 100 natural numbers is
(a) Pythagoras (b) Newton (c) Gauss (d) Euclid

12. The 13^{th} term from the end of the A.P:20,13,6,-1,…,-148
(A) 57 (B) -57 (C) 64 (D) -64

13. Assertion (A) : If the nth term of an A.P. is 7- 4n, then its common differences is -4.
Reason (R) : Common differences of an A.P .is given by d=$a_{n+1}-a_n$

14. Assertion (A) : 184 is the5^{th}term of the sequence 3, 7,11,…….

Reason (R) : The nth term of A.P .is given by a_n=a+(n−1)d

1)A	2)D	3)D	4)B	5)B	6)B	7)B	8)D	9)C	10)B	11) C	12) d	13)A	14)D	

Some more problems from previous year papers

1. **Find the 13^{th} term from the last term of the AP : 20, 13, 6,-1,. . ., -148.[CBSE-2023]**

Sol: $a = 20,\ d = 13 - 20 = -7$

$a_n = l = -148$

n^{th}term from the end of the AP $= l - (n-1)d$

13^{th}term from the end of the AP $= -148 - 12 \times (-7) = -148 + 84 = -64$

2. **In an AP ., if the first term a=7, nth term a_n=84 andd the sum of first n terms s_n=2093/2, then find n. [CBSE-2024]**

$Sol: S_n = \frac{n}{2}[a + a_n]$

$\frac{2093}{2} = \frac{n}{2}[7 + 84]$

$2093 = n \times 91$

$n = \frac{2093}{91} = 23$

3. The sum of first and eight terms of an A.P. is 32 and their product is 60. Find the first term and common difference of the A.P. Hence, also find the sum of its first 20 terms.

$Sol: First\ term + eight\ term\ of\ A.P = 32$

$a + a + 7d = 32$

$a + 7d = 32 - a \rightarrow (1)$

Product of first and eight terms=60

$a \times (a + 7d) = 60$

$a(32 - a) = 60$

$32a - a^2 = 60$

$a^2 - 32a + 60 = 0$

$(a - 2)(a - 30) = 0$

$a = 2\ or\ 30$

If $a = 2$ then $2 + 7d = 32 - 2 \Rightarrow 7d = 28 \Rightarrow d = 4$

$The\ sum\ of\ its\ first\ 20\ terms = \frac{n}{2}[2a + (n - 1)d] = \frac{20}{2}[2 \times 2 + 19 \times (4)] = 10[4 + 76]$

$= 10 \times 80 = 800$

If $a = 30$ then $30 + 7d = 32 - 30 \Rightarrow 7d = -28 \Rightarrow d = -4$

$The\ sum\ of\ its\ first\ 20\ terms = \frac{n}{2}[2a + (n - 1)d] = \frac{20}{2}[2 \times 30 + 19 \times (-4)] = 10[60 - 76]$

$= 10 \times (-16) = -160$

4. In an A.P of 40 terms, the sum of first 9 terms is 153 and the sum of last 6 terms is 687, deter main the first term and common difference of A.P. Also , find the sum of all terms of the A.P.[CBSE-2024]

$Sol: Sum\ of\ first\ 9\ terms\ in\ A.P = 153$

$$S_n = \frac{n}{2}[2a + (n - 1)d]$$

$\frac{9}{2}[2a + 8d] = 153$

$2a + 8d = \frac{153 \times 2}{9} = 34$

$2a + 8d = 34 \rightarrow (1)$

The sum of last 6 terms $= 687$

$a_{35} + a_{356} + a_{37} + a_{38} + a_{39} + a_{40} = 687$

$(a + 34d) + (a + 35d) + (a + 36d) + (a + 37d) + (a + 38d) + (a + 39d) = 687$

$6da + 219d = 687$

$2a + 73d = 229 \rightarrow (2)$

$From\ (2) - (1): 2a + 73d - 2a - 8d = 229 - 34$

$65d = 195$

$d = 3$

Substitute d=3 in (1)

$2a + 8 \times 3 = 34$

$2a = 34 - 24 = 10$

$a = 5$

$The\ sum\ of\ all\ terms\ of\ the\ A.P = \frac{40}{2}[2 \times 3 + 39 \times 3] = 20 \times (6 + 117) = 20 \times 223 = 4460$

5. Find the sum of first 20 terms of an A.P whose n^th^ term is given by $a_n = 5 - 2n$ [CBSE-2022]

Sol: $a_n = 5 - 2n$

$a = a_1 = 5 - 2 \times 1 = 5 - 2 = 3$

$a_{20} = 5 - 2 \times 20 = 5 - 40 = -35$

$S_n = \frac{n}{2}[a + a_n]$

$S_{20} = \frac{20}{2}[3 - 35] = 10 \times (-30) = -300$

6. Which term of the A.P $-\frac{11}{2}, -3, -\frac{1}{2}, \dots is\ \frac{49}{2}$?

Sol: $a = -\frac{11}{2};\ \ d = -3 + \frac{11}{2} = \frac{-6 + 11}{2} = \frac{5}{2}$

$a_n = \frac{49}{2}$

$a + (n - 1)d = \frac{49}{2}$

$-\frac{11}{2} + (n - 1) \times \frac{5}{2} = \frac{49}{2}$

$(n - 1) \times \frac{5}{2} = \frac{49}{2} + \frac{11}{2} = \frac{60}{2} = 30$

$n - 1 = 30 \times \frac{2}{5} = 12$

$n = 12 + 1 = 13$

7. Find a and b so that the numbers a,7,b,23 are in A.P[CBSE-2022]

Sol: a, 7, b, 23 are in A. P

$a_2 = 7 \Rightarrow a + d = 7 \rightarrow (1)$

$a_4 = 23 \Rightarrow a + 3d = 23 \rightarrow (2)$

$From\ (2) - (1): a + 3d - a - d = 23 - 7$

$2d = 16 \Rightarrow d = 8$

$From(1): a + 8 = 7 \Rightarrow a = 7 - 8 = -1$

$b = a + 2d = -1 + 16 = 15$