CHAPTER 4

X-MATHEMATICS-NCERT-2024-25

QUADRATIC EQUATIONS

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1. The polynomial equation whose highest degree is two is called a quadratic equation.
2. Any equation of the form $p(x) = 0$ where p(x) is polynomial of degree 2, is a **quadratic equation**.
3. **Standard form of quadratic equation** in variable x is $\boldsymbol{ax^2 + bx + c = 0}$, where a, b, c are real numbers and $a \neq 0$. We can write it as $y = ax^2 + \text{b}x + \text{c}$

coefficient of x^2 — *constant*

$$ax^2 + bx + c = 0$$

coefficient of x

Example 1 : Represent the following situations mathematically:

(i) John and Jivanti together have 45 marbles. Both of them lost 5 marbles each, and the product of the number of marbles they now have is 124. We would like to find out how many marbles they had to start with.

Sol: Let the number of marbles at John $= x$. Then the number of marbles at Jivanti $= 45 - x$.

The number of marbles left with John, when he lost 5 marbles $= x - 5$

The number of marbles left with Jivanti, when she lost 5 marbles $= 45 - x - 5 = 40 - x$

Therefore, their product $= 124$

$(x - 5)(40 - x) = 124$

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$40x - x^2 - 200 + 5x = 124$

$-x^2 + 45x - 200 = 124$

$-x^2 + 45x - 200 - 124 = 0$

$-x^2 + 45x - 324 = 0$

$x^2 - 45x + 324 = 0$

This is the required representation of the problem mathematically.

(ii) A cottage industry produces a certain number of toys in a day. The cost of production of each toy (in rupees) was found to be 55 minus the number of toys produced in a day. On a particular day, the total cost of production was ₹750. We would like to find out the number of toys produced on that day.

Sol: Let the number of toys produced on that day $= x$.

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The cost of production (in rupees) of each toy that day $= 55 - x$

So, the total cost of production (in rupees) that day $= x(55 - x)$

$x(55 - x) = 750$

$55x - x^2 = 750$

$-x^2 + 55x - 750 = 0$

$x^2 - 55x + 750 = 0$

This is the required representation of the problem mathematically.

Example 2 : Check whether the following are quadratic equations:

Standard form of quadratic equation in variable x is $\boldsymbol{ax^2 + bx + c = 0}$, where a, b, c are real numbers and $a \neq 0$.

i. $\boldsymbol{(x-2)^2 + 1 = 2x - 3}$

Sol: $(x-2)^2 + 1 = 2x - 3$

$\Rightarrow x^2 - 4x + 4 + 1 - 2x + 3 = 0$

$\Rightarrow x^2 - 6x + 8 = 0$

It is of the form $ax^2 + bx + c = 0$ $(a = 1, b = -6, c = 8)$

The given equation is a quadratic equation.

ii. $\boldsymbol{x(x+1) + 8 = (x+2)(x-2)}$

Sol: $x(x+1) + 8 = (x+2)(x-2)$

$\Rightarrow x^2 + x + 8 = x^2 - 2^2$

$\Rightarrow x^2 + x + 8 - x^2 + 4 = 0$

$\Rightarrow x + 12 = 0$

It is not of the form $ax^2 + bx + c = 0$

The given equation is not a quadratic equation.

iii. $\boldsymbol{x(2x+3) = x^2 + 1}$

Sol: $x(2x+3) = x^2 + 1$

$\Rightarrow 2x^2 + 3x - x^2 - 1 = 0$

$\Rightarrow x^2 + 3x - 1 = 0$

It is of the form $ax^2 + bx + c = 0$ $(a = 1, b = 3, c = -1)$

The given equation is a quadratic equation.

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iv. $(x+2)^3 = x^3 - 4$

Sol: $(x+2)^3 = x^3 - 4$

$(a+b)^3 = a^3 + b^3 + 3ab(a+b)$

$\Rightarrow x^3 + 2^3 + 3 \times x \times 2(x+2) = x^3 - 4$

$\Rightarrow x^3 + 8 + 6x(x+2) = x^3 - 4$

$\Rightarrow x^3 + 8 + 6x^2 + 12x - x^3 + 4 = 0$

$\Rightarrow 6x^2 + 12\text{x} + 12 = 0$

It is of the form $ax^2 + bx + c = 0$ $(a = 6, b = 12, c = 12)$

The given equation is a quadratic equation.

EXERCISE 4.1

1. **Check whether the following are quadratic equations :**

i. $(x+1)^2 = 2(x-3)$

Sol: $(x+1)^2 = 2(x-3)$

$\Rightarrow x^2 + 2x + 1 = 2x - 6$

$\Rightarrow x^2 + 2x + 1 - 2x + 6 = 0$

$\Rightarrow x^2 + 7 = 0$

It is of the form $ax^2 + bx + c = 0$ $(a = 1, b = 0, c = 7)$

The given equation is a quadratic equation.

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ii. $x^2 - 2x = (-2)(3-x)$

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Sol: $x^2 - 2x = (-2)(3-x)$

$\Rightarrow x^2 - 2x = -6 + 2x$

$\Rightarrow x^2 - 2x + 6 - 2x = 0$

$\Rightarrow x^2 - 4x + 6 = 0$

It is of the form $ax^2 + bx + c = 0$ $(a = 1, b = -4, c = 6)$

The given equation is a quadratic equation.

iii. $(x-2)(x+1) = (x-1)(x+3)$

Sol: $(x-2)(x+1) = (x-1)(x+3)$

$\Rightarrow x^2 + x - 2x - 2 = x^2 + 3x - x - 3$

$\Rightarrow x^2 - x - 2 = x^2 + 2x - 3$

$\Rightarrow x^2 - x - 2 - x^2 - 2x + 3 = 0$

$\Rightarrow -3x + 1 = 0$

It is not of the form $ax^2 + bx + c = 0$

The given equation is not a quadratic equation.

iv. $(x - 3)(2x + 1) = x(x + 5)$

Sol: $(x - 3)(2x + 1) = x(x + 5)$

$\Rightarrow 2x^2 + x - 6x - 3 = x^2 + 5x$

$\Rightarrow 2x^2 - 5x - 3 - x^2 - 5x = 0$

$\Rightarrow x^2 - 10x - 3 = 0$

It is of the form $ax^2 + bx + c = 0 \ (a = 1, b = -10, c = -3)$

The given equation is a quadratic equation.

v. $(2x - 1)(x - 3) = (x + 5)(x - 1)$

Sol: $(2x - 1)(x - 3) = (x + 5)(x - 1)$

$\Rightarrow 2x^2 - 6x - x + 3 = x^2 - x + 5x - 5$

$\Rightarrow 2x^2 - 7x + 3 = x^2 + 4x - 5$

$\Rightarrow 2x^2 - 7x + 3 - x^2 - 4x + 5 = 0$

$\Rightarrow x^2 - 11x + 8 = 0$

It is of the form $ax^2 + bx + c = 0 \ (a = 1, b = -11, c = 8)$

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The given equation is a quadratic equation.

vi. $x^2 + 3x + 1 = (x - 2)^2$

Sol: $x^2 + 3x + 1 = (x - 2)^2$

$\Rightarrow x^2 + 3x + 1 = x^2 - 4x + 4$

$\Rightarrow x^2 + 3x + 1 - x^2 + 4x - 4 = 0$

$\Rightarrow 7x - 3 = 0$

It is not of the form $ax^2 + bx + c = 0$

The given equation is not a quadratic equation.

vii. $(x + 2)^3 = 2x(x^2 - 1)$

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Sol: $(x+2)^3 = 2x(x^2-1)$

$\Rightarrow x^3 + 2^3 + 3 \times x \times 2(x+2) = 2x^3 - 2x$

$\Rightarrow x^3 + 8 + 6x(x+2) = 2x^3 - 2x$

$\Rightarrow x^3 + 8 + 6x^2 + 12x - 2x^3 + 2x = 0$

$\Rightarrow -x^3 + 6x^2 + 14x + 8 = 0$

It is not of the form $ax^2 + bx + c = 0$

The given equation is not a quadratic equation.

viii. $x^3 - 4x^2 - x + 1 = (x-2)^3$

Sol: $x^3 - 4x^2 - x + 1 = (x-2)^3$

$\Rightarrow x^3 - 4x^2 - x + 1 = x^3 - 2^3 - 3 \times x \times 2(x-2)$

$\Rightarrow x^3 - 4x^2 - x + 1 = x^3 - 8 - 6x^2 + 12x$

$\Rightarrow x^3 - 4x^2 - x + 1 - x^3 + 8 + 6x^2 - 12x = 0$

$\Rightarrow 2x^2 - 13x + 9 = 0$

It is of the form $ax^2 + bx + c = 0$ $(a = 2, b = -13, c = 9)$

The given equation is a quadratic equation.

2. **Represent the following situations in the form of quadratic equations :**

i. The area of a rectangular plot is 528 m². The length of the plot (in metres) is one more than twice its breadth. We need to find the length and breadth of the plot.

Sol: Let breadth of rectangular plot $(b) = x\ m$

Length of rectangular plot $(l) = (2x+1)m$

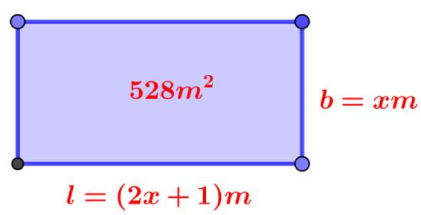
Given area of the rectangular plot $= 528\ m^2$

$l \times b = 528$

$(2x+1) \times x = 528$

$2x^2 + x - 528 = 0$

This is the required quadratic equation.



ii. The product of two consecutive positive integers is 306. We need to find the integers.

Sol: Let the two consecutive positive integers be $x, x+1$

Given the product of two consecutive positive integers $= 306$

$x \times (x+1) = 306 \Longrightarrow x^2 + x - 306 = 0$

This is the required quadratic equation

iii. Rohan's mother is 26 years older than him. The product of their ages after 3 years will be 360 years. We need to find Rohan's present age.

Sol: Let Rohan's age=x years

Rohan's mother age=$(x+26)years$

	Rohan	Rohan's mother
Present age(years)	x	$x+26$
Age after 3 years	$x+3$	$x+29$

After 3 years

Rohan's age=$x+3$ years

Rohan's mother age=$(x+26+3) = (x+29)years$

Given the product of their ages after 3 years=360 years

$\Rightarrow (x+3)(x+29) = 360$

$\Rightarrow x^2 + 29x + 3x + 87 - 360 = 0$

$\Rightarrow x^2 + 32x - 273 = 0$

This is the required quadratic equation.

iv. A train travels a distance of 480 km at a uniform speed. If the speed had been 8 km/h less, then it would have taken 3 hours more to cover the same distance. We need to find the speed of the train.

Sol: Let the speed of the train= $x\ km/h$

Distance= 480 km

$\text{Time}(T_1) = \frac{\text{Distance}}{\text{Speed}} = \frac{480}{x}\ h.$

If the speed had been 8 km/h less, then the speed=$(x-8)\ km/h$

$\text{Time}(T_2) = \frac{\text{Distance}}{\text{Speed}} = \frac{480}{x-8}h$

Difference of the times= $(T_2 - T_1)$ =3h

$$\frac{480}{x-8} - \frac{480}{x} = 3$$

$$480\left(\frac{1}{x-8} - \frac{1}{x}\right) = 3$$

$$\frac{x-(x-8)}{x(x-8)} = \frac{3}{480}$$

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$$\frac{x-x+8}{x^2-8x}=\frac{1}{160}$$

$$\frac{8}{x^2-8x}=\frac{1}{160}$$

$$x^2-8x=160\times 8$$

$$x^2-8x=1280$$

$$x^2-8x-1280=0$$

This is the required quadratic equation to find the speed of the train.

Solution of a Quadratic Equation by Factorisation

(i) A real number α is called a root of the quadratic equation $ax^2 + bx + c = 0$, if $a\alpha^2 + b\,\alpha + c = 0$ We also say that x = α is a solution of the quadratic equation. (i.e) the real value of for which the quadratic equation $ax^2 + bx + c = 0$is satisfied is called its solution.

(ii) The zeroes of the quadratic polynomial $ax^2 + bx + c$ and the roots of the quadratic equation $ax^2 + bx + c = 0$ are the same.

Example-3. Find the roots of the equation $2x^2 - 5x + 3 = 0$, by factorisation.

Sol: $2x^2-5x+3=0$

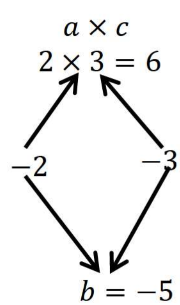
$2x^2-2x-3x+3=0$

$2x(x-1)-3(x-1)=0$

$(x-1)(2x-3)=0$

$x-1=0 \quad or \quad 2x-3=0$

$x=1 \quad or \quad x=\frac{3}{2}$



$\therefore 1\ and\ \frac{3}{2}$ are the roots of the equation $2x^2-5x+3=0$

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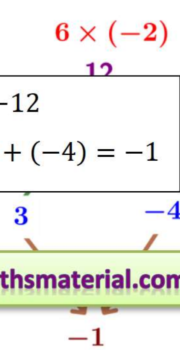
Example 4 : Find the roots of the quadratic equation $6x^2 - x - 2 = 0$.

Sol: $6x^2-x-2=0$

$6x^2+3x-4x-2=0$

$3x(2x+1)-2(2x+1)=0$

$(2x+1)(3x-2)=0$



$6\times(-2)=-12$

$3\times(-4)=-12\ and\ 3+(-4)=-1$

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$2x + 1 = 0 \; or \; 3x - 2 = 0$

$x = -\frac{1}{2} \; or \; x = \frac{2}{3}$

The roots of $6x^2 - x - 2 = 0 \; are \; -\frac{1}{2} \; and \; \frac{2}{3}$

Example 5 : Find the roots of the quadratic equation $3x^2 - 2\sqrt{6}x + 2 = 0$

Sol: $3x^2 - 2\sqrt{6}x + 2 = 0$

$3 \times 2 = 6$

$(-\sqrt{6})(-\sqrt{6}) = 6$

$-\sqrt{6} - \sqrt{6} = -2\sqrt{6}$

$3x^2 - \sqrt{6}x - \sqrt{6}x + 2 = 0$

$\sqrt{3} \times \sqrt{3}x^2 - \sqrt{3} \times \sqrt{2}x - \sqrt{3} \times \sqrt{2}x + \sqrt{2} \times \sqrt{2} = 0$

$\sqrt{3}x(\sqrt{3}x - \sqrt{2}) - \sqrt{2}(\sqrt{3}x - \sqrt{2}) = 0$

$(\sqrt{3}x - \sqrt{2})(\sqrt{3}x - \sqrt{2}) = 0$

$\sqrt{3}x - \sqrt{2} = 0 \Rightarrow x = \frac{\sqrt{2}}{\sqrt{3}} = \sqrt{\frac{2}{3}}$

The roots of $3x^2 - 2\sqrt{6}x + 2 = 0 \; are \; \sqrt{\frac{2}{3}}, \sqrt{\frac{2}{3}}$

Example 6 : Find the dimensions of the prayer hall having a carpet area of 300 square metres with its length one metre more than twice its breadth.

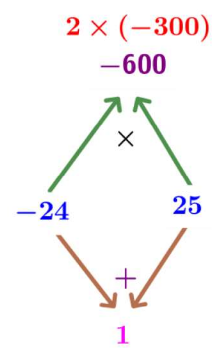
Sol: Let breadth of the hall = x metres

Length $= (2x + 1)$ metres.

Area of the hall $= (2x + 1).x \, m^2 = (2x^2 + x)m^2$

$2x^2 + x = 300 \; (Given)$

$2x^2 + x - 300 = 0$



$2x^2 - 24x + 25x - 300 = 0$

$2 \times (-300) = -600$

$(-24) \times 25 = -600 \; and \; -24 + 25 = 1 + (-4)$

$2x(x - 12) + 25(x - 12) = 0$

$i.e., (x - 12)(2x + 25) = 0$

$x - 12 = or \; 2x + 25 = 0$

$x = 12 \; or \; x = \frac{-25}{2}$

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Since x is the breadth of the hall, it cannot be negative

$\therefore x = 12$

Length $= 2x + 1 = 2 \times 12 + 1 = 24 + 1 = 25m$ and breadth $= x = 12\ m$

The dimensions of the hall are 25 m and 12m

EXERCISE 4.2

1. Find the roots of the following quadratic equations by factorisation:

$(i) x^2 - 3x - 10 = 0$

Sol: $x^2 - 3x - 10 = 0$

$x^2 - 2x + 5x - 10 = 0$

$x(x-2) + 5(x-2) = 0$

$(x-2)(x+5) = 0$

$x - 2 = 0 \quad or \quad x + 5 = 0$

$x = 2 \ or \ x = -5$

The roots of $x^2 - 3x - 10 = 0$ are 2 and -5

$(ii) 2x^2 + x - 6 = 0$

Sol: $2x^2 + x - 6 = 0$

$2x^2 - 3x + 4x - 6 = 0$

$x(2x-3) + 2(2x-3) = 0$

$(2x-3)(x+2) = 0$

$2x - 3 = 0 \quad or \quad x + 2 = 0$

$x = \frac{3}{2} \quad or \quad x = -2$

The roots of $2x^2 + x - 6 = 0$ are $\frac{3}{2}$ and -2

$(iii)\ \sqrt{2}x^2 + 7x + 5\sqrt{2} = 0$

Sol: $\sqrt{2}x^2 + 7x + 5\sqrt{2} = 0$

$\sqrt{2}x^2 + 2x + 5x + 5\sqrt{2} = 0$

$\sqrt{2}x(x + \sqrt{2}) + 5(x + \sqrt{2}) = 0$

$(x + \sqrt{2})(\sqrt{2}x + 5) = 0$

$x + \sqrt{2} = 0 \ \ or \ \ \sqrt{2}x + 5 = 0$

$\sqrt{2} \times 5\sqrt{2} = 5 \times 2 = 10$
$2 \times 5 = 10$
$2 + 5 = 7$

$x = -\sqrt{2} \quad or \quad x = \frac{-5}{\sqrt{2}}$

The roots of $\sqrt{2}x^2 + 7x + 5\sqrt{2} = 0$ $are\ -\sqrt{2}\ and\ \frac{-5}{\sqrt{2}}$

$(iv)\ \ 2x^2 - x + \frac{1}{8} = 0$

Sol: $2x^2 - x + \frac{1}{8} = 0$

Multiply with '8'

$8 \times 2x^2 - 8 \times x + 8 \times \frac{1}{8} = 8 \times 0$

$16x^2 - 8x + 1 = 0$

$16x^2 - 4x - 4x + 1 = 0$

$4x(4x - 1) - 1(4x - 1) = 0$

$(4x - 1)(4x - 1) = 0$

$4x - 1 = 0 \quad or \quad 4x - 1 = 0$

$x = \frac{1}{4} \quad or \ x = \frac{1}{4}$

The roots of $2x^2 - x + \frac{1}{8} = 0$ $are\ \frac{1}{4}\ and\ \frac{1}{4}$.

$(v) 100x^2 - 20x + 1 = 0$

Sol: $100x^2 - 20x + 1 = 0$

$100x^2 - 10x - 10x + 1 = 0$

$10x(10x - 1) - 1(10x - 1) = 0$

$(10x - 1)(10x - 1) = 0$

$10x - 1 = 0 \quad or \quad 10x - 1 = 0$

$x = \frac{1}{10} \quad or \ x = \frac{1}{10}$

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The roots of $100x^2 - 20x + 1 = 0$ $are\ \frac{1}{10}\ and\ \frac{1}{10}$

2. (i) John and Jivanti together have 45 marbles. Both of them lost 5 marbles each, and the product of the number of marbles they now have is 124. We would like to find out how many marbles they had to start with.

Sol: Total marbles=45

	John	Jivanti
Number of marbles	x	$45 - x$
Number of marbles when he lost 5 marbles	$x - 5$	$45 - x - 5 = 40 - x$

Given that the product of marbles when they lost 5 marbles =124

$(x - 5)(40 - x) = 124$

$40x - x^2 - 200 + 5x = 124$

$-x^2 + 45x - 200 = 124$

$-x^2 + 45x - 200 - 124 = 0$

$-x^2 + 45x - 324 = 0$

$x^2 - 45x + 324 = 0$

$x^2 - 36x - 9x + 324 = 0$

$x(x - 36) - 9(x - 36) = 0$

$(x - 36)(x - 9) = 0$

$x = 36 \text{ or } x = 9$

If $x = 36$ then John's marbles=36 and Jivanti's marbles=9

If $x = 9$ then John's marbles=9 and Jivanti's marbles=36

(ii) A cottage industry produces a certain number of toys in a day. The cost of production of each toy (in rupees) was found to be 55 minus the number of toys produced in a day. On a particular day, the total cost of production was ₹ 750. We would like to find out the number of toys produced on that day.

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Sol: Let the number of toys produced on that day be x. Therefore, the cost of production (in rupees) of each toy that day = $55 - x$

Given the total cost of production (in rupees) that day $= 750$

$x(55 - x) = 750$

$55x - x^2 = 750$

$-x^2 + 55x - 750 = 0$

$x^2 - 55x + 750 = 0$

$(-25) \times (-30) = 750$
$-25 - 30 = -55$

$(x - 25)(x - 30) = 0$

$x = 25 \text{ or } x = 30$

$\therefore$ The number of toys produced on that day = 25 or 30

3. Find two numbers whose sum is 27 and product is 182.

Sol: Let one number $= x$, The second number $= 27 - x$

Product of numbers=182

$x(27 - x) = 182$

$27x - x^2 = 182$

$-x^2 + 27x - 182 = 0$

$x^2 - 27x + 182 = 0$

$x^2 - 13x - 14x + 182 = 0$

$x(x - 13) - 14(x - 13) = 0$

$(x - 13)(x - 14) = 0$

$x - 13 = 0 \quad or \quad x - 14 = 0$

$x = 13 \quad or \quad x = 14$

If $x = 13$ the required numbers are 13 and 14.

If $x = 14$ the required numbers are 14 and 13.

4. Find two consecutive positive integers, sum of whose squares is 365.

Sol: Let the two consecutive positive integers be $x, x + 1$.

Sum of whose squares $= 365$

$x^2 + (x + 1)^2 = 365$

$x^2 + x^2 + 2x + 1 - 365 = 0$

$2x^2 + 2x - 364 = 0$

$x^2 + x - 182 = 0$

$x^2 - 13x + 14x - 182 = 0$

$x(x - 13) + 14(x - 17) = 0$

$(x - 13)(x + 14) = 0$

$x = 13 \quad or \quad x = -14$

$\therefore x = 13$ (since x is a positive integer so $x \neq -14$)

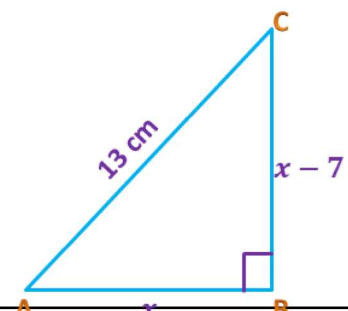
The required two consecutive positive integers are 13 and 14.

5. The altitude of a right triangle is 7 cm less than its base. If the hypotenuse is 13 cm, find the other two sides.

Sol: Base of a right triangle (AB)$=x$

The altitude (BC) $= x - 7$ cm

The hypotenuse (AC)$= 13$ cm



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From Pythagoras theorem

$AB^2 + BC^2 = AC^2$

$x^2 + (x-7)^2 = 13^2$

$x^2 + x^2 - 14x + 49 - 169 = 0$

$2x^2 - 14x - 120 = 0$

$x^2 - 7x - 60 = 0$

$x^2 - 12x + 5x - 60 = 0$

$x(x-12) + 5(x-12) = 0$

$(x-12)(x+5) = 0$

$x - 12 = 0 \quad or \quad x + 5 = 0$

$x = 12 \quad or \quad x = -5$

$\therefore x = 12$ (since side of a triangle is positive integer so $x \neq -5$)

The other two sides are $12\ cm, (12-7)\ cm\ \ i.e\ 12\ cm, 5\ cm.$

6. A cottage industry produces a certain number of pottery articles in a day. It was observed on a particular day that the cost of production of each article (in rupees) was 3 more than twice the number of articles produced on that day. If the total cost of production on that day was ` 90, find the number of articles produced and the cost of each article.

Sol: Let the number of articles produced=x

The cost of each article=$Rs\ (2x+3)$

Given the total cost of production on that day=Rs 90

$x(2x+3) = 90$

$2x^2 + 3x - 90 = 0$

$2x^2 - 12x + 15x - 90 = 0$

$2x(x-6) + 15(x-6) = 0$

$(x-6)(2x+15) = 0$

$x - 6 = 0 \quad or \quad 2x + 15 = 0$

$x = 6 \quad or \quad x = \frac{-15}{2}$

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$\therefore\ x = 6$ (Number of articles is always can't be negative)

The number of articles produced= x =6

The cost of each article= $(2x + 3)$= $(2 \times 6 + 3)$ = ₹ 15.

Quadratic formula (formula for finding the roots of a quadratic equation)

The the roots of quadratic equation $ax^2 + bx + c = 0$ are given by $\boldsymbol{x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}}$

Nature of Roots

The nature of roots of a quadratic equation $ax^2 + bx + c = 0$ depends on $\boldsymbol{b^2 - 4ac}$

$\boldsymbol{b^2 - 4ac}$ is called the discriminant of this quadratic equation.

A quadratic equation $ax^2 + bx + c = 0$ has

(i) two distinct real roots, if $b^2 - 4ac > 0$.

(ii) two equal real roots, if $b^2 - 4ac = 0$

(iii) no real roots, if $b^2 - 4ac < 0$.

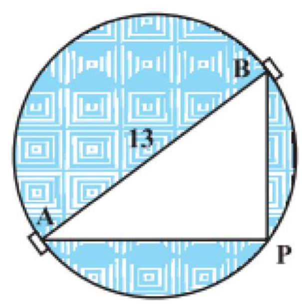
Example 7: Find the discriminant of the quadratic equation $2x^2 - 4x + 3 = 0$, and hence find the nature of its roots.

Sol: Given Q. E is $2x^2 - 4x + 3 = 0$; $a = 2, b = -4, c = 3$

$b^2 - 4ac = (-4)^2 - 4 \times 2 \times 3 = 16 - 24 = -8 < 0$

So, the given equation has no real roots.

Example 8 : A pole has to be erected at a point on the boundary of a circular park of diameter 13 metres in such a way that the differences of its distances from two diametrically opposite fixed gates A and B on the boundary is 7 metres. Is it possible to do so? If yes, at what distances from the two gates should the pole be erected?



Sol: Let P be the required location of the pole.

Let BP = x m then AP = $(x + 7)$m and AB = 13 m

We know that angle in semicircle=90^0. So $\angle APB = 90^0$

$AP^2 + BP^2 = AB^2$ (By Pythagoras theorem)

$(x + 7)^2 + x^2 = 13^2$

$x^2 + 14x + 49 + x^2 - 169 = 0$

$2x^2 + 14x - 120 = 0$

$x^2 + 7x - 60 = 0$

$-5 \times 12 = -60$
$-5 + 12 = 7$

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$(x-5)(x+12)=0$

$x-5=0$ or $x+12=0$

$x=5$ or $x=-12$

$\therefore x=5$ (distance can't be negative)

Thus, the pole has to be erected on the boundary of the park at a distance of 5m from the gate B and 12m from the gate A.

Example-9. Find the discriminant of the equation $3x^2-2x+\frac{1}{3}=0$ and hence find the nature of its roots. Find them, if they are real.

Sol: Given Q.E is $3x^2-2x+\frac{1}{3}=0$: $a=3, b=-2, c=\frac{1}{3}$

$$b^2-4ac=(-2)^2-4\times 3\times\frac{1}{3}=4-4=0$$

So, the roots are equal and real.

The roots are $\frac{-b}{2a},\frac{-b}{2a}\Rightarrow\frac{2}{2\times3},\frac{2}{2\times3}\Rightarrow\frac{1}{3},\frac{1}{3}$.

EXERCISE 4.3

1. Find the nature of the roots of the following quadratic equations. If the real roots exist, find them:

(i) $2x^2-3x+5=0$

Sol: $a=2, b=-3, c=5$

$b^2-4ac=(-3)^2-4\times2\times5=9-40=-31<0$

So, the Q.E has no real roots.

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(ii) $3x^2-4\sqrt{3}x+4=0$

Sol: $a=3, b=-4\sqrt{3}, c=4$

$$b^2-4ac=\left(-4\sqrt{3}\right)^2-4\times3\times4=48-48=0$$

So, the roots are real and equal

The roots are $\frac{-b}{2a},\frac{-b}{2a}\Rightarrow\frac{4\sqrt{3}}{2\times3},\frac{4\sqrt{3}}{2\times3}\Rightarrow\frac{2\sqrt{3}}{3},\frac{2\sqrt{3}}{3}$

(iii) $2x^2-6x+3=0$

Sol: $a=2, b=-6, c=3$

$b^2-4ac=(-6)^2-4\times2\times3=36-24=12>0$

So, the roots are real and distinct.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{6 \pm \sqrt{12}}{2 \times 2} = \frac{6 \pm 2\sqrt{3}}{4} = \frac{2(3 \pm \sqrt{3})}{4} = \frac{3 \pm \sqrt{3}}{2}$$

The roots are $\frac{3+\sqrt{3}}{2}, \frac{3-\sqrt{3}}{2}$

2. Find the values of k for each of the following quadratic equations, so that they have two equal roots.

(i) $2x^2 + kx + 3 = 0$

Sol: $a = 2, b = k, c = 3$

If the Q.E has equal roots then $b^2 - 4ac = 0$

$k^2 - 4 \times 2 \times 3 = 0$

$k^2 = 24 \Rightarrow k = \pm\sqrt{24} = \pm\sqrt{4 \times 6} = \pm 2\sqrt{6}$

(ii) $kx(x-2) + 6 = 0$

Sol: $kx^2 - 2kx + 6 = 0$

$a = k, b = -2k, c = 6$

If the Q.E has equal roots then $b^2 - 4ac = 0$

$(-2k)^2 - 4 \times k \times 6 = 0$

$4k^2 - 24k = 0$

$4k(k-6) = 0$

$4k = 0 \ \ or \ \ k - 6 = 0$

$k = 0 \ \ or \ \ k = 6$

$\therefore k = 6 \ (if \ k = 0 \ then \ a = 0, b = 0 \ it \ is \ not \ a \ Q.E)$

3. Is it possible to design a rectangular mango grove whose length is twice its breadth, and the area is 800 m2 ? If so, find its length and breadth.

Sol: Let the breadth $(b) = x\ m$

Length $(l) = 2x\ m$

Given area of the rectangular grove=800m²

$x \times 2x = 800$

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$x^2 = \frac{800}{2} = 400 \Rightarrow x = \sqrt{400} = 20$

Yes , it is possible

Length of the mango grove$=2 \times 20 = 40m$

Breadth of the mango grove$=20\ m$.

4. **Is the following situation possible? If so, determine their present ages. The sum of the ages of two friends is 20 years. Four years ago, the product of their ages in years was 48.**

Sol: The sum of ages of two friends=20 years

	First friend	Second friend
Present age(in years)	x	$20 - x$
Four years ago age	$x - 4$	$20 - x - 4 = 16 - x$

Four years ago, the product of their ages=48

$(x - 4)(16 - x) = 48$

$16x - x^2 - 64 + 4x - 48 = 0$

$-x^2 + 20x - 112 = 0$

$x^2 - 20x + 112 = 0$

$a = 1, b = -20, c = 112$

$b^2 - 4ac = (-20)^2 - 4 \times 1 \times 112 = 400 - 448 = -48 < 0$

The roots are not real. So, the situation is not possible.

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5. **Is it possible to design a rectangular park of perimeter 80 m and area 400 m² ? If so, find its length and breadth.**

Sol: Let length of rectangular park (l)=$x\ m$

Perimeter of the park=80 m

$2(l + b) = 80$

$x + b = \frac{80}{2} = 40 \Rightarrow b = 40 - x$

Area of park=400 m^2

$x(40 - x) = 400$

$40x - x^2 - 400 = 0$

$x^2 - 40x + 400 = 0$

$(x - 20)(x - 20) = 0$

$x - 20 = 0 \Rightarrow x = 20$

Length of the rectangular park=20 m

Breadth of the rectangular park=$40 - 20$ =20m

CASE STUDY BASED QUESTINS

1) **Raj and Ajay are very close friends. Both the families decide to go to Ranikhet by their own cars. Raj's car travels at a speed of x km/h while Ajay's car travels 5 km/h faster than Raj's car. Raj took 4 hours more than Ajay to complete the journey of 400**


(i) What will be the distance covered by Ajay's car in two hours?

a) 2(x +5)km b) (x – 5)km c) 2(x + 10)km d) (2x + 5)km

(ii) Which of the following quadratic equation describe the speed of Raj's car?

a) x^2 - 5 x - 500 = 0 b) x^2 + 4x - 400 = 0 c) x^2 + 5x - 500 = 0 d) x^2 - 4x + 400 = 0

(iii) What is the speed of Raj's car?

a) 20 km/hour b) 15 km/hour c) 25 km/hour d) 10 km/hour

(iv) How much time took Ajay to travel 400 km?

a) 20 hour b) 40 hour c) 25 hour d) 16 hour

Sol:

Speed of Raja's car= $x\ km/h$

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Speed of Ajay's car= $(x + 5)\ km/h$

(i) a

Sol: The distance covered by Ajay's car in two hours=$2 \times (x + 5)\ km = 2(x + 5)\ km$

(ii) c

Sol: Distance=400km

$$Time\ taken\ by\ Raj's\ car(T_1) = \frac{Distance}{Speed} = \frac{400}{x}\ h$$

$$Time\ taken\ by\ Ajay's\ car(T_2) = \frac{Distance}{Speed} = \frac{400}{x+5}\ h$$

$$According\ to\ problem : T_1 - T_2 = 4$$

$$\frac{400}{x} - \frac{400}{x+5} = 4$$

$$400\left(\frac{1}{x} - \frac{1}{x+5}\right) = 4$$

$$\frac{x+5-x}{x(x+5)} = \frac{4}{400}$$

$$\frac{5}{x^2+5x} = \frac{1}{100}$$

$$x^2 + 5x = 500$$

$$x^2 + 5x - 500 = 0$$

(iii) a

Sol:

$$x^2 + 5x - 500 = 0$$

$$x^2 + 25x - 20x - 500 = 0$$

$$x(x+25) - 20(x+25) = 0$$

$$(x+25)(x-20) = 0$$

$$x = 20\ or -25$$

$$x = 20(Speed\ can\ never\ be\ negative)$$

The speed of Raj's car=20 km/h

(iv) d

Sol: $Time\ taken\ by\ Ajay's\ car = \frac{400}{x+5} = \frac{400}{20+5} = \frac{400}{25} = 16\ hours$

(v) c

Sol: Speed of Ajay's car= $(x+5) = (20+5) = 25\ km/h$

2) **The speed of a motor boat is 20 km/hr. For covering the distance of 15 km the boat took 1 hour more for upstream than downstream.**

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(i) **Let speed of the stream be x km/hr. then speed of the motorboat in upstream will be**

a) $20\ km/hr$ b) $(20 + x)\ km/hr$ c) $(20 - x)\ km/hr$ d) $2\ km/hr$

(ii) **What is the relation between speed ,distance and time?**

a) speed = (distance)/time b) distance = (speed)/time c) time = speed x distance d) speed = distance x time

(iii) **Which is the correct quadratic equation for the speed of the current ?**

a) $x^2 + 30x - 200 = 0$ b) $x^2 + 20x - 400 = 0$ c) $x^2\ 30x - 400 = 0$ d) $x^2 - 20x - 400 = 0$

(iv) **What is the speed of current ?**

a) 20 km/hour b) 10 km/hour c) 15 km/hour d) 25 km/hour

(v) **How much time boat took in downstream?**

a) 90 minute b) 15 minute c) 30 minute d) 45 minute

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Sol: The speed of a motor boat = 20 km/hr

Speed of the stream= x km/hr

Speed of the motorboat in upstream=$(20 - x)$ km/h

Speed of the motorboat in downstream=$(20 + x)$ km/h

Distance=15 km

Time taken for upstream$(T_1) = \frac{\text{Distance}}{\text{Speed}} = \frac{15}{20 - x}\ h$

Time taken for downstream$(T_2) = \frac{\text{Distance}}{\text{Speed}} = \frac{15}{20 + x}\ h$

According to problem : $T_1 - T_2 = 1\ h$

$$\frac{15}{20 - x} - \frac{15}{20 + x} = 1$$

$$15\left(\frac{1}{20 - x} - \frac{1}{20 + x}\right) = 1$$

$$\frac{20 + x - 20 + x}{(20 - x)(20 + x)} = \frac{1}{15}$$

$$\frac{2x}{400-x^2}=\frac{1}{15}$$

$30x = 400 - x^2$

$x^2 + 30x - 400 = 0$

$x^2 + 40x - 10x - 400 = 0$

$x(x + 40) - 10(x + 40) = 0$

$(x + 40)(x - 10) = 0$

$x + 40 = 0 \; or \; x - 10 = 0$

$x = -40 \; or \; 10$

$x = 10(Speed\ can\ never\ be\ negative)$

Speed of the stream= $x = 10$ km/hr

Time taken for downstream$(T_2) = \frac{15}{20+x} = \frac{15}{20+10} = \frac{15}{30} = \frac{1}{2}h = 30\ minutes$

(i) ***c***) $(20 - x) km/hr$

(ii) ***b***) $distance = (speed)/\ time$

(iii) ***c***) $x2 + 30x - 400 = 0$

(iv) ***b***) $10\ km/hour$

(v) ***c***) $30\ minutes$

3) A rectangular floor area can be completely tiled with 200 square tiles . If the side length of each tile is increased by 1 unit, it would take only 128 tiles to cover the floor

(i) Assuming the original length of each side of a tile be x units, make a quadratic equation from the above information.

(ii) Write the corresponding quadratic equation in standard form.

(iii) (a) Find the value of x , the length of side of a tile by factorisation. (OR)

(b) Solve the quadratic equation for x using quadratic formula.

Sol: Let the original length of each side of tile $= x\ units$

Area of each tile $= x \times x = x^2\ sq\ units$

Area of floor $= Area\ of\ 200\ tiles = 200x^2\ sq\ units \rightarrow (1)$

The length of tile if the side is increased by 1 unit $= (x + 1)\ units$

Area of each new tile $= (x + 1) \times (x + 1) = (x + 1)^2\ sq\ units$

Area of floor $= Area\ of\ 128\ new\ \ tiles = 128(x + 1)^2\ sq\ units \rightarrow (2)$

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$From\ (1) and\ (2): 200x^2 = 128(x+1)^2$

$200x^2 = 128(x^2 + 2x + 1)$

$200x^2 - 128x^2 - 256x - 128 = 0$

$200x^2 - 256x - 128 = 0$

$9x^2 - 32x - 16 = 0$

(i) The required quadratic equation: $200x^2 = 128(x+1)^2$

(ii) The required quadratic equation in standard form: $9x^2 - 32x - 16 = 0$

(iii)a) $9x^2 - 32x - 16 = 0$

$9x^2 - 36x + 4x - 16 = 0$

$9x(x-4) + 4(x-4) = 0$

$(x-4)(9x+4) = 0$

$(x-4) = 0\ or\ (9x+4) = 0$

$x = 4\ or\ x = \frac{-4}{9}$

$x = 4$

$b)$ Using quadratic formula

$a = 9, b = -32, c = -16$

$b^2 - 4ac = (-32)^2 - 4 \times 9 \times (-16) = 1024 + 576 = 1600$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{32 \pm \sqrt{1600}}{2 \times 9} = \frac{32 \pm 40}{18}$

$x = \frac{32+40}{18}\ or\ \frac{32-40}{18}$

$x = \frac{72}{18}\ or\ \frac{-8}{18}$

$x = 4\ or\ \frac{-4}{9}$

$Hence\ x = 4$

Some more problems for brain boosting:

1. Does $(x - 1)^2 + 2(x + 1) = 0$ have a real root? Justify your answer.
2. Find the roots of the quadratic equation $x^2 - 2\sqrt{2}x - 6 = 0$ using the quadratic formula.
3. Find the roots of the quadratic equation $2x^2 - \sqrt{5}x - 2 = 0$ using the quadratic formula
4. Find the roots of $6x^2 - \sqrt{2}x - 2 = 0$ by the factorisation of the corresponding quadratic polynomial.
5. Find the roots of $3\sqrt{2}x^2 - 5x - \sqrt{2} = 0$ by the factorisation of the corresponding quadratic polynomial.

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6. Had Ajita scored 10 more marks in her mathematics test out of 30 marks, 9 times these marks would have been the square of her actual marks. How many marks did she get in the test?
7. A natural number, when increased by 12, equals 160 times its reciprocal. Find the number.
8. If Zeba were younger by 5 years than what she really is, then the square of her age (in years) would have been 11 more than five times her actual age. What is her age now?
9. A train travels at a certain average speed for a distance 63 km and then travels a distance of 72 km at an average speed of 6 km/hr more than the original speed. If it takes 3 hours to complete total journey, what is its original average speed?(CBSE-2023)
10. two water taps together can fill a tank in 15/ 8 hours .the tap with longer diameter takes 2 hours less than the smaller one to fill the tank separately .find the time in which each tap can fill the tank separately?(CBSE-2023).
11. Find the sum and product of the roots of the equation $2x^2 - 9x + 4$.(CBSE-Delhi-2023)
12. Find the discriminant of the quadratic equation $4x^2 - 5 = 0$ and hence comment on the nature of roots of the equation. .(CBSE-Delhi-2023)
13. Solve the quadratic equation:$x^2 - 2ax + (a^2 - b^2) = 0\ for\ x$ (CBSE-2022)
14. The sum of two numbers is 34. If 3 is subtracted from one number and 2 is added to another, the product of these two numbers becomes 260. Find the numbers?(CBSE-2022)
15. The hypotenuse (in cm) of a right angled triangle is 6 cm more than twice the length of the length of the shortest side. If the length of third side is 6 cm less than thrice the length of shortest side, then find the dimensions of the triangle.(CBSE-2022)
16. If $\alpha\ and\ \beta$ are the zeroes of the quadratic polynomial$f(x) = x^2 - x - 4$, find the value of $\frac{1}{\alpha} + \frac{1}{\beta} - \alpha\beta$
17. Solve the quadratic equation $(x-1)^2 - 5(x-1) - 6 = 0$
18. If the roots of the quadratic equation $(a-b)x^2 + (b-c)x + (c-a) = 0$ are equal, prove that 2a=b+c
19. Find the value of k, if the quadratic equation $(k+4)x^2 + (k+1)x + 1 = 0$ has equal roots.
20. If the equation $(1+m^2)x^2 + 2mcx + c^2 - a^2 = 0$ has equal roots, show that $c^2 = a^2(1+m^2)$
21.

Answers:

1) The discriminant of the equation is less than zero. Therefore, the equation has no real roots.
2) The roots of the equation are √2 and -3√2
3) the roots are (√5 + √21)/4 and (√5 - √21)/4.
4) the roots of the equation are -√2/3 and √2/2
5) the roots of the equation are √2 and -1/3√2.
6) Ajita scored 15 marks in the examination

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7) the natural number is x = 8

8) the present age of Zeba is 14 years.

9) 42km/h

10) 5 hours and 3 hours.

11) 9/2 and 2

12) Discriminant=70 and roots are real distinct

13) x= a+b or a-b

14) 16,18 or 23,11

15) 26cm,24cm and 10cm

16) 15/4

17) x=0 or 7

18)

19) k=-3,5

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20)

MCQ

1. Which of the following is not a quadratic equation?

(A) $2(x-1)^2 = 4x^2 - 2x + 1$ (B) $2x - x^2 = x^2 + 5$ (C) $(\sqrt{2}x + \sqrt{3})^2 + x^2 = 3x^2 - 5x$

(D) $(x^2 + 2x)^2 = x^4 + 3 + 4x^3$

2. Which of the following equations has 2 as a root?

(A) $x^2 - 4x + 5 = 0$ (B) $x^2 + 3x - 12 = 0$ (C) $2x^2 - 7x + 6 = 0$ (D) $3x^2 - 6x - 2 = 0$

3. If $\frac{1}{2}$ is a root of the equation $x^2 + kx - \frac{5}{4} = 0$, then the value of k is

(A) 2 (B) – 2 (C) $\frac{1}{4}$ (D) $\frac{1}{2}$

4. Values of k for which the quadratic equation $2x^2 - kx + k = 0$ has equal roots is

(A) 0 only (B) 4 (C) 8 only (D) 0, 8

5. The quadratic equation $2x^2 - \sqrt{5}x + 1 = 0$ has

(A) Two distinct real roots (B) two equal real roots (C) no real roots (D) more than 2 real roots

6. $(x^2 + 1)^2 - x^2 = 0$ has(11)

(A) four real roots (B) two real roots (C) no real roots (D) one real root.

7. The quadratic equation $x^2 + 3x + 2\sqrt{2} = 0$ has

(A) two distinct real roots (B) two equal real roots (C) no real roots (D) more than 2 real roots

8. The quadratic equation $2x^2 - 3\sqrt{2}x + \frac{9}{4} = 0$ has

(A) two distinct real roots (B) two equal real roots (C) no real roots (D) more than 2 real roots

9. A quadratic equation whose roots are $(2 + \sqrt{3})$ and $(2 - \sqrt{3})$ is

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$(A) x^2 - 4x + 1$ $(B) x^2 + 4x + 1$ $(C) 4x^2 - 3$ $(D) x^2 - 1$

10. The root of the equation $x^2 - 3x - m(m + 3) = 0$, where m is constant are

(A) m,m+3 (B) –m,m+3 (C) m,-(m+3) (D) -m,-(m+3)

11. If 1 is a root of the equations $my^2 + my + 3 = 0$ and $x^2 + x + n = 0$ then $mn=$

A) 3 B) $-\frac{7}{2}$ C) 6 D) -3

12. If the roots of equation $ax^2 + bx + c = 0, a \neq 0$ are real and equal, then which of the following relation is true(CBSE-2024)

A) $a = \frac{b^2}{c}$ B) $b^2 = ac$ C) $ac = \frac{b^2}{4}$ D) $c = \frac{b^2}{a}$

1)C	2)C	3)A	4)C	5)C	6)C	7)C	8)B	9)A	10)B	11)D	12)C

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1. Assertion: If one root of the quadratic equation $6x^2 - x - k = 0$ is 2/3, then the value of k is 2.

   Reason: The quadratic equation $ax^2 + bx + c = 0, a \neq 0$ has almost two roots.

2. Assertion: $(2x - 1)^2 - 4x^2 + 5 = 0$ is not a quadratic equation.

   Reason: An equation of the form $ax^2 + bx + c = 0, a \neq 0$, where $a, b, c \in R$ is called a quadratic equation.

3. Assertion: The roots of the quadratic equation $x^2 + 2x + 2 = 0$ are imaginary

   Reason: If discriminant $D = b^2 - 4ac < 0$ then the roots of quadratic equation $ax^2 + bx + c = 0$ are imaginary.

4. Assertion: $3x^2 - 6x + 3 = 0$ has equal roots.

   Reason: The quadratic equation $ax^2 + bx + c = 0$ have equal roots if discriminant $D > 0$.

5. Assertion: The quadratic equation $4x^2+6x+3$ has no real roots.

   Reason: The value of the discriminant is -12

6. Assertion : The values of x are $-\frac{a}{2}$, a for a quadratic equation $2x^2 - ax - a^2 = 0$

   Reason : For quadratic equation $ax^2 + bx + c = 0$; $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

7. Assertion : $4x^2 - 12x + 9 = 0$ has repeated roots.

Reason: The quadratic equation $ax^2 + bx + c = 0$ have repeated roots if discriminant $D > 0$.

1)B	2)A	3)A	4)C	5)A	6) A	7)C

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