CHAPTER 3

X-MATHEMATICS-NCERT-2024-25

PAIR OF LINEAR EQUATIONS IN TWO VARIABLES

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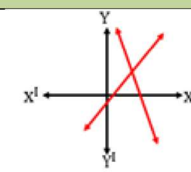
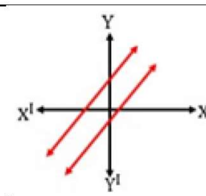
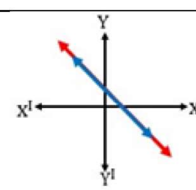
https://sureshmathsmaterial.com

1. An equation of the form $ax + b = 0$ where a, b are real numbers and $a \neq 0$ is called **linear equation in one variable** x.
2. An equation of the form $ax + by + c = 0$ where a, b, c are real numbers and where at least one of a or b is not zero (i.e. $a^2 + b^2 \neq 0$), is called **a linear equation in two variables x and y**
3. Two linear equations in the same two variables are called a pair of linear equations in two variables
   $a_1x + b_1y + c_1 = 0({a_1}^2 + {b_1}^2 \neq 0)$
   $a_2x + b_2y + c_2 = 0({a_2}^2 + {b_2}^2 \neq 0)$
4. The values of x and y satisfying each one of given pair of linear equations are called their **solution**.
5. **Consistent and Inconsistent Systems:** A pair of linear equations can have at least one common solution; it's called a consistent system. If there are no common solutions, it's an inconsistent system.
6. The graph of linear equation is a straight line.

Comparison of ratios	Graphical representation	Algebraic interpretation	Solution	Graph
$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$	Intersecting lines	Consistent	Unique solution	
$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$	Parallel lines	In consistent	No solution	
$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$	Coincident lines	Consistent	Infinite number of solutions	

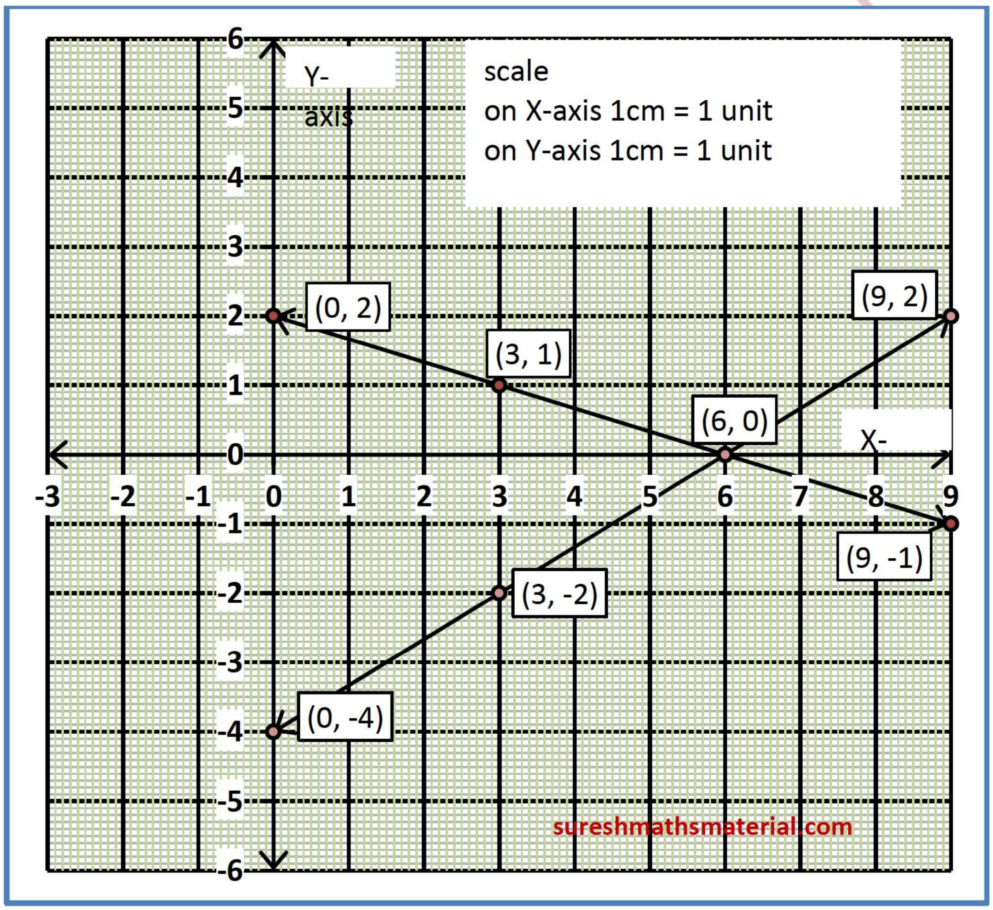
3.2 Graphical Method of Solution of a Pair of Linear Equations.

Example 1 : Check graphically whether the pair of equations
$x + 3y = 6$ and $2x - 3y = 12$ is consistent. If so, solve them graphically.

Sol:

$x + 3y = 6 \Rightarrow y = \frac{6-x}{3}$		
x	$y = \frac{6-x}{3}$	(x, y)
0	$y = \frac{6-0}{3} = \frac{6}{3} = 2$	$(0,2)$
3	$y = \frac{6-3}{3} = \frac{3}{3} = 1$	$(3,1)$
6	$y = \frac{6-6}{3} = \frac{0}{3} = 0$	$(6,0)$
9	$y = \frac{6-9}{3} = \frac{-3}{3} = -1$	$(9,-1)$

$2x - 3y = 12 \Rightarrow y = \frac{2x-12}{3}$		
x	$y = \frac{2x-12}{3}$	(x, y)
0	$y = \frac{2(0)-12}{3} = \frac{-12}{3}$ $= -4$	$(0,-4)$
3	$y = \frac{2(3)-12}{3} = \frac{-6}{3} = -2$	$(3,-2)$
6	$y = \frac{2(6)-12}{3} = \frac{0}{3} = 0$	$(6,0)$
9	$y = \frac{2(9)-12}{3} = \frac{6}{3} = 2$	$(9,2)$



Both the lines intersect at (6,0)

So, the solution of the pair of linear equations is x = 6 and y = 0

i.e., the given pair of equations is consistent.

Example 2 : Graphically, find whether the following pair of equations has no solution, unique solution or infinitely many solutions:

$5x - 8y + 1 = 0$ *and* $3x - \frac{24}{5}y + \frac{3}{5} = 0$

Sol:

$5x - 8y + 1 = 0 \rightarrow (1)$

$3x - \frac{24}{5}y + \frac{3}{5} = 0 \rightarrow (2)$

Multiply by $\frac{5}{3}$ *, we get*

$\Rightarrow \frac{5}{3} \times 3x - \frac{5}{3} \times \frac{24}{5}y + \frac{5}{3} \times \frac{3}{5} = \frac{5}{3} \times 0$

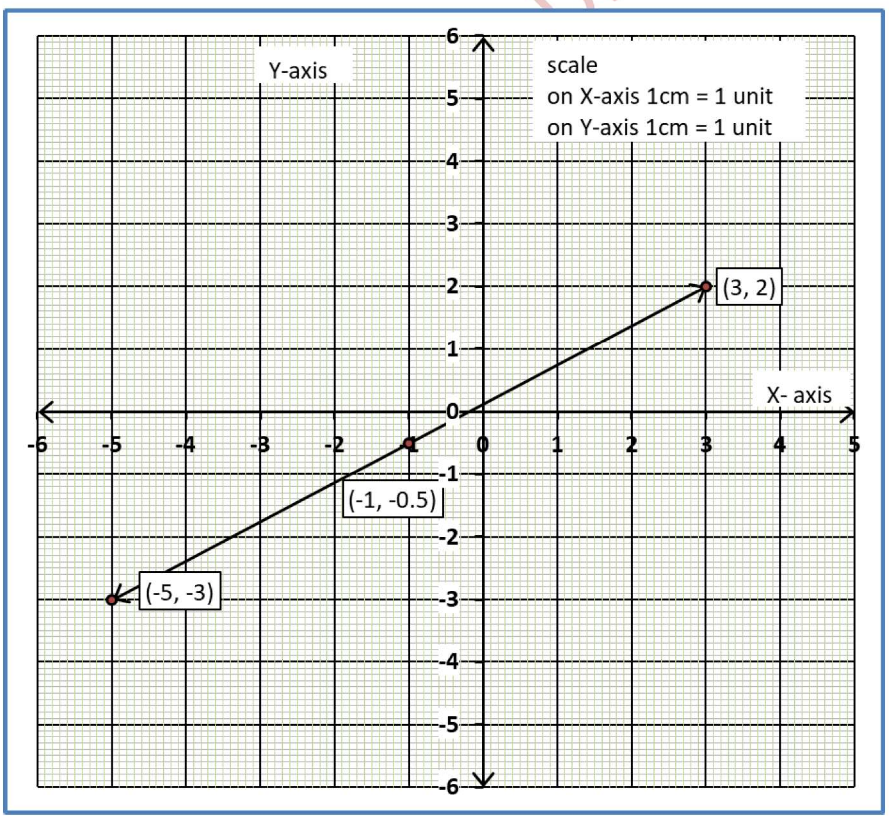
$\Rightarrow 5x - 8y + 1 = 0 \rightarrow (2)$

(1) and (2) are same equations.

Given equations are coincident.

Equations (1) and (2) have infinitely many solutions.

$5x - 8y = -1 \Rightarrow 8y = 5x + 1 \Rightarrow y = \frac{5x+1}{8}$		
x	$y = \frac{5x+1}{8}$	(x, y)
-5	$y = \frac{5(-5)+1}{8} = \frac{-25+1}{8} = \frac{-24}{8} = -3$	$(-5, -3)$
-1	$y = \frac{5(-1)+1}{8} = \frac{-5+1}{8} = \frac{-4}{8} = -0.5$	$(-1, -0.5)$
3	$y = \frac{5(3)+1}{8} = \frac{15+1}{8} = \frac{16}{8} = 2$	$(3,2)$



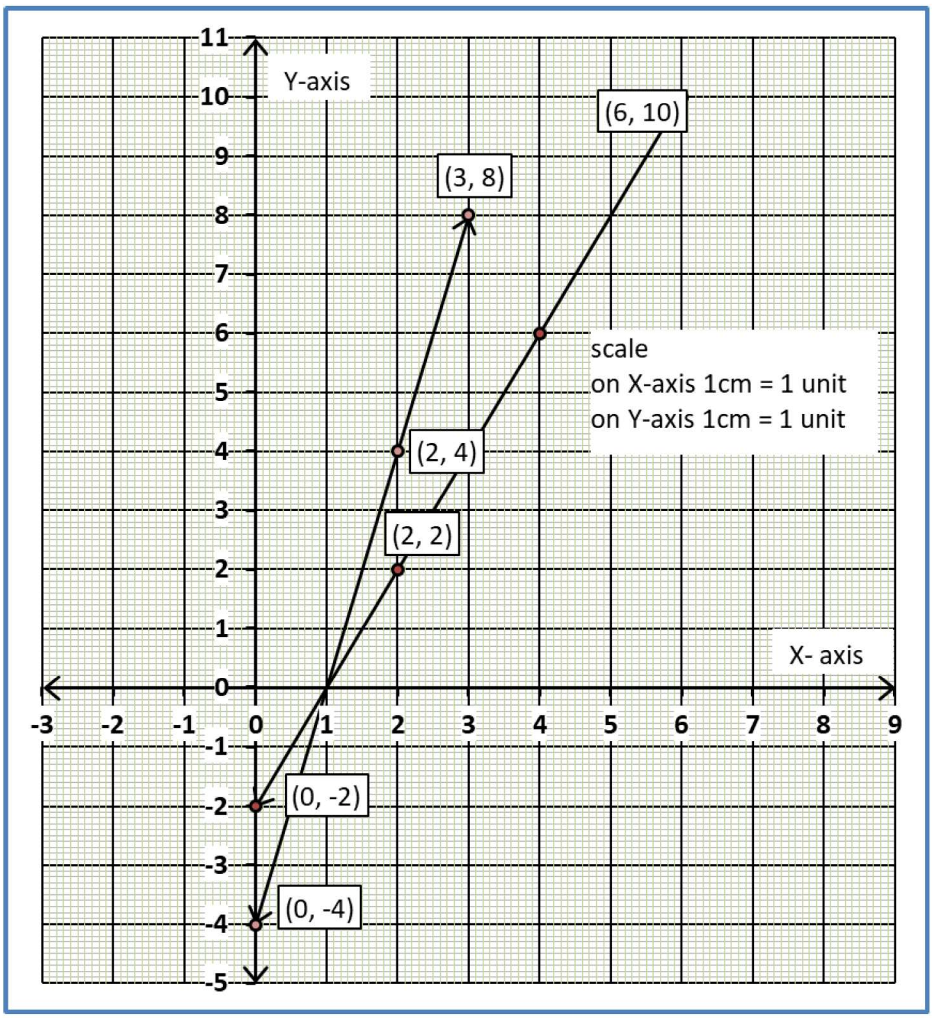
Example 3 : Champa went to a 'Sale' to purchase some pants and skirts. When her friends asked her how many of each she had bought, she answered, "The number of skirts is two less than twice the number of pants purchased. Also, the number of skirts is four less than four times the number of pants purchased". Help her friends to find how many pants and skirts Champa bought.

Solution : Let the number of pants =x and the number of skirts = y.

Then the equations formed are : y = 2x – 2 and y = 4x – 4

$\boldsymbol{y = 2x - 2}$		
x	$y = 2x - 2$	(x, y)
0	$y = 2(0) - 2 = 0 - 2 = -2$	$(0, -2)$
2	$y = 2(2) - 2 = 4 - 2 = 2$	$(2,2)$
4	$y = 4(2) - 2 = 8 - 2 = 6$	$(4,6)$
6	$y = 6(2) - 2 = 12 - 2 = 10$	$(6,10)$

$\boldsymbol{y = 4x - 4}$		
x	$y = 4x - 4$	(x, y)
0	$y = 4(0) - 4 = 0 - 4 = -4$	$(0, -4)$
1	$y = 4(1) - 4 = 4 - 4 = 0$	$(1,0)$
2	$y = 4(2) - 4 = 8 - 4 = 4$	$(2,4)$
3	$y = 4(3) - 4 = 12 - 4 = 8$	$(3,8)$



The two lines intersect at the point (1, 0)

So, x = 1, y = 0 is the required solution of the pair of linear equations.

i.e., the number of pants she purchased is 1 and she did not buy any skirt.

EXERCISE 3.1

1. **Form the pair of linear equations in the following problems, and find their solutions graphically.**
   (i) 10 students of Class X took part in a Mathematics quiz. If the number of girls is 4 more than the number of boys, find the number of boys and girls who took part in the quiz.

Sol: Let the number of boys=x and number of girls=y

Total number of students=10

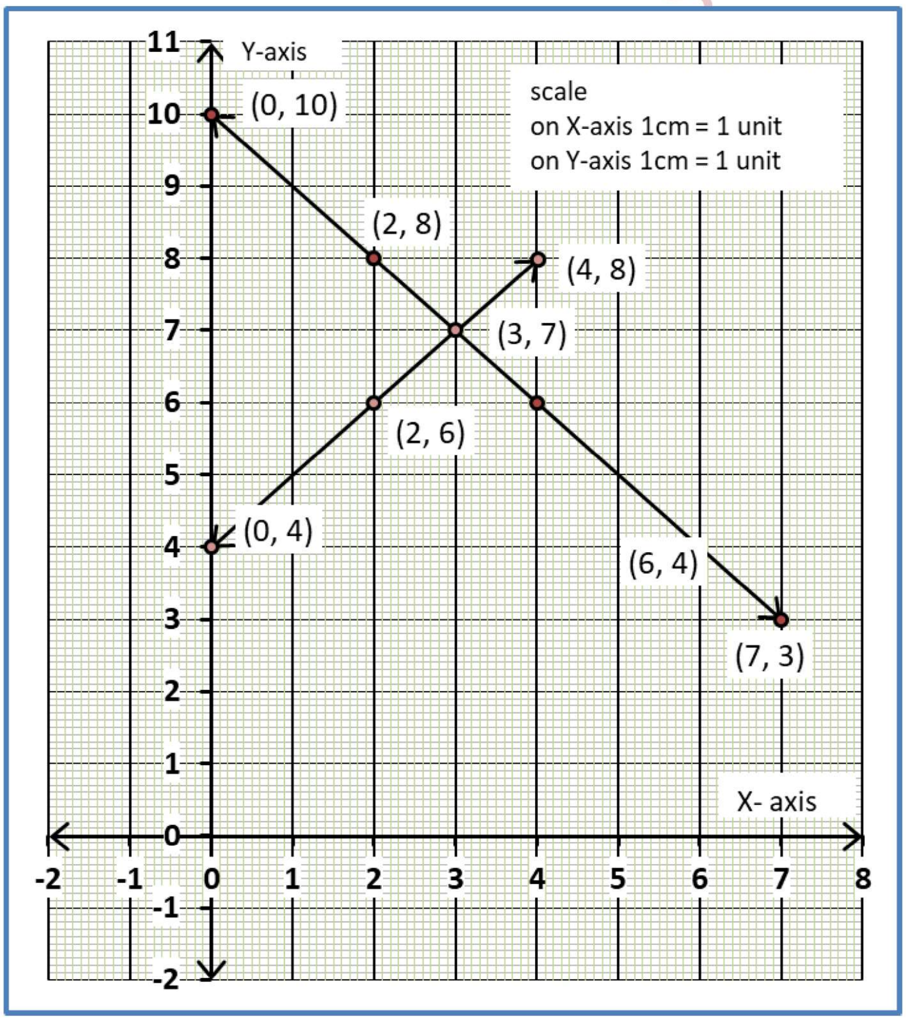
$\Rightarrow x + y = 10 \rightarrow (1)$

The number of girls is 4 more than the number of boys.

$\Rightarrow y = x + 4 \Rightarrow x - y = -4 \rightarrow (2)$

$x + y = 10 \Rightarrow y = 10 - x$		
x	$y = 10 - x$	(x, y)
2	$y = 10 - 2 = 8$	(2,8)
4	$y = 10 - 4 = 6$	(4,6)
6	$y = 10 - 6 = 4$	(6,4)
7	$y = 10 - 7 = 3$	(7,3)

$x - y = -4 \Rightarrow y = x + 4$		
x	$y = x + 4$	(x, y)
2	$y = 2 + 4 = 6$	(2,6)
3	$y = 3 + 4 = 7$	(3,7)
4	$y = 4 + 4 = 8$	(4,8)
6	$y = 6 + 4 = 10$	(6,10)



The two lines intersect at the point (3, 7)

So, x = 3, y = 7 is the required solution of the pair of linear equations.

i.e., the number of boys=3 and the number of girls=7.

(ii) 5 pencils and 7 pens together cost ₹ 50, whereas 7 pencils and 5 pens together cost ₹ 46. Find the cost of one pencil and that of one pen.

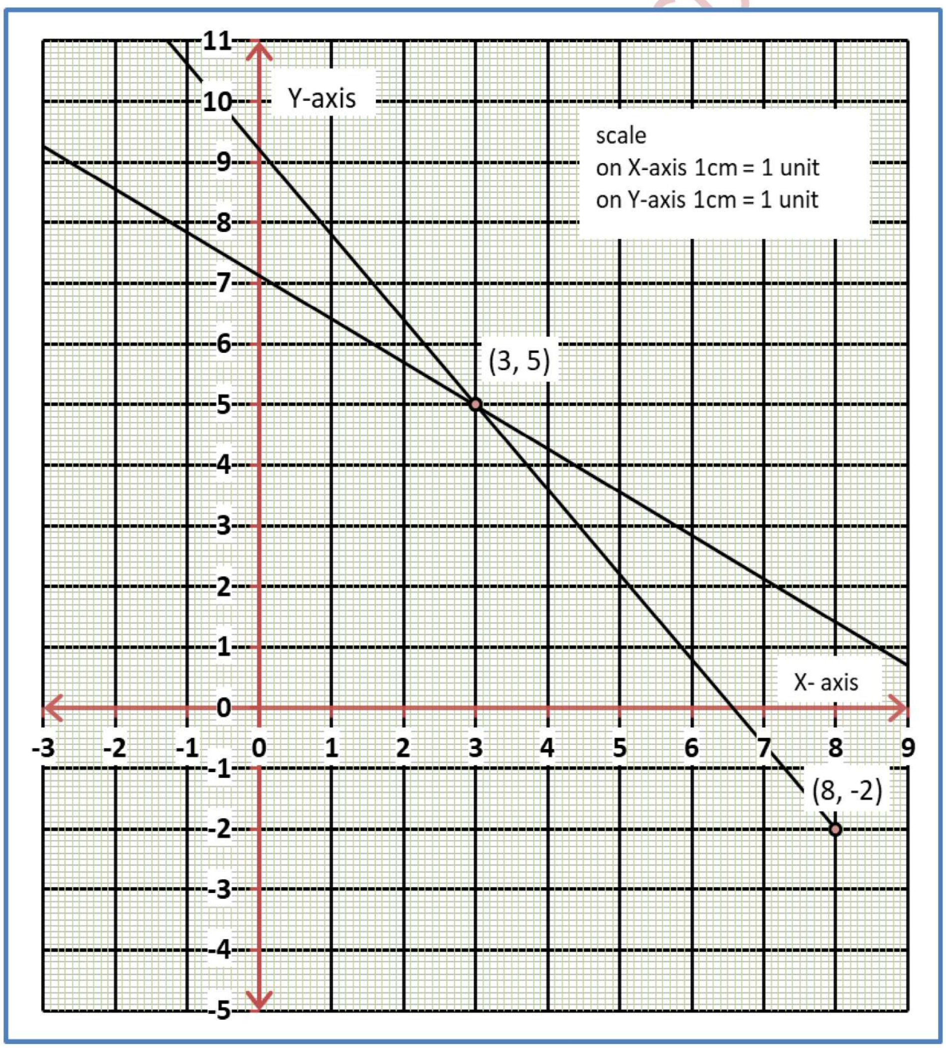
Sol: Let the cost of 1 pencil=₹ x and the cost of 1 pen=₹y

5 pencils + 7 pens = ₹ 50 ⇒ 5x + 7y = 50 → (1)

7 pencils + 5 pens = ₹ 46 ⇒ 7x + 5y = 46 → (2)

$5x + 7y = 50 \Rightarrow y = \frac{50-5x}{7}$		
x	$y = \frac{50-5x}{7}$	(x, y)
-4	$y = \frac{50-5(-4)}{7} = \frac{70}{7} = 10$	$(-4,10)$
3	$y = \frac{50-5(3)}{7} = \frac{35}{7} = 5$	(3,5)
10	$y = \frac{50-5(10)}{7} = \frac{0}{7} = 0$	(10,0)

$7x + 5y = 46 \Rightarrow 5y = 46 - 7x \Rightarrow y = \frac{46-7x}{5}$		
x	$y = \frac{46-7x}{5}$	(x, y)
-2	$y = \frac{46-7(-2)}{5} = \frac{60}{5} = 12$	$(-2,12)$
3	$y = \frac{46-7(3)}{5} = \frac{25}{5} = 5$	(3,5)
8	$y = \frac{46-7(8)}{5} = \frac{-10}{5} = -2$	$(8,-2)$



The two lines intersect at the point (3, 5)
So, x = 3, y = 7 is the required solution of the pair of linear equations.

i.e., the cost of 1 pencil=₹3 and the cost of 1 pen=₹5

2. **On comparing the ratios $\frac{a_1}{a_2}$, $\frac{b_1}{b_2}$ *and* $\frac{c_1}{c_2}$ find out whether the lines represented by the following pairs of linear equations equations intersect at a point, are parallel or are coincident.**

(*i*) $5x - 4y + 8 = 0$
$7x + 6y - 9 = 0$

Sol: $5x - 4y + 8 = 0$; $(a_1 = 5, \quad b_1 = -4, c_1 = 8)$

$7x + 6y - 9 = 0$; $(a_2 = 7, \; b_2 = 6, \; c_2 = -9)$

$\frac{a_1}{a_2} = \frac{5}{7}$; $\frac{b_1}{b_2} = \frac{-4}{6} = \frac{-2}{3}$; $\frac{c_1}{c_2} = \frac{8}{-9} = \frac{-8}{9}$

$\frac{a_1}{a_2} \neq \frac{b_1}{b_2} \Rightarrow$ Given pairs of linear equations intersect at a point.

(*ii*) $9x + 3y + 12 = 0$
$18x + 6y + 24 = 0$

Sol: $9x + 3y + 12 = 0$; $a_1 = 9, \quad b_1 = 3, c_1 = 12$

$18x + 6y + 24 = 0$; $a_2 = 18, \; b_2 = 6, \; c_2 = 24$

$\frac{a_1}{a_2} = \frac{9}{18} = \frac{1}{2}$; $\frac{b_1}{b_2} = \frac{3}{6} = \frac{1}{2}$; $\frac{c_1}{c_2} = \frac{12}{24} = \frac{1}{2}$

$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} \Rightarrow$ Given pairs of linear equations are coincident

(*iii*) $6x - 3y + 10 = 0$
$2x - y + 9 = 0$

Sol: $6x - 3y + 10 = 0$; $a_1 = 6, \quad b_1 = -3, c_1 = 10$

$2x - y + 9 = 0$; $a_2 = 2, \; b_2 = -1, \; c_2 = 9$

$\frac{a_1}{a_2} = \frac{6}{2} = 3$; $\frac{b_1}{b_2} = \frac{-3}{-1} = 3$; $\frac{c_1}{c_2} = \frac{10}{9}$

$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2} \Rightarrow$ Given pairs of linear equations are parallel

3. **On comparing the ratios $\frac{a_1}{a_2}$, $\frac{b_1}{b_2}$ *and* $\frac{c_1}{c_2}$ find out whether the lines represented by the following pairs of linear equations are consistent, or inconsistent**

(*i*) $3x + 2y = 5$; $2x - 3y = 7$

Sol: $3x + 2y - 5 = 0$; $a_1 = 3, \quad b_1 = 2, c_1 = -5$

$2x - 3y - 7 = 0$; $a_2 = 2, \; b_2 = -3, \; c_2 = -7$

$\frac{a_1}{a_2} = \frac{3}{2}$; $\frac{b_1}{b_2} = \frac{2}{-3}$; $\frac{c_1}{c_2} = \frac{-5}{-7} = \frac{5}{7}$

$\frac{a_1}{a_2} \neq \frac{b_1}{b_2} \Rightarrow$ Given pairs of lines are intersecting and have one solution.

The pair of given equations are consistent.

(ii) $2x - 3y = 8$; $4x - 6y = 9$

Sol: $2x - 3y - 8 = 0$; ; $a_1 = 2$, $b_1 = -3, c_1 = -8$

$4x - 6y - 9 = 0$; $a_2 = 4$, $b_2 = -6$, $c_2 = -9$

$\frac{a_1}{a_2} = \frac{2}{4} = \frac{1}{2}$; $\frac{b_1}{b_2} = \frac{-3}{-6} = \frac{1}{2}$; $\frac{c_1}{c_2} = \frac{-8}{-9} = \frac{8}{9}$

$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2} \Rightarrow$ Given pairs of linear equations are parallel

The pair of given equations are inconsistent.

(iii) $\frac{3}{2}x + \frac{5}{3}y = 7$; $9x - 10y = 14$

Sol: $\frac{3}{2}x + \frac{5}{3}y = 7 \Rightarrow 6 \times \frac{3}{2}x + 6 \times \frac{5}{3}y - 6 \times 7 = 0$

$9x + 10y - 42 = 0$; ($a_1 = 9$, $b_1 = 10$, $c_1 = -42$)

$9x - 10y - 14 = 0$; ($a_2 = 9$, $b_2 = -10$, $c_2 = -14$)

$\frac{a_1}{a_2} = \frac{9}{9} = 1$; $\frac{b_1}{b_2} = \frac{10}{-10} = -1$; $\frac{c_1}{c_2} = \frac{-42}{-14} = 3$

$\frac{a_1}{a_2} \neq \frac{b_1}{b_2} \Rightarrow$ Given pairs of linear equations are intersecting and have one solution

The pair of given equations are consistent

(iv) $5x - 3y = 11$; $-10x + 6y = -22$

Sol: $5x - 3y - 11 = 0$; $a_1 = 5$, $b_1 = -3, c_1 = -11$

$-10x + 6y + 22 = 0$; $a_2 = -10$, $b_2 = 6$, $c_2 = 22$

$\frac{a_1}{a_2} = \frac{5}{-10} = \frac{-1}{2}$; $\frac{b_1}{b_2} = \frac{-3}{6} = \frac{-1}{2}$; $\frac{c_1}{c_2} = \frac{-11}{22} = \frac{-1}{2}$

$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

$\Rightarrow$ Given pairs of linear equations are coincident and have infinitely many solutions.

The pair of given equations are consistent

(v) $\frac{4}{3}x + 2y = 8$; $2x + 3y = 12$

$\frac{4}{3}x + 2y - 8 = 0$; $a_1 = \frac{4}{3}$, $b_1 = 2, c_1 = -8$

$2x + 3y - 12 = 0$; $a_2 = 2$, $b_2 = 3$, $c_2 = -12$

$$\frac{a_1}{a_2} = \frac{\frac{4}{3}}{2} = \frac{4}{3 \times 2} = \frac{2}{3}; \qquad \frac{b_1}{b_2} = \frac{2}{3}; \qquad \frac{c_1}{c_2} = \frac{-8}{-12} = \frac{2}{3}$$

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

⇒ Given pairs of linear equations are coincident and have infinitely many solutions.

The pair of given equations are consistent

4. Which of the following pairs of linear equations are consistent /inconsistent? If consistent, obtain the solution graphically:

(i) $x + y = 5, 2x + 2y = 10$

Sol: $x + y - 5 = 0$; $a_1 = 1$, $b_1 = 1, c_1 = -5$

$2x + 2y - 10 = 0$; $a_2 = 2$, $b_2 = 2$, $c_2 = -10$

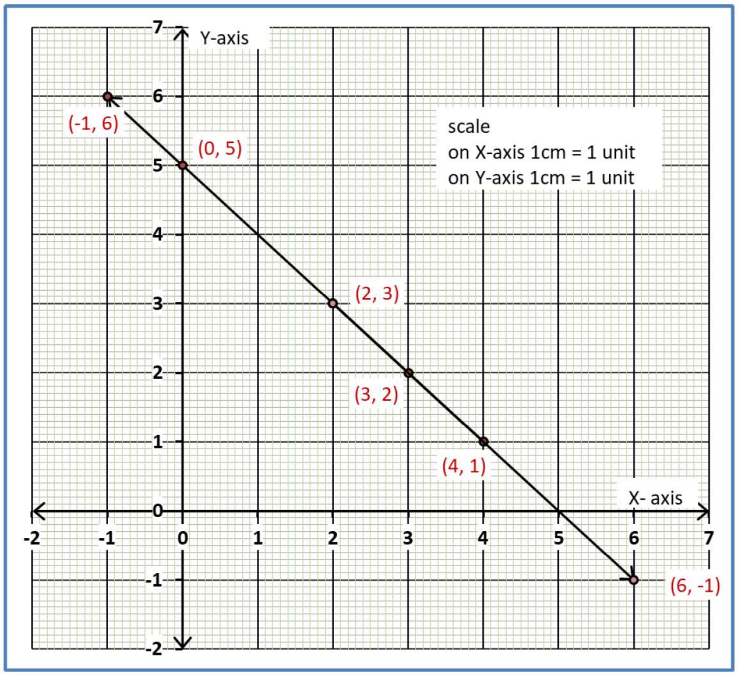
$$\frac{a_1}{a_2} = \frac{1}{2}; \qquad \frac{b_1}{b_2} = \frac{1}{2}; \qquad \frac{c_1}{c_2} = \frac{-5}{10} = \frac{1}{2}$$

$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$ ⇒ lines are coincident and have infinitely many solutions.

The pair of given equations are consistent

$x + y = 5 \Rightarrow y = 5 - x$		
x	$y = 5 - x$	(x, y)
2	$y = 5 - 2 = 3$	(2,3)
4	$y = 5 - 4 = 1$	(4,1)
6	$y = 5 - 6 = -1$	$(6, -1)$

$2x + 2y = 10 \Rightarrow y = \frac{10 - 2x}{2}$		
x	$y = \frac{10 - 2x}{2}$	(x, y)
0	$y = \frac{10 - 2(0)}{2} = 5$	(0,5)
-1	$y = \frac{10 - 2(-1)}{2} = 6$	$(-1,6)$
3	$y = \frac{10 - 2(3)}{2} = 2$	(3,2)



(ii) $x - y = 8, 3x - 3y = 16$

Sol: $x - y - 8 = 0$; $a_1 = 1,\quad b_1 = -1, c_1 = -8$

$3x - 3y - 16 = 0$; $a_2 = 3,\ b_2 = -3,\ c_2 = -16$

$$\frac{a_1}{a_2} = \frac{1}{3};\qquad \frac{b_1}{b_2} = \frac{-1}{-3} = \frac{1}{3};\qquad \frac{c_1}{c_2} = \frac{-8}{-16} = \frac{1}{2}$$

$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2} \Rightarrow$ lines are parallel and have no solution.

The pair of given equations are inconsistent.

(iii) $2x + y - 6 = 0, 4x - 2y - 4 = 0$

Sol: $2x + y - 6 = 0$; $a_1 = 2,\quad b_1 = 1, c_1 = -6$

$4x - 2y - 4 = 0$; $a_2 = 4,\ b_2 = -2,\ c_2 = -4$

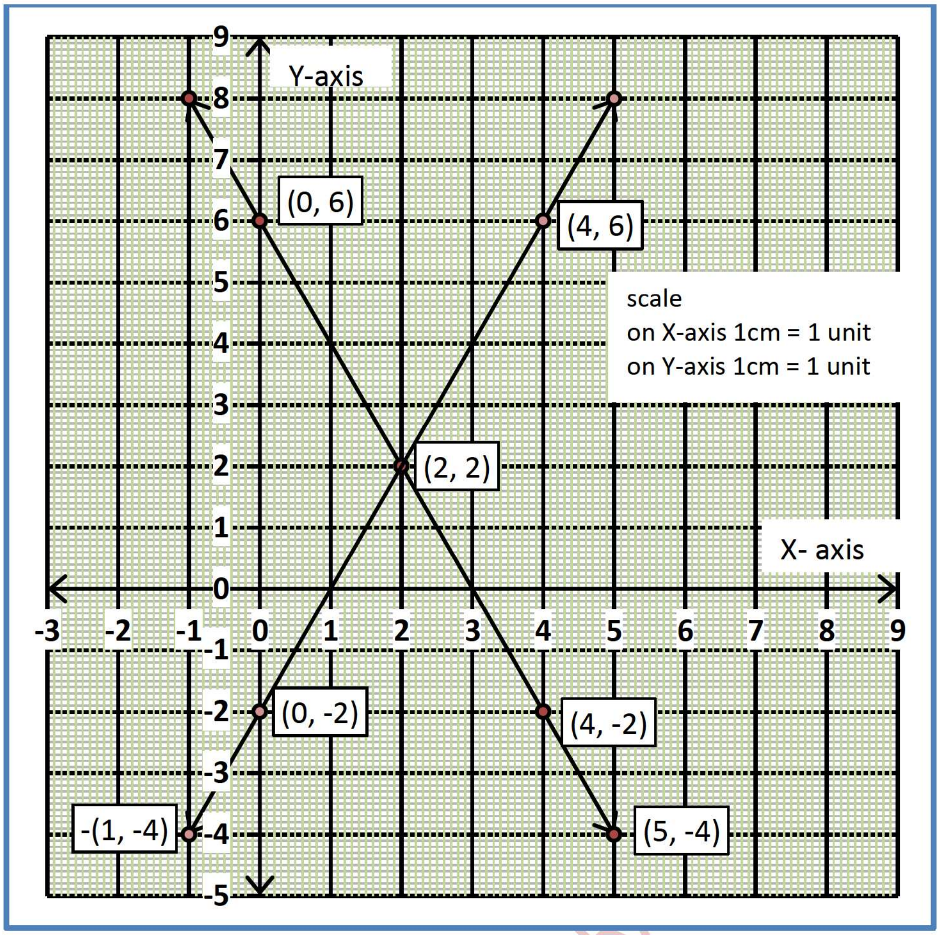
$$\frac{a_1}{a_2} = \frac{2}{4} = \frac{1}{2};\qquad \frac{b_1}{b_2} = \frac{1}{-2} = \frac{-1}{2};\qquad \frac{c_1}{c_2} = \frac{-6}{-4} = \frac{3}{2}$$

$\frac{a_1}{a_2} \neq \frac{b_1}{b_2} \Rightarrow$ lines are intersecting and have one solution.

The pair of given equations are consistent

$2x + y - 6 = 0 \Rightarrow y = 6 - 2x$		
x	$y = 6 - 2x$	(x, y)
0	$y = 6 - 2 \times 0 = 6 - 0 = 6$	(0,6)
2	$y = 6 - 2 \times 2 = 6 - 4 = 2$	(2,2)
4	$y = 6 - 2 \times 4 = 6 - 8 = -2$	$(4, -2)$

$4x - 2y - 4 = 0 \Rightarrow y = 2x - 2$		
x	$y = 2x - 2$	(x, y)
0	$y = 2 \times 0 - 2 = 0 - 2 = -2$	$(0, -2)$
2	$y = 2 \times 2 - 2 = 4 - 2 = 2$	(2,2)
4	$y = 2 \times 4 - 2 = 8 - 2 = 6$	(4,6)



Graphs intersect at (2,2)

Solution:x=2 and y=2

$(iv) 2x - 2y - 2 = 0, 4x - 4y - 5 = 0$

Sol: $2x - 2y - 2 = 0$; $a_1 = 2,$ $b_1 = -2, c_1 = -2$

$4x - 4y - 5 = 0$; $a_2 = 4,$ $b_2 = -4,$ $c_2 = -5$

$$\frac{a_1}{a_2} = \frac{2}{4} = \frac{1}{2}; \quad \frac{b_1}{b_2} = \frac{-2}{-4} = \frac{1}{2}; \quad \frac{c_1}{c_2} = \frac{-2}{-5} = \frac{2}{5}$$

$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2} \Rightarrow$ lines are parallel and have no solution.

The pair of given equations are inconsistent.

5. **Half the perimeter of a rectangular garden, whose length is 4 m more than its width, is 36 m. Find the dimensions of the garden.**

Sol: Let length of the garden $= x$ and breadth $= y$

Given: length=width+4

$x = y + 4$

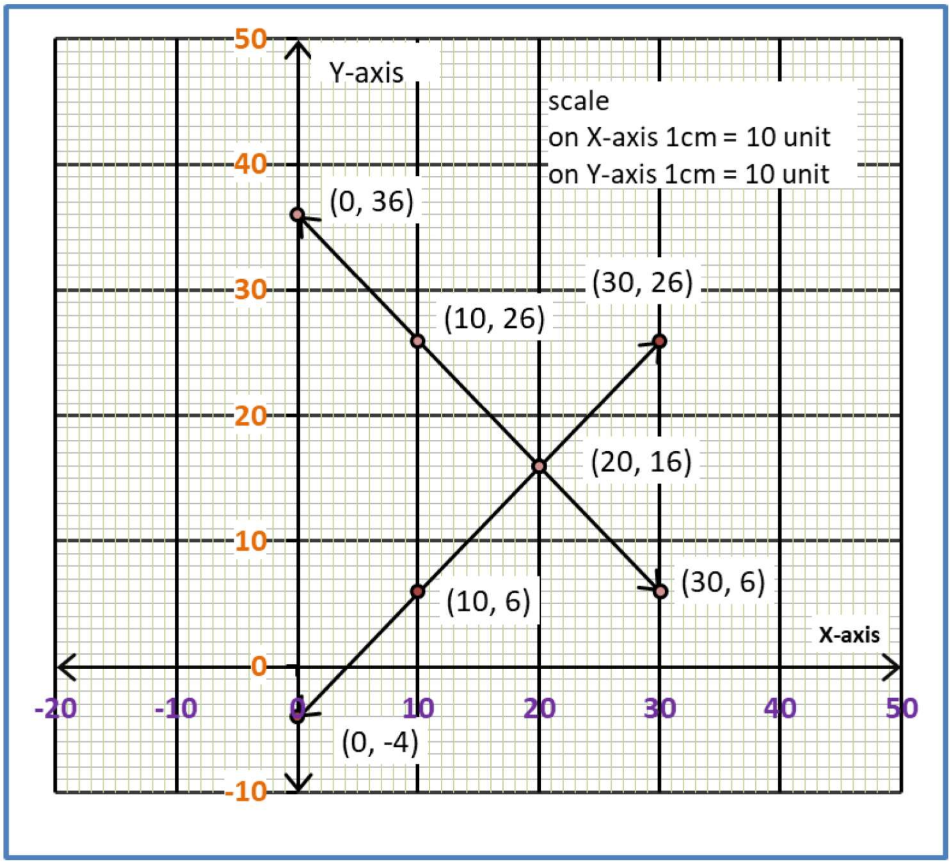
$x - y = 4 \rightarrow (1)$

Half the perimeter of the rectangle =36 m

$x + y = 36 \rightarrow (2)$

$x - y = 4 \Rightarrow y = x - 4$		
x	$y = x - 4$	(x, y)
0	$y = 0 - 4 = -4$	$(0, -4)$
10	$y = 10 - 4 = 6$	$(10,6)$
20	$y = 20 - 4 = 16$	$(20,16)$
30	$y = 30 - 4 = 26$	$(30,26)$

$x + y = 36 \Rightarrow y = 36 - x$		
x	$y = 36 - x$	(x, y)
0	$y = 36 - 0 = 36$	$(0,36)$
10	$y = 36 - 10 = 26$	$(10,26)$
20	$y = 36 - 20 = 16$	$(20,16)$
30	$y = 36 - 30 = 6$	$(30,6)$



The two lines intersect at the point (20, 16)
So, x = 20, y = 16 is the required solution of the pair of linear equations.

i.e., Length=20m and the breadth=16m.

6. **Given the linear equation 2x + 3y – 8 = 0, write another linear equation in two variables such that the geometrical representation of the pair so formed is:**
(i) intersecting lines (ii) parallel lines (iii) coincident lines

Sol: Given the linear equation 2x + 3y – 8 = 0

(i) Intersecting lines:

Condition: $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

$4x - 5y + 7 = 0$

(ii) Parallel lines:

Condition: $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

$4x + 6y + 7 = 0$

(iii) Coincident lines

Condition: $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

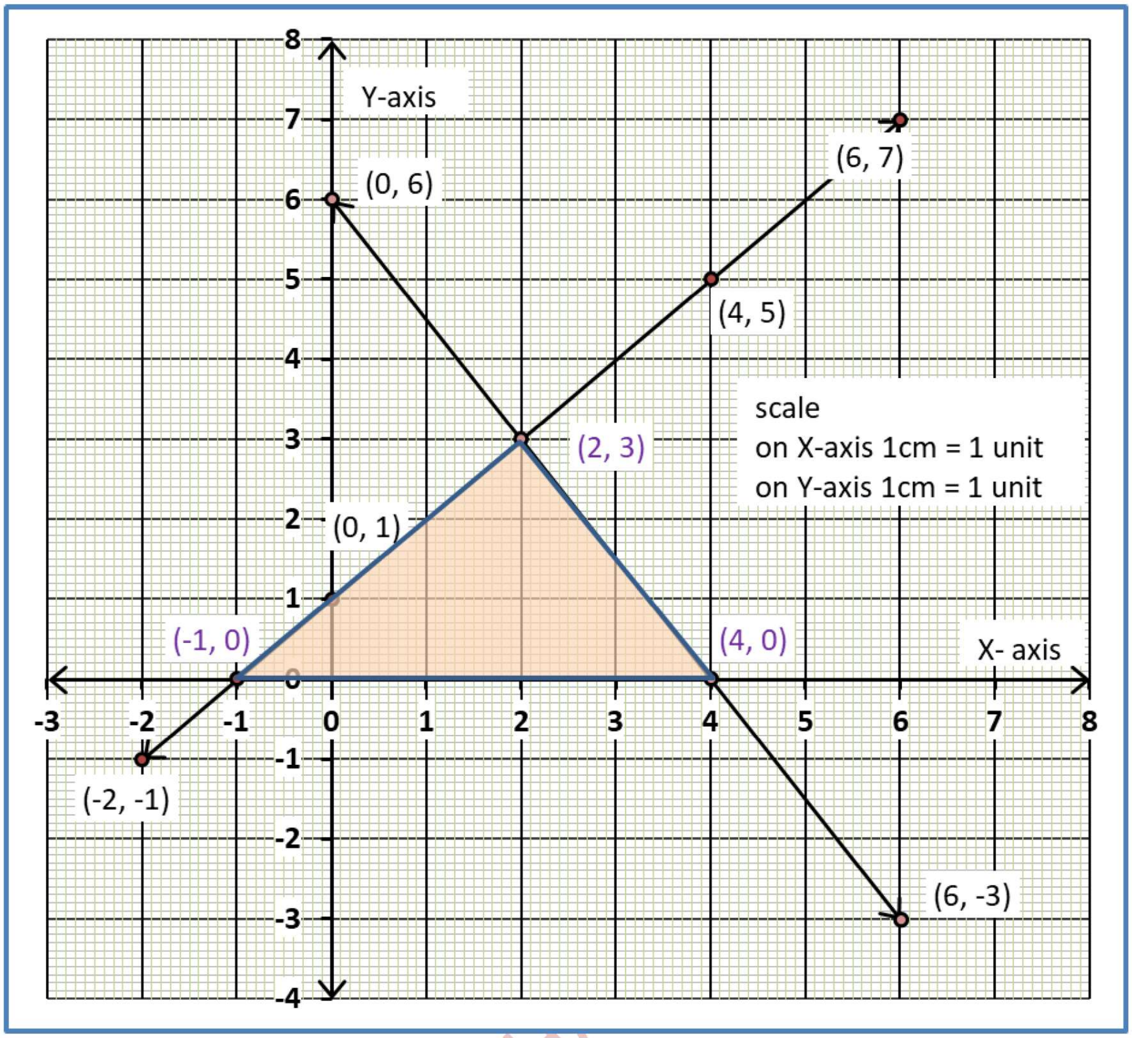
$8x + 12y - 32 = 0$

7. Draw the graphs of the equations x – y + 1 = 0 and 3x + 2y – 12 = 0. Determine the coordinates of the vertices of the triangle formed by these lines and the x-axis, and shade the triangular region.

Sol:

$x - y + 1 = 0 \Rightarrow y = x + 1$		
x	$y = x + 1$	(x, y)
-2	$y = -2 + 1 = -1$	$(-2, -1)$
-1	$y = -1 + 1 = 0$	$(-1, 0)$
0	$y = 0 + 1 = 1$	$(0, 1)$
4	$y = 4 + 1 = 5$	$(4, 5)$
6	$y = 6 + 1 = 7$	$(6, 7)$

$3x + 2y = 12 \Rightarrow y = \frac{12 - 3x}{2}$		
x	$y = \frac{12 - 3x}{2}$	(x, y)
0	$y = \frac{12 - 0}{2} = 6$	$(0,6)$
2	$y = \frac{12 - 6}{2} = 3$	$(2,3)$
4	$y = \frac{12 - 12}{2} = 0$	$(4,0)$
6	$y = \frac{12 - 18}{2} = -3$	$(6, -3)$



Vertices of the triangle are A(-1,0), B(4,0) and C(2,3)

ALGEBRAIC METHODS OF SOLVING A PAIR OF LINEAR EQUATIONS

3.3.1 Substitution Method

Example 4 : Solve the following pair of equations by substitution method: 7x – 15y = 2, x + 2y = 3

Sol: $7x - 15y = 2 \rightarrow (1)$

$x + 2y = 3 \rightarrow (2)$

From (2): $x = 3 - 2y \rightarrow (3)$

Substitute the value of $x = 3 - 2y$ in Equation (1).

$7(3 - 2y) - 15y = 2$

$21 - 14y - 15y = 2$

$-29y = -19$

$y = \frac{19}{29}$

Substituting this value of y in Equation (3)

$x = 3 - 2\left(\frac{19}{29}\right) = 3 - \frac{38}{29} = \frac{87-38}{29} = \frac{49}{29}$

The solution is $x = \frac{49}{29}, y = \frac{19}{29}$

Example 5 : Solve the following question—Aftab tells his daughter, "Seven years ago, I was seven times as old as you were then. Also, three years from now, I shall be three times as old as you will be." (Isn't this interesting?) Represent this situation algebraically and graphically by the method of substitution.

Sol:

	Aftab's age	daughter's age
Present	x	y
Seven years ago	$x-7$	$y-7$
Three years from now	$x+3$	$y+3$

Seven years ago:

Aftab's age=7× daughter's age

$x - 7 = 7(y - 7)$

$x - 7 = 7y - 49$

$x - 7y = -42 \longrightarrow (1)$

Three years from now:

Aftab's age=3× daughter's age

$x + 3 = 3(y + 3)$

$x + 3 = 3y + 9$

$x - 3y = 6 \rightarrow (2)$

From(2): $x = 6 + 3y \rightarrow (3)$

Substitute the value of x in Equation (1).

$6 + 3y - 7y = -42$

$6 - 4y = -42$

$-4y = -42 - 6 = -48$

$y = \frac{-48}{-4} = 12$

Substitute the value of y in equation (3)

$x = 6 + 3y = 6 + 3 \times 12 = 6 + 36 = 42$

So, Aftab and his daughter are 42 and 12 years old, respectively.

Example 6 : In a shop the cost of 2 pencils and 3 erasers is ₹9 and the cost of 4 pencils and 6 erasers is ₹18. Find the cost of each pencil and each eraser.

Sol: Let cost of 1 pencil $= x$ and cost of 1 eraser $= y$

2 pencils + 3 erasers = ₹9 $\Rightarrow 2x + 3y = 9 \longrightarrow (1)$

4 pencils + 6 erasers = ₹18 $\Rightarrow 4x + 6y = 18 \longrightarrow (2)$

$From\ the\ equ(1): x = \dfrac{9 - 3y}{2}$

Substitute the value of x in Equation (2)

$4\left(\dfrac{9 - 3y}{2}\right) + 6y = 18$

$18 - 6y + 6y = 18$

$18 = 18$

This statement is true for all values of y, the given equations are the same

Therefore, Equations (1) and (2) have infinitely many solutions.

We cannot find a unique cost of a pencil and an eraser,

Example 7 : Two rails are represented by the equations x + 2y – 4 = 0 and 2x + 4y – 12 = 0. Will the rails cross each other?

Solution : The pair of linear equations formed were:

$x + 2y - 4 = 0 \longrightarrow (1)$

$2x + 4y - 12 = 0 \longrightarrow (2)$

From Equ(1): $x = 4 - 2y$

Substitute this value of x in Equ (2)

$2(4 - 2y) + 4y - 12 = 0$

$8 - 4y + 4y - 12 = 0$

$-4 = 0$

This is a false statement.

Therefore, the equations do not have a common solution. So, the two rails will not cross each other.

EXERCISE 3.2

1. **Solve the following pair of linear equations by the substitution method.**

$(i) x + y = 14; x - y = 4$

Sol: $x + y = 14 \longrightarrow (1)$

$x - y = 4 \longrightarrow (2)$

From (1): $x = 14 - y \longrightarrow (3)$

Substitute this value of x in Equ (2)

$14 - y - y = 4$

$-2y = 4 - 14$

$-2y = -10$

$y = \frac{-10}{-2} = 5$

Substitute the value of y in equation (3)

$x = 14 - 5 = 9$

The solution is $x = 9, y = 5$

$(ii) s - t = 3; \frac{s}{3} + \frac{t}{2} = 6$

Sol: $s - t = 3 \rightarrow (1)$

$\frac{s}{3} + \frac{t}{2} = 6 \Longrightarrow 6 \times \frac{s}{3} + 6 \times \frac{t}{2} = 6 \times 6$

$2s + 3t = 36 \rightarrow (2)$

From equ (1): $s = 3 + t \rightarrow (3)$

Substitute this value of s in Equ (2)

$2(3 + t) + 3t = 36$

$6 + 2t + 3t = 36$

$5t = 36 - 6 = 30$

$t = \frac{30}{5} = 6$

Substitute the value of t in equation (3)

$s = 3 + 6 = 9$

$The\ solution\ is\ s = 9, t = 6$

$(iii) 3x - y = 3; 9x - 3y = 9$

Sol: $3x - y = 3 \rightarrow (1)$

$9x - 3y = 9 \rightarrow (2)$

$From (1): y = 3x - 3$

$Substitute\ this\ value\ of\ y\ in\ Equ$ (2)

$9x - 3(3x - 3) = 9$

$9x - 9x + 9 = 9$

$9 = 9$

This statement is true for all values of x, the given equations are the same

Therefore, Equations (1) and (2) have infinitely many solutions

$(iv)\ 0.2x + 0.3y = 1.3;\ 0.4x + 0.5y = 2.3$

Sol: $0.2x + 0.3y = 1.3$

Multiply with 10

$2x + 3y = 13 \rightarrow (1)$

$0.4x + 0.5y = 2.3$

Multiply with 10

$4x + 5y = 23 \rightarrow (2)$

From(1): $2x = 13 - 3y$

$\Rightarrow x = \frac{13-3y}{2} \rightarrow (3)$

Substitute the value of x in equation (2) we get

$4\left(\frac{13-3y}{2}\right) + 5y = 23$

$26 - 6y + 5y = 23$

$-y = 23 - 26 = -3$

$y = 3$

Substitute $y = 3$ in (3)

$x = \frac{13-3y}{2} = \frac{13-3\times 3}{2} = \frac{13-9}{2} = \frac{4}{2} = 2$

The required solution is $x = 2$ *and* $y = 3$

$(v)\ \sqrt{2}x + \sqrt{3}y = 0;\ \sqrt{3}x - \sqrt{8}y = 0$

Sol: $\sqrt{2}x + \sqrt{3}y = 0 \rightarrow (1)$

$\sqrt{3}x - \sqrt{8}y = 0 \rightarrow (2)$

From equ(1): $\sqrt{3}y = -\sqrt{2}x$

$y = \frac{-\sqrt{2}x}{\sqrt{3}} \rightarrow (3)$

Substituting the value of y in equation (2) we get

$\sqrt{3}x - \sqrt{8}\left(\frac{-\sqrt{2}x}{\sqrt{3}}\right) = 0$

$\Rightarrow \sqrt{3}x + \frac{4x}{\sqrt{3}} = 0 \Rightarrow \frac{3x+4x}{\sqrt{3}} = 0$

$3x + 4x = 0$

$7x = 0$

$x = 0$

$y = \frac{-\sqrt{2}x}{\sqrt{3}} = 0$

The required solution is $x = 0$ *and* $y = 0$.

(vi) $\frac{3x}{2} - \frac{5y}{3} = -2;\ \frac{x}{3} + \frac{y}{2} = \frac{13}{6}$

Sol: $\frac{3x}{2} - \frac{5y}{3} = -2 \Rightarrow 6 \times \frac{3x}{2} - 6 \times \frac{5y}{3} = 6 \times (-2)$

$9x - 10y = -12 \rightarrow (1)$

$\frac{x}{3} + \frac{y}{2} = \frac{13}{6} \Rightarrow 6 \times \frac{x}{3} + 6 \times \frac{y}{2} = 6 \times \frac{13}{6}$

$2x + 3y = 13 \rightarrow (2)$

$From\ (1): x = \frac{10y - 12}{9} \rightarrow (3)$

Substitute this value of x *in Equ* (2)

$2\left(\frac{10y - 12}{9}\right) + 3y = 13$

$\frac{20y - 24}{9} + 3y = 13$

$20y - 24 + 27y = 117$

$47y = 117 + 24 = 141$

$y = \frac{141}{47} = 3$

Substituting the value of y in equation (3)

$x = \frac{10(3) - 12}{9} = \frac{30 - 12}{9} = \frac{18}{9} = 2$

The solution is $x = 2, y = 3$

2. Solve 2x + 3y = 11 and 2x – 4y = – 24 and hence find the value of 'm' for which y = mx + 3.

Sol: $2x + 3y = 11 \rightarrow (1)$

$2x - 4y = -24 \rightarrow (2)$

From (1): $x = \frac{11 - 3y}{2} \rightarrow (3)$

Substituting the value of y in equation (2)

$2\left(\frac{11 - 3y}{2}\right) - 4y = -24$

$11 - 3y - 4y = -24$

$-7y = -24 - 11$

$-7y = -35$

$y = 5$

Substituting the value of y in equation (3)

$$x = \frac{11 - 3(5)}{2} = \frac{11 - 15}{2} = \frac{-4}{2} = -2$$

$The\ solution\ is\ x = -2, y = 5$

$Substituting\ x = -2, y = 5\ in\ y = mx + 3$

$5 = m(-2) + 3$

$5 = -2m + 3$

$2m = 3 - 5$

$2m = -2$

$m = -1$

The value of 'm' $= -1$

3. Form the pair of linear equations for the following problems and find their solution by substitution method.

(i) The difference between two numbers is 26 and one number is three times the other. Find them.

Sol: Let the two numbers are x and y(x>y)

From problem

$x - y = 26 \rightarrow (1)$

$x = 3y \rightarrow (2)$

Substituting the value of x in equation (1)

$3y - y = 26$

$2y = 26$

$y = 13$

Substitute y=13 in (2)

$x = 3 \times 13 = 39$

The required numbers are 39 and 13

(ii) The larger of two supplementary angles exceeds the smaller by 18 degrees. Find them.

Sol: Let the two supplementary angles be x and y $(x > y)$

From problem

$x + y = 180^0 \rightarrow (1)$

$x - y = 18^0 \rightarrow (2)$

$From\ (1): x = 180^0 - y \rightarrow (3)$

Substitute x value in (2)

$180^0 - y - y = 18^0$

$2y = 180^0 - 18^0$

$2y = 162^0$

$y = 81^0$

Substitute $y = 81^0\ in\ equ\ (3)$

$x = 180^0 - 81^0 = 99^0$

The angles are $99^0\ \ and\ \ 81^0$

(iii) The coach of a cricket team buys 7 bats and 6 balls for ₹ 3800. Later, she buys 3 bats and 5 balls for ₹ 1750. Find the cost of each bat and each ball.

Sol: let the cost of each bat = ₹x and ball = ₹y

From problem

$7x + 6y = 3800 \rightarrow (1)$

$3x + 5y = 1750 \rightarrow (2)$

From (1): $y = \dfrac{3800 - 7x}{6} \rightarrow (3)$

Substituting the value of y in (2)

$3x + 5\left(\dfrac{3800 - 7x}{6}\right) = 1750$

$3x + \dfrac{19000 - 35x}{6} = 1750$

$18x + 19000 - 35x = 10500$

$-17x = 10500 - 19000$

$-17x = -8500$

$x = \dfrac{-8500}{-17} = 500$

Substituting the value of x=500 in(3)

$y = \dfrac{3800 - 7(500)}{6} = \dfrac{3800 - 3500}{6} = \dfrac{300}{6} = 50$

The cost of each bat=₹ 500 and each ball=₹50

(iv) The taxi charges in a city consist of a fixed charge together with the charge for the distance covered. For a distance of 10 km, the charge paid is ₹ 105 and for a journey of 15 km, the charge paid is ₹ 155. What are the fixed charges and the charge per km? How much does a person have to pay for travelling a distance of 25 km?

Sol: Let the fixed charge = ₹x and the charge for 1km = ₹y

The charge paid for 10 km=₹105

$x + 10y = 105 \rightarrow (1)$

The charge paid for 15 km=₹155

$x + 15y = 155 \rightarrow (2)$

$from\ (1): x = 105 - 10y \rightarrow (3)$

Substituting the value of y in (3)

$105 - 10y + 15y = 155$

$105 + 5y = 155$

$5y = 155 - 105 = 50$

$y = \frac{50}{5} = 10$

Substitute y=10 in (3)

$x = 105 - 10 \times 10$

$x = 105 - 100$

$x = 5$

∴ Fixed charge= $x = ₹5$

Charge for 1 km= $y = ₹10$

Charge for 25 km $= \boldsymbol{x + 25y} = 5 + 25 \times 10 = 5 + 250 = ₹255$

(v) A fraction becomes $\frac{9}{11}$, if 2 is added to both the numerator and the denominator. If, 3 is added to both the numerator and the denominator it becomes $\frac{5}{6}$. Find the fraction.

Sol: Let the fraction $= \frac{x}{y}$

From the problem

$\frac{x+2}{y+2} = \frac{9}{11}$ $\quad and \quad$ $\frac{x+3}{y+3} = \frac{5}{6}$

$11(x+2) = 9(y+2) \quad and \quad 6(x+3) = 5(y+3)$

$11x + 22 = 9y + 18 \ and\ 6x + 18 = 5y + 15$

$11x - 9y = 18 - 22\ and\ 6x - 5y = 15 - 18$

$11x - 9y = -4 \rightarrow (1)\ and\ 6x - 5y = -3 \rightarrow (2)$

From(1): $x = \frac{9y - 4}{11} \rightarrow (3)$

Substituting the value of x in (2)

$6\left(\frac{9y-4}{11}\right) - 5y = -3$

$\frac{54y - 24}{11} - 5y = -3$

$54y - 24 - 55y = -33$

$-y = -33 + 24$

$-y = -9$

$y = 9$

Substitute y=9 in (3)

$$x = \frac{9(9)-4}{11} = \frac{81-4}{11} = \frac{77}{11} = 7$$

The required fraction $= \frac{7}{9}$

(vi) Five years hence, the age of Jacob will be three times that of his son. Five years ago, Jacob's age was seven times that of his son. What are their present ages?

Sol: Let the age of Jacob $= x$ and his son $= y$

From problem

$x + 5 = 3(y + 5)$

$x + 5 = 3y + 15$

$x - 3y = 15 - 5$

$x - 3y = 10 \rightarrow (1)$

$x - 5 = 7(y - 5)$

$x - 5 = 7y - 35$

$x - 7y = -35 + 5$

$x - 7y = -30 \rightarrow (2)$

$From\ (1): x = 10 + 3y \rightarrow (3)$

Substituting the value of x in (2)

$10 + 3y - 7y = -30$

$10 - 4y = -30$

$-4y = -30 - 10$

$-4y = -40$

$y = 10$

Substituting y=10 in (3)

$x = 10 + 3(10) = 10 + 30 = 40$

The presen age of Jacob's $= 40\ years$ and his son is $10\ years$

Elimination Method

Example 8 : The ratio of incomes of two persons is 9 : 7 and the ratio of their expenditures is 4 : 3. If each of them manages to save ₹ 2000 per month, find their monthly incomes.

Sol: The ratio of incomes of two persons = 9 : 7

Let their incomes be $9x$ and $7x$

The ratio of their expenditures = 4 : 3

Let their expenditures be $4y$ and $3y$

Given each of them manages to save ₹2000 per month

$9x - 4y = 2000 \rightarrow (1)$

$7x - 3y = 2000 \rightarrow (2)$

$3 \times (1) \Rightarrow 27x - 12y = 6000$

$4 \times (2) \Rightarrow 28x - 12y = 8000$

$\quad (-) \quad (+) \quad (-)$

$-x = -2000$

$x = 2000$

Substitute $x = 2000$ in (1)

$9(2000) - 4y = 2000$

$18000 - 4y = 2000$

$-4y = 2000 - 18000$

$-4y = -16000$

$4y = 16000$

$y = \frac{16000}{4} = 4000$

Their incomes are $9 \times 2000 \quad and \quad 7 \times 2000$

$\Rightarrow$ ₹18000 *and* ₹ 14000

Example 9 : Use elimination method to find all possible solutions of the following pair of linear equations :$2x + 3y = 8; 4x + 6y = 7$

Sol: $2x + 3y = 8 \rightarrow (1)$

$4x + 6y = 7 \rightarrow (2)$

$2 \times (1) \Rightarrow 4x + 6y = 16$

$1 \times (2) \Rightarrow 4x + 6y = 7$

$(-) \quad (-) \quad (-)$

Subtract $0 = 9$ it is not possible.

So, the given pair of equations has no solutions.

Example 10 : The sum of a two-digit number and the number obtained by reversing the digits is 66. If the digits of the number differ by 2, find the number. How many such numbers are there?

Sol: Let the unit place digit be x and tens place digit be y

The number$=10y + x$

The number obtained by reversing the digits$=10x + y$

From the problem

$(10y + x) + (10x + y) = 66$

$11x + 11y = 66$

$x + y = 6 \rightarrow (1)$

Given the digits of the number differ by 2

$x - y = 2 \rightarrow (2)$

$(1) \Rightarrow x + y = 6$

$(2) \Rightarrow x - y = 2$

Adding $2x = 8$

$x = \frac{8}{2} = 4$

Substitute $x = 4$ in (1)

$4 + y = 6$

$y = 6 - 4 = 2$

The number is $10y + x$ and $10x + y$

$\Rightarrow 10 \times 2 + 4 \quad and \quad 10 \times 4 + 2$

$\Rightarrow 24 \ and \ 42$

EXERCISE 3.3

1. Solve the following pair of linear equations by the elimination method and the substitution method :

$(i) x + y = 5 \ and \ 2x - 3y = 4$

Sol: Elimination method:

$x + y = 5 \rightarrow (1)$

$2x - 3y = 4 \rightarrow (2)$

$3 \times equ(1) \Longrightarrow 3x + 3y = 15$

$equ(2) \Longrightarrow 2x - 3y = 4$

Adding $5x = 19$

$x = \frac{19}{5}$

Substitute $x = \frac{19}{5}$ in equation (1)

$\frac{19}{5} + y = 5$

$y = 5 - \frac{19}{5} = \frac{25-19}{5} = \frac{6}{5}$

$\therefore$The solution is $x = \frac{19}{5} \ and \ y = \frac{6}{5}$

Substitution method:

$x + y = 5 \rightarrow (1)$

$2x - 3y = 4 \rightarrow (2)$

$From\ (1): x = 5 - y \rightarrow (3)$

Substitute $x = 5 - y$ in equation (2)

$2(5 - y) - 3y = 4$

$10 - 2y - 3y = 4$

$-5y = 4 - 10 = -6$

$y = \frac{-6}{-5} = \frac{6}{5}$

Substitute $x = \frac{6}{5}$ in equation (1)

$x = 5 - \frac{6}{5} = \frac{25-6}{5} = \frac{19}{5}$

$\therefore$The solution is $x = \frac{19}{5} \ and \ y = \frac{6}{5}$

$(ii) 3x + 4y = 10 \ and \ 2x - 2y = 2$

Sol: **Elimination method**

$3x + 4y = 10 \rightarrow (1)$

$2x - 2y = 2 \rightarrow (2)$

$equ(1) \Longrightarrow 3x + 4y = 10$

$2 \times equ(2) \Longrightarrow 4x - 4y = 4$

Adding $7x = 14$

$x = \frac{14}{7} = 2$

Substitute $x = 2$ in equation (1)

$3 \times 2 + 4y = 10$

$4y = 10 - 6 = 4$

$y = \frac{4}{4} = 1$

$\therefore$ The solution is $x = 2$ and $y = 1$

Substitution method:

$3x + 4y = 10 \rightarrow (1)$

$2x - 2y = 2 \rightarrow (2)$

$From\ (2): 2x = 2 + 2y$

$x = 1 + y \rightarrow (3)$

Substitute $x = 1 + y$ in equation (1)

$3(1 + y) + 4y = 10$

$3 + 3y + 4y = 10$

$3 + 7y = 10$

$7y = 7$

$y = 1$

Substitute $y = 1$ in equation (3)

$x = 1 + 1 = 2$

$\therefore$ The solution is $x = 2$ and $y = 1$

(*iii*) $3x - 5y - 4 = 0$ *and* $9x = 2y + 7$

Sol: **Elimination method**

$3x - 5y = 4 \rightarrow (1)$

$9x - 2y = 7 \rightarrow (2)$

$3 \times equ(1) \Longrightarrow 9x - 15y = 12$

$equ(2) \Longrightarrow 9x - 2y = 7$

$(-) \quad (+) \quad (-)$

Subtracting $-13y = 5$

$y = \frac{-5}{13}$

Substitute $y = \frac{-5}{13}$ in equation (1)

$$3x - 5 \times \left(\frac{-5}{13}\right) = 4$$

$$3x + \frac{25}{13} = 4$$

$$3x = 4 - \frac{25}{13} = \frac{27}{13}$$

$$x = \frac{27}{3 \times 13} = \frac{9}{13}$$

$\therefore$ The solution is $x = \frac{9}{13}$ and $y = \frac{-5}{13}$

Substitution method:

$3x - 5y = 4 \rightarrow (1)$

$9x - 2y = 7 \rightarrow (2)$

$From\ (1): 3x = 4 + 5y$

$$x = \frac{4 + 5y}{3} \rightarrow (3)$$

Substitute x $value$ in equation (2)

$$9\left(\frac{4 + 5y}{3}\right) - 2y = 7$$

$$12 + 15y - 2y = 7$$

$$12 + 13y = 7$$

$$13y = 7 - 12 = -5$$

$$y = \frac{-5}{13}$$

Substitute $y = \frac{-5}{13}$ in equation (3)

$$x = \frac{4 + 5y}{3} = \frac{4 + 5\left(\frac{-5}{13}\right)}{3} = \frac{52 - 25}{3 \times 13} = \frac{27}{3 \times 13} = \frac{9}{13}$$

$\therefore$ The solution is $x = \frac{9}{13}$ and $y = \frac{-5}{13}$

$(iv)\ \frac{x}{2} + \frac{2y}{3} = -1$ and $x - \frac{y}{3} = 3$

Sol: **Elimination method**

$$\frac{x}{2} + \frac{2y}{3} = -1 \Rightarrow 3x + 4y = -6 \rightarrow (1)$$

$$x - \frac{y}{3} = 3 \Rightarrow 3x - y = 9 \rightarrow (2)$$

$equ(1) \Rightarrow 3x + 4y = -6$

$equ(2) \Rightarrow 3x - y = 9$

$\quad (-) \quad (+) \quad (-)$

Subtracting $\quad 5y = -15$

$$y = \frac{-15}{5} = -3$$

$Fropm\ (2): y = 3x - 9$

Substitute $y = -3$ in equation (1)

$3x + 4(-3) = -6$

$3x - 12 = -6$

$3x = -6 + 12 = 6$

$x = 2$

$\therefore$ The solution is $x = 2\ and\ y = -3$

Substitution method:

$3x + 4y = -6 \rightarrow (1)$

$3x - y = 9 \rightarrow (2)$

$Fropm\ (2): y = 3x - 9$

Substitute $y = 3x - 9$ in equation (1)

$3x + 4(3x - 9) = -6$

$3x + 12x - 36 = -6$

$15x = -6 + 36 = 30$

$x = 2$

$\therefore$ The solution is $x = 2\ and\ y = -3$

2. Form the pair of linear equations in the following problems, and find their solutions (if they exist) by the elimination method :

(i) If we add 1 to the numerator and subtract 1 from the denominator, a fraction reduces to 1. It becomes $\frac{1}{2}$ if we only add 1 to the denominator. What is the fraction?

Sol: Let the fraction $= \frac{x}{y}$

From problem

$\frac{x+1}{y-1} = 1\ and\ \frac{x}{y+1} = \frac{1}{2}$

$x + 1 = y - 1\ and\ 2x = y + 1$

$x - y = -1 - 1\ and\ 2x - y = 1$

$x - y = -2 \rightarrow (1)\ and\ 2x - y = 1 \rightarrow (2)$

$$\begin{array}{rl} (1) \Rightarrow & x - y = -2 \\ -1 \times (2) \Rightarrow & -2x + y = -1 \\ \hline Adding & -x \quad = -3 \\ \hline \end{array}$$

$x = 3$

Substitute x=3 in (1)

$3 - y = -2$

$-y = -2 - 3$

$y = 5$

Required fraction $= \frac{3}{5}$

(ii) Five years ago, Nuri was thrice as old as Sonu. Ten years later, Nuri will be twice as old as Sonu. How old are Nuri and Sonu?

Sol:

	Nuri's age	Sonu's age
Present	x	y
5 years ago	$x-5$	$y-5$
10 years later	$x+10$	$y+10$

From problem

$x-5=3(y-5)$ *and* $x+10=2(y+10)$

$x-5=3y-15$ *and* $x+10=2y+20$

$x-3y=-15+5$ *and* $x-2y=20-10$

$x-3y=-10 \rightarrow (1)$ *and* $x-2y=10 \rightarrow (2)$

$2\times(1) \Rightarrow \quad 2x-6y=-20$

$-3\times(2) \Rightarrow -3x+6y=-30$

$Adding \quad -x \qquad = -50$

$x=50$

Substitute x=50in equ (1)

$50-3y=-10$

$-3y=-10-50$

$-3y=-60$

$y=20$

Age of Nuri=50 years and age of Sonu=20 years

(iii) The sum of the digits of a two-digit number is 9. Also, nine times this number is twice the number obtained by reversing the order of the digits. Find the number.

Sol: Let the unit place of digit be x and tens place digit be y

The number=$10y+x$

The number obtained by reversing the digits=$10x+y$

From the problem

$x+y=9 \rightarrow (1)$

$9(10y+x)=2(10x+y)$

$90y+9x=20x+2y$

$20x+2y-90y-9x=0$

$11x-88y=0$

$11(x-8y)=0$

$x-8y=0 \rightarrow (2)$

From (1)-(2)

$x+y-x+8y=9-0$

$9y=9$

$y=1$

Substitute y=1 in (1)

$x+1=9$

$x=8$

The number=10y+x=10×1+8=18

(iv) Meena went to a bank to withdraw ₹ 2000. She asked the cashier to give her ₹ 50 and ₹ 100 notes only. Meena got 25 notes in all. Find how many notes of ₹ 50 and ₹ 100 she received.

Sol: Let the number of ₹50 notes $= x$

The number of ₹100 notes $= y$

Total notes=25

$x + y = 25 \rightarrow (1)$

Value of notes=₹ 2000

$50x + 100y = 2000$

$x + 2y = 40 \rightarrow (2)$

$(2) \Rightarrow x + 2y = 40$

$(1) \Rightarrow x + \ \ y = 25$

$(-)\ (-)\ \ (-)$

$y = 15$

Substitute y=15 in equ (1)

$x + 15 = 25$

$x = 25 - 15 = 10$

∴ Meena received ten ₹50 notes and fifteen ₹100 rupee notes.

(v) A lending library has a fixed charge for the first three days and an additional charge for each day thereafter. Saritha paid ₹ 27 for a book kept for seven days, while Susy paid ₹ 21 for the book she kept for five days. Find the fixed charge and the charge for each extra day.

Sol: Let the fixed charge for the first three days $= x$

Charge per extra day$= y$

Saritha paid ₹ 27 for a book kept for seven days.

$x + 4y = 27 \rightarrow (1)$Susy paid ₹ 21 for the book she kept for five days.

$x + 2y = 21 \rightarrow (2)$

From (1)-(2)

$x + 4y - x - 2y = 27 - 21$

$2y = 6$

$y = 3$

Substituting y = 3 in equation (1)

$x + 4(3) = 27$

$x = 27 - 12$

$x = 15$

The fixed charge=₹ 15 and the charge for each extra day=₹3

Some more problems for brain boosting

1. For the pair of equations λx + 3y = –7 and 2x + 6y = 14 to have infinitely many solutions, the value of λ should be 1. Is the statement true? Give reasons.
2. For all real values of c, the pair of equations x – 2y = 8 and 5x – 10y = c have a unique solution. Justify whether it is true or false.

3. For which value(s) of k will the pair of equations $kx + 3y = k - 3$ and $12x + ky = k$ have no solution?
4. Draw the graph of the pair of equations $2x + y = 4$ and $2x - y = 4$. Write the vertices of the triangle formed by these lines and the y-axis. Also find the area of this triangle
5. Draw the graphs of the pair of linear equations $x - y + 2 = 0$ and $4x - y - 4 = 0$. Calculate the area of the triangle formed by the lines so drawn and the x-axis
6. Two straight paths are represented by the equations $x - 3y = 2$ and $-2x + 6y = 5$. Check whether the paths cross each other or not.
7. Two years ago, Salim was thrice as old as his daughter and six years later, he will be four years older than twice her age. How old are they now?
8. The age of the father is twice the sum of the ages of his two children. After 20 years, his age will be equal to the sum of the ages of his children. Find the age of the father.
9. Two numbers are in the ratio 5 : 6. If 8 is subtracted from each of the numbers, the ratio becomes 4 : 5. Find the numbers
10. The cost of 4 pens and 4 pencil boxes is Rs 100. Three times the cost of a pen is Rs 15 more than the cost of a pencil box. Form the pair of linear equations for the above situation. Find the cost of a pen and a pencil box.
11. Solve the following system of linear equations $7x - 2y = 5$ *and* $8x + 7y = 15$ and verify your answer [CBSE 2024]
12. Three years ago, Rashmi was thrice as old as Nazma. Ten years later, Rashmi will be twise as old as Nazma. How old are Rashmi and Nazma now? [CBSE 2024]

Answers

1. No
2. False
3. k=-6
4. (2,0), (0, 4), (0, –4); 8 sq. units.
5. 6 sq units.
6. Do not cross each other
7. Salim's age = 38 years, Daughter's age = 14 years
8. 40 years
9. 40,48
10. $4x + 4y = 100, 3x = y + 15$, where Rs x and Rs y are the costs of a pen and a pencil box respectively; Rs 10, Rs 15
11. x=1 and y=1
12. Rashmi=42 years and Nazma=16 years

MCQ

1. The pair of equations $5x - 15y = 8$ and $3x - 9y = 24/5$ has
   (A) one solution (B) two solutions (C) infinitely many solutions (D) no solution
2. The sum of the digits of a two-digit number is 9. If 27 is added to it, the digits of the number get reversed. The number is
   (A) 25 (B) 72 (C) 63 (D) 36
3. The pair of equations $x + 2y + 5 = 0$ and $-3x - 6y + 1 = 0$ have
   (A) a unique solution (B) exactly two solutions (C) infinitely many solutions (D) no solution

4. For what value of k, do the equations 3x – y + 8 = 0 and 6x – ky = –16 represent coincident lines?
(A) 1 /2 (B) -1/ 2 (C) 2 (D) –2

5. If the lines given by 3x + 2ky = 2 and 2x + 5y + 1 = 0 are parallel, then the value of k is
(A) –5/ 4 (B) 2/ 5 (C) 15/ 4 (D) 3/ 2

6. The value of c for which the pair of equations cx – y = 2 and 6x – 2y = 3 will have infinitely many solutions is
(A) 3 (B) – 3 (C) –12 (D) no value

7. One equation of a pair of dependent linear equations is –5x + 7y = 2. The second equation can be
(A) 10x + 14y + 4 = 0 (B) –10x – 14y + 4 = 0 (C) –10x + 14y + 4 = 0 (D) 10x – 14y = –4

8. If x = a, y = b is the solution of the equations x – y = 2 and x + y = 4, then the values of a and b are, respectively
(A) 3 and 5 (B) 5 and 3 (C) 3 and 1 (D) –1 and –3

9. Aruna has only Re 1 and Rs 2 coins with her. If the total number of coins that she has is 50 and the amount of money with her is Rs 75, then the number of Re 1 and Rs 2 coins are, respectively
(A) 35 and 15 (B) 35 and 20 (C) 15 and 35 (D) 25 and 25

10.

1) C	2) D	3)D	4)C	5)C	6)D	7)D	8)C	9)D	10)

Case Study-based Questions

1) A test consists of 'True' or 'False' questions. One mark is awarded for every correct answer while ¼ mark is deducted for every wrong answer. A student knew answers to some of the questions. Rest of the questions he attempted by guessing. He answered 120 questions and got 90 marks.

Type of Question	Marks given for correct answer	Marks deducted for wrong answer
True/False	1	0.25

(i) If answer to all questions he attempted by guessing were wrong, then how many questions did he answer correctly?
(ii) How many questions did he guess?
(iii) If answer to all questions he attempted by guessing were wrong and answered 80 correctly, then how many marks he got?
(iv) If answer to all questions he attempted by guessing were wrong, then how many questions answered correctly to score 95 marks?

Sol: (i) Let correctly answered questions=x and wrong answered questions=y

Total number of questions=120

$x + y = 120 \rightarrow (1)$

Total marks=90

$1 \times x - 0.25 \times y = 90$

Multiplying with 4

$4x - y = 360 \rightarrow (2)$

From (1)+(2)

$x + y + 4x - y = 120 + 360$

$5x = 480$

$x = \frac{480}{5} = 96$

$Substitute\ x = 96\ in\ (1)$

$96 + y = 120$

$y = 120 - 96 = 24$

$\therefore x = 96\ and\ y = 24$

The number of questions student answered correctly=96

(ii) The number of questions by guessing were wrong=24

(iii) If student attempted by guessing were wrong and answered 80 correctly then

$student\ got\ the\ marks = 1 \times 80 - 0.25 \times 40 = 80 - 10 = 70$

(iv) Let correctly answered questions=x then wrong answered questions=120-x

$x - 0.25(120 - x) = 95$

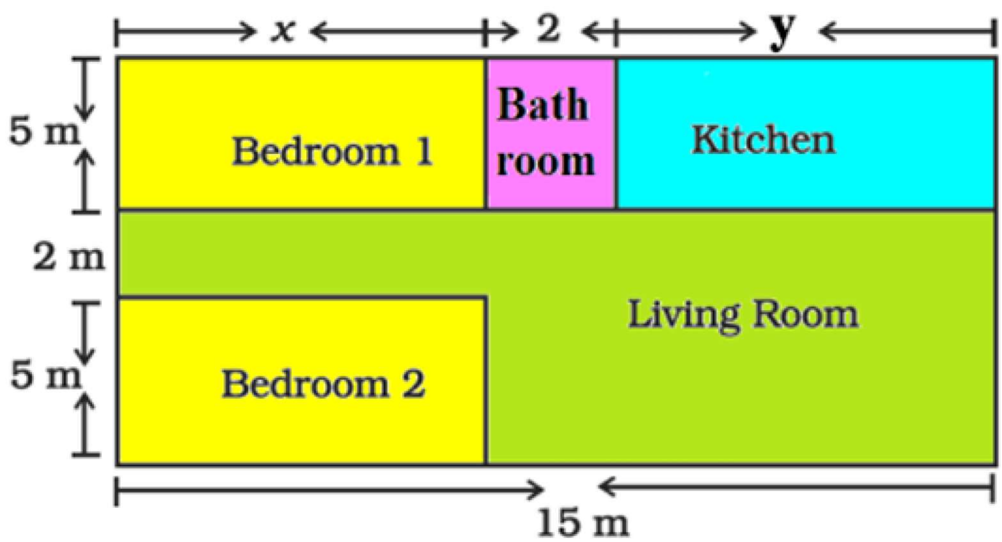
Multiply by 4

$4x - 120 + x = 380$

$5x = 500$

$x = 100$

2) Amit is planning to buy a house and the layout is given below. The design and the measurement has been made such that areas of two bedrooms and kitchen together is 95 sq.m.



Based on the above information, answer the following questions:

(i) Form the pair of linear equations in two variables from this situation.

(ii) Find the length of the outer boundary of the layout.

(iii) Find the area of each bedroom and kitchen in the layout.
(iv) Find the area of living room in the layout.
(v) Find the cost of laying tiles in kitchen at the rate of ₹50 per sq.m

Sol: (i) Total length=15 m

$x + 2 + y = 15$

$x + y = 13 \rightarrow (1)$

The areas of two bedrooms and kitchen together is 95 sq.m

$2 \times (5x) + 5y = 95$

$2x + y = 19 \rightarrow (2)$

The required pair of linear equations are $x + y = 13$ *and* $2x + y = 19$

(ii) The length of the outer boundary of the layout=2(15 m+12 m)=2×27 m=54 m

(iii) From (2)-(1)

$2x + y - x - y = 19 - 13$

$x = 6$

$Substitute\ x = 6\ in\ (1)$

$6 + y = 13$

$y = 13 - 6 = 7$

$\therefore x = 6\ and\ y = 7$

$Area\ of\ each\ bedroom = 5x = 5 \times 6 = 30\ sq\ m$

$Area\ of\ kitchen = 5y = 5 \times 7 = 35\ sq.m$

(iv) Area of living room $= 15 \times 7 -$ Area of bedroom $= 105 - 30 = 75\ sq.m.$

(v) Area of kitchen $= 35\ sq.m$

The cost of laying tiles in kitchen at the rate of ₹ 50 per sq.m=₹50×35=₹1750

Previous year problems:

1. **A shopkeeper gives books on rent for reading. He has variety of books in his store related to fiction, stories and quizzes etc .He takes a fixed charge for the first two days, and an additional charge for subsequent day. Amruta paid ₹22 for a book kept for six days, while Radhika paid ₹16 for the book kept for four days. Assume that the fixed charge be ₹ x and additional charge (per day) be ₹ y .**

Based on the above information, answer any four of the following questions

a) **The situation of amount paid by Radhika, is algebraically represented by.**

$Sol: x + 2y = 16 \rightarrow (1)$

b) **The situation of amount paid by Amruta, is algebraically represented by**

$Sol: x + 4y = 22 \rightarrow (2)$

c) **What are the fixed charges of the book?**

$Sol: 2 \times (1) - (2) \Rightarrow 2x + 4y - x - 4y = 32 - 22$

$\Rightarrow x = 10$

The fixed charges of the book =₹10

d) What are the additional charges for each subsequent day for a book?

$Sol: Substitute\ x = 10\ in\ (1)$

$10 + 2y = 16$

$2y = 16 - 10 = 6$

$y = \frac{6}{2} = 3$

The additional charges for each subsequent day for a book=₹3

e) What is the total amount paid by both, if both of them have kept the book for 2 more days?

$Sol: (22 + 2 \times 3) + (16 + 2 \times 3) = 28 + 22 = ₹50$

2. Two Schools 'P' and 'Q' decided to award prizes to their students for two games of Hockey ₹x per student and Cricket ₹y per student. School 'P' decided to award a total of 9,500 for the two games to 5 and 4 students respectively; while school 'Q' decided to award 7,370 for the two games to 4 and 3 students respectively. Based on the above information, answer the following questions

a) Represent the given information algebraically(in x and y)

$Sol: 5x + 4y = 9500 \rightarrow (1)\ and\ 4x + 3y = 7{,}370 \rightarrow (2)$

b) What is the prize amount for hockey?

Sol:

$4 \times (2) \Rightarrow 16x + 12y = 29{,}480$

$3 \times (1) \Rightarrow 15x + 12y = 28{,}500$

$On\ Subtracting : x = 980$

The prize amount for hockey=₹980

c) Prize amount of which game is more and by how much?

Sol: Substitute x=980 in (1)

$5 \times 980 + 4y = 9500$

$4900 + 4y = 9500$

$4y = 9500 - 4900 = 4600$

$y = \frac{4600}{4} = 1150$

The prize amount for cricket=₹1150

The prize amount of cricket is more and it is (1150-980)=₹170

d) What will be the total amount prize if there are 2 students each from 2 games?

$Sol: 2(x + y) = 2(980 + 1150) = 2 \times 2130 = ₹4260$