CHAPTER 2

X-MATHEMATICS-NCERT-2024-25

Polynomials

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1.Constant: A number having a fixed numerical value is called a constant.

Ex: $5, -3, \frac{1}{2}, -\sqrt{3}, \ldots$

2.Variable: A number which can take various numerical values is known as variable.

$Ex: x, y, z, a, b, p, q, \ldots$

3.Algebraic Expression: An algebraic expression is an expression made up of variables and constants along with mathematical operators.

Example: $8x + 7, -5x^2 - 13, 5x^3 + \frac{8}{x^2} + y$, etc.

4.Polynomial: A polynomial is an algebraic expression in which the exponent on any variable is a whole number.

Example: $2x + 5, 3x^2 + 5x + 6, -5y \ldots$

Polynomials	Not polynomials
$2x$	$4x^{\frac{1}{2}}$
$\frac{1}{3}x - 4$	$3x^2 + 4x^{-1} + 5$
$x^2 - 2x - 1$	$4 + \frac{1}{x}$

5.Degree of a polynomial: The highest power of x *in a polynomial* $p(x)$ is called the degree of the polynomial $p(x)$.

Polynomial	Degree o0f the polynomial
$3x - 5$	1
$3x^2 - 10x + 6$	2
$-2x^3 - 3x + 12$	3

6.Linear polynomial: A polynomial of degree 1 is called a linear polynomial.

Example: $3x + 5, \quad 7x - 8, \quad -9x, \ldots$

The general form a linear polynomial in variable x *is* $ax + b$ $(a, b \in R, a \neq 0)$.

7.Quadratic polynomial : A polynomial of degree 2 is called a quadratic polynomial.

Example: $x^2 - 5x + 6, \quad 2x^2 - 5, \quad 7x^2, \ldots..$

The general form a quadratic polynomial in variable x *is* $ax^2 + bx + c$ $(a, b, c \in R, a \neq 0)$.

8.Cubic polynomial : A polynomial of degree 3 is called a cubic polynomial.

Example: $5x^3 - 4x^2 + x - 1, \quad 2x^3 - 3x + 5, \quad -3x^3 - 10, \ldots\ldots.$

The general form a cubic polynomial in variable x *is* $ax^3 + bx^2 + cx + d$ $(a, b, c, d \in R, a \neq 0)$.

9.The general form of n^{th} degree polynomial in one variable x:

$p(x) = a_0x^n + a_1x^{n-1} + a_2x^{n-2} + \cdots\cdots + a_{n-1}x + a_n$ is a polynomial of n^{th} degree , where $a_0, a_1, a_2, \ldots \ldots \ldots, a_{n-1}, a_n$ are real coefficients and $a_0 \neq 0$.

10. Value of a polynomial at a given point:

The value of $p(x)$ at $x = k$ is $p(k)$. (substitute k value in x place)

11. zero of a polynomial:

A real number k is said to be a zero of a polynomial $p(x), if\ p(k) = 0$

12. The zero of the linear polynomial $ax + b$ is $\frac{-b}{a} = \frac{-(\text{Constant term})}{\text{Coefficientof } x}$

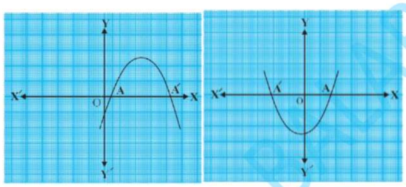
13. The graph of $y = ax + b$ is a straight line which intersects the $x -$ axis at exactly one point $\left(\frac{-b}{a}, 0\right)$

14. The x-coordinate of the point where the graph of $y = ax + b$ intersects the x-axis is the zero of the polynomial $ax + b$

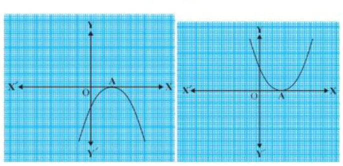
2.1 Geometrical Meaning of the Zeroes of a Polynomial

A real number k is a zero of the polynomial $p(x)$ if $p(k) = 0$

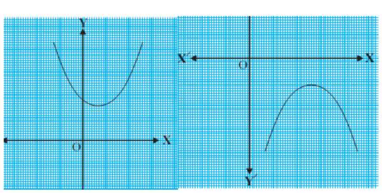
1. The graph of $y = ax^2 + bx + c (a \neq 0)$ either opens upwards like ∪ (if $a > 0$) or opens downwards like ∩ (if $a < 0$). The shape of these curves are called **parabolas.**
2. The graph of $y = ax^2 + bx + c (a \neq 0)$ intersects X-axis at two points $(\alpha, 0)$ and $(\beta, 0)$ then $\boldsymbol{\alpha}, \boldsymbol{\beta}$ are the zeroes of the polynomial $ax^2 + bx + c$



3. The graph of $y = ax^2 + bx + c (a \neq 0)$ touches X-axis one point at $(\alpha, 0)$ then $'\boldsymbol{\alpha}'$ is only one zero of the polynomial $ax^2 + bx + c$.



4. The graph of $y = ax^2 + bx + c (a \neq 0)$ does not intersects X-axis then the polynomial $ax^2 + bx + c$ has no real zeroes.

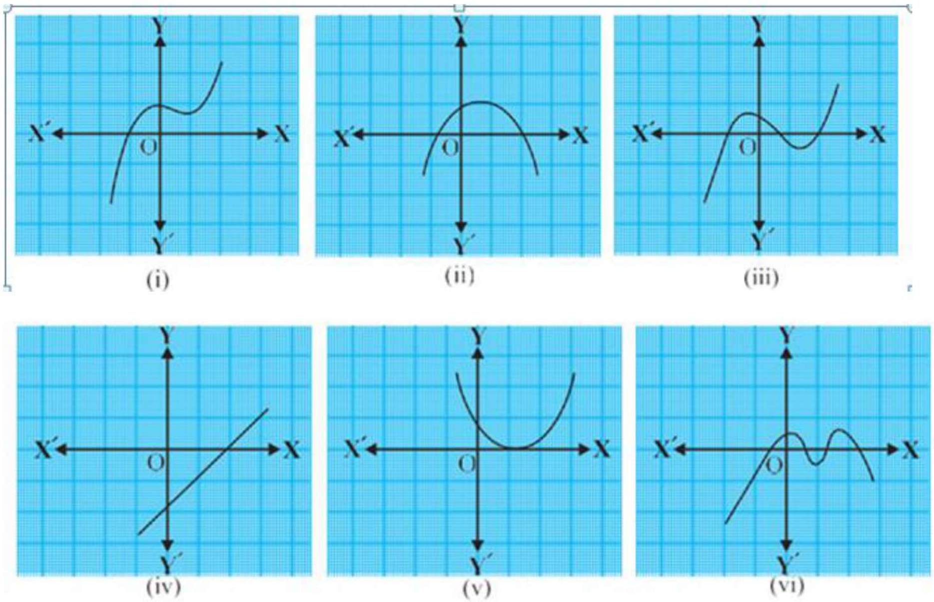


5. Every linear polynomial have at most one zero.

6. Every quadratic polynomial have at most two zeroes.

7. Every cubic polynomial have at most three zeroes

Example 1 : Look at the graphs in Fig. 2.9 given below. Each is the graph of y = p(x), where p(x) is a polynomial. For each of the graphs, find the number of zeroes of p(x).



Solution:

(i) The graph intersects the *x*-axis at one point only. So, the number of zeroes is 1.

(ii) The graph intersects the *x*-axis at two points. So, the numbers of zeroes is 2.

(iii) The graph intersects the *x*-axis at three points. So, the numbers of zeroes is 3.

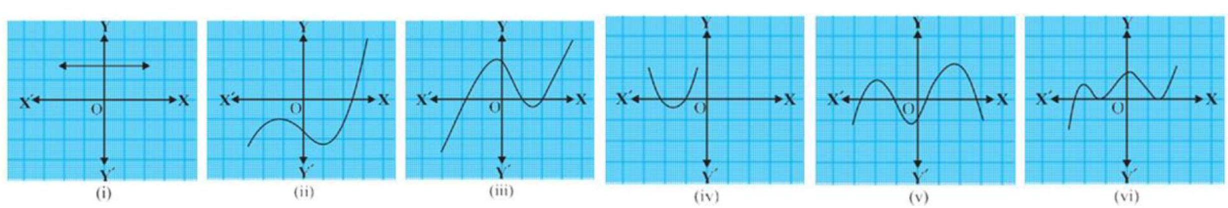
(iv) The graph intersects the *x*-axis at one point only. So, the number of zeroes is 1

(v) The graph intersects the *x*-axis at one point only. So, the number of zeroes is 1

(vi) The graph intersects the *x*-axis at four points. So, the number of zeroes is 4

EXERCISE 2.1

1. **The graphs of $y = p(x)$ are given in Fig below, for some polynomials p(x). Find thenumber of zeroes of $p(x)$, in each case.**



(i) The graph does not intersect the *x*-axis. So, the number of zeroes is 0.

(ii) The graph intersects the *x*-axis at one point only. So, the numbers of zeroes is 1.

(iii) The graph intersects the *x*-axis at three points. So, the numbers of zeroes is 3.

(iv) The graph intersects the x-axis at two points. So, the number of zeroes is 2.

(v) The graph intersects the x-axis at four points. So, the number of zeroes is 4.

(vi) The graph intersects the x-axis at three points. So, the number of zeroes is 3.

Relationship between Zeroes and Coefficients of a Polynomial.

1. If α and β are the zeroes of the quadratic polynomial $ax^2 + bx + c$ $(a, b, c \in R, a \neq 0)$ then

   (i) Sum of zeroes $= \alpha + \beta = \frac{-(\text{coefficient of } x)}{\text{coefficient of} x^2} = \frac{-b}{a}$

   (ii) Product of zeroes $= \alpha\beta = \frac{\text{constant term}}{\text{coefficient of} x^2} = \frac{c}{a}$

   Coefficient of $x^2 = a$
   Coefficient of $x = b$
   Constant term = c

2. If α and β are the zeroes of the quadratic polynomial then the quadratic polynomial

   $= k[x^2 - (\alpha + \beta)x + \alpha\beta]$

   $= k[x^2 - (\text{sum of zeroes})x + \text{product of zeroes}]$ (Where k is constant)

Example 2 : Find the zeroes of the quadratic polynomial $x^2 + 7x + 10$, and verify the relationship between the zeroes and the coefficients.

Coefficient of $x^2 = a = 1$
Coefficient of $x = b = 7$
Constant term $= c = 10$

Sol: $p(x) = x^2 + 7x + 10$

$= x^2 + 5x + 2x + 10$

$= x(x + 5) + 2(x + 5)$

$= (x + 5)(x + 2)$

To find zeroes let $p(x) = 0$

$(x + 5)(x + 2) = 0$

$x + 5 = 0$ or $x + 2 = 0$

$x = -5$ or $x = -2$

The zeroes of the polynomial $p(x) = x^2 + 7x + 10$ are -5 and -2

Sum of zeroes $= (-5) + (-2) = -7 = \frac{-(7)}{1} = \frac{-(\text{coefficient of } x)}{\text{coefficient of } x^2} = \frac{-b}{a}$

Product of the zeroes $= (-5) \times (-2) = 10 = \frac{10}{1} = \frac{\text{constant term}}{\text{coefficient of } x^2} = \frac{c}{a}$

Example 3 : Find the zeroes of the polynomial $x^2 - 3$ and verify the relationship between the zeroes and the coefficients.

Coefficient of $x^2 = a = 1$
Coefficient of $x = b = 0$
Constant term $= c = -3$

Sol: $(x) = x^2 - 3$

$= x^2 - \sqrt{3}^2$

$= (x + \sqrt{3})(x - \sqrt{3})$

To find zeroes let $p(x) = 0$

$(x+\sqrt{3})(x-\sqrt{3}) = 0$

$x+\sqrt{3} = 0 \quad or \quad x-\sqrt{3} = 0$

$x = -\sqrt{3} \quad or \quad x = \sqrt{3}$

The zeroes of the polynomial $p(x) = x^2 - 3$ are $-\sqrt{3}$ and $\sqrt{3}$

Sum of the zeroes $= (-\sqrt{3}) + \sqrt{3} = 0 = \frac{-(0)}{1} = \frac{-(coefficient\ of\ x)}{coefficient\ of\ x^2} = \frac{-b}{a}$

Product of the zeroes $= (-\sqrt{3}) \times \sqrt{3} = -3 = \frac{-3}{1} = \frac{constant\ term}{coefficient\ of\ x^2} = \frac{c}{a}$

Example 4 : Find a quadratic polynomial, the sum and product of whose zeroes are–3 and 2, respectively.

Sol: Sum of the zeroes $= \alpha + \beta = -3$

Product of the zeroes $= \alpha\beta = 2$

Quadratic polynomial $= k[x^2 - (\alpha+\beta)x + \alpha\beta]$

$= k[x^2 - (-3)x + 2]$

$= k[x^2 + 3x + 2]$

One quadratic polynomial $= [x^2 + 3x + 2]$ (When k=1)

CUBIC POLYNOMIALS

1. *If* α, β, γ are the zeroes of the cubic polynomial $ax^3 + bx^2 + cx + d$ $(a, b, c, d \in R, a \neq 0)$ then

(i) $\alpha + \beta + \gamma = \frac{-(\text{coefficient of } x^2)}{\text{coefficient of } x^3} = \frac{-b}{a}$

(ii) $\alpha\beta + \beta\gamma + \gamma\alpha = \frac{\text{coefficient of } x}{\text{coefficient of } x^3} = \frac{c}{a}$

(iii) $\alpha\beta\gamma = \frac{-(\text{constant term})}{\text{coefficient of } x^3} = \frac{-d}{a}$

2. If α, β, γ are the zeroes of the cubic polynomial then the cubic polynomial is

$= k[x^3 - (\alpha+\beta+\gamma)x^2 + (\alpha\beta+\beta\gamma+\gamma\alpha)x - \alpha\beta\gamma]$

Example 5: Verify that $3, -1, -\frac{1}{3}$ are the zeroes of the cubic polynomial $p(x) = 3x^3 - 5x^2 - 11x - 3$, and then verify the relationship between the zeroes and the coefficients.

Sol: $P(x) = 3x^3 - 5x^2 - 11x - 3$ Here $a = 3, b = -5, c = -11$ *and* $d = -3$

$P(3) = 3(3)^3 - 5(3)^2 - 11(3) - 3$

$= 3 \times 27 - 5 \times 9 - 33 - 3$

$= 81 - 45 - 33 - 3$

$= 81 - 81 = 0$

$P(-1) = 3(-1)^3 - 5(-1)^2 - 11(-1) - 3$

$= 3 \times (-1) - 5 \times 1 + 11 - 3$

$= -3 - 5 + 11 - 3$

$= -11 + 11 = 0$

$P\left(-\frac{1}{3}\right) = 3(-\frac{1}{3})^3 - 5\left(-\frac{1}{3}\right)^2 - 11(-\frac{1}{3}) - 3$

$= 3\left(-\frac{1}{27}\right) - 5\left(\frac{1}{9}\right) + \frac{11}{3} - 3$

$= -\frac{1}{9} - \frac{5}{9} + \frac{11}{3} - 3$

$= \frac{-1 - 5 + 33 - 27}{9}$

$= \frac{-33 + 33}{9} = 0$

$P(3) = 0$, $P(-1) = 0$, and $P\left(-\frac{1}{3}\right) = 0$

So ,$3, -1, -\frac{1}{3}$ are the zeroes of the cubic polynomial $P(x) = 3x^3 - 5x^2 - 11x - 3$

we take $\alpha = 3, \beta = -1, \gamma = -\frac{1}{3}$

$\alpha + \beta + \gamma = 3 + (-1) + \left(\frac{-1}{3}\right) = 2 - \frac{1}{3} = \frac{5}{3} = \frac{-(-5)}{3} = \frac{-b}{a}$

$\alpha\beta + \beta\gamma + \gamma\alpha = 3 \times (-1) + (-1) \times \left(\frac{-1}{3}\right) + \left(\frac{-1}{3}\right) \times 3 = -3 + \frac{1}{3} - 1 = \frac{-11}{3} = \frac{c}{a}$

$\alpha\beta\gamma = 3 \times (-1) \times \left(\frac{-1}{3}\right) = 1 = \frac{-(-3)}{3} = \frac{-d}{a}$

EXERCISE 2.2

1. Find the zeroes of the following quadratic polynomials and verify the relationship between the zeroes and the coefficients.

(*i*) $x^2 - 2x - 8$

Coefficient of $x^2 = a = 1$
Coefficient of $x = b = -2$
Constant term $= c = -8$

Sol: $p(x) = x^2 - 2x - 8$; $a = 1, b = -2, c = -8$

$= x^2 - 4x + 2x - 8$

$= x(x - 4) + 2(x - 4)$

$= (x - 4)(x + 2)$

To find zeroes let $p(x) = 0$

$(x - 4)(x + 2) = 0$

$x - 4 = 0 \quad or \quad x + 2 = 0$

$x = 4 \quad or \quad x = -2$

The zeroes of the polynomial $p(x) = x^2 - 2x - 8$ are 4 and -2

Sum of the zeroes $= (4) + (-2) = 2 = \frac{-(-2)}{1} = \frac{-(coefficient\ of\ x)}{coefficient\ of x^2} = \frac{-b}{a}$

Product of the zeroes $= (4) \times (-2) = -8 = \frac{-8}{1} = \frac{constant\ term}{coefficient\ of x^2} = \frac{c}{a}$

(ii) $4s^2 - 4s + 1$

Sol: $p(s) = 4s^2 - 4s + 1$

$= 4s^2 - 2s - 2s + 1$

$= 2s(2s - 1) - 1(2s - 1)$

$= (2s - 1)(2s - 1)$

Coefficient of $s^2 = a = 4$
Coefficient of $s = b = -4$
Constant term $= c = 1$

To find zeroes let $p(s) = 0$

$(2s - 1)(2s - 1) = 0$

$2s - 1 = 0 \quad or \quad 2s - 1 = 0$

$2s = 1 \quad or \quad 2s = 1$

$s = \frac{1}{2} \quad or \quad s = \frac{1}{2}$

The zeroes of the polynomial $p(s) = 4s^2 - 4s + 1$ are $\frac{1}{2}$ and $\frac{1}{2}$

Sum of the zeroes $= \frac{1}{2} + \frac{1}{2} = \frac{2}{2} = 1 = \frac{-(-4)}{4} = \frac{-(\text{coefficient of } s)}{\text{coefficient of } s^2} = \frac{-b}{a}$

Product of the zeroes $= \frac{1}{2} \times \frac{1}{2} = \frac{1}{4} = \frac{constant\ term}{coefficient\ of s^2} = \frac{c}{a}$

(iii) $6x^2 - 3 - 7x$

Sol: $p(x) = 6x^2 - 7x - 3$

$= 6x^2 - 9x + 2x - 3$

$= 3x(2x - 3) + 1(2x - 3)$

$= (2x - 3)(3x + 1)$

Coefficient of $x^2 = a = 6$
Coefficient of $x = b = -7$
Constant term $= c = -3$

To find zeroes let $p(x) = 0$

$(2x - 3)(3x + 1) = 0$

$2x - 3 = 0 \quad or \quad 3x + 1 = 0$

$2x = 3 \quad or \quad 3\,x = -1$

$x = \frac{3}{2} \quad or \quad x = \frac{-1}{3}$

The zeroes of the polynomial $p(x) = 6x^2 - 7x - 3$ are $\frac{3}{2}$ and $\frac{-1}{3}$

Sum of the zeroes $= \left(\frac{3}{2}\right) + \left(\frac{-1}{3}\right) = \frac{9-2}{6} = \frac{7}{6} = \frac{-(-7)}{6} = \frac{-(coefficient\ of\ x)}{coefficient\ of\ x^2} = \frac{-b}{a}$

Product of the zeroes $= \left(\frac{3}{2}\right) \times \left(\frac{-1}{3}\right) = \frac{-3}{6} = \frac{constant\ term}{coefficient\ of\ x^2} = \frac{c}{a}$

(iv) $4u^2 + 8u$

Coefficient of $u^2 = a = 4$

Coefficient of $u = b = 8$

Constant term $= c = 0$

Sol: $P(u) = 4u^2 + 8u$

$= 4u(u+2)$

To find zeroes let $P(u) = 0$

$4u(u+2) = 0$

$4u = 0 \quad or \quad u + 2 = 0$

$u = 0 \quad or \quad u = -2$

The zeroes of the polynomial $p(u) = 4u^2 + 8u$ are 0 and -2

Sum of the zeroes $= 0 + (-2) = -2 = \frac{-8}{4} = \frac{-(coefficient\ of\ u)}{coefficient\ of\ u^2} = \frac{-b}{a}$

Product of the zeroes $= 0 \times (-2) = 0 = \frac{0}{4} = \frac{constant\ term}{coefficient\ of\ u^2} = \frac{c}{a}$

(v) $t^2 - 15$

Coefficient of $t^2 = a = 1$

Coefficient of $t = b = 0$

Constant term $= c = -15$

Sol: $p(t) = t^2 - 15$

$= t^2 - \left(\sqrt{15}\right)^2$

$= (t + \sqrt{15})(t - \sqrt{15})$

To find zeroes let $P(t) = 0$

$\left(t + \sqrt{15}\right)\left(t - \sqrt{15}\right) = 0$

$t + \sqrt{15} = 0 \quad or \quad t - \sqrt{15} = 0$

$t = -\sqrt{15} \quad or \quad t = \sqrt{15}$

The zeroes of the polynomial $p(t) = t^2 - 15$ are $\sqrt{15}$ *and* $-\sqrt{15}$

Sum of the zeroes $= \sqrt{15} + (-\sqrt{15}) = 0 = \frac{-0}{1} = \frac{-(coefficient\ of\ t)}{coefficient\ of\ t^2} = \frac{-b}{a}$

Product of the zeroes $= \sqrt{15} \times (-\sqrt{15}) = -15 = \frac{-15}{1} = \frac{constant\ term}{coefficient\ of\ t^2} = \frac{c}{a}$

(vi) $3x^2 - x - 4$

Coefficient of $x^2 = a = 3$
Coefficient of $x = b = -1$
Constant term $= c = -4$

Sol: $p(x) = 3x^2 - x - 4$

$= 3x^2 - 4x + 3x - 4$

$= x(3x - 4) + 1(3x - 4)$

$= (3x - 4)(x + 1)$

To find zeroes let $p(x) = 0$

$(3x - 4)(x + 1) = 0$

$3x - 4 = 0 \quad or \quad x + 1 = 0$

$3x = 4 \quad or \quad x = -1$

$x = \frac{4}{3} \quad or \quad x = -1$

The zeroes of the polynomial $p(x) = 3x^2 - x - 4$ are $\frac{4}{3}$ and -1

Sum of the zeroes $= \left(\frac{4}{3}\right) + (-1) = \frac{4-3}{3} = \frac{1}{3} = \frac{-(-1)}{3} = \frac{-(coefficient\ of\ x)}{coefficient\ of\ x^2} = \frac{-b}{a}$

Product of the zeroes $= \left(\frac{4}{3}\right) \times (-1) = \frac{-4}{3} = \frac{constant\ term}{coefficient\ of\ x^2} = \frac{c}{a}$

2. Find a quadratic polynomial each with the given numbers as the sum and product of its zeroes respectively.

(i) $\frac{1}{4}, -1$

Sol: sum of the zeroes $= \alpha + \beta = \frac{1}{4}$

Product of zeroes $= \alpha\beta = -1$

The quadratic polynomial $= k[x^2 - (\alpha + \beta)x + \alpha\beta]$

$$= k[x^2 - \left(\frac{1}{4}\right)x + (-1)] = k[x^2 - \frac{1}{4}x - 1]$$

When k = 4 , one quadratic polynomial $= 4 \times [x^2 - \frac{1}{4}x - 1] = 4x^2 - x - 4$

(ii) $\sqrt{2}, \frac{1}{3}$

Sol: sum of the zeroes $= \alpha + \beta = \sqrt{2}$

Product of zeroes $= \alpha\beta = \frac{1}{3}$

The quadratic polynomial $=$ k$[x^2 - (\alpha + \beta)x + \alpha\beta]$

$= k[x^2 - \sqrt{2}x + \left(\frac{1}{3}\right)]$

Quadratic polynomial $= 3 \times \left[x^2 - \sqrt{2}x + \left(\frac{1}{3}\right)\right]$ $(when\ k = 3)$

$= 3x^2 - 3\sqrt{2}x + 1$

(iii) **$0, \sqrt{5}$**

Sol: sum of the zeroes $= \alpha + \beta = 0$

Product of zeroes $= \alpha\beta = \sqrt{5}$

The quadratic polynomial is $=$ k$[x^2 - (0)x + \sqrt{5}]$

$= k[x^2 + \sqrt{5}]$

Quadratic polynomial $= x^2 + \sqrt{5}$ $(when\ k = 1)$

(iv) **$1, 1$**

Sol: Sum of the zeroes $= \alpha + \beta = 1$

Product of the zeroes $= \alpha\beta = 1$

Quadratic polynomial $= k[x^2 - (\alpha + \beta)x + \alpha\beta]$

$= k[x^2 - (1)x + 1]$

$= k[x^2 - x + 1]$

Required one quadratic polynomial $= [x^2 - x + 1]$ $(when\ k = 1)$

(v) **$-\frac{1}{4}, \frac{1}{4}$**

Sol: Sum of the zeroes $= \alpha + \beta = -\frac{1}{4}$

Product of the zeroes$= \alpha\beta = \frac{1}{4}$

Quadratic polynomial $= k[x^2 - (\alpha + \beta)x + \alpha\beta]$

$= k\left[x^2 - \left(-\frac{1}{4}\right)x + \frac{1}{4}\right]$

$= k\left[x^2 + \frac{1}{4}x + \frac{1}{4}\right]$

Required quadratic polynomial $= 4 \times \left[x^2 + \frac{1}{4}x + \frac{1}{4}\right]$ $(when\ k = 4)$

$= 4x^2 + x + 1$

(vi) 4, 1

Sol: Sum of the zeroes $= \alpha + \beta = 4$

Product of the zeroes$= \alpha\beta = 1$

Quadratic polynomial $= k[x^2 - (\alpha + \beta)x + \alpha\beta]$

$= k[x^2 - (4)x + 1]$

$= k[x^2 - 4x + 1]$

One quadratic polynomial $= [x^2 - 4x + 1]$ $(when\ k = 1)$

Some more problems for boosting brain

1. Find the zeroes of the following polynomials by factorisation method and verify the relations between the zeroes and the coefficients of the polynomials:

   $(i)\ 4x^2 - 3x - 1$ $(ii)\ 5t^2 + 12t + 7$ $(iii) 4x^2 + 5\sqrt{2}x - 3$ $(iv)\ v^2 + 4\sqrt{3}v - 15$

2. Find a quadratic polynomial, the sum and product of whose zeroes are $\sqrt{2}$ and $-\frac{3}{2}$, respectively. Also find its zeroes.

3. For each of the following, find a quadratic polynomial whose sum and product respectively of the zeroes are as given. Also find the zeroes of these polynomials by factorisation.

   $(i)\ \frac{-8}{3}, \frac{4}{3}$ $(ii) - 2\sqrt{3}, -9$ $(iii) \frac{-3}{2\sqrt{5}}, -\frac{1}{2}$

4. Verify 1,-1 and -3 are the zeroes of the cubic polynomial $x^3 + 3x^2 - x - 3$ and check the relation between the zeroes and the coefficients.

5. Given that the zeroes of the cubic polynomial $x^3 - 6x^2 + 3x + 10$ are of the form a, a + b, a + 2b for some real numbers a and b, find the values of a and b as well as the zeroes of the given polynomial.

6. Given that $\sqrt{2}$ is a zero of the cubic polynomial $6x^3 + 2x^2 - 10x - 4 2$, find its other two zeroes.

7. Find the zeroes of the quadratic polynomial $\sqrt{3}x^2 - 8x + 4\sqrt{3}$

8. Find a quadratic polynomial, the sum and product of whose zeroes are 6 and 9 respectively.

   Hence find the zeroes.

9. If α and β are the zeroes of the polynomial $f(x) = x^2 - 4x - 5$ then find the value of $\alpha^2 + \beta^2$

10. Solve, for x : $\sqrt{3}x^2 + 10x + 7\sqrt{3}$

11. If α and β are the zeroes of the polynomial $x^2 + 2x + 1$ *then* $\frac{1}{\alpha} + \frac{1}{\beta}$

12. If $x = -2$ is a root of the equation $3x^2 + 7x + p = 0$ find the value of k so that the roots of the equation $x^2 + k(4x + k - 1) + p = 0$are equal.

13. If α and β are the zeroes of the quadratic polynomial $f(x) = x^2 - x - 4$, find the value of $\frac{1}{\alpha} + \frac{1}{\beta} - \alpha\beta$

14. Find the value of k such that the polynomial $x^2-(k+6)x+2(2k-1)$ has sum of its zeroes equal to half of their product.

15. .If α and β are the zeroes of $(x)=6x^2-7x+2$. Find the quadratic polynomial whose zeroes are $\frac{1}{\alpha}$ and $\frac{1}{\beta}$

16. If one zero of the polynomial $p(x)=6x^2+37x-(k-2)$ is reciprocal of the other, then find the value of k.

17. The zeroes of the polynomial$x^2 + px + q$ are twice the zeroes of the polynomial $4x^2 - 5x - 6$ then find the value of p.

MCQ

1. If one zero of the quadratic polynomial $x^2 + 3x + k$ is 2, then the value of k is
   (A) 10 (B)–10 (C) 5 (D) –5
2. Given that two of the zeroes of the cubic polynomial $ax^3 + bx^2 + cx + d$ are 0, the third zero is
   $(A)\ \frac{-b}{a}$ $(B)\frac{b}{a}$ $(C)\ \frac{c}{a}$ $(D)\frac{-d}{a}$
3. If one of the zeroes of the quadratic polynomial $(k-1)x^2 + kx + 1$ is –3, then the value of k is
   $(A)\ \frac{4}{3}$ $(B)\frac{-4}{3}$ $(C)\ \frac{2}{3}$ $(D)\frac{-2}{3}$
4. A quadratic polynomial, whose zeroes are –3 and 4, is
   $(A)x^2 - x + 12$ $(B)x^2 + x + 12$ $(C)\ \frac{x^2}{2} - \frac{x}{2} - 6$ $(D)2x^2 + 2x - 24$

5. If the zeroes of the quadratic polynomial $x^2 + (a + 1)x + b$ are 2 and –3, then
(A) a = –7, b = –1 (B) a = 5, b = –1 (C) a = 2, b = – 6 (D) a = 0, b = – 6

6. If α and β are the roots of $ax^2-bx+c=0 (a\neq 0)$ then value of α +β is
(A) $\frac{b}{a}$ (B) $\frac{a}{b}$ (C) $\frac{-b}{a}$ (D) $\frac{-c}{a}$

7. The zeroes of polynomial $ax^2-bx+c=0$ are reciprocal of each other if
(A) $b^2= a$ (B) c =b (C) b =a (D) c= a

8. If -1 is a zero of the polynomial $p(x)=kx^2-4x+k$, the value of k is
(A) –4 (B) –2 (C) 2 (D) 4

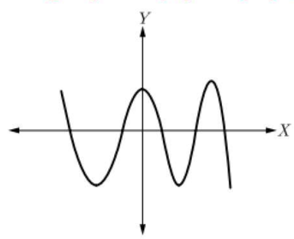
9. If one of the zeroes of the cubic polynomial $x^3 + ax^2 + bx + c$ is –1, then the product of the other two zeroes is
(A) b – a + 1 (B) b – a – 1 (C) a – b + 1 (D) a – b –1

10. The zeroes of the quadratic polynomial $x^2 + 99x + 127$ are
(A) both positive (B) both negative (C) one positive and one negative (D) both equal

11. The zeroes of the quadratic polynomial $x^2 + kx + k, k \neq 0$
(A) cannot both be positive (B) cannot both be negative (C) are always unequal (D) are always equal

12. If the zeroes of the quadratic polynomial $ax^2 + bx + c, c \neq 0$ are equal, then
(A) c and a have opposite signs (B) c and b have opposite signs (C) c and a have the same sign (D) c and b have the same sign.

13. The graph of y p = (x) , where p (x) is a polynomial in variable x , is as follows.



The number of zeroes of p x( ) is ....
(A) 2 (B) 3 (C) 4 (D) 5

14. The zeroes of the quadratic polynomial $\sqrt{3}x^2 - 8x + 4\sqrt{3}$ *are*
(A) $2\sqrt{3}$ *and* $\sqrt{3}$ (B) $2\sqrt{3}$ *and* $\frac{1}{\sqrt{3}}$ (C) $\frac{1}{\sqrt{3}}$ *and* $\sqrt{3}$ (D) $\frac{2}{\sqrt{3}}$ *and* $2\sqrt{3}$

15. If α and β are the zeroes of the polynomial $2x^2 - 13x + 6$ then α+ β is equal to
(A) -3 (B) 3 (C) $\frac{13}{2}$ (D) $-\frac{13}{2}$

16. **Assertion** : If both zeros of the quadratic polynomial $x^2 - 2kx + 2$ are equal in magnitude but opposite in sign then value of k is ½.
Reason : Sum of zeros of a quadratic polynomial $ax^2 + bx + c$ *is* $\frac{-b}{a}$

17. **Assetion(A):**If the sum of the zeroes of the polynomial $x^2-2kx+8$ is 2 then value of k is 1
Reason(R): Sum of zeroes of a quadratic polynomial ax^2+bx+c is -b/a

18. **Assertion(A)** : $x^3 + x$ has only one real zero.

Reason(R) : A polynomial of n^{th} degree must have n real zeroes.

19. **Assetion(A):** If the graph of the polynomial touches X=axis at only one point, then the polynomial cannot be a quadratic polynomial.
Reason(R) :A polynomial of degree n(>1) can have at most n zeroes.

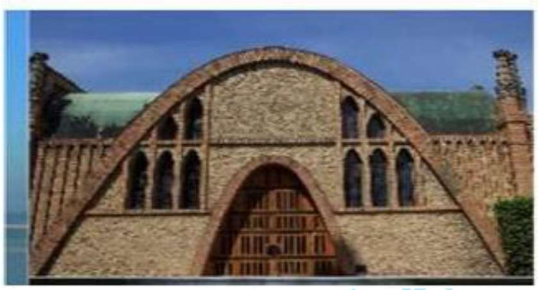
3.

1)	2)	3)	4)	5)	6)	7)	8)	9)	10)	11)	12)	13)	14)	15)	16)	17)	18)	19)
B	A	A	C	D	A	D	B	A	B	C	C	C	D	C	D	A	D	A

Case study based questions.

1. The below picture are few natural examples of parabolic shape which is represented by a quadratic polynomial. A parabolic arch is an arch in the shape of a parabola. In structures, their curve represents an efficient method of load, and so can be found in bridges and in architecture in a variety of forms.






(i) **In the standard form of quadratic polynomial $ax^2 + bx + c$, a, b and c are**

a) All are real numbers. b) All are rational numbers.

c) 'a' is a non-zero real number and b and c are any real numbers. d) All are integers.

(ii) **If the roots of the quadratic polynomial are equal, where the discriminant D $= b^2 - 4ac$, then**

a) $D > 0$ b) $D < 0$ c) D d) $D = 0$

(iii) ***If α and $\frac{1}{\alpha}$ are the zeroes of the quadratic polynomial $2x^2 - x + 8k$, then k is***

a) 4 b) 1/4 c) -1/4 d) 2

(iv) **The graph of $x^2+1=0$**

a) Intersects x-axis at two distinct points. b) Touches x-axis at a point. c) Neither touches nor intersects x-axis. d) Either touches or intersects x- axis.

(v) **If the sum of the roots is –p and product of the roots is -1/p , then the quadratic polynomial is**

(a) $k\left(-px^2 + \frac{x}{p} + 1\right)$ (b) $k\left(px^2 - \frac{x}{p} - 1\right)$ (c) $k\left(x^2 + px - \frac{1}{p}\right)$ (d) $k\left(x^2 - px + \frac{1}{p}\right)$

Sol:

(i) c) 'a' is a non-zero real number and b and c are any real numbers.

(ii) d) $D = 0$

(iii) $Product\ of\ zeroes = \frac{c}{a}$

$\alpha \times \frac{1}{\alpha} = \frac{8k}{2} \Rightarrow 8k = 2 \Rightarrow k = \frac{1}{4}$

Option (b) is correct

(iv) $D = b^2 - 4ac = 0 - 4 \times 1 \times 1 = -4 < 0$

Roots are not real. So, the graph neither touches nor intersects x-axis.

Option (c) is correct

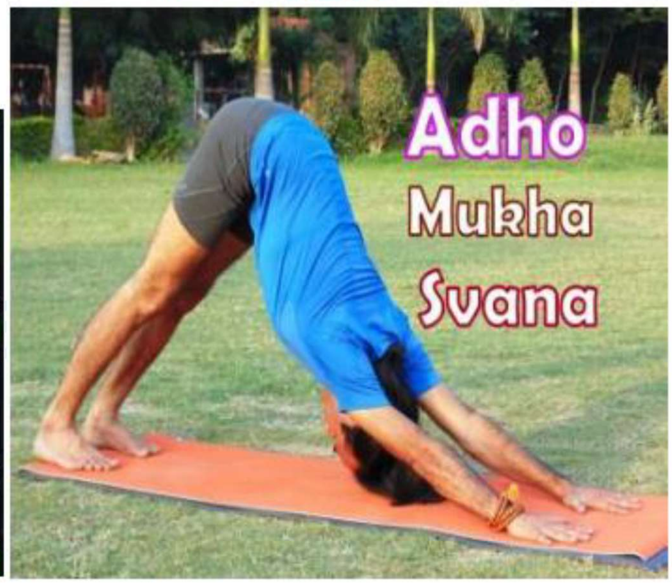
(v) $\alpha + \beta = -p\ and\ \alpha \times \beta = \frac{-1}{p}$

$The\ quadratic\ polynomial = k(x^2 - (\alpha + \beta)x + \alpha\beta)$

$= k\left(x^2 + px - \frac{1}{p}\right)$

Option (c) is correct

2. An asana is a body posture, originally and still a general term for a sitting meditation pose, and later extended in hatha yoga and modern yoga as exercise, to any type of pose or position, adding reclining, standing, inverted, twisting, and balancing poses. In the figure, one can observe that poses can be related to representation of quadratic polynomial.



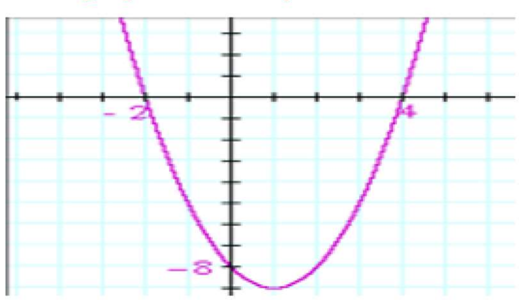
(i) The shape of the poses shown is

a) Spiral b) Ellipse c) Linear d) Parabola

(ii) The graph of parabola opens downwards, if ______

a) $a \geq 0$ b) a = 0 c) a < 0 d) a > 0

(iii) In the graph, how many zeroes are there for the polynomial?



a) 0 b) 1 c) 2 d) 3

(iv) **The two zeroes in the above shown graph are**

a) 2, 4 b) -2, 4 c) -8, 4 d) 2,-8

(v) **The zeroes of the quadratic polynomial $4\sqrt{3}x^2 + 5x - 2\sqrt{3}$ are**

$(a)\ \frac{2}{\sqrt{3}}, \frac{\sqrt{3}}{4}$ $(b)\ -\frac{2}{\sqrt{3}}, \frac{\sqrt{3}}{4}$ $(c)\ \frac{2}{\sqrt{3}}, -\frac{\sqrt{3}}{4}$ $(d)\ -\frac{2}{\sqrt{3}}, -\frac{\sqrt{3}}{4}$

Sol:

(i) (d) parabola

(ii) (c) a<0

(iii) The graph intersects X-axis at two points, therefore it should have two zeroes. Option (c) is correct

(iv) (b) -2,4

(v) $4\sqrt{3}x^2 + 5x - 2\sqrt{3} = 4\sqrt{3}x^2 + 8x - 3x - 2\sqrt{3} = 4x(\sqrt{3}x + 2) - \sqrt{3}(\sqrt{3}x + 2)$

$= (\sqrt{3}x + 2)(4x - \sqrt{3})$

$\sqrt{3}x + 2 = 0, 4x - \sqrt{3} = 0$

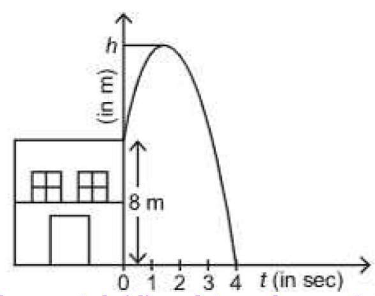
$x = \frac{-2}{\sqrt{3}}, \frac{\sqrt{3}}{4}$

Option (b) is correct

3. Sukriti throws a ball upwards from a rooftop which is 8 meters high from ground level. The ball reaches a maximum height and then returns and hits the ground. The height of the ball at time (t) (in seconds) is represented by (h) (in meters), and the equation of its path is given as: [$h = -t^2 + 2t + 8$]Let's answer some questions based on the information provided:

(i) **The maximum height achieved by the ball is:**

(a)7 m (b) 8 m (c) 9 m (d) 10 m



Sol: The maximum height achieved by the ball at $t = \frac{-b}{2a}$

$= \frac{-2}{2 \times (-1)} = \frac{-2}{-2} = 1$

The maximum height $= -1^2 + 2 \times 1 + 8 = -1 + 10 = 9\ m$

(ii) The polynomial represented by the above graph is:

(a) linear polynomial (b) quadratic polynomial (c) constant polynomial (d) cubic polynomial

Sol: (b)

(iii) The time taken by the ball to reach the maximum height is

(a) 2 sec. (b) 4 sec. (c) 1 sec. (d) 2 min

Sol: (c) 1 sec

(iv) The number of zeros of the polynomial whose graph is given is:

(a)1 (b) 2 (c) 0 (d) 3

Sol: (b) 2

(v) The zeros of the polynomial are:

(a) 4 (b) -2, 4 (c) 2, 4 (d) 0, 4

Sol: (b)

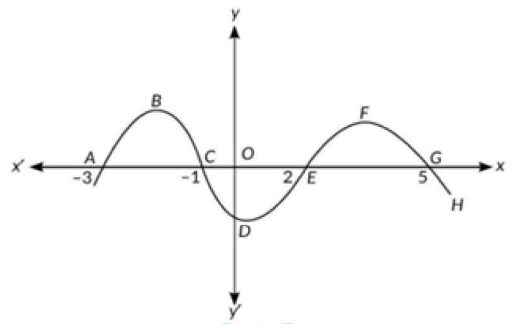
$-t^2 + 2t + 8 = 0$

$t^2 - 2t - 8 = 0$

$(t-4)(t+2) = 0$

$t = 4 \; and \; -2$

4. A car moves on a highway. The path it traces is given below:



Based on the above information, attempt any 4 questions

(i) What is the shape of the curve EFG?

(a) Parabola (b) Ellipse (c) Straight line (d) Circle

(ii) If the curve ABC is represented by the polynomial $-(x^2+4x+3)$, then its zeroes are

(a) 1 and -3 (b) -1 and 3 (c) 1 and 3 (d) -1 and -3

(iii) If the path traced by the car has zeroes at -1 and 2, then it is given by

(a) x^2+x+2 (b) x^2-x+2 (c) x^2-x-2 (d) x^2+x-2

(iv) The number of zeroes of the polynomial representing the whole curve, is

(a) 4 (b) 3 (c) 2 (d) 1

(v) The distance between C and G is

(a) 4 units (b) 6 units (c) 8 units (d) 7 units

(vi) The quadratic polynomial, the sum of whose zeroes is -5 and their product is 6, is

(a) x^2+5x+6 (b) x^2-5x+6 (c) x^2-5x-6 (d) $-x^2+5x+6$

Sol:

(i) (a) parabola

(ii) $x^2 + 4x + 3 = 0 \Rightarrow (x+1)(x+3) = 0 \Rightarrow x = -1 \; and \; -3$

Option (d) is correct

(iii) $x^2 - (\alpha+\beta)x + \alpha\beta = x^2 - 1 + 2x + (-1 \times 2) = x^2 - x - 2$

Option (c) is correct

(iv) The graph intersects X-axis at 4 points. So, the number of zeroes of the polynomial is 4.

Option (a) is correct.

(v) Distance between C and G=5-(-1)=5+1=6 units.

Option (b) is correct.

(vi) $\alpha+\beta=-5$ *and* $\alpha\beta=6$

The quadratic polynomial=$x^2-(\alpha+\beta)x+\alpha\beta=x^2-(-5)x+6=x^2+5x+6$

Option (a) is correct

Some more problems

1) If zeroes of the polynomial $x^2+4x+2a$ are α and $\frac{2}{\alpha}$, then find the value of a .

Sol: $Product\ of\ zeroes=\dfrac{Constant\ term}{coefficint\ of\ x^2}$

$$\alpha\times\frac{2}{\alpha}=\frac{2a}{1}$$

$$2a=2$$

$$a=1$$

2) Find the value of k such that the polynomial $x^2-(k+6)x+2(2k-1)$has sum of its zeroes equal to half of their product.[CBSE-2019]

Sol: $\alpha+\beta=\dfrac{-b}{a}=\dfrac{k+6}{1}=k+6$

$$\alpha\beta=\frac{c}{a}=\frac{2(2k+1)}{1}=2(2k-1)$$

$From\ problem: \alpha+\beta=\frac{1}{2}\times\alpha\beta$

$$k+6=\frac{1}{2}\times2(2k-1)$$

$$k+6=2k-1$$

$$k=7$$

3) Find the zeroes of the quadratic polynomial $7y^2-\frac{11}{3}y-\frac{2}{3}$ and verify the relationship between the zeroes and the coefficients.

$Sol: p(y)=7y^2-\dfrac{11}{3}y-\dfrac{2}{3}=\dfrac{1}{3}(21y^2-11y-2)=\dfrac{1}{3}(21y^2-14y+3y-2)$

$$\frac{1}{3}[7y(3y-2)+1(3y-2)]=\frac{1}{3}(3y-2)(7y+1)$$

To find zeroes let p(y)=0

$$\frac{1}{3}(3y-2)(7y+1)=0$$

$$3y-2=0 \text{ and } 7y+1=0$$

$$y=\frac{2}{3} \text{ and } \frac{-1}{7}$$

The zeroes of the given polynomial are $\frac{2}{3}$ and $\frac{-1}{7}$

$$Sum\ of\ zeroes=\frac{2}{3}+\left(\frac{-1}{7}\right)=\frac{14-3}{21}=\frac{11}{21}$$

$$\frac{-(coefficient\ of\ y)}{coefficient\ of\ y^2}=\frac{-\left(-\frac{11}{3}\right)}{7}=\frac{11}{7\times 3}=\frac{11}{21}$$

$$\therefore Sum\ of\ zeroes=\frac{-(coefficient\ of\ y)}{coefficient\ of\ y^2}$$

$$\text{Product of the zeroes }=\frac{2}{3}\times\left(\frac{-1}{7}\right)=\frac{-2}{21}$$

$$\frac{constant\ term}{coefficient\ of y^2}=\frac{-\frac{2}{3}}{7}=-\frac{2}{3\times 7}=-\frac{2}{21}$$

$$\therefore \text{Product of the zeroes }=\frac{constant\ term}{coefficient\ of y^2}$$

4) The zeroes of the polynomial x^2+px+q are twice the zeroes of the polynomial $4x^2-5x-6$ then find the value of p.[CBSE-2024]

Sol: *Let α,β are the zeroes of* $4x^2-5x-6$

$$\alpha+\beta=\frac{-b}{a}=\frac{5}{4} \text{ and } \alpha\beta=\frac{c}{a}=\frac{-6}{4}=\frac{-3}{2}$$

The polynomial with the zeroes 2α *and* $2\beta=\ x^2-(2\alpha+2\beta)x+(2\alpha)(2\beta)$

$$=x^2-2(\alpha+\beta)x+4(\alpha\beta)$$

$$=x^2-2\left(\frac{5}{4}\right)x+4\left(\frac{-3}{2}\right)$$

$$=x^2-\frac{5}{2}x-6=x^2+px+q$$

$$p=-\frac{5}{2} \text{ and } q=-6$$

5) If the sum of the zeroes of the polynomial $p(x)=2x^2-k\sqrt{2}x+1$ is $\sqrt{2}$ then value of k is:

Sol: $p(x) = 2x^2 - k\sqrt{2}x + 1\ ; \quad a = 2, b = -k\sqrt{2}, c = 1$

$Sum\ of\ the\ zeroes = \sqrt{2}$

$\frac{-b}{a} = \sqrt{2}$

$\frac{k\sqrt{2}}{2} = \sqrt{2}$

$k = 2$

6) If α and β are zeroes of the polynomial $5x^2 + 3x - 7$, the value of $\frac{1}{\alpha} + \frac{1}{\beta}$ is:

Sol: $\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha + \beta}{\alpha\beta} = \frac{\frac{-b}{a}}{\frac{c}{a}} = \frac{-b}{c} = \frac{-3}{-7} = \frac{3}{7}$

7) The zeroes of the quadratic polynomial $2x^2 - 3x - 9$ are [C]

(A) $3, \frac{-3}{2}$ (B) $-3, \frac{-3}{2}$ (C) $-3, \frac{3}{2}$ (D) $3, \frac{3}{2}$

8) Find the zeroes of the quadratic polynomial x^2 -15, and verify the relationship between the zeroes and the coefficients.[CBSE-2024]

Sol: $Let\ p(x) = x^2 - 15$

$= x^2 - (\sqrt{15})^2$

$= (x + \sqrt{15})(x - \sqrt{15})$

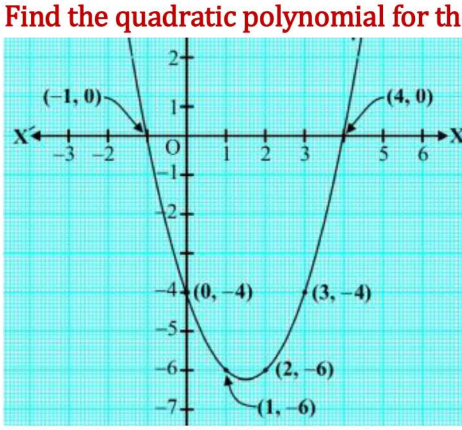
$\therefore Zeroes\ of\ p(x)\ are -\sqrt{15}\ and\ \sqrt{15}$

Verification:

Sum of the zeroes $= (-\sqrt{15}) + \sqrt{15} = 0 = \frac{-(0)}{1} = \frac{-(coefficient\ of\ x)}{coefficient\ of x^2}$

Product of the zeroes $= (-\sqrt{15}) \times \sqrt{15} = -15 = \frac{-15}{1} = \frac{constant\ term}{coefficient\ of x^2}$

9) Find the quadratic polynomial for the below graph



Sol: The graph intersects X − axis at $(-1,0)$ *and* $(4,0)$

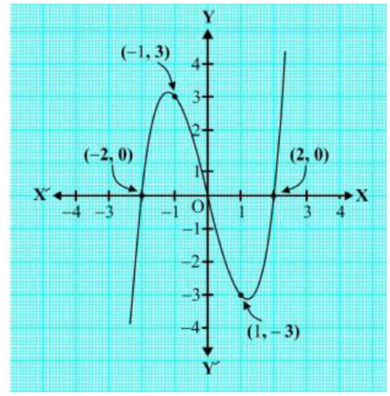
$The\ zeroes\ of\ the\ polynomial\ are\ \alpha = -1\ and\ \beta = 4$

$Required\ polynomial\ P(x) = x^2 - (\alpha + \beta)x + (\alpha\beta)$

$= x^2 - (-1 + 4)x + (-1 \times 4)$

$= x^2 - 3x - 4$

10)Find the cubic polynomial for the adjacent graph



Sol: The graph intersects X − axis at $(-2,0)$, $(0,0)$ and $(2,0)$

$The\ zeroes\ of\ the\ polynomial\ are\ \alpha = -2\ , \beta = 0\ and\ \gamma = 2$

$Required\ polynomial\ P(x)$

$= x^3 - (\alpha + \beta + \gamma)x^2 + (\alpha\beta + \beta\gamma + \gamma\alpha)x^2 - (\alpha\beta\gamma)$

$= x^3 - (-2 + 0 + 2)x^2 + [-2 \times 0 + 0 \times 2 + 2 \times (-2)]x - (-2 \times 0 \times 2)$

$= x^3 - 4x$

11) $\boldsymbol{If\ p(x) = x^2 - 5x + 6\ find\ p(1) + p(4)}$

Sol: $p(x) = x^2 - 5x + 6$

$p(1) = (1)^2 - 5(1) + 6 = 1 - 5 + 6 = 7 - 5 = 2$

$p(4) = (4)^2 - 5(4) + 6 = 16 - 20 + 6 = 22 - 20 = 2$

$p(1) + p(4) = 2 + 2 = 4$

12) $\boldsymbol{If\ \alpha, \beta\ are\ the\ zeroes\ of\ the\ polynomial\ x^2 + x - 2, then\ find\ the\ value\ of\ \frac{\alpha}{\beta} + \frac{\beta}{\alpha}}$.

Sol: $p(x) = x^2 + x - 2$

$\alpha + \beta = \frac{-b}{a} = \frac{-1}{1} = -1$

$\alpha\beta = \frac{c}{a} = \frac{-2}{1} = -2$

$\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{\alpha^2 + \beta^2}{\alpha\beta} = \frac{(\alpha + \beta)^2 - 2\alpha\beta}{\alpha\beta}$

$= \frac{(-1)^2 - 2(-2)}{-2} = \frac{1 + 4}{-2} = \frac{-5}{2}$

13) $\boldsymbol{If\ \alpha, \beta\ are\ the\ zeroes\ of\ the\ polynomial\ p(x) = kx^2 - 30x + 45k\ and}$ $\alpha + \beta = \alpha\beta$, $\boldsymbol{then\ find\ the\ value\ of\ k}$.

Sol: $p(x) = kx^2 - 30x + 45k$

$\alpha + \beta = \frac{-b}{a} = \frac{30}{k}$

$$\alpha\beta = \frac{c}{a} = \frac{45k}{k} = 45$$

$$If\ \alpha + \beta = \alpha\beta\ then \frac{30}{k} = 45$$

$$k = \frac{30}{45} = \frac{2}{3}$$

14) If α, β are the zeroes of the polynomial $5x^2 - 6x + 1$, then find the value of $\alpha + \beta + \alpha\beta$.

$$Sol: \alpha + \beta = \frac{-b}{a} = \frac{6}{5}\ and\ \alpha\beta = \frac{c}{a} = \frac{1}{5}$$

$$\alpha + \beta + \alpha\beta = \frac{6}{5} + \frac{1}{5} = \frac{7}{5}$$

15) If one zero of the polynomial $6x^2 + 37x - (k - 2)$ is reciprocal of the other, then what is the value of the k?

Sol: $p(x) = 6x^2 + 37x - (k - 2)$

$$Let\ the\ zeroes\ are\ \alpha, \frac{1}{\alpha}$$

$$product\ of\ zeroes = \frac{c}{a}$$

$$\alpha \times \frac{1}{\alpha} = \frac{-(k-2)}{6}$$

$$k - 2 = -6$$

$$k = -6 + 2 = -4$$