CHAPTER 1

X-MATHEMATICS-NCERT-2024-25

REAL NUMBERS

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1. **Natural Numbers (N):** Counting numbers {1, 2, 3, ...}.
2. **Whole Numbers (W):** Natural numbers along with zero {0, 1, 2, 3, ...}.
3. **Integers (Z):** Whole numbers and their negatives {... -3, -2, -1, 0, 1, 2, 3, ...}.
4. **Rational Numbers (Q):** Numbers that can be expressed as the quotient or fraction p/q, where p and q are integers and q ≠0
5. Let $x = \frac{p}{q}$ be a rational number, such that the prime factorization of q is of the form $2^n \times 5^m$, where n and m are non-negative integers. Then x has a decimal expansion which terminates.

   Ex: $(i)\ \frac{16}{125} = \frac{16}{5^3} = 0.128 \rightarrow$ Terminating decimal.

   $(ii)\ \frac{7}{8} = \frac{7}{2^3} = 0.875 \rightarrow$ Terminating decimal.

   $(iii)\ \frac{7}{20} = \frac{7}{2^2 \times 5} = 0.35 \rightarrow$ Terminating decimal.

6. Let $x = \frac{p}{q}$ be a rational number, such that the prime factorization of q is not of the form $2^n \times 5^m$, where n and m are non $-$ negative integers. Then, x has a decimal expansion which is non-terminating repeating (recurring).

   Ex: $(i)\ \frac{11}{12} = \frac{11}{2^2 \times 3} = 0.91666\ldots. = 0.91\overline{6} \rightarrow \text{Non} - \text{terminating, repeating decimal.}$

   $(ii)\ \frac{5}{14} = \frac{5}{2 \times 7} = 0.35714285714285 71\ldots = 0.3\overline{571428} \rightarrow \text{Nonterminating, repeating decimal.}$

   $(iii)\ \frac{11}{30} = \frac{11}{2 \times 3 \times 5} = 0.36666\ldots. = 0.3\overline{6} \rightarrow \text{Non} - \text{terminating, repeating decimal.}$

7. If the prime factorisation of q is of the form $2^m.5^n$

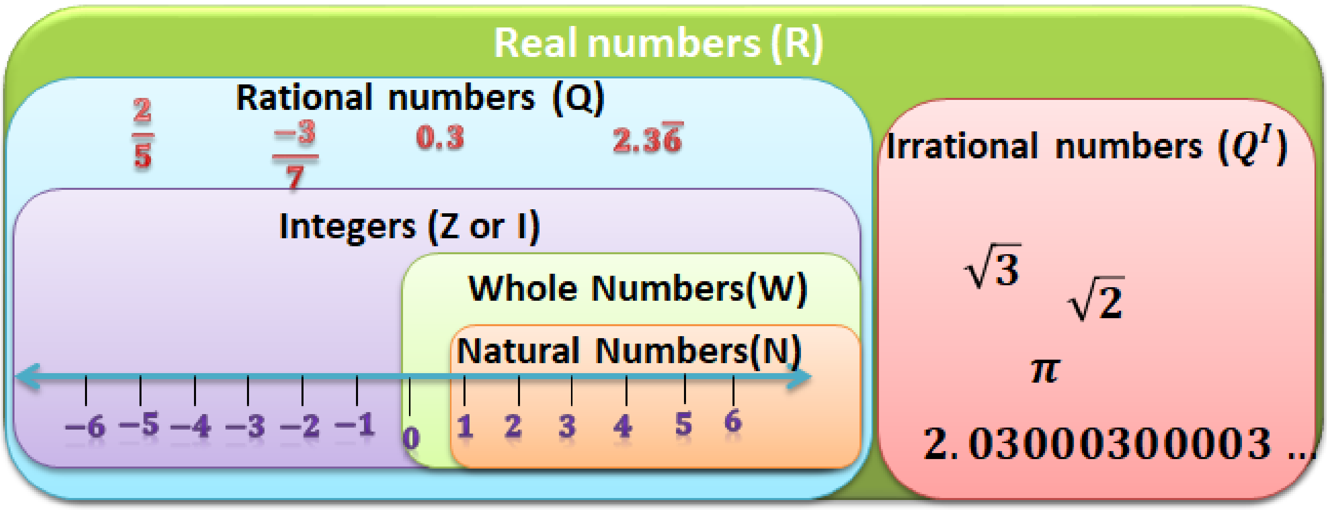
   Then the decimal expansion of the rational number $\frac{p}{q}$, $(p\ and\ q\ are\ co-prime)$

   will terminate after m decimalplace if $m > n$, after n decimalplace if $n > m$

   Ex: $(i)\ \frac{33}{2^2 \times 5}$ will terminate after two decimal places.

   $(ii)\ \frac{14587}{1250} = \frac{14587}{2 \times 5^4}$ will terminate after 4 decimal places.

8. **Irrational Numbers**: Numbers that cannot be expressed as fractions, such as the square root of 2 (√2) or π (pi).
9. Irrational Numbers are non-terminating and non-repeating decimals.
10. **Real Numbers**: Real numbers are a set of numbers that include all rational and irrational numbers. They can be positive, negative, or zero. Examples of real numbers include integers, fractions, decimals, and square roots of non-perfect squares (irrational numbers).



11. Theorem 1.1 (**Fundamental Theorem of Arithmetic**) : Every composite number can be expressed (factorised) as a product of primes, and this factorisation is unique, apart from the order in which the prime factors occur.
12. The prime factorisation of a natural number is unique, except for the order of its factors.
13. In general, given a composite number x, we factorise it as $x = p_1 p_2 \ldots p_n$, where $p_1, p_2, \ldots p_n$ are primes and written in ascending order, i.e., $p_1 \leq p_2 \leq \ldots \leq p_n$. If we combine the same primes, we will get powers of primes.
14. The prime factorisation of a composite number contains 5 and 2 it ends with 0.

Example 1 : Consider the numbers 4^n, where n is a natural number. Check whether there is any value of n for which 4^n ends with the digit zero.

Sol: $4^n = (2^2)^n = 2^{2n}$

5 is not in prime factorisation of 4^n

So, there is no natural number n for which 4^n ends with the digit zero.

HCF (Highest Common Factor):

Product of the smallest power of each common prime factor of the numbers.

LCM (Lowest Common Multiple):

Product of the greatest power of each prime factor of the numbers

Example 2 : Find the LCM and HCF of 6 and 20 by the prime factorisation method.

Sol: $6 = 2^1 \times 3^1$

$20 = 2 \times 2 \times 5 = 2^2 \times 5^1$.

$\text{HCF}(6, 20) = 2^1 = 2$

$\text{LCM}(6, 20) = 2^2 \times 3^1 \times 5^1 = 60$

2	6
3	3
	1

2	20
2	10
5	5
	1

Relationship between two positive integers and their HCF and LCM:

For any two positive integers a and b

$\mathbf{HCF(a, b) \times LCM(a, b) = a \times b}$

$$\mathrm{HCF(a,b)} = \frac{a \times b}{\mathrm{LCM(a,b)}} \quad \text{and} \quad \mathrm{LCM(a,b)} = \frac{a \times b}{\mathrm{HCF(a,b)}}$$

Example 3: Find the HCF of 96 and 404 by the prime factorisation method. Hence,find their LCM.

$$\begin{array}{r|l} 2 & 96 \\ \hline 2 & 48 \\ \hline 2 & 24 \\ \hline 2 & 12 \\ \hline 2 & 6 \\ \hline 3 & 3 \\ \hline & 1 \end{array} \qquad \begin{array}{r|l} 2 & 404 \\ \hline 2 & 202 \\ \hline 101 & 101 \\ \hline & 1 \end{array}$$

Sol: $96 = 2^5 \times 3,$

$404 = 2^2 \times 101$

$\mathrm{HCF}(96,404) = 2^2 = 4$

$$\mathrm{LCM}(96,404) = \frac{96 \times 404}{\mathrm{HCF}(96,404)} = \frac{96 \times 404}{4} = 96 \times 101 = 9696$$

Example 4 : Find the HCF and LCM of 6, 72 and 120, using the prime factorisation method.

$$\begin{array}{r|l} 2 & 6 \\ \hline 3 & 3 \\ \hline & 1 \end{array} \qquad \begin{array}{r|l} 2 & 72 \\ \hline 2 & 36 \\ \hline 2 & 18 \\ \hline 3 & 9 \\ \hline 3 & 3 \\ \hline & 1 \end{array} \qquad \begin{array}{r|l} 2 & 120 \\ \hline 2 & 60 \\ \hline 2 & 30 \\ \hline 3 & 15 \\ \hline 5 & 5 \\ \hline & 1 \end{array}$$

Sol: $6 = 2 \times 3$

$72 = 2^3 \times 3^2$

$120 = 2^3 \times 3 \times 5$

$\mathrm{HCF}\ (6, 72, 120) = 2^1 \times 3^1 = 2 \times 3 = 6$

$\mathrm{LCM}\ (6, 72, 120) = 2^3 \times 3^2 \times 5^1 = 8 \times 9 \times 5 = 360$

Note: The product of three numbers is not equal to the product of their HCF and LCM.

EXERCISE1.1

1. Express each number as a product of its prime factors:

(i) 140

Sol:

$$\begin{array}{r|l} 2 & 140 \\ \hline 2 & 70 \\ \hline 5 & 35 \\ \hline 7 & 7 \\ \hline & 1 \end{array}$$

$140 = 2 \times 2 \times 5 \times 7 = 2^2 \times 5 \times 7$

(ii) 156

Sol:

$$\begin{array}{r|l} 2 & 156 \\ \hline 2 & 78 \\ \hline 3 & 39 \\ \hline 13 & 13 \\ \hline & 1 \end{array}$$

$156 = 2 \times 2 \times 3 \times 13 = 2^2 \times 3 \times 13$

(iii) 3825

Sol:

$$\begin{array}{r|l} 3 & 3825 \\ \hline 3 & 1275 \\ \hline 5 & 425 \\ \hline 5 & 85 \\ \hline 17 & 17 \\ \hline & 1 \end{array}$$

$3825 = 3 \times 3 \times 5 \times 5 \times 17 = 3^2 \times 5^2 \times 17$

(iv) 5005

Sol:

$$\begin{array}{r|l} 5 & 5005 \\ \hline 7 & 1001 \\ \hline 11 & 143 \\ \hline 13 & 13 \\ \hline & 1 \end{array}$$

$5005 = 5 \times 7 \times 11 \times 13$

(v) 7429

Sol:

$$\begin{array}{r|l} 17 & 7429 \\ \hline 19 & 437 \\ \hline 23 & 23 \\ \hline & 1 \end{array}$$

$7429 = 17 \times 19 \times 23$

2. Find the LCM and HCF of the following pairs of integers and verify that LCM × HCF = product of the two numbers.

(i) 26 and 91

$$\begin{array}{r|l} 7 & 91 \\ \hline & 13 \end{array} \qquad \begin{array}{r|l} 2 & 26 \\ \hline & 13 \end{array}$$

Sol: $26 = 2 \times 13$

$91 = 7 \times 13$

HCF $(26, 91) = 13$

LCM $(26, 91) = 2 \times 7 \times 13 = 182$

LCM $(26, 91) \times$ HCF $(26, 91) = 182 \times 13 = 2366$

Product of two numbers $= 26 \times 91 = 2366$

$\therefore$ LCM $(26, 91) \times$ HCF $(26, 91) = 26 \times 91$

(ii) 510 and 92

$$\begin{array}{r|l} 2 & 510 \\ \hline 3 & 255 \\ \hline 5 & 85 \\ \hline & 17 \end{array} \qquad \begin{array}{r|l} 2 & 92 \\ \hline 2 & 46 \\ \hline & 23 \end{array}$$

Sol: $510 = 2 \times 3 \times 5 \times 17$

$92 = 2^2 \times 23$

HCF$(510,92)=2$

$\text{LCM}(510,92) = 2^2 \times 3 \times 5 \times 17 \times 23 = 4 \times 3 \times 5 \times 17 \times 23 = 23460$

$\text{LCM}(510,92) \times \text{HCF}(510,92) = 23460 \times 2 = 46920$

Product of two numbers $= 510 \times 92 = 46920$

$\text{LCM}(510,92) \times \text{HCF}(510,92) = 510 \times 92$

(iii) 336 and 54.

$$\begin{array}{r|l} 2 & 336 \\ \hline 2 & 168 \\ \hline 2 & 84 \\ \hline 2 & 42 \\ \hline 3 & 21 \\ \hline & 7 \end{array} \qquad \begin{array}{r|l} 2 & 54 \\ \hline 3 & 27 \\ \hline 3 & 9 \\ \hline & 3 \end{array}$$

Sol: $336 = 2 \times 2 \times 2 \times 2 \times 3 \times 7 = 2^4 \times 3 \times 7$

$54 = 2 \times 3 \times 3 \times 3 = 2 \times 3^3$

HCF $(336,54) = 2 \times 3 = 6$

$\text{LCM}(336,54) = 2^4 \times 3^3 \times 7 = 16 \times 27 \times 7 = 3024$

$\text{LCM}(336,54) \times \text{HCF}(336,54) = 3024 \times 6 = 18144$

Product of two numbers $= 336 \times 54 = 18144$

$\text{LCM}(336,54) \times \text{HCF}(336,54) = 336 \times 54$

3. Find the LCM and HCF of the following integers by applying the prime factorisation method.

(i) 12, 15 and 21

Sol: $12 = 2^2 \times 3^1$

$15 = 3^1 \times 5^1$

$21 = 3^1 \times 7^1$

$\text{L.C.M of 12,15 and 21} = 2^2 \times 3^1 \times 5^1 \times 7^1 = 4 \times 3 \times 5 \times 7 = 420$

$\text{H.C.F of 12, 15 and 21} = 3^1 = 3$

(ii) 17, 23 and 29

Sol: $17 = 17^1$

$23 = 23^1$

$29 = 29^1$

$\text{L.C.M of 17,23 and 29} = 17 \times 23 \times 29 = 11339$

$\text{H.C.F of 17,23 and 29} = 1 \ (\text{H.C.F of co} - \text{primes} = 1)$

(iii) 8, 9 and 25

Sol: $8 = 2^3$

$9 = 3^2$

$25 = 5^2$

$\text{L.C.M of 8,9 } and \text{ 25} = 2^3 \times 3^2 \times 5^2 = 8 \times 9 \times 25 = 1800$

$\text{H.C.F of 8,9 } and \text{ 25} = 1 \ (\text{H.C.F of co} - \text{primes} = 1)$

4. Given that HCF (306, 657) = 9, find LCM (306, 657).

Sol: $\text{LCM}(a,b) = \dfrac{a \times b}{\text{HCF}(a,b)}$

$$\text{LCM }(306, 657) = \frac{306 \times 657}{\text{HCF }(306, 657)} = \frac{306 \times 657}{9} = 34 \times 657 = 22{,}338$$

5. Check whether 6^n can end with the digit 0 for any natural number n.

Sol : If the prime factorisation of a number contain 2 and 5 then the number ends with the digit zero.

$6^n = (2 \times 3)^n = 2^n \times 3^n$

since 5 is not present in prime factorisation of 6^n.

So 6^n cannot end with the digit zero for any natural number n.

6. Explain why $7 \times 11 \times 13 + 13$ and $7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 + 5$ are composite numbers.

Sol: $7 \times 11 \times 13 + 13$

$= 13 \times (7 \times 11 + 1)$

$= 13 \times (77 + 1)$

$= 13 \times 78 = 2 \times 3 \times 13^2$.

2,3 and 13 are the factors of 7x11x13+13 .

So, 7×11×13+13 is a composite number.

$7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 \ +5$

$=5 \times (7 \times 6 \times 4 \times 3 \times 2 \times 1 + 1)$

$=5 \times (1008 + 1)$

$=5 \times 1009$

5 ,1009are factories of $7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 \ +5$.

So, $7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 \ +5$ is a composite number.

7. There is a circular path around a sports field. Sonia takes 18 minutes to drive one round of the field, while Ravi takes 12 minutes for the same. Suppose they both start at the same point and at the same time, and go in the same direction. After how many minutes will they meet again at the starting point?

Sol: Time taken by Sonia to drive one round=18 minutes.

Time taken by Ravi to drive one round=12 minutes.

Time taken by both to meet again=LCM (18,12)

$18 = 2 \times 3^2$

$12 = 2^2 \times 3$

$\text{LCM}(18,12) = 2^2 \times 3^2 = 4 \times 9 = 36$

After 36 minutes they will meet again at the starting point.

Revisiting Irrational Numbers

(i) **Rational Numbers (Q):** Numbers that can be expressed as the quotient or fraction p/q, where p and q are integers and q is not equal to zero.

(ii) **Irrational Numbers(Q'):** Numbers that cannot be expressed as p/q where p,q are integers.

$\sqrt{2}, \sqrt{3}, \sqrt{15}, \pi, -\frac{\sqrt{2}}{\sqrt{3}}, 0.10110111011110111.. \ etc$

(iii) $\sqrt{p}$ is irrational, where p is a prime.

Theorem 1.2 : Let p be a prime number. If p divides a², then p divides a, where a is a positive integer.

Sol: $a = \ p_1 \, p_2 ... p_n$, where $p_1, p_2, ... p_n$. are primes, not necessarily distinct.

$a^2 = (p_1 \, p_2 ... p_n)(p_1 \, p_2 ... p_n) = {p_1}^2 \, {p_2}^2 ... {p_n}^2$.

If p divides a^2

From the Fundamental Theorem of Arithmetic, it follows that p is one of the prime factors of a^2

So p is one of $p_1, p_2, ... p_n$

$\Rightarrow p \; divides \; a$

Theorem 1.3 : Prove that $\sqrt{2}$ is irrational.

Proof:Let us assume $\sqrt{2}$ is rational.

Then $\sqrt{2} = \frac{a}{b}$ $(\, a, b \; are \; coprimes)$

Squaring on both sides we get

$2 = \frac{a^2}{b^2} \Rightarrow 2b^2 = a^2 \rightarrow (1)$

$\Rightarrow \; b^2 = \frac{a^2}{2}$

$\Rightarrow 2 \; divides \; a^2$

$\Rightarrow \; 2 \; divides \; a$

p *be a prime number .*

If p *dividesa*^{2} *then* p *divides* a

We can write $a = 2c \;\; for \; some \; integer \; c$

$\Rightarrow a^2 = 4c^2$

$\Rightarrow \; 2b^2 = \; 4c^2$ (from (1))

$\Rightarrow \; b^2 = \; 2c^2$

$\Rightarrow \; c^2 = \frac{b^2}{2}$

$\Rightarrow 2 \; divides \; b^2$

$\Rightarrow \; 2 \; divides \; b$

Therefore , both $a \; and \; b$ have 2 as a common factor.

But this contradicts the fact that $a \; and \; b$ are co-prime.

Thus our assumption is false.

So,we conclude that $\sqrt{2}$ is irrational.

Example 5 : Prove that $\sqrt{3}$ is irrational.

Proof:Let us assume $\sqrt{3}$ is rational.

Then $\sqrt{3} = \frac{a}{b}$ $(\, a, b \; are \; coprimes)$

Squaring on both sides we get

$3 = \frac{a^2}{b^2} \Rightarrow 3b^2 = a^2 \rightarrow (1)$

$\Rightarrow \; b^2 = \frac{a^2}{3}$

$\Rightarrow 3 \; divides \; a^2$

$\Rightarrow \; 3 \; divides \; a$

p *be a prime number .*

If p *dividesa*^{2} *then* p *divides* a

We can write $a = 3c$ $for\ some\ integer\ c$

$\Rightarrow a^2 = 9c^2$

$\Rightarrow 3b^2 = 9c^2$ (from (1))

$\Rightarrow b^2 = 3c^2$

$\Rightarrow c^2 = \frac{b^2}{3}$

$\Rightarrow 3\ divides\ b^2$

$\Rightarrow 3\ divides\ b$

Therefore , both $a\ and\ b$ have 3 as a common factor.

But this contradicts the fact that $a\ and\ b$ are co-prime.

Thus our assumption is false.

So,we conclude that $\sqrt{3}$ is irrational.

Example 6 : Show that 5– $\sqrt{3}$ is irrational.

Solution: Let us assume that 5 – $\sqrt{3}$ is rational.

$5 - \sqrt{3} = \frac{a}{b}$ $(\ a, b\ are\ coprimes)$

$5 - \frac{a}{b} = \sqrt{3}$

$\sqrt{3} = 5 - \frac{a}{b} = \frac{5b - a}{b} \ldots \ldots \ldots \ldots (1)$

$Since\ 5, a\ and\ b\ are\ integers\ the\ R.H.S\ of\ (1)\ i.e\ \frac{5b-a}{b}\ is\ rational$

$so\ the\ L.H.S = \sqrt{3}\ also\ rational$

But this contradicts the fact that $\sqrt{3}$ is irrational.

Thus our assumption is false.

So,we conclude that 5 – $\sqrt{3}$ is irrational .

Example 7 : Show that $3\sqrt{2}$ is irrational.

Solution: Let us assume that $\mathbf{3\sqrt{2}}$ is rational.

Let $3\sqrt{2} = \frac{a}{b}$ $(\ a, b\ are\ coprimes)$

$\sqrt{2} = \frac{a}{3b} \ldots \ldots \ldots \ldots \ldots (1)$

$Since\ 3, a\ and\ b\ are\ integers\ the\ R.H.S\ of\ (1) ie\ \frac{a}{3b} is\ rational$

$so\ the\ L.H.S\ \sqrt{2}\ also\ rational$

But this contradicts the fact that $\sqrt{2}$ is irrational.

Thus our assumption is false.

So,we conclude that $3\sqrt{2}$ is irrational .

EXERCISE1.2

1. **Prove that $\sqrt{5}$ is irrational.**

Proof: Let us assume $\sqrt{5}$ is rational.

Then $\sqrt{5} = \frac{a}{b}$ $(\ a, b\ are\ coprimes)$

Squaring on both sides we get

$p\ be\ a\ prime\ number$.
$If\ p\ divides\ a^2\ \ then\ p\ divides\ a$

$$5 = \frac{a^2}{b^2} \Rightarrow 5b^2 = a^2 \rightarrow (1)$$

$$\Rightarrow b^2 = \frac{a^2}{5}$$

$\Rightarrow$ 5 divides a^2

$\Rightarrow$ 5 divides a

We can write $a = 5c\ \ for\ some\ integer\ c$

$\Rightarrow a^2 = 25c^2$

$\Rightarrow 5b^2 = 25c^2$ (from (1))

$\Rightarrow b^2 = 5c^2$

$$\Rightarrow c^2 = \frac{b^2}{5}$$

$\Rightarrow$ 5 divides b^2

$\Rightarrow$ 5 divides b

Therefore , both $a\ and\ b$ have 5 as a common factor.

But this contradicts the fact that $a\ and\ b$ are co-prime.

Thus our assumption is false.

So,we conclude that $\sqrt{5}$ is irrational.

2. **Prove that $3 + 2\sqrt{5}$ is irrational.**

Solution: Let us assume that $3 + 2\sqrt{5}$ is rational.

Let $3 + 2\sqrt{5} = \frac{a}{b}$ $(\ a, b\ are\ coprimes)$

$$2\sqrt{5} = \frac{a}{b} - 3 = \frac{a - 3b}{b}$$

$$\sqrt{5} = \frac{a-3b}{2b} \rightarrow (1)$$

Since $2,3, a$ *and* b *are integers the* $R.H.S$ *of* (1) *ie* $\frac{a-3b}{2b}$ *is rational*

So the L. H. S $\sqrt{5}$ *also rational*

But this contradicts the fact that $\sqrt{5}$ is irrational.

Thus our assumption is false.

So,we conclude that $3 + 2\sqrt{5}$ is irrational .

3. Prove that the following are irrationals :

$(i)\ \frac{1}{\sqrt{2}}$

Sol: Let us assume that $\frac{1}{\sqrt{2}}$ is rational.

Let $\frac{1}{\sqrt{2}} = \frac{a}{b}$ $(\,a, b$ *are coprimes*$)$

$$\sqrt{2} = \frac{b}{a}$$

Since a and b are integers, $\frac{b}{a}$ is rational , and so $\sqrt{2}$ is rational.

But this contradicts the fact that $\sqrt{2}$ is irrational.

Thus our assumption is false.

So,we conclude that $\frac{1}{\sqrt{2}}$ is irrational .

$(ii)\ 7\sqrt{5}$

Solution: Let us assume that $7\sqrt{5}$ is rational.

Let $7\sqrt{5} = \frac{a}{b}$ $(\,a, b$ *are coprimes*$)$

$$\sqrt{5} = \frac{a}{7b} \ldots\ldots\ldots\ldots\ldots (1)$$

Since $7, a$ *and* b *are integers the* $R.H.S$ *of* (1) *ie* $\frac{a}{7b}$ *is rational*

So, the L. H. S $\sqrt{5}$ also rational.

But this contradicts the fact that $\sqrt{5}$ is irrational.

Thus our assumption is false.

So,we conclude that $7\sqrt{5}$ is irrational .

$(iii) 6+\sqrt{2}$

Sol: Let us assume that $6+\sqrt{2}$ is rational .

$Let\ 6+\sqrt{2}=\frac{a}{b}\ \ (a, b\ are\ coprimes)$

$$\sqrt{2}=\frac{a}{b}-6=\frac{a-6b}{b}$$

Since $6, a\ and\ b$ are integers, $\frac{a-6b}{b}$ is rational ,and so $\sqrt{2}$ is rational .

But this contradicts the fact that $\sqrt{2}$ is irrational.

Thus our assumption is false.

So,we conclude that $6+\sqrt{2}$ is irrational .

Some more problems

1. **Prove that $5-2\sqrt{3}$ is an irrational number. It is given that $\sqrt{3}$ is an irrational number.**[CBSE-2024]

Sol: Assuming **$5-2\sqrt{3}$ to be a rational number.**

$Let\ 5-2\sqrt{3}=\frac{a}{b}\ \ (a, b\ are\ coprimes)$

$$2\sqrt{3}=5-\frac{a}{b}=\frac{5b-a}{b}$$

$$\sqrt{3}=\frac{5b-a}{2b}$$

Here RHS is rational but LHS is irrational.

Therefore our assumption is wrong.

Hence, 5 – $2\sqrt{3}$ is an irrational number

2. **Show that the number 5×11×17+3×11 is a composite number.**[CBSE-2024]

Sol: $5\times 11\times 17+3\times 11=11\times(5\times 17+3)=11\times 88=11\times 11\times 2^3$

It means the number can be expressed as a product of two factors other than 1, therefore the given number is a composite number.

3. **In a teachers' workshop, the number of teachers teaching French, Hindi and English are 48, 80 and 144 respectively. Find the minimum number of rooms required if in each room the same number of teachers are seated and all of them are of the same subject.**[CBSE-2024]

Sol: Minimum number of rooms required means there should be maximum number of teachers in a room.

We have to find HCF of 48, 80 and 144.

$48 = 2^4 \times 3$

$80 = 2^4 \times 5$

$144 = 2^4 \times 3^2$

HCF (48, 80, 144) = $2^4 = 16$

Therefore, total number of rooms required $= \frac{48}{16} + \frac{80}{16} + \frac{144}{16} = 3 + 5 + 9 = 17$

4. If the HCF(2520,6600)=40 and LCM(2520,6600)=252×k[CBSE-2024]

Sol: HCF(2520,6600)× LCM(2520,6600)=2520×6600

40 ×252×k=2520×6600

$$k = \frac{2520 \times 6600}{40 \times 252} = 1650$$

5. Can two numbers have 18 as their HCF and 380 as their LCM? Give reasons.

Sol: No.

Reason: The HCF of the numbers is a factor of the LCM of the numbers. In given problem HCF of two numbers is 18 not a factor of LCM of the numbers 380.

6. Use Euclid's division algorithm to find the HCF of 441, 567, 693.

Sol: 693=1×567+126

567=4×126+63

126=2×63

HCF(693,567)=63

441=7×63

HCF(441,63)=63

∴HCF(693,567,441)=63

7. Show that 12^n cannot end with the digit 0 or 5 for any natural number n.

Sol: $12^n = (2^2 \times 3)^n = 2^{2n} \times 3^n$

since 5 is not present in prime factorisation of 12^n.

So, 12^n cannot end with the digit zero or 5 for any natural number n.

8. Prove that $\sqrt{2} + \sqrt{3}$ is irrational.

Solution : Let us suppose that $\sqrt{2} + \sqrt{3}$ is rational.

Let $\sqrt{2}+\sqrt{3}=a$, where a is rational.

$$\sqrt{2}=a-\sqrt{3}$$

Squaring on both sides,

$$2=a^2+3-2a\sqrt{3}$$

$$\sqrt{3}=\frac{a^2+1}{2a}$$

Which is a contradiction as the right hand side is a rational number while $\sqrt{3}$ is irrational.

Hence $\sqrt{2}+\sqrt{3}$ is irrational

9. **Write the denominator of the rational number $\frac{257}{5000}$ in the form $2^m \times 5^n$, where m, n are non-negative integers. Hence, write its decimal expansion, without actual division.**

Sol: $\frac{257}{5000}=\frac{257}{5\times 10^3}=\frac{257}{5\times 2^3\times 5^3}=\frac{257}{2^3\times 5^4}=\frac{257}{2^3\times 5^4}\times\frac{2}{2}=\frac{514}{2^4\times 5^4}=\frac{514}{10^4}=0.0514$

MCQ

1. After how many decimal places will the decimal expansion of the number $\frac{47}{2^2\times 5^2}$ terminate?
   (a) 5 (b) 2 (c) 3 (d) 1
2. The decimal expansion of the rational number $\frac{33}{2^2\times 5}$ will terminate after.
   (a) one decimal place (b) two decimal places (c) three decimal places (d) more than 3 decimal places
3. Euclid's division lemma states that for two positive integers a and b, there exist unique integers q and r such that $a = bq + r$, where r must satisfy
   (a) $1 < r < b$ (b) $0 < r \le b$ (c) $0 \le r < b$ (d) $0 < r < b$
4. For some integer m, every even integer is of the form
   (a) m (b) m + 1 (c) 2m (d) 2m + 1
5. $n^2 - 1$ is divisible by 8, if n is
   (a) an integer (b) a natural number (c) an odd integer (d) an even integer
6. The largest number which divides 70 and 125, leaving remainders 5 and 8, respectively, is
   (a) 13 (b) 65 (c) 875 (d) 1750
7. The largest number which divides 318 and 739 leaving remainders 3 and 4, respectively is
   (a) 110 (b) 7 (c) 35 (d) 105
8. The LCM of smallest two-digit composite number and smallest composite number is
   a) 12 b) 4 c) 20 d) 44
9. If two positive integers a and b are written as $a = x^3y^2$ and $b = xy^3$; x, y are prime numbers, then HCF (a, b) is
   (a) xy (b) xy^2 (c) x^3y^3 (d) x^2y^2
10. If two positive integers p and q can be expressed as $p = ab^2$ and $q = a^3b$; a, b being prime numbers, then LCM (p, q) is
    (a) ab (b) a^2b^2 (c) a^3b^2 (d) a^3b^3
11. The least number that is divisible by all the numbers from 1 to 10 (both inclusive) is

(a) 10 (b) 100 (c) 504 (d) 2520

12. The decimal expansion of the rational number $\frac{14587}{1250}$ will terminate after:

(a) one decimal place (b) two decimal places (c) three decimal places (d) four decimal places

13. The decimal expantion of $\frac{63}{72\times175}$ is

(a) terminating (b)non-terminating(c) non termination and repeating (d)an irritional number

14. If the HCF of 65 and 117 is expressible in the form 65m – 117, then the value of m is

(a) 4 (b) 2 (c) 1 (d) 3

15. If two positive integers p and q can be expressed as $p=18a^2b^4$ and $q=20a^3b^2$, wherw a,b are prime numbers, then LCM(p,q)[CBSE-2024]

$(a)\ 2a^2b^2$ $(b)180a^2b^2$ $(c)12a^2b^2$ $(d)180a^3b^4$

16. The HCF and LCM of 12,21,15 are

(a) 3,140 (b) 12,420 (c)3, 420 (d) 420,3

17. If 3 is the least prime factor of number a and 7 is the least prime factor of number b, then the least prime factor of (a+b) is

(a) 2 (b) 3 (c) 5 (d) 10

18.

19.

1)b	2)b	3)c	4)c	5)c	6)a	7)d	8)c	9)b	10)c	11)d	12)d	13)a	14)b	15)d
16)c	17)a													

Assertion (A) and Reason(R)

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

(c) Assertions (A) is true but reason (R) is false.

(d) Assertions (A) is false but reason (R) is true.

1. **Assertion (A):** The HCF of two numbers is 5 and their product is 150. Then their LCM is 40.

   Reason(R): For any two positive integers a and b, HCF$(a,b)\times$ LCM$(a,b) = a \times b$ a b c d

2. **Assertion:** 13/3125 is a terminating decimal fraction.

   Reason: If $q = 2^n.5^m$ where n and m are non-negative integers, then p/q is a terminating decimal fraction.

3. **Assertion:** When a positive integer a is divided by 3, the values of remainder can be 0, 1 or 2.

   Reason: According to Euclid's Division Lemma $a = bq + r$, where $0 \leq r < b$ and r is an integer.

4. **Assertion:** $\mathbf{12^n}$ ends with the digit zero, where n is any natural number.

   Reason: Any number ends with digit zero, if its prime factor is of the form $2^m \times 5^n$, where m an n are natural numbers

5. **Assertion:** (18, 25) is a pair of co-primes.

   Reason: Pair of co-prime has a common factor 2.

6. **Assertion:** The decimal expansion of the rational number $33/2^2 \times 5$ will terminate after two decimal place.

   Reason: The rational numbers are those numbers that exist in the form of p/q, where p and q are any integers and q≠0.

7. **Assertion:** $(2-\sqrt{5})$ is an irrational number.

   Reason: The sum or difference of a rational and an irrational number is irrational.

1)d	2)a	3)a	4)d	5)c	6)b	7)a

Case Study-based Questions

1. **To enhance the reading skills of grade X students, the school nominates you and two of your friends to set up a class library. There are two sections- section A and section B of grade X. There are 32 students in section A and 36 students in section B.**

(i) What is the minimum number of books you will acquire for the class library, so that they can be distributed equally among students of Section A or Section B?

a) 144 b) 128 c) 288 d) 272

(ii) If the product of two positive integers is equal to the product of their HCF and LCM is true then, the HCF (32 , 36) is

a) 2 b) 4 c) 6 d) 8

(iii) 36 can be expressed as a product of its primes as

$a)\ 2^2 \times 3^2$ $b)\ 2^1 \times 3^3$ $c)\ 2^3 \times 3^1$ $d)\ 2^0 \times 3^0$

(iv) $7 \times 11 \times 13 \times 15 + 15$ is a

a) Prime number b) Composite number c) Neither prime nor composite d) None of the above

(v) If p and q are positive integers such that p = ab^2 and q= a^2b, where a , b are prime numbers, then the LCM (p, q) is

Sol: $32 = 2 \times 2 \times 2 \times 2 \times 2 = 2^5$

$36 = 2 \times 2 \times 3 \times 3 = 2^2 \times 3^2$

(i) The minimum number of books you will acquire for the class library=LCM (32,36)

$= 2^5 \times 3^2 = 32 \times 9 = 288$

$(ii) HCF(32,36) = \frac{32 \times 36}{LCM(32,36)} = \frac{32 \times 36}{32 \times 9} = 4$

$(iii) 36 = 2^2 \times 3^2$

$(iv)\ 7 \times 11 \times 13 \times 15 + 15 = 15(7 \times 11 \times 13 + 1) = 15 \times 1002$

15,1002 are factors of $7 \times 11 \times 13 \times 15 + 15$. So, it is a composite number

$(v)\ LCM(p,q) = a^2 \times b^2$

2. A seminar is being conducted by an Educational Organisation, where the participants will be educators of different subjects. The number of participants in Hindi, English and Mathematics are 60, 84 and 108 respectively.

(i) In each room the same number of participants are to be seated and all of them being in the same subject, hence maximum number participants that can accommodated in each room are

a) 14 b) 12 c) 16 d) 18

(ii) What is the minimum number of rooms required during the event?

a) 11 b) 31 c) 41 d) 21

(iii) The LCM of 60, 84 and 108 is

a) 3780 b) 3680 c) 4780 d) 4680

(iv) The product of HCF and LCM of 60,84 and 108 is

a) 55360 b) 35360 c) 45500 d) 45360

(v) 108 can be expressed as a product of its primes as

$a)\ 2^3 \times 3^2$ $b) 2^3 \times 3^3$ $c)\ 2^2 \times 3^2$ $d) 2^2 \times 3^3$

Sol: $60 = 2^2 \times 3 \times 5$

$84 = 2^2 \times 3 \times 7$

$108 = 2^2 \times 3^3$

$HCF = 2^2 \times 3 = 12$

$LCM = 2^2 \times 3^3 \times 5 \times 7 = 3780$

$(i)\ Maximum\ number\ of\ participants\ in\ each\ room = HCF(60,84,108) = 12$

(ii) $Minimum\ number\ of\ rooms = \frac{Total\ participants}{Maximum\ number\ of\ participants\ in\ each\ room} = \frac{252}{12}$

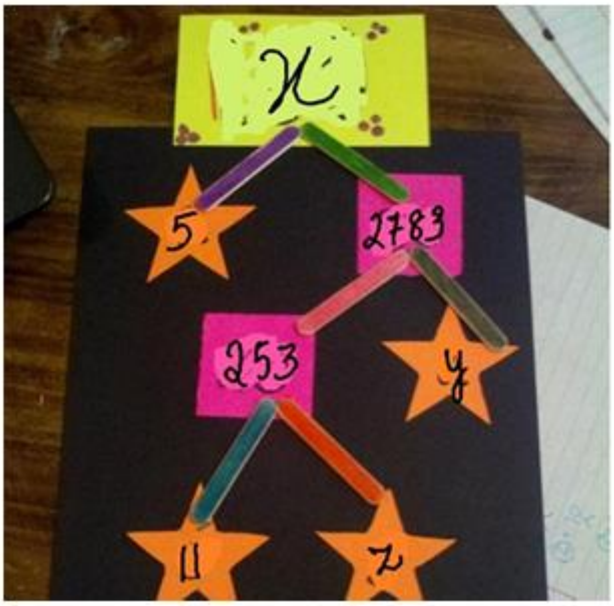
$= 21$

(iii) LCM$(60,84,108) = 3780$

(iv) The product of HCF and LCM of 60,84 and 108 $= 12 \times 3780 = 45360$

(v) $108 = 2^2 \times 3^3$

3. A Mathematics Exhibition is being conducted in your School and one of your friends is making a model of a factor tree. He has some difficulty and asks for your help in completing a quiz for the audience. Observe the following factor tree and answer the following:



(i) What will be the value of x?

a) 15005 b) 13915 c) 56920 d) 17429

(ii) What will be the value of y?

a) 23 b) 22 c) 11 d) 19

(iii) What will be the value of z?

a) 22 b) 23 c) 17 d) 19

(iv) According to Fundamental Theorem of Arithmetic 13915 is a

a) Composite number b) Prime number c) Neither prime nor composite d) Even number

(v) The prime factorisation of 13915 is

$(a)\ 5 \times 11^3 \times 13^2$ $(b) 5 \times 11^3 \times 33^2$ $(c)\ 5 \times 11^2 \times 23$ $(d)\ 5 \times 11^2 \times 13^2$

Sol: (i) $x = 5 \times 2783 = 13915$

(ii) $y = \frac{2783}{253} = 11$

(iii) $z = \frac{253}{11} = 23$

(iv) $a)$ $composite\ number$

$(v)\ 13915 = 5 \times 11 \times 11 \times 23 = 5 \times 11^2 \times 23$

Previous years questions:

4. **Khushi, being health-conscious, has decided to serve only fruits at her birthday party being health conscious. She decided to serve only fruits in her birthday party. She bought 36 apples and 60 bananas and decided to distribute the fruits equally among all her guests.**
Based on the above information, answer the following questions:

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(i) How many guests Khushi can invite at the most?

$Sol: 36 = 2^2 \times 3^2\ and\ 60 = 2^2 \times 3 \times 5$

(To determine the maximum number of guests Khushi can invite, we need to find the **HCF)**

$HCF(36,60) = 2^2 \times 3 = 4 \times 3 = 12$

Khushi can invite **up to 12 guests**

(ii) How many apples and bananas will each guest get?

$Sol: 36 \div 12 = 3\ and\ 60 \div 12 = 5$

Each guest will receive 3 apples and 5 bananas.

(iii)
(A) If Khushi decides to add 42 mangoes how many guests Khushi can invite at the most?

$Sol: 42 = 2 \times 3 \times 7$

$HCF(36,60\ and\ 42) = 2 \times 3 = 6$

Khushi can invite up to 6 guests

(B) If the cost of 1 dozen of babnanas is ₹60, the cost of 1 apple is ₹15 and cost of 1 mango ₹20, find the total amount spent on 60 bananas, 36 apples and 42 mangoes.

$Sol: Total\ ammount\ spent = 5 \times ₹60 + 36 \times ₹15 + 42 \times ₹20 = ₹(300 + 540 + 840) = ₹1680$

5. **A sweet shopkeeper prepares 396 gulab jamuns and 342 ras-gullas. He packs them into containers. Each container consists of either gulab jamun or ras-gullas but have equal number of pieces. Find the number of pieces he should put in each box so that number of boxes are least.**

Sol: $Number\ of\ gulabjamuns = 396 = 2^2 \times 3^2 \times 11$

$Number\ of\ rasgullas = 342 = 2 \times 3^2 \times 19$

$HCF(396,342) = 2 \times 3^2 = 18$

The shopkeeper should put 18 sweets in each container to minimize the number of boxes required

6. **An army contingent of 678 soldiers is to march behind an army band of 36 members in a Republic Day parade. The two groups are to march in the same number of columns. What is the maximum number of columns they can march?**

Sol: Number of soldiers in an army contingent $= 678 = 2 \times 3 \times 113$

Number of members in an army band $= 36 = 2^2 \times 3^2$

$\text{HCF}(678,36) = 2 \times 3 = 6$

So, the maximum number of columns they can march is 6.

7. **Find the least number which when divided by 12,16 and 24 leaves remainder 7 in each case.**

Sol: $12 = 2^2 \times 3;\ \ 16 = 2^4;\ \ 24 = 2^3 \times 3$

$\text{LCM}(12,16,24) = 2^4 \times 3 = 16 \times 3 = 48$

Required least number $= 48 + 7 = 55$

8. **Two numbers are in the ratio 2:3 and their LCM is 180. What is the HCF of these numbers?**

Sol: Ratio of two numbers $= 2:3$

Let the two numbers be $2x$ and $3x$

$LCM(2x, 3x) = 2 \times 3 \times x = 6x = 180$

$$x = \frac{180}{6} = 30$$

$HCF(2x, 3x) = x = 30$

9. **Find the largest possible positive integer that divides 125, 162 and 259 leaving remainder 5, 6 and 7 respectively.**

Sol: $125 - 5 = 120, 162 - 6 = 156, 259 - 7 = 252$

$120 = 2^3 \times 3 \times 5;\ \ 156 = 2^2 \times 3 \times 13;\ \ 252 = 2^2 \times 3^2 \times 7$

$HCF(120,156,252) = 2^2 \times 3 = 12$

Hence, the required largest number is 12

10. Find the largest number which on dividing 1251,9377 and 15628 leaving remainder 1, 2 and 3 respectively

$$\begin{array}{r|l} 2 & 1250 \\ \hline 5 & 625 \\ \hline 5 & 125 \\ \hline 5 & 25 \\ \hline & 5 \end{array} \qquad \begin{array}{r|l} 3 & 9375 \\ \hline 5 & 3125 \\ \hline 5 & 625 \\ \hline 5 & 125 \\ \hline 5 & 25 \\ \hline & 5 \end{array} \qquad \begin{array}{r|l} 5 & 15625 \\ \hline 5 & 3125 \\ \hline 5 & 625 \\ \hline 5 & 125 \\ \hline 5 & 25 \\ \hline & 5 \end{array}$$

Sol: $1251 - 1 = 1250, 9377 - 2 = 9375, 15628 - 3 = 15625$

$1250 = 2 \times 5^4$

$9375 = 3 \times 5^5$

$15625 = 5^6$

Required largest number=HCF(1250,9375,15625)=$5^4 = 625$

11. How many irrational numbers lie between $\sqrt{2}$ *and* $\sqrt{3}$? Write any two of them

Sol: There are infinite irrational numbers between $\sqrt{2}$ *and* $\sqrt{3}$

Example: $\sqrt{2.4}, \sqrt{2.5}, \sqrt{2.8}$

12. Three alarams clocks ring their alarams at regular intervals of 20min,25 min and 30 min respectively. If they first beep together at 12 noon, at what time will they beep again for the first time?

Sol: $20 = 2^2 \times 5; 25 = 5^2; 30 = 2 \times 3 \times 5$

$\text{LCM}(20,25,30) = 2^2 \times 3 \times 5^2 = 300\ min = 5\ hours$

Therefore, the three alarm clocks will ring together again for the first time at 5:00 pm

13. The traffic lights at three different road crossings change after every 48 seconds 72 seconds and 108 seconds respectively if they change simultaneously at 7:00 a.m. at what time in the change together next?[CBSE-2023]

Sol:

$$\begin{array}{r|l} 2 & 48 \\ \hline 2 & 24 \\ \hline 2 & 12 \\ \hline 2 & 6 \\ \hline 3 & 3 \\ \hline & 1 \end{array} \qquad \begin{array}{r|l} 2 & 72 \\ \hline 2 & 36 \\ \hline 2 & 18 \\ \hline 3 & 9 \\ \hline 3 & 3 \\ \hline & 1 \end{array} \qquad \begin{array}{r|l} 2 & 108 \\ \hline 2 & 54 \\ \hline 3 & 27 \\ \hline 3 & 9 \\ \hline 3 & 3 \\ \hline & 1 \end{array}$$

$48 = 2^4 \times 3$

$72 = 2^3 \times 3^2$

$108 = 2^2 \times 3^3$

$LCM\ of\ 48,72\ and\ 108 = 2^4 \times 3^3 = 16 \times 27 = 432$

After 432 seconds=7 min 12 sec , the lights will change simultaneously.

Hence the lights will change next simultaneously at 7:00 +7 min 12 sec=7:07:12 am

14.
15.
16.
17.