CHAPTER 6

VIII CLASS-NCERT (2024-25)

Cubes and Cube Roots **(Notes)**

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1. Number obtained when a number is multiplied by itself three times are called cube number
2. A natural number is called a perfect cube if it is the cube of some natural number
3. If $m = n^3$, then m is a perfect cube where m and n are natural numbers.
4. $a^3 = a \times a \times a$

   $x^3 = x \times x \times x$

$1^3 = 1$	$8^3 = 512$	$15^3 = 3375$
$2^3 = 8$	$9^3 = 729$	$16^3 = 4096$
$3^3 = 27$	$10^3 = 1000$	$17^3 = 4913$
$4^3 = 64$	$11^3 = 1331$	$18^3 = 5832$
$5^3 = 125$	$12^3 = 1728$	$19^3 = 6859$
$6^3 = 216$	$13^3 = 2197$	$20^3 = 8000$
$7^3 = 343$	$14^3 = 2744$	

5. **How many perfect cubes are there from 1 to 100?**

Sol: Four (1,8,27,64)

6. **How many perfect cubes are there from 1 to 1000?**

Sol: Ten(1,8,27,64,125,216,343,512,729,1000)

The one's digit of the number	The one's digit of cube of the number	The one's digit of the number	The one's digit of cube of the number
1	1	6	6
2	8	7	3
3	7	8	2
4	4	9	9
5	5	0	0

Hardy – Ramanujan Number: 1729

1729 is the smallest number that can be expressed as a sum of two cubes in two different ways:

$1729 = 1728 + 1 = 12^3 + 1^3$

$1729 = 1000 + 729 = 10^3 + 9^3$

Some this type of numbers:

1) $4104 = 8 + 4096 = 2^3 + 16^3$; $4104 = 729 + 3375 = 9^3 + 15^3$

2). $13832 = 8 + 13824 = 2^3 + 24^3$; $13832 = 5832 + 8000 = 18^3 + 20^3$

TRY THESE

Find the one's digit of the cube of each of the following numbers.

Number	the one's digit of the cube of the number	Number	the one's digit of the cube of the number
(i) 3331	1	(v) 1024	4
(ii) 8888	2	(vi) 77	3
(iii) 149	9	(vii) 5022	8
(iv) 1005	5	(viii) 53	7

Some interesting patterns

Adding consecutive odd numbers

$1 = 1 = 1^3$
$3 + 5 = 8 = 2^3$
$7 + 9 + 11 = 27 = 3^3$
$13 + 15 + 17 + 19 = 64 = 4^3$
$21 + 23 + 25 + 27 + 29 = 125 = 5^3$

TRY THESE

Express the following numbers as the sum of odd numbers using the above pattern?

$(a)\ 6^3 = 216 = 31 + 33 + 35 + 37 + 39 + 41$

$(b)\ 8^3 = 512 = 57 + 59 + 61 + 63 + 65 + 67 + 69 + 71$

$(c)\ 7^3 = 343 = 43 + 45 + 47 + 49 + 51 + 53 + 55$

Consider the following pattern.

$2^3 - 1^3 = 1 + 2 \times 1 \times 3$

$3^3 - 2^3 = 1 + 3 \times 2 \times 3$

$4^3 - 3^3 = 1 + 4 \times 3 \times 3$

Using the above pattern, find the value of the following.

$(i)\ 7^3 - 6^3 = 1 + 7 \times 6 \times 3$

$(ii)\ 12^3 - 11^3 = 1 + 12 \times 11 \times 3$

$(iii)\ 20^3 - 19^3 = 1 + 20 \times 19 \times 3$

$(vi)\ 51^3 - 50^3 = 1 + 51 \times 50 \times 3$

$$n^3 - (n-1)^3 = 1 + n \times (n-1) \times 3$$

Cubes and their prime factors:

If a number can be expressed as a product of three equal factors then it is said to be a perfect cube or cubic number.

Example 1: Is 243 a perfect cube?

Sol: $243 = (3 \times 3 \times 3) \times 3 \times 3$

After grouping 3×3 remains.

3	243
3	81
3	9
3	27
	3

Therefore, 243 is not a perfect cube.

TRY THESE

Which of the following are perfect cubes?

1. 400

Sol: $400 = \underline{2 \times 2 \times 2} \times 2 \times 5 \times 5$

Here, 2 and 5 do not appear in groups of three.

Hence 400 is not a perfect cube.

2	400
2	200
2	100
2	50
5	25
	5

2. 3375

Sol: $3375 = \underline{3 \times 3 \times 3} \times \underline{5 \times 5 \times 5}$

The prime factors can be grouped in triples.

Hence 3375 is a perfect cube.

$3375 = (3 \times 5)^3 = 15^3$

3	3375
3	1125
3	375
5	125
5	25
	5

3. 8000

Sol: $8000 = (2 \times 2 \times 2) \times (2 \times 2 \times 2) \times (5 \times 5 \times 5)$

The prime factors can be grouped in triples

Hence 8000 is a perfect cube.

$8000 = (2 \times 2 \times 5)^3 = 20^3$

2	8000
2	4000
2	2000
2	1000
2	500
2	250
5	125
5	25
	5

4. 15625

Sol: $15625 = (5 \times 5 \times 5) \times 5 \times 5$

Here, 5 do not appear in groups of three.

Hence 15625 is not a perfect cube.

5	15625
5	3125
5	625
5	25
	5

5. 9000

Sol: $9000 = (2 \times 2 \times 2) \times 3 \times (5 \times 5 \times 5) \times 5$

Here, after grouping 3 and 5 do not appear in groups of three.

Hence 9000 is not a perfect cube.

2	9000
2	4500
2	2250
3	1125
5	375
5	125
5	25
	5

6. 6859

Sol: $6859 = 19 \times 19 \times 19$

The prime factors can be grouped in triples

Hence 6859 is a perfect cube.

$6859 = (19)^3$

19	6859
19	361
	19

7. 2025

Sol: $2025 = (3 \times 3 \times 3) \times 3 \times 5 \times 5$

Here, after grouping 3 and 5 do not appear in groups of three

Hence 2025 is not a perfect cube.

5	2025
5	405
3	81
3	27
3	9
	3

8. 10648

2	10648
2	5324
2	2662
11	1331
11	121
	11

Sol: $10648 = (2 \times 2 \times 2) \times (11 \times 11 \times 11)$

The prime factors can be grouped in triples

Hence 10648 is a perfect cube.

$10648 = (2 \times 11)^3 = 22^3$

Example 2: **Is 392 a perfect cube? If not, find the smallest natural number by which 392 must be multiplied so that the product is a perfect cube**

2	392
2	196
7	98
7	49
	7

Sol: $392 = (2 \times 2 \times 2) \times 7 \times 7$

The prime factor 7 does not appear in a group of three.

Hence, the smallest number by which 392 should be multiplied to make it a perfect cube is 7.

Example 3: **Is 53240 a perfect cube? If not, then by which smallest natural number should 53240 be divided so that the quotient is a perfect cube?**

2	53240
2	26620
2	1331
5	2662
11	1331
11	121
	11

Sol: $53240 = (2 \times 2 \times 2) \times (11 \times 11 \times 11) \times 5$

After grouping 5 remains

Hence the smallest number by which 53240 should be divided to make it a perfect cube is 5

$53240 \div 5 = 2 \times 2 \times 2 \times 11 \times 11 \times 11$

$10648 = 22^3$

Example 4: **Is 1188 a perfect cube? If not, by which smallest natural number should 1188 be divided so that the quotient is a perfect cube?**

2	1188
2	594
3	297
3	99
3	33
	11

Sol: $1188 = 2 \times 2 \times (3 \times 3 \times 3) \times 11$

After grouping remaining $2 \times 2 \times 11 = 44$

The smallest natural number should 1188 be divided by 44 to make it a perfect cube.

$1188 \div 44 = (3 \times 3 \times 3)$

$27 = 3^3$

Example 5: Is 68600 a perfect cube? If not, find the smallest number by which 68600 must be multiplied to get a perfect cube.

2	68600
2	34300
2	17150
5	8575
5	1715
7	343
7	49
	7

Sol: $68600 = (2 \times 2 \times 2) \times 5 \times 5 \times (7 \times 7 \times 7)$.

After grouping remaining 5×5

Required smallest number=5

$68600 \times 5 = (2 \times 2 \times 2) \times (5 \times 5 \times 5) \times (7 \times 7 \times 7)$.

$343000 = (2 \times 5 \times 7)^3 = 70^3$

THINK, DISCUSS AND WRITE

Check which of the following are perfect cubes.

(i) **2700** –Not a perfect cube

(ii) **16000** –Not a perfect cube

(iii) **64000** - perfect cube

(iv) **900** –Not a perfect cube

(v) **125000**- perfect cube

(vi) **36000**–Not a perfect cube

(vii) **21600** –Not a perfect cube

(viii) **10,000** –Not a perfect cube

(ix) **27000000** - perfect cube

(x) **1000** - Perfect cube

What pattern do you observe in these perfect cubes?

Sol: We observe that the numbers above which are perfect cubes have the number of zeros in multiple of 3

EXERCISE 6.1

1. **Which of the following numbers are not perfect cubes?**

(i) 216

Sol: $216 = (2 \times 2 \times 2) \times (3 \times 3 \times 3)$

The prime factors can be grouped in triples

Hence 216 is a perfect cube.

$216 = (2 \times 3)^3 = 6^3$

2	216
2	108
2	54
3	27
3	9
	3

(ii) 128

Sol: $128 = (2 \times 2 \times 2) \times (2 \times 2 \times 2) \times 2$

2 is not in triple

128 is not a perfect cube

2	128
2	64
2	32
2	16
2	8
2	4
	2

2	1000
2	500
2	250
5	125
5	25
	5

2	100
2	50
5	25
	5

(iii) 1000

Sol: $1000 = (2 \times 2 \times 2) \times (5 \times 5 \times 5)$

The prime factors can be grouped in triples

Hence 1000 is a perfect cube.

$1000 = (2 \times 5)^3 = 10^3$

(iv) 100

Sol: $100 = 2 \times 2 \times 5 \times 5$

2,5 are not in triplets

100 is not a perfect cube

(v) 46656

Sol: $46656 = (2 \times 2 \times 2) \times (2 \times 2 \times 2) \times (3 \times 3 \times 3) \times (3 \times 3 \times 3)$

The prime factors can be grouped in triples

Hence 1000 is a perfect cube.

2	46656
2	23328
2	11664
2	5832
2	2916
2	1458
3	729
3	243
3	81
3	27
3	9
	3

$46656 = (2 \times 2 \times 3 \times 3)^3 = 36^3$

2. Find the smallest number by which each of the following numbers must be multiplied to obtain a perfect cube.

(i) 243

Sol: $243 = (3 \times 3 \times 3) \times 3 \times 3$

Here, after grouping remaining is 3×3

We multiply 243 by 3 to make it a perfect cube.

3	243
3	81
3	9
3	27
	3

(ii) 256

Sol: $256 = (2 \times 2 \times 2) \times (2 \times 2 \times 2) \times 2 \times 2$

Here, after grouping remaining is 2×2

We multiply 256 by 2 to make it a perfect cube

2	256
2	128
2	64
2	32
2	16
2	8
2	4
	2

(iii) 72

Sol: $72 = (2 \times 2 \times 2) \times 3 \times 3$

Here, after grouping remaining is 3×3

We multiply 72 by 3 to make it a perfect cube

2	72
2	36
2	18
3	9
	3

(iv) 675

Sol: $675 = (3 \times 3 \times 3) \times 5 \times 5$

Here, after grouping remaining is 5×5

We multiply 675 by 5 to make it a perfect cube.

3	675
3	225
3	75
5	25
	5

(v) 100

Sol: $100 = 2 \times 2 \times 5 \times 5$

We multiply 100 by $2 \times 5 = 10$ to make it a perfect cube.

3. **Find the smallest number by which each of the following numbers must be divided to obtain a perfect cube.**

(i) 81

Sol: $81 = (3 \times 3 \times 3) \times 3$

Here, after grouping remaining is 3

We divided 81 by 3 to make it a perfect cube .

3	81
3	27
3	9
	3

(ii) 128

Sol: $128 = (2 \times 2 \times 2) \times (2 \times 2 \times 2) \times 2$

Here, after grouping remaining is 2 .

We divided 128 by 2 to make it a perfect cube.

2	128
2	64
2	32
2	16
2	8
2	4
	2

(iii) 135

Sol: $135 = (3 \times 3 \times 3) \times 5$

Here, after grouping remaining is 5 .

We divided 135 by 5 to make it a perfect cube.

3	135
3	45
3	15
	5

2	192
2	96
2	48
2	24
2	12
2	6

(iv) 192

Sol: $192 = (2 \times 2 \times 2) \times (2 \times 2 \times 2) \times 3$

Here, after grouping remaining is 3 .

We divided 192 by 3 to make it a perfect cube.

(v) 704

2	704
2	352
2	176
2	88
2	44
2	22
	11

Sol: $704 = (2 \times 2 \times 2) \times (2 \times 2 \times 2) \times 11$

Here, after grouping remaining is 11 .

We divided 704 by 11 to make it a perfect cube.

4. **Parikshit makes a cuboid of plasticine of sides 5 cm, 2 cm, 5 cm.**

How many such cuboids will he need to form a cube?

Sol: Dimensions of cube=5×2×5

2,5 are not in triples . So, the number must be multiply by 2×2×5 make it perfect cube.

Hence we will need 2×2×5=20 cuboids.

Cube Roots

The cube root is the inverse operation of finding cube

The symbol $\sqrt[3]{\ }$ denotes 'cube-root.'

Cubes	Cube roots	Cubes	Cube roots
$1^3 = 1$	$\sqrt[3]{1} = 1$	$11^3 = 1331$	$\sqrt[3]{1331} = 11$
$2^3 = 8$	$\sqrt[3]{8} = 2$	$12^3 = 1728$	$\sqrt[3]{1728} = 12$
$3^3 = 27$	$\sqrt[3]{27} = 3$	$13^3 = 2197$	$\sqrt[3]{2197} = 13$
$4^3 = 64$	$\sqrt[3]{64} = 4$	$14^3 = 2744$	$\sqrt[3]{2744} = 14$
$5^3 = 125$	$\sqrt[3]{125} = 5$	$15^3 = 3375$	$\sqrt[3]{3375} = 15$
$6^3 = 216$	$\sqrt[3]{216} = 6$	$16^3 = 4096$	$\sqrt[3]{4096} = 16$
$7^3 = 343$	$\sqrt[3]{343} = 7$	$17^3 = 4913$	$\sqrt[3]{4913} = 17$
$8^3 = 512$	$\sqrt[3]{512} = 8$	$18^3 = 5832$	$\sqrt[3]{5832} = 18$
$9^3 = 729$	$\sqrt[3]{729} = 9$	$19^3 = 6859$	$\sqrt[3]{6859} = 19$
$10^3 = 1000$	$\sqrt[3]{1000} = 10$	$20^3 = 8000$	$\sqrt[3]{8000} = 20$

Example 6: **Find the cube root of 8000.**

2	8000
2	4000
2	2000
2	1000
2	500
2	250
5	125
5	25
	5

Sol: $8000 = (2 \times 2 \times 2) \times (2 \times 2 \times 2) \times (5 \times 5 \times 5)$

$\sqrt[3]{8000} = 2 \times 2 \times 5 = 20$

(or)

$8000 = 8 \times 1000 = 2^3 \times 10^3$

$\sqrt[3]{8000} = 2 \times 10 = 20$

Example 7: **Find the cube root of 13824 by prime factorisation method.**

Sol: $13824 = (\underline{2 \times 2 \times 2}) \times (\underline{2 \times 2 \times 2}) \times (\underline{2 \times 2 \times 2}) \times (\underline{3 \times 3 \times 3})$.

$\sqrt[3]{13824} = 2 \times 2 \times 2 \times 3 = 24$

2	13824
2	6912
2	3456
2	1728
2	864
2	432
2	216
2	108
2	54
3	27
3	9
	3

EXERCISE 7.2

1. **Find the cube root of each of the following numbers by prime factorisation method.**

(i) 64

Sol: $64 = (2 \times 2 \times 2) \times (2 \times 2 \times 2)$

$\sqrt[3]{64} = 2 \times 2 = 4$

(ii) 512

Sol: $512 = (2 \times 2 \times 2) \times (2 \times 2 \times 2) \times (2 \times 2 \times 2)$

$\sqrt[3]{512} = 2 \times 2 \times 2 = 8$

(iii) 10648

Sol: $10648 = (2 \times 2 \times 2) \times (11 \times 11 \times 11)$

$\sqrt[3]{10648} = 2 \times 11 = 22$

(iv) 27000

Sol: $27000 = (2 \times 2 \times 2) \times (3 \times 3 \times 3) \times (5 \times 5 \times 5)$

$\sqrt[3]{27000} = 2 \times 3 \times 5 = 30$

(v) 15625

Sol: $15625 = (5 \times 5 \times 5) \times (5 \times 5 \times 5)$

$\sqrt[3]{15625} = 5 \times 5 = 25$

(vi) 13824

Sol: $13824 = (2 \times 2 \times 2) \times (2 \times 2 \times 2) \times (2 \times 2 \times 2) \times (3 \times 3 \times 3)$

$\sqrt[3]{13824} = 2 \times 2 \times 2 \times 3 = 24$

(vii) 110592

Sol: $110592 = (2 \times 2 \times 2) \times (2 \times 2 \times 2) \times (2 \times 2 \times 2) \times (2 \times 2 \times 2) \times (3 \times 3 \times 3)$

(viii) 46656

Sol: $46656 = (2 \times 2 \times 2) \times (2 \times 2 \times 2) \times (3 \times 3 \times 3) \times (3 \times 3 \times 3)$

$\sqrt[3]{46656} = 2 \times 2 \times 3 \times 3 = 36$

(ix) 175616

2	10648
2	5324
2	2662
11	1331
11	121
	11

2	512
2	256
2	128
2	64
2	32
2	16
2	8
2	4
	2

2	64
2	32
2	16
2	8
2	4
	2

5	15625
5	3105
5	625
5	125
5	25
	5

2	27000
2	13500
2	6750
3	3375
3	1125
3	375
5	125
5	25
	5

2	13824
2	6912
2	3456
2	1728
2	864
2	432
2	216
2	108
2	54
3	27
3	9
	3

Sol: $175616 = (2 \times 2 \times 2) \times (2 \times 2 \times 2) \times (2 \times 2 \times 2) \times (7 \times 7 \times 7)$

$\sqrt[3]{175616} = 2 \times 2 \times 2 \times 7 = 56$

(x) 91125

Sol: $91125 = 5^3 \times 3^3 \times 3^3$

$\sqrt[3]{91125} = 5 \times 3 \times 3 = 45$

5	91125
5	18225
5	3645
3	729
3	243
3	81
3	9
3	27
	3

2	175616
2	87808
2	43904
2	21952
2	10976
2	5488
2	2744
2	1372
2	686
7	343
7	49
	7

2	110592
2	55296
2	27648
2	13824
2	6912
2	3456
2	1728
2	864
2	432
2	216
2	108
2	54
3	27
3	9
	3

2	46656
2	23328
2	11664
2	5832
2	2916
2	1458
3	729
3	243
3	81
3	27
3	9
	3

2	13824
2	6912
2	3456
2	1728
2	864
2	432
2	216
2	108
2	54
3	27
3	9
	3

5	91125
5	18225
5	3645
3	729
3	243
3	81
3	9
3	27
	3

2. **State true or false**

(i) Cube of any odd number is even. → False

(ii) A perfect cube does not end with two zeros. → False

(iii) If square of a number ends with 5, then its cube ends with 25. → True

(iv) There is no perfect cube which ends with 8. → False

(v) The cube of a two digit number may be a three digit number. → False

(vi) The cube of a two digit number may have seven or more digits. → False

(vii) The cube of a single digit number may be a single digit number→ False

Cross Number Puzzle

Down

1. Cube of 9.

2. Missing number to make 12, ___, 37, a pythagorean triplet.

4. Smallest number by which 248 be multiplied to make the resultant a perfect cube number.

5. Square of 75.

6. Smallest square number that is divisible by each of 5 and 11

9. Without adding, find the sum of $1 + 3 + 5 + 7 + 9 + 11$.

10. Smallest number which when added to 7669 makes the resultant a perfect square.

Across

2. Square of 19.

3. Look at the numbers given below and find the number which cannot be a perfect square. 81, 100, 144, 25000

7. Square root of 4489

8. Smallest natural number other than 1 which is a perfect square as well as a perfect cube number.

10. Cube root of 357911.

11. Smallest number which when subtracted from 374695 makes the resultant a perfect square number.

1	2			6
3	5			
4	7	10		
			9	
11			8