CHAPTER 5

AP VIII CLASS-CBSE (2024-25)

SQUARES AND SQUARE ROOTS (Notes)

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1. If a natural number m can be expressed as n ^{2} , where n is also a natural number, then m is a square number.
2. $a^2 = a \times a$

Number(n)	Square(n^2)	Number(n)	Square(n^2)	Number(n)	Square(n^2)
1	1	11	121	21	441
2	4	12	144	22	484
3	9	13	169	23	529
4	16	14	196	24	576
5	25	15	225	25	625
6	36	16	256	26	676
7	49	17	279	27	729
8	64	18	324	28	784
9	81	19	361	29	841
10	100	20	400	30	900

3. The numbers 1, 4, 9, 16,.....are square numbers. These numbers are also called **Perfect squares**
4. All square numbers end with 0, 1, 4, 5, 6 or 9 at units place.
5. Square numbers can only have even number of zeros at the end.
6. The square number does not end with 2, 3, 7 or 8 at unit's place.

TRY THESE

1. **Find the perfect square numbers between (i) 30 and 40 (ii) 50 and 60**

Sol: (i) 36 (ii) There is no perfect square between 50 and 60

TRY THESE

1. **Can we say whether the following numbers are perfect squares? How do we know?**

   The square number does not end with 2,3,7 or 8 at unit's place

(i) 1057

Sol: Unit digit is 7 . So, it is not a perfect square

(ii) 23453

Sol: Unit digit is 3 . So, it is not a perfect square

(iii) 7928

Sol: Unit digit is 8 . So, it is not a perfect square

(iv) 222222

Sol: Unit digit is 2 . So, it is not a perfect square

(v) 1069

Sol: Unit digit is 9 . We don't say 1069 is a perfect square are not

(vi) 2061

Sol: Unit digit is 1 . We don't say 2061 is a perfect square are not

TRY THESE

Which of 123^2 , 77^2 , 82^2 , 161^2 , 109^2 would end with digit 1?

Sol: 123^2 is end with digit 9

77^2 is end with digit 9

82^2 is end with digit 4

161^2 is end with digit 1

109^2 is end with digit 1

So, 161^2 , 109^2 would end with digit 1.

TRY THESE

Which of the following numbers would have digit 6 at unit place.

(i) 19^2 (ii) 24^2 (iii) 26^2 (iv) 36^2 (v) 34^2

Sol: (ii) 24^2 (iii) 26^2 (iv) 36^2 (v) 34^2

TRY THESE

What will be the "one's digit" in the square of the following numbers?

(i) The unit digit in the square of 1234 is 6

(ii) The unit digit in the square of 26387 is 9

(iii) The unit digit in the square of 52698 is 4

(iv) The unit digit in the square of 99880 is 0

(v) The unit digit in the square of 21222 is 4

(vi) The unit digit in the square of 9106 is 6

TRY THESE

1. **The square of which of the following numbers would be an odd number/an even number? Why?**

   The square of an even number is an even number and the square of an odd number is an odd

   (i) 727^2 is an odd number

   (ii) 158^2 is an even number

   (iii) 269^2 is an odd number

   (iv) 1980^2 is an even number

Numbers between square numbers

i) Between n^2 and $(n + 1)^2$ there are $2n$ numbers which is 1 less than the difference of two squares.

ii) There are $2n$ non perfect square numbers between the squares of the numbers n and $(n + 1)$

TRY THESE

1. How many natural numbers lie between 9^2 and 10^2 ? Between 11^2 and 12^2 ?

Sol: The number of natural numbers between 9^2 and 10^2 is $2 \times 9 = 18$

The number of natural numbers between 11^2 and 12^2 is $2 \times 11 = 22$

2. How many non-square numbers lie between the following pairs of numbers

(i) Number of non square numbers lie between 100^2 and 101^2 is $2 \times 100 = 200$

(ii) Number of non square numbers lie between 90^2 and 91^2 is $2 \times 90 = 180$

(iii) Number of non square numbers lie between 1000^2 and 1001^2 is $2 \times 1000 = 2000$

TRY THESE

Find whether each of the following numbers is a perfect square or not?

(i) 121 is a perfect square $(121 = 11^2)$

(ii) 55 is not a perfect squarer

(iii) 81 is a perfect square$(81 = 9^2)$

(iv) 49 is a perfect square$(49 = 7^2)$

(v) 69 is not a perfect squarer

A sum of consecutive natural numbers

1) we can express the square of any odd number as the sum of two consecutive positive integers.

2) n is an odd number . $n^2 = \frac{n^2-1}{2} + \frac{n^2+1}{2}$

Ex: $9^2 = 81 = 40 + 41$

$11^2 = 121 = 60 + 61$

$15^2 = 225 = 112 + 113$

TRY THESE

1. Express the following as the sum of two consecutive integers.

(i) $\mathbf{21^2} = 441 = 220 + 221$

(ii) $\mathbf{13^2} = 169 = 84 + 85$

(iii) $\mathbf{11^2} = 121 = 60 + 61$

(iv) $\mathbf{19^2} = 361 = 180 + 181$

2. Do you think the reverse is also true, i.e., is the sum of any two consecutive positive integers is perfect square of a number? Give example to support your answer.

Sol: No, the reverse is not true.

Example: the two consecutive number 10 and 11 gives a sum of 21. But we know that 21 is not a perfect square.

Product of two consecutive even or odd natural numbers:

Some more patterns in square numbers

$$1^2 = 1$$
$$11^2 = 121$$
$$111^2 = 12321$$
$$1111^2 = 1234321$$
$$11111^2 = 123454321$$
$$111111^2 = 12345654321$$
$$111111^2 = 1234567654321$$
$$1111111^2 = 123456787654321$$
$$11111111^2 = 12345678987654321$$

Another interesting pattern.

$$7^6 = 49$$
$$67^2 = 4489$$
$$667^2 = 444889$$
$$6667^2 = 44448889$$
$$66667^2 = 4444488889$$
$$666667^2 = 444444888889$$
$$6666667^2 = 44444448888889$$
$$66666667^2 = 4444444488888889$$

EXERCISE-5.1

1. **What will be the unit digit of the squares of the following numbers?**

Number	Unit digit of the square	Number	Unit digit of the square
(i) 81	1	(vi)26387	9
(ii) 272	4	(vii)52698	4
(iii) 799	1	(viii)99880	0
(iv) 3853	9	(ix) 12796	6
(v)1234	6	(x) 55555	5

2. **The following numbers are obviously not perfect squares. Give reason.**

Perfect squares are does not end with 2,3,7 or 8 at unit's place. Perfect square numbers can only have even number of zeros at the end.

(i) **1057** is end with 7 .

So, 1057 is not a perfect square .

(ii) 23453 is end with 3 .

So, 23453 is not a perfect square

(iii) 7928 is end with 8 .

So, 7928 is not a perfect square

(iv) 222222 is end with 2 .

So, 222222 is not a perfect square

(v) 64000 has odd zeroes at the end .

So, 64000 is not a perfect square

(vi) 89722 is end with 2 .

So, 89722 is not a perfect square

(vii) 222000 has odd zeroes at the end.

So, 222000 is not a perfect square

(viii) 505050 has odd zeroes at the end.

So, 505050 is not a perfect square

3. The squares of which of the following would be odd numbers?

The square of an even number is an even number and the square of an odd number is an odd

(i) The square of 431 is an odd number.

(ii) The square of 2826 is an even number.

(iii) The square of 7779 is an odd number.

(iv) The square of 82004 is an odd number.

4. Observe the following pattern and find the missing digits

11^2	121
101^2	10201
1001^2	1002001
100001^2	10000200001
10000001^2	100000020000001

5. Observe the following pattern and supply the missing numbers.

11^2	121
101^2	10201
10101^2	102030201
1010101^2	1020304030201
101010101^2	10203040504030201

6. Using the given pattern, find the missing numbers.

$1^2 + 2^2 + 2^2 = 3^2$

$n^2 + (n+1)^2 + [n(n+1)]^2 = [n(n+1)+1]^2$

$2^2 + 3^2 + 6^2 = 7^2$

$3^2 + 4^2 + 12^2 = 13^2$

$4^2 + 5^2 + 20^2 = 21^2$

$5^2 + 6^2 + 30^2 = 31^2$

$6^2 + 7^2 + 42^2 = 43^2$

7. Without adding, find the sum.

The sum of first n odd numbers=n^2

(i) $1 + 3 + 5 + 7 + 9 = 5^2 = 25$

(ii) $1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17 + 19 = 10^2 = 100$

(i) $1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17 + 19 + 21 + 23 = 12^2 = 144$

8. (i) Express 49 as the sum of 7 odd numbers.

Sol: 49=1+3+5+7+9+11+13

(ii) Express 121 as the sum of 11 odd numbers.

Sol: 121=1+3+5+7+9+11+13+15+17+19+21

9. How many numbers lie between squares of the following numbers?

There are '2n' numbers lie between n^2 and $(n + 1)^2$.

(i) 12 and 13

Sol: $2 \times 12 = 24$ numbers lie between 12^2 and 13^2

(ii) 25 and 26

Sol: $2 \times 25 = 50$ numbers lie between 25^2 and 26^2

(iii) 99 and 100

Sol: $2 \times 99 = 198$ numbers lie between 99^2 and 100^2

FINDING THE SQUARE OF A NUMBER CONTAINING 5 IN UNIT'S PLACE.

$(a5)^2 = a(a + 1)hundreds + 25$

$15^2 = (1 \times 2)hundreds + 25 = 200 + 25 = 225$

$25^2 = (2 \times 3)hundreds + 25 = 600 + 25 = 625$

$35^2 = (3 \times 4)hundreds + 25 = 1200 + 25 = 1225$

$45^2 = (4 \times 5)hundreds + 25 = 2000 + 25 = 2025$

$55^2 = (5 \times 6)hundreds + 25 = 3000 + 25 = 3025$

$65^2 = (6 \times 7)hundreds + 25 = 4200 + 25 = 4225$

$75^2 = (7 \times 8)hundreds + 25 = 5600 + 25 = 5625$

$85^2 = (8 \times 9)hundreds + 25 = 7200 + 25 = 7225$

$95^2 = (9 \times 10)hundreds + 25 = 9000 + 25 = 9025$

$105^2 = (10 \times 11)hundreds + 25 = 11000 + 25 = 11025$

$205^2 = (20 \times 21)hundreds + 25 = 42000 + 25 = 42025$

Pythagorean triplets

a, b, c *are positive integers . If* $\mathbf{a^2 + b^2 = c^2}$ *then* (a, b, c) *are said tobe* pythagorean triplet

Example: (i) $3^2 + 4^2 = 9 + 16 = 25 = 5^2$

$(3,4,5)$ *is a* pythagorean triplet

(ii) $5^2 + 12^2 = 25 + 144 = 169 = 13^2$

$(5,12,13)$ *is a* pythagorean triplet

Fore any natural number m>1 , we have $(2m)^2 + (m^2 - 1)^2 = (m^2 + 1)^2$.

So, $\mathbf{2m, m^2 - 1}$ **and** $\mathbf{m^2 + 1}$ *form a Pythagorean triplet.*

Example 2: Write a Pythagorean triplet whose smallest member is 8.

Sol: Let 2m=8

$$m = \frac{8}{2} = 4$$

$$m^2 - 1 = 4^2 - 1 = 16 - 1 = 15$$

$$m^2 + 1 = 4^2 + 1 = 16 + 1 = 17$$

The triplet is 8, 15, 17 with 8 as the smallest member.

Example 3: Find a Pythagorean triplet in which one member is 12.

Sol: Let 2m=12

$$m = \frac{12}{2} = 6$$

$$m^2 - 1 = 6^2 - 1 = 36 - 1 = 35$$

$$m^2 + 1 = 6^2 + 1 = 36 + 1 = 37$$

The required triplet is 12, 35, 37.

EXERCISE-5.2

1. **Find the square of the following numbers.**

(i) **32**

Sol: $32^2 = (30 + 2)^2 = (30 + 2)(30 + 2)$

$= 30(30 + 2) + 2(30 + 2)$

$= 30^2 + 30 \times 2 + 2 \times 30 + 2^2$

$= 900 + 60 + 60 + 4$

$= 1024$

(ii) **35**

Sol : $35^2 = (3 \times 4) hundreds + 25 = 1200 + 25 = 1225$

(iii) 86

Sol: $86^2 = (80 + 6)^2 = (80 + 6)(80 + 6)$

$= 80(80 + 6) + 6(80 + 6)$

$= 80^2 + 80 \times 6 + 6 \times 80 + 6^2$

$= 6400 + 480 + 480 + 36$

$= 7396$

Shortcut: $86^2 = (80 + 6)^2$

$= 80^2 + 2 \times 80 \times 6 + 6^2$

$= 6400 + 960 + 36$

$= 7396$

(iv) 93

Sol: $93^2 = (90 + 3)^2 = (90 + 3)(90 + 3)$

$= 90(90 + 3) + 6(90 + 3)$

$= 90^2 + 90 \times 3 + 3 \times 90 + 3^2$

$= 8100 + 270 + 270 + 9$

$= 8649$

(v) 71

Sol: $71^2 = (70 + 1)^2 = (70 + 1)(70 + 1)$

$= 70(70 + 1) + 1(70 + 1)$

$= 70^2 + 70 \times 1 + 1 \times 70 + 1^2$

$= 4900 + 70 + 70 + 1$

$= 5041$

(vi) 46

Sol: $46^2 = (40 + 6)^2 = (40 + 6)(40 + 6)$

$= 40(40 + 6) + 6(40 + 6)$

$= 40^2 + 40 \times 6 + 6 \times 40 + 6^2$

$= 1600 + 240 + 240 + 36$

$= 2116$

Shortcut: $46^2 = (40 + 6)^2$

$= 40^2 + 2 \times 40 \times 6 + 6^2$

$= 1600 + 480 + 36$

$= 2116$

2. **Write a Pythagorean triplet whose one member is.**

(i) 6

Sol: We know that $2m, m^2 - 1$ and $m^2 + 1$ form a Pythagorean triplet

Let $2m = 6 \Rightarrow m = 3$

$m^2 - 1 = 3^2 - 1 = 9 - 1 = 8$

$m^2 + 1 = 3^2 + 1 = 9 + 1 = 10$

The triplet is 6, 8, 10

Let $m^2 - 1 = 6 \Rightarrow m^2 = 7 \Rightarrow$ *The value of m will not be an integer*

Let $m^2 + 1 = 6 \Rightarrow m^2 = 5 \Rightarrow$ *The value of m will not be an integer*

(ii) 14

Sol: We know that $2m, m^2 - 1$ and $m^2 + 1$ form a Pythagorean triplet

Let $2m = 14 \Rightarrow m = 7$

We get $m^2 - 1 = 7^2 - 1 = 49 - 1 = 48$

$m^2 + 1 = 7^2 + 1 = 49 + 1 = 50$

The triplet is 14, 48, 50

Let $m^2 - 1 = 14 \Rightarrow m^2 = 15 \Rightarrow The\ value\ of\ m\ will\ not\ be\ an\ integer$

Let $m^2 + 1 = 14 \Rightarrow m^2 = 13 \Rightarrow The\ value\ of\ m\ will\ not\ be\ an\ integer$

(iii) 16

Sol: We know that $2m, m^2 - 1$ and $m^2 + 1$ form a Pythagorean triplet

Let $2m = 16 \Rightarrow m = 8$

$m^2 - 1 = 8^2 - 1 = 64 - 1 = 63$

$m^2 + 1 = 8^2 + 1 = 64 + 1 = 65$

The triplet is 16, 63, 65

Let $m^2 - 1 = 16 \Rightarrow m^2 = 17 \Rightarrow The\ value\ of\ m\ will\ not\ be\ an\ integer$

Let $m^2 + 1 = 16 \Rightarrow m^2 = 15 \Rightarrow The\ value\ of\ m\ will\ not\ be\ an\ integer$

(vi) 18

Sol: We know that $2m, m^2 - 1$ and $m^2 + 1$ form a Pythagorean triplet

Let $2m = 18 \Rightarrow m = 9$

$m^2 - 1 = 9^2 - 1 = 81 - 1 = 80$

$m^2 + 1 = 9^2 + 1 = 81 + 1 = 82$

The triplet is 18, 80, 82

Let $m^2 - 1 = 14 \Rightarrow m^2 = 15 \Rightarrow The\ value\ of\ m\ will\ not\ be\ an\ integer$

Let $m^2 + 1 = 14 \Rightarrow m^2 = 13 \Rightarrow The\ value\ of\ m\ will\ not\ be\ an\ integer$

Square Roots

If a square number is expressed, as the product of two equal factors, then one the factors is called the square root of that square number.

Since $9^2 = 81\ and\ (-9)^2 = 81$. *We say that the* ***square root of 81 are 9 and − 9***

In this chapter, we shall take up only positive square root of a natural number.

Symbol used for square root is $\sqrt{\ }$.

Statement	Inference	Statement	Inference
$1^2 = 1$	$\sqrt{1} = 1$	$11^2 = 121$	$\sqrt{121} = 11$
$2^2 = 4$	$\sqrt{4} = 2$	$12^2 = 144$	$\sqrt{144} = 12$
$3^2 = 9$	$\sqrt{9} = 3$	$13^2 = 169$	$\sqrt{169} = 13$
$4^2 = 16$	$\sqrt{16} = 4$	$14^2 = 196$	$\sqrt{196} = 14$
$5^2 = 25$	$\sqrt{25} = 5$	$15^2 = 225$	$\sqrt{225} = 15$
$6^2 = 36$	$\sqrt{36} = 6$	$16^2 = 256$	$\sqrt{256} = 16$
$7^2 = 49$	$\sqrt{49} = 7$	$17^2 = 289$	$\sqrt{289} = 7$
$8^2 = 64$	$\sqrt{64} = 8$	$18^2 = 324$	$\sqrt{324} = 18$

$9^2 = 81$	$\sqrt{81} = 9$	$19^2 = 361$	$\sqrt{361} = 19$
$10^2 = 100$	$\sqrt{100} = 10$	$20^2 = 400$	$\sqrt{400} = 20$

Finding square root through repeated subtraction

TRY THESE (Page-100)

By repeated subtraction of odd numbers starting from 1, find whether the following numbers are perfect squares or not? If the number is a perfect square then find its square root.

(i) 121

Sol: Step 1: $121 - 1 = 120$

Step 2: $120 - 3 = 117$

Step 3: $117 - 5 = 112$

Step 4: $112 - 7 = 105$

Step 5: $105 - 9 = 96$

Step 6: $96 - 11 = 85$

Step 7: $85 - 13 = 72$

Step 8: $72 - 15 = 57$

Step 9: $57 - 17 = 40$

Step 10: $40 - 19 = 21$

Step 11: $21 - 21 = 0$

From 121 we have subtracted successive odd numbers starting from 1 and obtained 0 at 11^{th} step. Therefore $\sqrt{121} = 11$

(ii) 55

Sol: Step 1: $55 - 1 = 54$

Step 2: $54 - 3 = 51$

Step 3: $51 - 5 = 46$

Step 4: $46 - 7 = 39$

Step 5: $39 - 9 = 30$

Step 6: $30 - 11 = 19$

Step 7: $19 - 13 = 6$

The result is not zero 55 is not a perfect square.

(iii) 36

Sol: Step 1: $36 - 1 = 35$

Step 2: $35 - 3 = 32$

Step 3: $32 - 5 = 27$

Step 4: $27 - 7 = 20$

Step 5: $20 - 9 = 11$

Step 6: $11 - 11 = 0$

From 36 we have subtracted successive odd numbers starting from 1 and obtained 0 at 6th step. Therefore $\sqrt{36} = 6$

(iv) 49

Sol: Step 1: $49 - 1 = 48$

Step 2: $48 - 3 = 45$

Step 3: $45 - 5 = 40$

Step 4: $40 - 7 = 33$

Step 5: $33 - 9 = 24$

Step 6: $24 - 11 = 13$

Step 7: $13 - 13 = 0$

From 49 we have subtracted successive odd numbers starting from 1 and obtained 0 at 6th step. Therefore $\sqrt{49} = 7$

(v) 90

Sol: Step 1: $90 - 1 = 89$

Step 2: $89 - 3 = 86$

Step 3: $86 - 5 = 81$

Step 4: $81 - 7 = 74$

Step 5: $74 - 9 = 65$

Step 6: $65 - 11 = 54$

Step 7: $54 - 13 = 41$

Step 8: $41 - 15 = 26$

Step 9: $26 - 17 = 9$

The result is not zero 90 is not a perfect square.

Finding square root through prime factorisation

$$Prime\ numbers = \{2,3,5,7,11,13,17,19,23,29, \ldots.\}$$

2	324
2	162
3	81
3	27
3	9
	3

2	6400
2	3200
2	1600
2	800
2	400
2	200
2	100
2	50
5	25
	5

Ex: **Find the square root of 324.**

Sol: $324 = (2 \times 2) \times (3 \times 3) \times (3 \times 3)$

$\sqrt{324} = 2 \times 3 \times 3$

$\sqrt{324} = 18$

Example 4: Find the square root of 6400.

Sol: $6400 = (2 \times 2) \times (2 \times 2) \times (2 \times 2) \times (2 \times 2) \times (5 \times 5)$

$\sqrt{6400} = 2 \times 2 \times 2 \times 2 \times 5$

$\sqrt{6400} = 80$

Example 5: Is 90 a perfect square?

Sol: $90 = 2 \times \underline{3 \times 3} \times 5$

The prime factors 2 and 5 do not occur in pairs.

Therefore, 90 is not a perfect square.

Example 6: **Is 2352 a perfect square? If not, find the smallest multiple of 2352 which is a perfect square. Find the square root of the new number.**

Sol: $2352 = \underline{2 \times 2} \times \underline{2 \times 2} \times 3 \times \underline{7 \times 7}$

The prime factor 3 has no pair. 2352 is not a perfect square.

Required smallest multiple is 3.

So, we multiply 2352 by 3

$2352 \times 3 = \underline{2 \times 2} \times \underline{2 \times 2} \times \underline{3 \times 3} \times \underline{7 \times 7}$

$\sqrt{7056} = 2 \times 2 \times 3 \times 7 = 84$

Example 7: **Find the smallest number by which 9408 must be divided so that the quotient is a perfect square. Find the square root of the quotient**.

Sol: $9408 = \underline{2 \times 2} \times \underline{2 \times 2} \times \underline{2 \times 2} \times 3 \times \underline{7 \times 7}$

The prime factor 3 has no pair.

If we divide 9408 by the factor 3, then

$9408 \div 3 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 7 \times 7$ which is a perfect square.

$3136 = \underline{2 \times 2} \times \underline{2 \times 2} \times \underline{2 \times 2} \times \underline{7 \times 7}$

$\sqrt{3136} = 2 \times 2 \times 2 \times 7 = 56$

Example 8: **Find the smallest square number which is divisible by each of the numbers 6, 9 and 15.**

Sol: The least number divisible by each one of 6, 9 and 15=LCM(6,9,15)

$=2 \times 3 \times 3 \times 5 = 90$ is not a perfect square.

To get perfect square 90 should be multiplied by 2×5, i.e., 10.

Required square number=90×10=900

2	6, 9, 15
3	3, 9, 15
3	1, 3, 5
5	1, 1, 5
	1, 1, 1

EXERCISE 5.3

1. **What could be the possible 'one's' digits of the square root of each of the following numbers?**

Number	The possible 'one's' digits of the square root of the number
(i) 9801	1 or 9 (since $1^2 = 1$ *and* $9^2 = 1$)
(ii) 99856	4 or 6 (since $4^2 = 16$ *and* $6^2 = 36$)
(iii) 998001	1 or 9 (since $1^2 = 1$ *and* $9^2 = 1$)
(iv) 657666025	5 (since $5^2 = 5$)

2. **Without doing any calculation, find the numbers which are surely not perfect squares**.

(**The perfect square number does not end with 2, 3, 7 or 8 at unit's place**)

(i) 153 → Not a perfect square

(ii) 257 → Not a perfect square

(iii) 408 → Not a perfect square

(iv) $441 = 21^2 \rightarrow$ 441 is a perfect square

3. **Find the square roots of 100 and 169 by the method of repeated subtraction**.

(i) Step 1: $100 - 1 = 99$

Step 2: $99 - 3 = 96$

Step 3: $96 - 5 = 91$

Step 4: $91 - 7 = 84$

Step 5: $84 - 9 = 75$

Step 6: $75 - 11 = 64$

Step 7: $64 - 13 = 51$

Step 8: $51 - 15 = 36$

Step 9: $36 - 17 = 19$

Step 10: $19 - 19 = 0$

From 100 we have subtracted successive odd numbers starting from 1 and obtained 0 at 10th step. Therefore $\sqrt{100} = 10$

(ii) Step 1: $169 - 1 = 168$

Step 2: $168 - 3 = 165$

Step 3: $165 - 5 = 160$

Step 4: $160 - 7 = 153$

Step 5: $153 - 9 = 144$

Step 6: $144 - 11 = 133$

Step 7: $133 - 13 = 120$

Step 8: $120 - 15 = 105$

Step 9: $105 - 17 = 88$

Step 10: $88 - 19 = 69$

Step 11: $69 - 21 = 48$

Step 12: $48 - 23 = 25$

Step 13: $25 - 25 = 0$

From 169 we have subtracted successive odd numbers starting from 1 and obtained 0 at 13th step. Therefore $\sqrt{169} = 13$

4. **Find the square roots of the following numbers by the Prime Factorisation Method.**

(i) **729**

Sol: 729=<u>3×3</u>×<u>3×3</u>×<u>3×3</u>

$\sqrt{729} = 3 \times 3 \times 3$

3	729
3	243
3	81
3	27
3	9
	3

$\sqrt{729} = 27$

(ii) **400**

Sol: 400=2×2×2×2×5×5

$\sqrt{400} = 2 \times 2 \times 5$

$\sqrt{400} = 20$

2	400
2	200
2	100
2	50
5	25
	5

(iii) **1764**

Sol: 1764=2×2×3×3×7×7

$\sqrt{1764} = 2 \times 3 \times 7$

$\sqrt{1764} = 42$

2	1764
2	882
3	441
3	147
7	49
	7

(iv) **4096**

Sol: 4096=2×2×2×2×2×2×2×2×2×2×2×2

$\sqrt{4096} = 2 \times 2 \times 2 \times 2 \times 2 \times 2$

$\sqrt{4096} = 64$

2	4096
2	2048
2	1024
2	512
2	256
2	128
2	64
2	32
2	16
2	8
2	4
	2

(v) **7744**

Sol: 7744=2×2×2×2×2×2×11×11

$\sqrt{7744} = 2 \times 2 \times 2 \times 11$

$\sqrt{7744} = 88$

2	7744
2	3872
2	1936
2	968
2	484
2	242
11	121
	11

(vi) **9604**

Sol: 9604=2×2×7×7×7×7

$\sqrt{9604} = 2 \times 7 \times 7$

$\sqrt{9604} = 98$

2	9604
2	4802
7	2401
7	343
7	49
	7

(vii) **5929**

Sol: $5929 = 7 \times 7 \times 11 \times 11$

$\sqrt{5929} = 7 \times 11$

$\sqrt{5929} = 77$

7	5929
7	847
11	121
	11

(viii) **9216**

Sol: 9216=2×2×2×2×2×2×2×2×2×2×3×3

$\sqrt{9216} = 2 \times 2 \times 2 \times 2 \times 2 \times 3$

$\sqrt{9216} = 96$

2	9216
2	4608
2	2304
2	1152
2	576
2	288
2	144
2	72
2	36
2	18
3	9
	3

(ix) **529**

Sol: $529 = 23 \times 23$

23	529
	23

$\sqrt{529} = 23$

(x) **8100**

Sol: 8100=2×2×3×3×3×3×5×5

2	8100
2	4050
3	2025
3	675
3	225
3	75
5	25
	5

$\sqrt{8100} = 2 \times 3 \times 3 \times 5$

$\sqrt{8100} = 90$

5. **For each of the following numbers, find the smallest whole number by which it should be multiplied so as to get a perfect square number. Also find the square root of the square number so obtained.**

(i) **252**

$$\begin{array}{r|l} 2 & 252 \\ \hline 2 & 126 \\ \hline 3 & 63 \\ \hline 3 & 21 \\ \hline & 7 \end{array}$$

Sol: $252 = 2 \times 2 \times 3 \times 3 \times 7$

The prime factor 7 has no pair.

So, we multiply 252 by 7 to get a perfect square.

$252 \times 7 = 2 \times 2 \times 3 \times 3 \times 7 \times 7$

$1724 = \underline{2\times2}\times\underline{3\times3}\times\underline{7\times7}$

$\sqrt{1724} = 2 \times 3 \times 7$

$\sqrt{1724} = 42$

(ii) **180**

$$\begin{array}{r|l} 2 & 180 \\ \hline 2 & 90 \\ \hline 3 & 45 \\ \hline 3 & 15 \\ \hline & 5 \end{array}$$

Sol: $180 = 2 \times 2 \times 3 \times 3 \times 5$

The prime factor 5 has no pair.

So, we multiply 180 by 5 to get a perfect square.

$180 \times 5 = 2 \times 2 \times 3 \times 3 \times 5 \times 5$

$900 = \underline{2\times2}\times\underline{3\times3}\times\underline{5\times5}$

$\sqrt{900} = 2 \times 3 \times 5 = 30$

(iii) **1008**

$$\begin{array}{r|l} 2 & 1008 \\ \hline 2 & 504 \\ \hline 2 & 252 \\ \hline 2 & 126 \\ \hline 3 & 63 \\ \hline 3 & 21 \\ \hline & 7 \end{array}$$

Sol: $1008 = 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 7$

The prime factor 7 has no pair.

So, we multiply 1008 by 7 to get a perfect square

$1008 \times 7 = 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 7 \times 7$

$7056 = \underline{2\times2}\times\underline{2\times2}\times\underline{3\times3}\times\underline{7\times7}$

$\sqrt{7056} = 2 \times 2 \times 3 \times 7$

$\sqrt{7056} = 84$

(iv) **2028**

$$\begin{array}{r|l} 2 & 2028 \\ \hline 2 & 1014 \\ \hline 3 & 507 \\ \hline 13 & 169 \\ \hline & 13 \end{array}$$

Sol: $2028 = \underline{2\times2}\times3\times\underline{13\times13}$

The prime factor 3 has no pair.

So, we multiply 2028 by 3 to get a perfect square

$2028 \times 3 = 2 \times 2 \times 3 \times 3 \times 13 \times 13$

$6084 = \underline{2\times2}\times\underline{3\times3}\times\underline{13\times13}$

$\sqrt{6084} = 2 \times 3 \times 13$

$\sqrt{6084} = 78$

(v) **1458**

2	1458
3	729
3	243
3	81
3	27
3	9
	3

Sol: $1458 = 2 \times \underline{3 \times 3} \times \underline{3 \times 3} \times \underline{3 \times 3}$

The prime factor 2 has no pair.

So, we multiply 1458 by 2 to get a perfect square.

$1458 \times 2 = 2 \times 2 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3$

$2916 = \underline{2 \times 2} \times \underline{3 \times 3} \times \underline{3 \times 3} \times \underline{3 \times 3}$

$\sqrt{2916} = 2 \times 3 \times 3 \times 3$

$\sqrt{2916} = 54$

(vi) **768**

2	768
2	384
2	192
2	96
2	48
2	24
2	12
2	6
	3

Sol: $768 = \underline{2 \times 2} \times \underline{2 \times 2} \times \underline{2 \times 2} \times \underline{2 \times 2} \times 3$

The prime factor 3 has no pair.

So, we multiply 768 by 3 to get a perfect square.

$768 \times 3 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3$

$2304 = \underline{2 \times 2} \times \underline{2 \times 2} \times \underline{2 \times 2} \times \underline{2 \times 2} \times \underline{3 \times 3}$

$\sqrt{2304} = 2 \times 2 \times 2 \times 2 \times 3$

$\sqrt{2304} = 48$

6. For each of the following numbers, find the smallest whole number by which it should be divided so as to get a perfect square. Also find the square root of the square number so obtained.

(i) 252

2	252
2	126
3	63
3	21
	7

Sol: $252 = (2 \times 2) \times (3 \times 3) \times 7$

The prime factor 7 has no pair.

So, we divided 252 by 7 to get a perfect square.

$252 \div 7 = 2 \times 2 \times 3 \times 3$

$36 = \underline{2 \times 2} \times \underline{3 \times 3}$

$\sqrt{36} = 2 \times 3$

$\sqrt{36} = 6$

(ii) 2925

3	2925
3	975
5	325
5	65
	13

Sol: $2925 = \underline{3 \times 3} \times \underline{5 \times 5} \times 13$

The prime factor 13 has no pair.

So, we divided 2925 by 13 to get a perfect square.

$2925 \div 13 = 3 \times 3 \times 5 \times 5 \times 13 \div 13$

$225 = 3 \times 3 \times 5 \times 5$

$\sqrt{225} = 3 \times 5$

$\sqrt{225} = 15$

(iii) 396

Sol: $396 = 2 \times 2 \times 3 \times 3 \times 11$

2	396
2	198
3	99
3	33
	11

The prime factor 11 has no pair.

So, we divided 396 by 11 to get a perfect square.

$396 \div 11 = 2 \times 2 \times 3 \times 3$

$36 = 2 \times 2 \times 3 \times 3$

$\sqrt{36} = 2 \times 3$

$\sqrt{36} = 6$

(iv) 2645

Sol: $2645 = 5 \times 23 \times 23$

5	2645
23	529
	23

The prime factor 5 has no pair.

So, we divided 2645 by 5 to get a perfect square.

$2645 \div 5 = 23 \times 23$

$529 = 23 \times 23$

$\sqrt{529} = 23$

(v) 2800

Sol: $2800 = \underline{2 \times 2} \times \underline{2 \times 2} \times \underline{5 \times 5} \times 7$

2	2800
2	700
2	1400
2	350
5	175
5	35
	7

The prime factor 7 has no pair

So, we divided 2800 by 7 to get a perfect square.

$2800 \div 7 = 2 \times 2 \times 2 \times 2 \times 5 \times 5$

$400 = \underline{2 \times 2} \times \underline{2 \times 2} \times \underline{5 \times 5}$

$\sqrt{400} = 2 \times 2 \times 5$

$\sqrt{400} = 20$

(vi) 1620

Sol: $1620 = \underline{2 \times 2} \times \underline{3 \times 3} \times \underline{3 \times 3} \times 5$

2	1620
2	810
3	405
3	135
3	45
3	15
	5

The prime factor 5 has no pair

So, we divided 1620 by 5 to get a perfect square.

$1620 \div 5 = 2 \times 2 \times 3 \times 3 \times 3 \times 3$

$324 = \underline{2 \times 2} \times \underline{3 \times 3} \times \underline{3 \times 3}$

$\sqrt{324} = 2 \times 3 \times 3$

$\sqrt{324} = 18$

7. The students of Class VIII of a school donated ₹ 2401 in all, for Prime Minister's National Relief Fund. Each student donated as many rupees as the number of students in the class.

Find the number of students in the class

Sol: Let the total number of students=x

Amount donated by each student=x

Total donation=2401

$x \times x = 2401$

$x^2 = 2401$

$x = \sqrt{2401}$

$x = \sqrt{\underline{7 \times 7} \times \underline{7 \times 7}}$

$x = 7 \times 7 = 49$

The number of students in the class $= 49$

7 | 2401
7 | 343
7 | 49
  | 7

8. **2025 plants are to be planted in a garden in such a way that each row contains as many plants as the number of rows. Find the number of rows and the number of plants in each row.**

Sol: Let the number of rows=x

The number of plants in each row=x

Total plants=$x \times x = x^2$

Given total plants are planted=2025

$x^2 = 2025$

$x = \sqrt{2025}$

$x = \sqrt{\underline{3 \times 3} \times \underline{3 \times 3} \times \underline{5 \times 5}}$

$x = 3 \times 3 \times 5$

$x = 45$

∴ The number of rows=45 and

The number of plants in each row=45

3 | 2025
3 | 675
3 | 225
3 | 75
5 | 25
  | 5

9. **Find the smallest square number that is divisible by each of the numbers 4, 9 and 10**

Sol: The smallest square number divisible by 4,9 and 10=least multiple of LCM(4,9,10)

LCM(4,9,10)=$2 \times 2 \times 9 \times 5 = 2 \times 2 \times 3 \times 3 \times 5 = 180$

The prime factor 5 has no pair

So, we multiply 180 by 5 to get a perfect square

Required number=$180 \times 5 = 900$

2 | 4,9,10
  | 2,9,5

10. **Find the smallest square number that is divisible by each of the numbers 8, 15 and 20.**

Sol: The smallest square number divisible by 8,15 and 20=Multiple of LCM(8,15,20)

LCM(8,15,20)=$2 \times 2 \times 2 \times 3 \times 5 = 120$

The prime factors 2,3, 5 have no pairs

So, we multiply 120 by $2 \times 3 \times 5 = 30$ to get a perfect square

2 | 8,15,20
2 | 4,15,10
5 | 2,15, 5
  | 2,3, 1

Required number=$120\times 30 = 3600$

Finding square root by division method

Number	Square	
10	100	Which is the smallest 3-digit perfect square
31	961	Which is the greatest 3-digit perfect square
32	1024	Which is the smallest 4-digit perfect square
99	9801	Which is the greatest 3-digit perfect square

TRY THESE

Without calculating square roots, find the number of digits in the square root of the following numbers

(i) 25600

Sol: The number of digits in $\sqrt{25600}$ *is* $\frac{5+1}{2} = \frac{6}{2} = 3$

(ii) 100000000

Sol: The number of digits in $\sqrt{100000000}$ *is* $\frac{9+1}{2} = \frac{10}{2} = 5$

(iii) 36864

Sol: The number of digits in $\sqrt{36864}$ *is* $\frac{5+1}{2} = \frac{6}{2} = 3$

Example 9: **Find the square root of : (i) 729 (ii) 1296**

Sol:

(i) $\sqrt{729} = 27$

(ii) $\sqrt{1296} = 36$

	27
2	$\overline{7}\,\overline{29}$
	− 4
47	329
	329
	0

	36
3	$\overline{12}\,\overline{96}$
	− 9
66	396
	396
	0

Example 10: **Find the least number that must be subtracted from 5607 so as to get a perfect square. Also find the square root of the perfect square.**

Sol: To find $\sqrt{5607}$ by long division method. We get the remainder 131

The required perfect square is $5607 - 131 = 5476$

$\sqrt{5476} = 74$

	74
7	$\overline{56}\,\overline{07}$
	− 49
144	707
	−576
	131

Example 11: Find the greatest 4-digit number which is a perfect square.

Sol: Greatest number of 4-digits = 9999.

To find 9999 by long division method. The remainder is 198.

The required perfect square is $9999 - 198 = 9801$.

$\sqrt{9801} = 99$

	99
9	$\overline{99}\,\overline{99}$
	− 81
189	1899
	− 1701
	198

Example 12: **Find the least number that must be added to 1300 so as to get a perfect square. Also find the square root of the perfect square.**

Sol: To find $\sqrt{1300}$ by long division method. The remainder is 4.

Next perfect square number is $37^2 = 1369$. Hence, the number to be added is

$37^2 - 1300 = 1369 - 1300 = 69$

	36
3	$\overline{13}\,\overline{00}$
	−9
66	400
	−396
	4

Example 13: **Find the square root of 12.25.**

Sol: $\sqrt{12.25} = 3.5$

	3.5
3	$\overline{12}.\overline{25}$
	−9
65	325
	325
	0

Example 14: **Area of a square plot is 2304 m2 . Find the side of the square plot.**

Sol: Area of square plot = 2304 m^2

Side of the square plot = $\sqrt{2304}$ m =48 m

	48
4	$\overline{23}\,\overline{04}$
	-16
88	704
	704
	0

Example 15: **There are 2401 students in a school. P.T. teacher wants them to stand in rows and columns such that the number of rows is equal to the number of columns. Find the number of rows.**

Sol: Let the number of rows be x

So, the number of columns = x

Therefore, number of students = $x \times x = x^2$ Thus,

$x^2 = 2401$

$x = \sqrt{2401} = 49$

The number of rows = 49.

	49
4	$\overline{24}\,\overline{01}$
	-16
89	801
	801
	0

EXERCISE 5.4

1. Find the square root of each of the following numbers by Division method.

(i) **2304**

$\sqrt{2304} = 48$

	48
4	$\overline{23}\,\overline{04}$
	-16
88	704
	704
	0

(i) **4489**

$\sqrt{4489} = 67$

	67
6	$\overline{44}\,\overline{89}$
	-36
127	889
	889
	0

(ii) **3481**

$\sqrt{3481} = 59$

	59
5	$\overline{34}\,\overline{81}$
	-25
109	981
	981
	0

	23
2	$\overline{5}\,\overline{29}$
	-4
43	129
	-129
	0

	57
5	$\overline{32}\,\overline{49}$
	-25
107	749
	-749
	0

	37
3	$\overline{13}\,\overline{69}$
	-9
67	469
	-469
	0

(iii) **529**

$\sqrt{529} = 23$

(iv) **3249**

$\sqrt{3249} = 57$

	76
7	$\overline{57}\,\overline{76}$
	-49
146	876
	-876
	0

	89
8	$\overline{79}\,\overline{21}$
	-64
169	15 21
	-15 21
	0

	24
2	$\overline{5}\,\overline{76}$
	-4
44	176
	-176
	0

(v) **1369**

$\sqrt{1369} = 37$

(vi) **5776**

$\sqrt{5776} = 76$

	32
3	$\overline{10}\,\overline{24}$
	-9
62	124
	-124
	0

	56
5	$\overline{31}\,\overline{36}$
	-25
106	636
	636
	0

	30
3	$\overline{9}\,\overline{00}$
	-9
	0

(vii) **7921**

$\sqrt{7921} = 89$

(viii) **576**

$\sqrt{576} = 24$

(ix) **1024**

$\sqrt{1024} = 32$

(x) **3136**

$\sqrt{3136} = 56$

(xi) **900**

$\sqrt{900} = 30$

2. Find the number of digits in the square root of each of the following numbers (without any calculation).

If a perfect square is of n-digits, then its square root will have

$(a)\ \frac{n}{2}\ digits\ if\ n\ is\ even \quad (b)\ \frac{n+1}{2}\ digits\ if\ n\ is\ odd$

(i) 64

Number of digits in 64=2 (ie) n=2 which is even.

$\frac{n}{2} = \frac{2}{2} = 1$

The number of digits in the square root of 64=1

(ii) **144**

Number of digits in 144=3 (ie) n=3 which is odd.

$\frac{n}{2} = \frac{2}{2} = 1$

The number of digits in the square root of 64=1

(iii) 4489

Number of digits in 4489=4 (ie) n=4 which is even.

$\frac{n}{2} = \frac{4}{2} = 2$

The number of digits in the square root of 4489=2

(iv) 27225

Number of digits in 27225=5 (ie) n=5 which is odd.

$\frac{n+1}{2} = \frac{5+1}{2} = \frac{6}{2} = 3$

The number of digits in the square root of 27225=3

(v) 390625

Number of digits in 390625=6 (ie) n=6 which is even.

$\frac{n}{2} = \frac{6}{2} = 3$

The number of digits in the square root of 390625=3

3. Find the square root of the following decimal numbers.

(i) 2.56

$\sqrt{2.56} = 1.6$

	1.6
1	$\overline{2}.\ \overline{56}$
	-1
26	156
	-156
	0

(ii) 7.29

$\sqrt{7.29} = 2.7$

	2.7
2	$\overline{7}.\overline{29}$
	-4
47	329
	-329
	0

(iii) 51.84

$\sqrt{51.84} = 7.$

	7.2
7	$\overline{51}.\ \overline{84}$
	-49
142	284
	-284
	0

(iv) 42.25

$\sqrt{42.25} = 6.5$

	6.5
6	$\overline{42}.\overline{25}$
	-36
125	625
	-625
	0

(v) 31.36

$\sqrt{31.36} = 5.6$

	5.6
5	$\overline{31}.\ \overline{36}$
	-25
106	636
	-636
	0

4. Find the least number which must be subtracted from each of the following numbers so as to get a perfect square. Also find the square root of the perfect square so obtained.

(i) 402

Sol:The remainder=2

(If we subtract the remainder 2 from 402, we get a perfect square).

The required perfect square=402-2=400

$\sqrt{400} = 20$

	20
2	$\overline{4}\,\overline{02}$
	-4
40	002
	-000
	2

(ii) 1989

Sol: The remainder=53

(If we subtract the remainder 53 from 1989,

	44
4	$\overline{19}\,\overline{89}$
	-16
84	389
	336
	53

we get a perfect square).

The required perfect square=1989-53=1936

$\sqrt{1936} = 44$

(iii) 3250

	57
5	$\overline{32}\,\overline{50}$
	-25
107	750
	749
	1

Sol: The remainder=1

(If we subtract the remainder 1 from 3250,

we get a perfect square).

The required perfect square=3250-1=3249

$\sqrt{3249} = 57$

(iv) 825

	28
2	$\overline{8}\,\overline{25}$
	-4
48	425
	-384
	41

Sol: The remainder=41

(If we subtract the remainder 41 from 825,

we get a perfect square).

The required perfect square=825-41=784

$\sqrt{784} = 28$

(v) 4000

	63
6	$\overline{40}\,\overline{00}$
	-36
123	400
	-369
	31

Sol: The remainder=31

(If we subtract 31 from 4000, we get a perfect square).

The require least number=31

The required perfect square= $4000 - 31 = 3969$

$\sqrt{3969} = 63$

5. Find the least number which must be added to each of the following numbers so as to get a perfect square. Also find the square root of the perfect square so obtained.

(i) 525

	22
2	$\overline{5}\,\overline{25}$
	-4
42	125
	$-\ 84$
	41

Sol: The remainder=41

$22^2 < 525$

The next perfect square number$=23^2 = 529$

The number to be added$=529 - 525=4$

The perfect square obtained=529 and $\sqrt{529} = 23$

Alternate method:

	23
2	$\overline{5}\,\overline{25}$
	-4
43	125
	$-\ 129$
	-4

Remainder$=-4$

If we add 4 to 525, we get a perfect square

The required least number=4

The perfect square=525+4=529 and $\sqrt{529} = 23$

(ii) 1750

Sol: Remainder$=-14$

If we add 14 to 1750, we get a perfect square

The required least number to be added $=14$

The perfect square$=1750+14=1764$ and $\sqrt{1764}=42$

	42
4	$\overline{17}\,\overline{50}$
	-16
42	150
	-164
	-14

(iii) 252

Sol: Remainder$=-4$

If we add 4 to 252, we get a perfect square

The required least number to be added $=4$

The perfect square$=252+4=256$ and $\sqrt{256}=16$

	16
1	$\overline{2}\,\overline{52}$
	-1
26	152
	-156
	-4

(iv) 1825

Sol: Remainder$=-24$

If we add 24 to 1825, we get a perfect square

The required least number to be added $=24$

The perfect square$=1825+24=1849$ and $\sqrt{1849}=43$

	43
4	$\overline{18}\,\overline{25}$
	-16
83	225
	-249
	-24

(v) 6412

Sol: Remainder$=-149$

If we add 149 to 6412, we get a perfect square

The required least number to be added $=149$

The perfect square$=6412+149=6561$ and $\sqrt{6561}=81$

	81
8	$\overline{64}\,\overline{12}$
	-64
161	12
	-161
	-149

6. Find the length of the side of a square whose area is 441 m².

Sol: Let side of the square$=x$

Area of the square$=side \times side = x \times x = x^2$

Given area of the square$=441$ m².

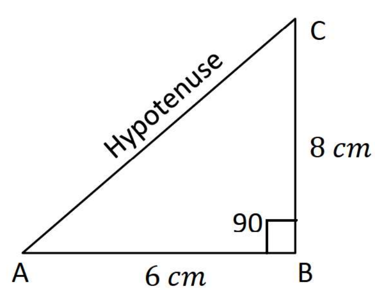
$x^2 = 441$

$x = \sqrt{441} = 21$

The length of the side of the square$=21$ m

	21
2	$\overline{4}\,\overline{41}$
	-4
41	41
	-41
	0

7. In a right triangle ABC, ∠B = 90°. (a) If AB = 6 cm, BC = 8 cm, find AC (b) If AC = 13 cm, BC = 5 cm, find AB



Sol: (a) $In\ \Delta ABC\ , \angle B = 90^0$

From Pythagoras theorem

$(Hypotenuse)^2 = (side)^2 + (side)^2$

$AC^2 = AB^2 + BC^2$

$AC^2 = 6^2 + 8^2 = 36 + 64 = 100$

$AC = \sqrt{100} = 10\ cm$

(b) $In\ \Delta ABC\ , \angle B = 90^0$

From Pythagoras theorem

$(Hypotenuse)^2 = (side)^2 + (side)^2$

$AC^2 = AB^2 + BC^2$

$13^2 = AB^2 + 5^2$

$169 = AB^2 + 25$

$AB^2 = 169 - 25 = 144$

$AB = \sqrt{144} = 12\ cm$