CHAPTER 1

VIII CLASS-NCERT(2024-25)
RATIONAL NUMBERS (Notes)
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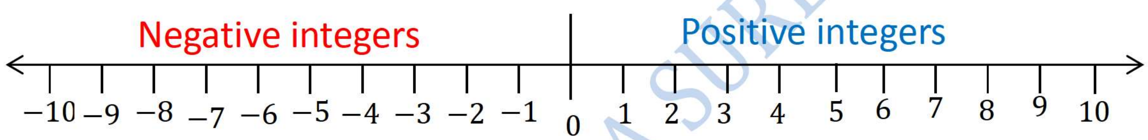
1. **Natural numbers**: The numbers which are used for counting are called Natural numbers and represented with letter N
   N={1,2,3,4,5,……}
2. **Whole numbers**: If '0' is added to Natural numbers then they are called Whole numbers. And is denoted by 'W'
   W={0,1,2,3,4,5,……}
3. **Integers**: Combination of positive and negative numbers Including 0 are called Integers and represented by 'Z' or 'I'.
   Z=$\{\dots\dots -4,-3,-2,-1,0,1,2,3,4,\dots\dots\dots\}$
4. Integers number line



5. **Addition of integers**:

(i) When two positive integers are added, we get a positive integer.
   e.g. $(+5)+(+6)=+11$

(ii) When two negative integers are added, we get a negative integer.
   e.g. $(-5)+(-6)=-11$

(iii) When one positive and one negative integer are added we subtract them as whole numbers by considering the numbers without their sign and then put the sign of the bigger number with the subtraction obtained.
   e.g. $(+8)+(-5)=3$, $(-8)+(+5)=-3$, $-7+5=-2$

6. **Multiplication of integers**:

(i) If the signs of two integers are same then the product is positive integer.
   e.g. $(+3)\times(+5)=15$, $(-4)\times(-3)=12$

(ii) If the signs of two integers are different then the product is negative integer.
   e.g. $(+3)\times(-5)=-15$, $(-3)\times(+5)=-15$, $(-4)\times(+3)=-12$, $(+4)\times(-3)=-12$

7. **Division of integers**:

(i) If the signs are same then the quotient is positive.
   e.g. $12\div 3=4$, $(-12)\div(-3)=4$

(ii) If the signs are different then the quotient is negative.

e. g. $(-12) \div 3 = -4, \quad 12 \div (-3) = -4$

8. Division by zero is not defined

$\frac{1}{0}, \quad \frac{3}{0}, \quad \frac{-51}{0}, \quad \frac{-8}{0}, \quad are\ not\ defined$

9. 0 ∈ W (0 belongs to whole numbers)

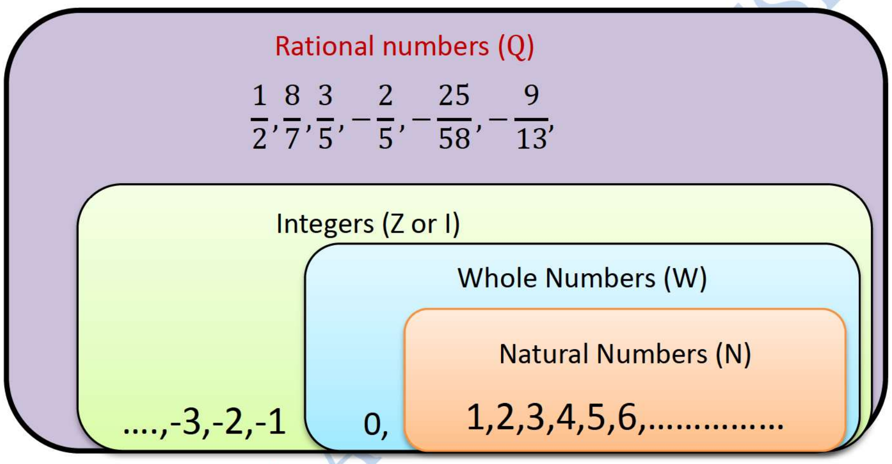
10. 0 ∉N (0 does not belong to natural numbers)

11. −3 ∈Z (−3 belongs to integers)

12. **Rational numbers**:

A number which can be written in the form $\frac{p}{q}$, where p and q are integers and q ≠ 0 is called a rational number.

Example: $-\frac{2}{3}, \frac{6}{7}, \frac{9}{-5}$ are all rational numbers. Since the numbers 0, −2, 4 can be written in the form $\frac{p}{q}$, they are also rational numbers.



1.Rational numbers are closed under addition i.e. $a, b \in R \Rightarrow a + b \in R$

$$e.g.\ a = \frac{3}{8}, b = \frac{-5}{7}\ are\ two\ rational\ numbers$$

$$a + b = \frac{3}{8} + \left(\frac{-5}{7}\right) = \frac{3 \times 7 + (-5 \times 8)}{56} = \frac{21 + (-40)}{56} = \frac{-19}{56}\quad is\ a\ rational\ number$$

2.Rational numbers are closed under subtraction i.e. $a, b \in R \Rightarrow a - b \in R$

$$e.g.\ a = \frac{3}{7}, b = \frac{-8}{5}\ are\ two\ rational\ numbers.$$

$$a + b = \frac{3}{7} - \left(\frac{-8}{5}\right) = \frac{3}{7} + \frac{8}{5} = \frac{3 \times 5 + 7 \times 8}{35} = \frac{15 + 56}{35} = \frac{71}{35}\quad is\ a\ rational\ number$$

3.Rational numbers are closed under multiplication i.e. $a, b \in R \Rightarrow a \times b \in R$

$$e.g.\ a = -\frac{4}{5}, b = \frac{-6}{11}\ are\ two\ rational\ numbers.$$

$$a \times b = \left(-\frac{4}{5}\right) \times \left(\frac{-6}{11}\right) = \frac{(-4) \times (-6)}{5 \times 11} = \frac{24}{55}\quad is\ a\ rational\ number$$

4.For any rational number a, $a \div 0 = \frac{a}{0}$ is not defined

5.Exclude zero then the collection of, all other rational numbers is closed under division

i.e. $a, b(\neq 0) \in R \Rightarrow a \div b = \frac{a}{b} \in R$

$e.g: \quad a = \frac{-3}{8}, b = \frac{-9}{2}$ *are two rational numbers*

$$a \div b = \frac{-3}{8} \div \frac{-9}{2} = \left(\frac{-3}{8}\right) \times \left(\frac{-2}{9}\right) = \frac{(-3) \times (-2)}{8 \times 9} = \frac{1}{12} \text{ is a rational number.}$$

6.Addition is commutative for rational numbers. i.e. $a, b \in Q \Rightarrow a + b = b + a$

$e.g: a = \frac{-6}{5}, b = \frac{-8}{3}$

$$a + b = \left(\frac{-6}{5}\right) + \left(\frac{-8}{3}\right) = \frac{(-6 \times 3) + (-8 \times 5)}{15} = \frac{(-18) + (-40)}{15} = \frac{-58}{15}$$

$$b + a = \left(\frac{-8}{3}\right) + \left(\frac{-6}{5}\right) = \frac{(-8 \times 5) + (-6 \times 3)}{15} = \frac{(-40) + (-18)}{15} = \frac{-58}{15}$$

$$\therefore a + b = b + a$$

7.Subtraction will not be commutative for rational numbers .i.e. $a, b \in R \Rightarrow a - b \neq b - a$

$e.g: a = \frac{2}{3}, b = \frac{5}{4}$

$$a - b = \frac{2}{3} - \frac{5}{4} = \frac{2 \times 4 - 5 \times 3}{12} = \frac{8 - 15}{12} = \frac{-7}{12}$$

$$b - a = \frac{5}{4} - \frac{2}{3} = \frac{5 \times 3 - 2 \times 4}{12} = \frac{15 - 8}{12} = \frac{7}{12}$$

$$a - b \neq b - a$$

8.Multiplication is commutative for rational numbers. i.e. $a, b \in Q \Rightarrow a \times b = b \times a$

$e.g: a = \frac{-8}{9}, b = \frac{-4}{7}$

$$a \times b = \left(\frac{-8}{9}\right) \times \left(\frac{-4}{7}\right) = \frac{(-8) \times (-4)}{(9) \times (7)} = \frac{32}{63}$$

$$b \times a = \left(\frac{-4}{7}\right) \times \left(\frac{-8}{9}\right) = \frac{(-4) \times (-8)}{(7) \times (9)} = \frac{32}{63}$$

$$\therefore a \times b = b \times a$$

9.Division is not commutative for rational numbers. i.e. $a, b \in Q \Rightarrow \frac{a}{b} \neq \frac{b}{a}$

$e.g: a = \frac{-5}{4}, b = \frac{3}{7}$

$$a \div b = \left(\frac{-5}{4}\right) \div \frac{3}{7} = \left(\frac{-5}{4}\right) \times \frac{7}{3} = \frac{(-5) \times 7}{4 \times 3} = \frac{-35}{12}$$

$$b \div a = \frac{3}{7} \div \left(\frac{-5}{4}\right) = \frac{3}{7} \times \left(\frac{-4}{5}\right) = \frac{3 \times (-4)}{7 \times 5} = \frac{-12}{35}$$

$\therefore a \div b \neq b \div a$

10. Additive is associative for rational numbers i.e. $a, b, c \in Q \Rightarrow a + (b + c) = (a + b) + c$

$e.g: a = \frac{-2}{3}, b = \frac{3}{5}, c = \frac{-5}{6}$

$$a + (b + c) = \frac{-2}{3} + \left[\frac{3}{5} + \left(\frac{-5}{6}\right)\right] = \frac{-2}{3} + \left[\frac{3 \times 6 + (-5 \times 5)}{30}\right] = \frac{-2}{3} + \left[\frac{18 - 25}{30}\right] = \frac{-2}{3} + \left(\frac{-7}{30}\right)$$

$$= \frac{(-2 \times 10) + (-7)}{30} = \frac{(-20) + (-7)}{30} = \frac{-27}{30}$$

$$(a + b) + c = \left[\frac{-2}{3} + \frac{3}{5}\right] + \left(\frac{-5}{6}\right) = \left[\frac{-2 \times 5 + 3 \times 3}{15}\right] + \left(\frac{-5}{6}\right) = \left[\frac{-10 + 9}{15}\right] + \left(\frac{-5}{6}\right)$$

$$= \left(\frac{-1}{15}\right) + \left(\frac{-5}{6}\right) = \frac{(-1 \times 2) + (-5 \times 5)}{30} = \frac{(-2) + (-25)}{30} = \frac{-27}{30}$$

$\therefore a + (b + c) = (a + b) + c$

11. Subtraction is not associative for rational numbers

$i.e. a, b, c \in Q \Rightarrow \boldsymbol{a - (b - c) \neq (a - b) - c}$

$e.g: a = \frac{-2}{3}, b = \frac{-4}{5}, c = \frac{1}{2}$

$a - (b - c)$

$$= \frac{-2}{3} - \left[\frac{-4}{5} - \frac{1}{2}\right]$$

$$= \frac{-2}{3} - \left[\frac{-4 \times 2 - 1 \times 5}{10}\right]$$

$$= \frac{-2}{3} - \left[\frac{-8 - 5}{10}\right]$$

$$= \frac{-2}{3} - \left(\frac{-13}{10}\right)$$

$$= \frac{-2}{3} + \frac{13}{10}$$

$$= \frac{-2 \times 10 + 13 \times 3}{30}$$

$$= \frac{-20 + 39}{30} = \frac{19}{30}$$

$(a - b) - c$

$$= \left[\frac{-2}{3} - \left(\frac{-4}{5}\right)\right] - \frac{1}{2}$$

$$= \left[\frac{-2}{3} + \frac{4}{5}\right] - \frac{1}{2}$$

$$= \left[\frac{-2 \times 5 + 4 \times 3}{15}\right] - \frac{1}{2}$$

$$= \left[\frac{-10 + 12}{15}\right] - \frac{1}{2}$$

$$= \frac{2}{15} - \frac{1}{2}$$

$$= \frac{2 \times 2 - 1 \times 15}{30}$$

$$= \frac{4 - 15}{30} = \frac{-11}{30}$$

$$a - (b - c) \neq (a - b) - c$$

12. Multiplication is associative for rational numbers

$i.e. a, b, c \in R \Rightarrow \boldsymbol{a \times (b \times c) = (a \times b) \times c}$

$e.g:\ a = \frac{-7}{3}, b = \frac{5}{4}, c = \frac{2}{9}$

$a \times (b \times c) = \frac{-7}{3} \times \left[\frac{5}{4} \times \frac{2}{9}\right] = \frac{-7}{3} \times \frac{10}{36} = \frac{-7 \times 10}{3 \times 36} = \frac{-7 \times 5}{3 \times 18} = \frac{-35}{54}$

$(a \times b) \times c = \left[\frac{-7}{3} \times \frac{5}{4}\right] \times \frac{2}{9} = \left(\frac{-35}{12}\right) \times \frac{2}{9} = \frac{-35 \times 2}{12 \times 9} = \frac{-35 \times 1}{6 \times 9} = \frac{-35}{54}$

$\therefore a \times (b \times c) = (a \times b) \times c$

13. Division is not associative for rational numbers

i.e. $a, b, c \in Q \Rightarrow a \div (b \div c) \neq (a \div b) \div c$

$e.g:\ a = \frac{1}{2}, b = \frac{-1}{3}, c = \frac{2}{5}$

$a \div (b \div c)$	$(a \div b) \div c$
$= \frac{1}{2} \div \left[\frac{-1}{3} \div \frac{2}{5}\right]$	$= \left[\frac{1}{2} \div \left(\frac{-1}{3}\right)\right] \div \frac{2}{5}$
$= \frac{1}{2} \div \left[\frac{-1}{3} \times \frac{5}{2}\right]$	$= \left[\frac{1}{2} \times \frac{-3}{1}\right] \div \frac{2}{5}$
$= \frac{1}{2} \div \left(\frac{-5}{6}\right)$	$= \frac{-3}{2} \div \frac{2}{5}$
$= \frac{1}{2} \times \left(\frac{-6}{5}\right)$	$= \frac{-3}{2} \times \frac{5}{2}$
$= \frac{-3}{5}$	$= \frac{-15}{4}$

$a \div (b \div c) \neq (a \div b) \div c$

14. Zero is called the identity for the addition of rational numbers.

$For\ a \in Q, a + 0 = 0 + a = a$

15. 1 is the multiplicative identity for rational numbers.

$For\ a \in Q, a \times 1 = 1 \times a = a$

16. For a rational number $\frac{a}{b}$, we have,

$\frac{a}{b} + \left(-\frac{a}{b}\right) = \left(-\frac{a}{b}\right) + \frac{a}{b} = 0$

We say that $\left(-\frac{a}{b}\right)$ is the additive inverse of $\frac{a}{b}$ and $\frac{a}{b}$ is the additive inverse of $\left(-\frac{a}{b}\right)$

17. $If\ \frac{a}{b} \times \frac{c}{d} = 1\ then\ \frac{c}{d}\ is\ called$ the reciprocal or multiplicative inverse of $\frac{a}{b}$

18. Distributivity of Multiplication over Addition and Subtraction.

For all rational numbers a, b, c

$a(b + c) = ab + ac$

$a(b - c) = ab - ac$

$e.g:\ a = \frac{-3}{4}, b = \frac{2}{3}, c = \frac{-5}{6}$

$a \times (b+c) = \frac{-3}{4} \times \left[\frac{2}{3} + \left(\frac{-5}{6}\right)\right]$

$= \frac{-3}{4} \times \left[\frac{2 \times 2 + (-5 \times 1)}{6}\right]$

$= \frac{-3}{4} \times \left(\frac{4 + (-5)}{6}\right)$

$= \frac{-3}{4} \times \left(\frac{-1}{6}\right) = \frac{3}{24} = \frac{1}{8}$

$a \times b + a \times c = \left(\frac{-3}{4} \times \frac{2}{3}\right) + \left(\frac{-3}{4} \times \frac{-5}{6}\right)$

$= \frac{-1}{2} + \frac{5}{8}$

$= \frac{(-1 \times 4) + 5}{8}$

$= \frac{-4+5}{8} = \frac{1}{8}$

$\therefore a \times (b+c) = a \times b + a \times c$

Properties of Rational numbers				
Property Name	Addition	Subtraction	Multiplication	Division
	$a, b, c \in Q$			a, b, c are non-zero rationale
Closure Property	$a + b \in Q$	$a - b \in Q$	$a \times b \in Q$	$a \div b \in Q$
Commutative law	$a + b = b + a$	$\mathbf{a - b \neq b - a}$	$a \times b = b \times a$	$a \div b \neq b \div a$
Associative Law	$(a+b)+c = a+(b+c)$	$(a-b)-c \neq a-(b-c)$	$(a \times b) \times c = a \times (b \times c)$	$(a \div b) \div c \neq a \div (b \div c)$
Identity Property	$a + 0 = a$ $0 + a = a$	Not applicable	$a \times 1 = a$ $1 \times a = a$	Not applicable
Inverse Property	$a + (-a) = 0$ *and* $(-a) + a = 0$ $(-a)$ *is additive inverse of* a a *is additive inverse of* $(-a)$		$a \times \frac{1}{a} = 1$ *and* $\frac{1}{a} \times a = 1$ $\frac{1}{a}$ *is multiplicative inverse of* a a *is multiplicative inverse of* $\frac{1}{a}$	
Distributive	$a(b+c) = ab + ac$		$a(b-c) = ab - ac$	

Example 1*: *Find $\frac{3}{7} + \left(\frac{-6}{11}\right) + \left(\frac{-8}{21}\right) + \left(\frac{5}{22}\right)$

***Sol*:** $\frac{3}{7} + \left(\frac{-6}{11}\right) + \left(\frac{-8}{21}\right) + \left(\frac{5}{22}\right)$

$= \frac{3 \times 66 + (-6) \times 42 + (-8) \times 22 + 5 \times 21}{462}$

$= \frac{198 - 252 - 176 + 105}{462}$

$= \frac{303 - 428}{462} = \frac{-125}{462}$

7	7,11,21,22
11	1, 11, 3, 22
	1,1,3, 2

L.C.M of 7,11,21,22 $= 7 \times 11 \times 3 \times 2$ =462

$462 \div 7 = 66;\quad 462 \div 11 = 42;$
$462 \div 21 = 22;\quad 462 \div 22 = 21$

Example 2*: *Find $\frac{-4}{5}\times\frac{3}{7}\times\frac{15}{16}\times\left(\frac{-14}{9}\right)$

Sol: $\frac{-4}{5}\times\frac{3}{7}\times\frac{15}{16}\times\left(\frac{-14}{9}\right)$

$=\left(\frac{-4}{5}\times\frac{15}{16}\right)\times\left[\frac{3}{7}\times\left(\frac{-14}{9}\right)\right]$

$=\left(\frac{-3}{4}\right)\times\left(\frac{-2}{3}\right)=\frac{1}{2}$

The role of zero (0)

Where a is a rational number then

$(i)\ a+0=0+a=a$

$(ii)\ a\times 0=0\times a=0$

'Zero' is called the identity for the addition of rational numbers.

The role of '1'

Where 'a' is a rational number then

$(i)\ a\times 1=1\times a=a$

1 is the multiplicative identity for rational numbers

TRY THESE

Find using distributivity

For all rational numbers $a, b\ and\ c$

$(i)\ a\times(b+c)=a\times b+a\times c$

$(ii)\ a\times(b-c)=a\times b-a\times c$

(i) $\left\{\frac{7}{5}\times\left(\frac{-3}{12}\right)\right\}+\left\{\frac{7}{5}\times\frac{5}{12}\right\}$

Sol: $\left\{\frac{7}{5}\times\left(\frac{-3}{12}\right)\right\}+\left\{\frac{7}{5}\times\frac{5}{12}\right\}=\frac{7}{5}\times\left(\frac{-3}{12}+\frac{5}{12}\right)=\frac{7}{5}\times\frac{2}{12}=\frac{7\times 1}{5\times 6}=\frac{7}{30}$

(ii) $\left\{\frac{9}{16}\times\frac{4}{12}\right\}+\left\{\frac{9}{16}\times\frac{-3}{9}\right\}$

Sol: $\left\{\frac{9}{16}\times\frac{4}{12}\right\}+\left\{\frac{9}{16}\times\frac{-3}{9}\right\}=\frac{9}{16}\times\left(\frac{4}{12}+\frac{-3}{9}\right)=\frac{9}{16}\times\left(\frac{1}{3}-\frac{1}{3}\right)=\frac{9}{16}\times 0=0$

Example 3*: *Find $\frac{2}{5}\times\frac{-3}{7}-\frac{1}{14}-\frac{3}{7}\times\frac{3}{5}$

Sol: $\frac{2}{5}\times\frac{-3}{7}-\frac{1}{14}-\frac{3}{7}\times\frac{3}{5}$

$=\frac{-3}{7}\times\frac{2}{5}+\left(-\frac{3}{7}\right)\times\frac{3}{5}-\frac{1}{14}$ *(by commutativity)*

$=\frac{-3}{7}\left(\frac{2}{5}+\frac{3}{5}\right)-\frac{1}{14}$

$$= \frac{-3}{7} \times \left(\frac{5}{5}\right) - \frac{1}{14}$$
$$= \frac{-3}{7} - \frac{1}{14}$$
$$= \frac{-3 \times 2 - 1}{14} = \frac{-6 - 1}{14} = \frac{-7}{14} = \frac{-1}{2}$$

EXERCISE 1.1

1. **Name the property under multiplication used in each of the following.**

$(i)\ \frac{-4}{5} \times 1 = 1 \times \frac{-4}{5} = \frac{-4}{5} \rightarrow$ Multiplicative identity

$(ii)\ -\frac{13}{17} \times \frac{-2}{7} = \frac{-2}{7} \times -\frac{13}{17} \rightarrow$ commutative under multiplication

$(iii)\ \frac{-19}{29} \times \frac{29}{-19} = 1 \rightarrow$ Multiplicative inverse

2. **Tell what property allows you to compute $\frac{1}{3} \times \left(6 \times \frac{4}{3}\right)$ as $\left(\frac{1}{3} \times 6\right) \times \frac{4}{3}$**

Sol: *Associative property under multiplication.*

3. The product of two rational numbers is always a **rational number**

Bits

1. A number can be expressed in the form $\frac{p}{q}$, where p and q are integers and q ≠ 0, is called a rational number.
2. The rational number 0 is the additive identity for rational numbers.
3. The rational number 1 is the multiplicative identity for rational numbers.
4. The additive inverse of the rational number $\frac{a}{b}$ is $\frac{-a}{b}$ and vice-versa.
5. The reciprocal or multiplicative inverse of the rational number $\frac{a}{b}$ is $\frac{b}{a}$.
6. Distributivity of rational numbers : For all rational numbers a, b and c
$$a(b + c) = ab + ac \quad and \quad a(b - c) = ab - ac$$
7. Between any two given rational numbers there are infinitely many rational numbers
8. If a and b are two rational numbers, then $\frac{a+b}{2}$ is a rational number between a and b
9. The reciprocal of a positive rational number is positive.
10. The reciprocal of a negative rational number is negative.
11. Zero has no reciprocal.
12. The numbers 1 and -1 are their own reciprocal.
13. The negative of a negative rational number is always a positive rational number.
14. The set of numbers which do not have any additive identity – Natural numbers(N)

15. The rational number that does not have any reciprocal is 0.

16. Commutative under addition: $a + b = b + a$

17. Commutative under multiplication: $a \times b = b \times a$

18. Associative property under addition: $a + (b + c) = (a + b) + c$

19. Associative property under multiplication: $a \times (b \times c) = (a \times b) \times c$

20. Division by zero is not defined

$$\frac{1}{0}, \quad \frac{3}{0}, \quad \frac{-51}{0}, \quad \frac{-8}{0}, \quad are\ not\ defined$$

21. 0 ∈ W (0 belongs to whole numbers)

22. 0 ∉N (0 does not belong to natural numbers)

23. −3 ∈Z (−3 belongs to integers)

24. A rational number and its additive inverse are opposite in their sign.

25. The multiplicative inverse of a rational number is its reciprocal.

26. Neither a positive nor a negative rational number is 0.

27. The equivalent of $\frac{5}{7}$, whose numerator is 45 is $\frac{45}{63}$

28. The equivalent rational number of $\frac{7}{9}$, whose denominator is 45 is $\frac{35}{45}$