

**CHAPTER**

**6**

VII-MATHEMATICS-NCERT-(2024-25)

**6. The Triangle and its Properties (Notes)**

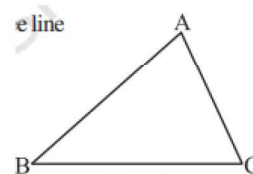
<https://sureshmathsmaterial.com/>

1. A simple closed curve made of three line segments is called a triangle.
2. Triangle has **three vertices**, **three sides** and **three angles**.

**Vertices:** A, B, C

**Sides:** AB, BC, CA

**Angles:**  $\angle BAC$ ,  $\angle ABC$ ,  $\angle BCA$



3. The side opposite to the vertex A is BC  
The side opposite to the vertex B is AC  
The side opposite to the vertex C is AB

**TRY THESE**

1. Write the six elements (i.e., the 3 sides and the 3 angles) of  $\triangle ABC$

**Sol:** Three sides of  $\triangle ABC$  are AB, BC, AC

Three angles of  $\triangle ABC$  are  $\angle BAC$ ,  $\angle ABC$ ,  $\angle BCA$  (or)  $\angle A$ ,  $\angle B$ ,  $\angle C$

2. Write the

(i) Side opposite to the vertex Q of  $\triangle PQR$

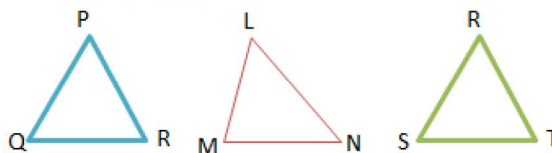
**Sol:** PR

(ii) Angle opposite to the side LM of  $\triangle LMN$

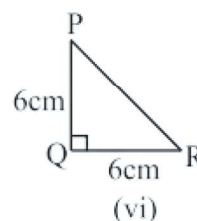
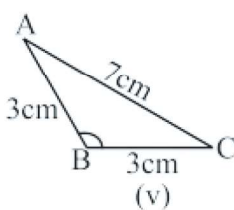
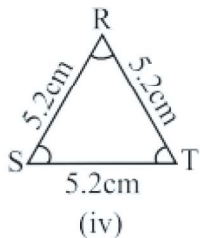
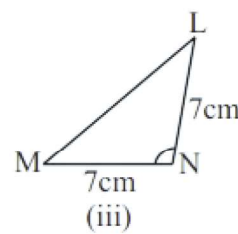
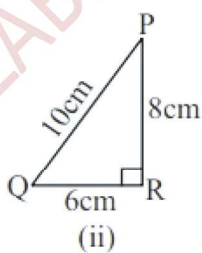
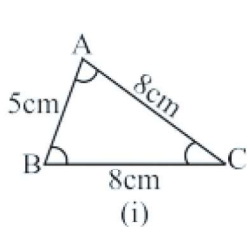
**Sol:**  $\angle N$

(iii) Vertex opposite to the side RT of  $\triangle RST$

**Sol:** S



3. Look at Fig 6.2 and classify each of the triangles according to its (a) Sides (b) Angles

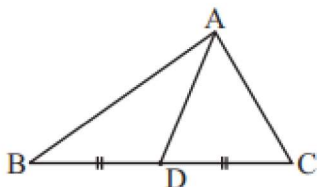


S.No	Based on sides	Based on angles
(i)	Isosceles triangle	Acute angled triangle
(ii)	Scalene triangle	Right angled triangle
(iii)	Isosceles triangle	Obtuse angled triangle

(iv)	Equilateral triangle	Acute angled triangle
(v)	Isosceles triangle	Obtuse angled triangle
(vi)	Isosceles triangle	Right angled triangle

### MEDIANS OF A TRIANGLE

The line segment joining a vertex of a triangle to the mid point of its opposite side is called a median of the triangle. A triangle has 3 medians



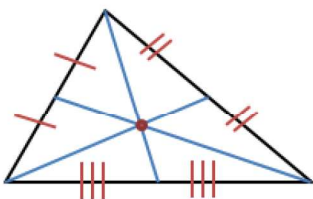
The line segment AD, joining the mid-point of BC to its opposite vertex A is called a median of the triangle.

### THINK, DISCUSS AND WRITE

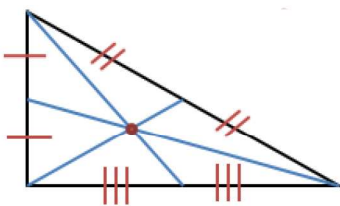
- How many medians can a triangle have?

Sol:3

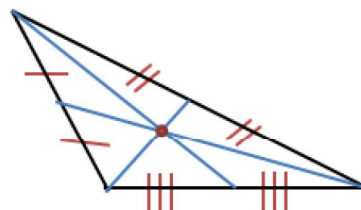
- Does a median lie wholly in the interior of the triangle? (If you think that this is not true, draw a figure to show such a case).



Acute angled triangle



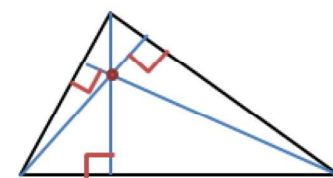
Right angled triangle



Obtuse angled triangle

### ALTITUDES OF A TRIANGLE

The perpendicular line segment from a vertex of a triangle to its opposite side is called an altitude of the triangle. A triangle has 3 altitudes.



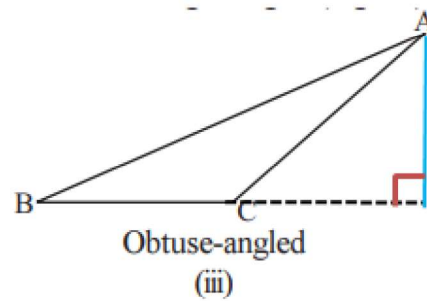
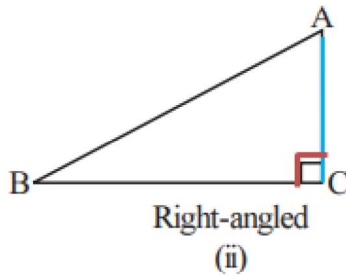
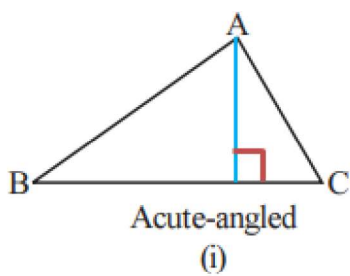
Acute angled triangle

### THINK, DISCUSS AND WRITE

- How many altitudes can a triangle have?

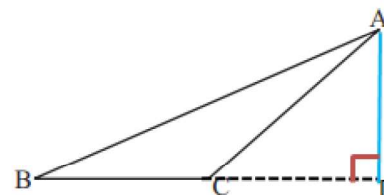
Sol: 3

- Draw rough sketches of altitudes from A to BC for the following triangles (Fig 6.6):



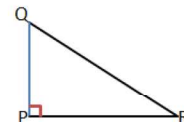
3. Will an altitude always lie in the interior of a triangle? If you think that this need not be true, draw a rough sketch to show such a case.

Sol: No, an altitude may lie outside of triangle also.



4. Can you think of a triangle in which two altitudes of the triangle are two of its sides?

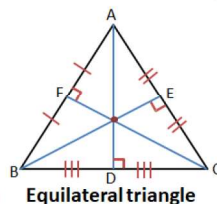
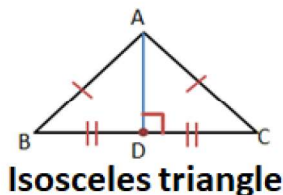
Sol: Yes, in right angled triangle two altitudes of the triangle are two of its sides.



5. Can the altitude and median be same for a triangle?

Sol: Yes, in an equilateral triangle both the median and the altitude are the same.

In an isosceles triangle one altitude and median be same

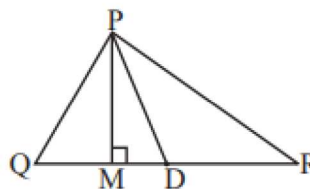


**DO THIS**

Take several cut-outs of (i) an equilateral triangle (ii) an isosceles triangle and (iii) a scalene triangle. Find their altitudes and medians. Do you find anything special about them? Discuss it with your friends.

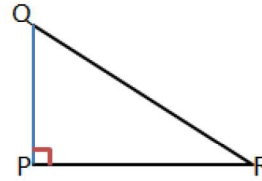
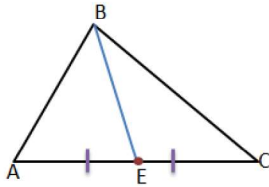
**EXERCISE 6.1**

1. In  $\Delta PQR$ ,  $D$  is the mid-point of  $\overline{QR}$   
 $\overline{PM}$  is altitude.  $\overline{PD}$  is median.  
 Is  $QM = MR$ ? NO



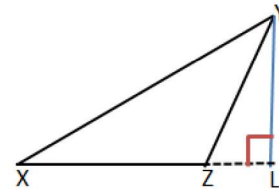
2. Draw rough sketches for the following:  
 (a) In  $\Delta ABC$ ,  $BE$  is a median.





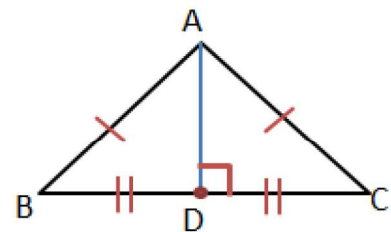
(b) In  $\Delta PQR$ ,  $PQ$  and  $PR$  are altitudes of the triangle.

(c) In  $\Delta XYZ$ ,  $YL$  is an altitude in the exterior of the triangle.



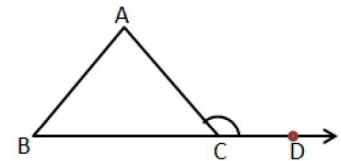
3. Verify by drawing a diagram if the median and altitude of an isosceles triangle can be same.

Sol:



### EXTERIOR ANGLE OF A TRIANGLE AND ITS PROPERTY

An exterior angle of a triangle is formed, when a side of a triangle is produced. At each vertex, you have two ways of forming an exterior angle.



**Exterior Angle Property of a triangle:**

**The measure of any exterior angle of a triangle is equal to the sum of the measures of its interior opposite angles**

**Given:** Consider  $\Delta ABC$ .  $\angle ACD$  is an exterior angle.

**To Show:**  $m \angle ACD = m \angle A + m \angle B$

**Proof:** Through C draw  $\overline{CE}$ , parallel to  $\overline{BA}$ .

$\angle 1 = \angle x$  ( $\overline{BA} \parallel \overline{CE}$  and  $\overline{AC}$  is a transversal. Therefore, alternate angles should be equal)

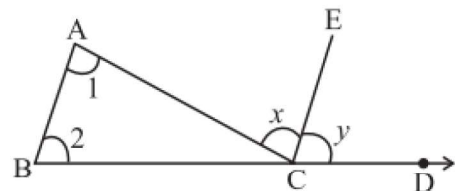
$\angle 2 = \angle y$  ( $\overline{BA} \parallel \overline{CE}$  and  $\overline{BD}$  is a transversal. Therefore, corresponding angles should be equal)

$$\angle 1 + \angle 2 = \angle x + \angle y$$

$$\angle x + \angle y = m \angle ACD \text{ (From Fig)}$$

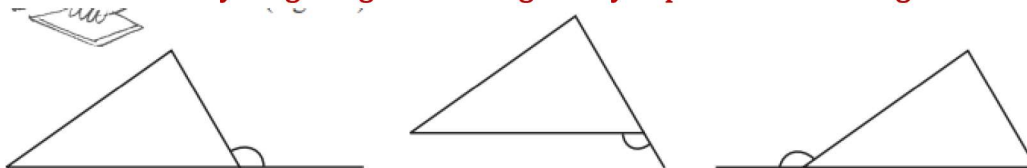
$$\text{Hence, } \angle ACD = \angle 1 + \angle 2$$

$$m \angle ACD = m \angle A + m \angle B$$

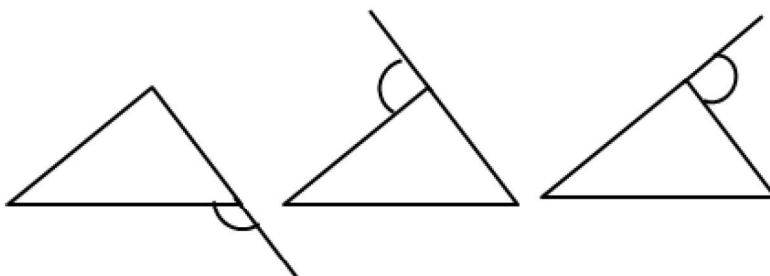


**THINK, DISCUSS AND WRITE**

1. Exterior angles can be formed for a triangle in many ways. Three of them are shown here. There are three more ways of getting exterior angles. Try to produce those rough sketches.



Sol:



2. Are the exterior angles formed at each vertex of a triangle equal?

Sol: No

3. What can you say about the sum of an exterior angle of a triangle and its adjacent interior angle?

Sol: The sum of an exterior angle of a triangle and its adjacent interior angle =  $180^\circ$

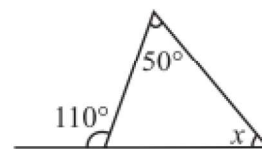
**Example 1:** Find angle  $x$  in Fig 6.11.

**Solution:** Sum of interior opposite angles = Exterior angle

$$50^\circ + x = 110^\circ$$

$$x = 110^\circ - 50^\circ$$

$$x = 60^\circ$$



### THINK, DISCUSS AND WRITE

- What can you say about each of the interior opposite angles, when the exterior angle is (i) a right angle? (ii) an obtuse angle? (iii) an acute angle?
- Can the exterior angle of a triangle be a straight angle?

Sol: No

#### TRY THESE

- An exterior angle of a triangle is of measure  $70^\circ$  and one of its interior opposite angles is of measure  $25^\circ$ . Find the measure of the other interior opposite angle.

Sol: Sum of interior opposite angles = Exterior angle

$$25^\circ + x = 70^\circ$$

$$x = 70^\circ - 25^\circ$$

$$x = 45^\circ$$

The other interior opposite angle =  $45^\circ$

2. The two interior opposite angles of an exterior angle of a triangle are  $60^\circ$  and  $80^\circ$ . Find the measure of the exterior angle.

**Sol:** Exterior angle = Sum of interior opposite angles

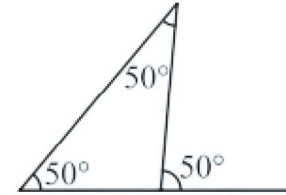
$$= 60^\circ + 80^\circ = 140^\circ$$

3. Is something wrong in this diagram (Fig 6.12)? Comment

**Sol:** Exterior angle =  $50^\circ$

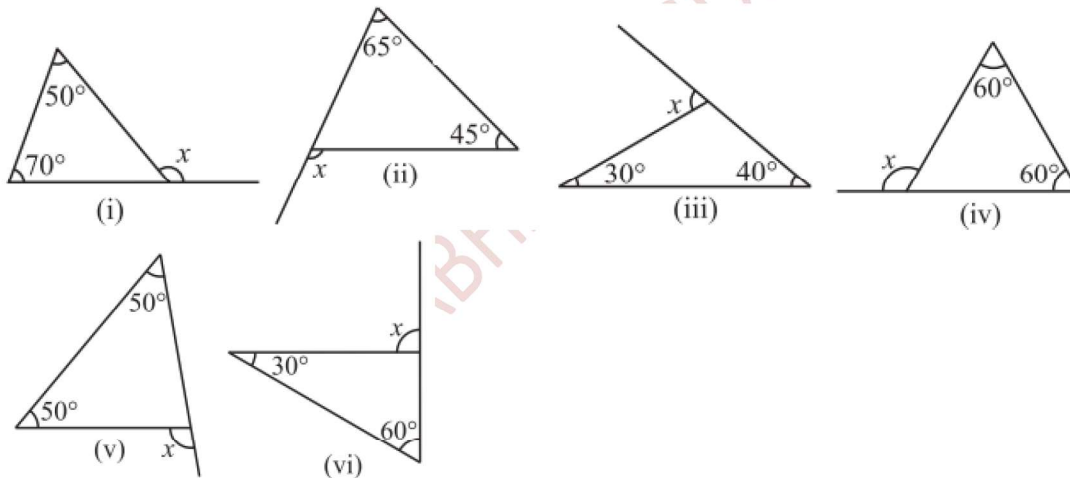
$$\text{Sum of interior opposite angles} = 50^\circ + 50^\circ = 100^\circ$$

Exterior angle  $\neq$  Sum of interior opposite angles



## EXERCISE 6.2

1. Find the value of the unknown exterior angle  $x$  in the following diagrams:



**Sol:** Exterior angle = Sum of interior opposite angles

$$(i) x = 50^\circ + 70^\circ = 120^\circ$$

$$(ii) x = 65^\circ + 45^\circ = 110^\circ$$

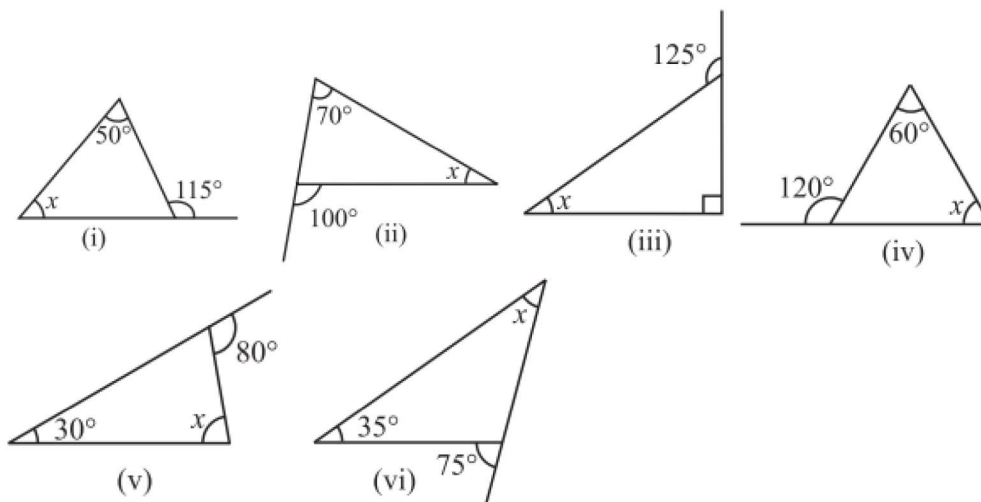
$$(iii) x = 30^\circ + 40^\circ = 70^\circ$$

$$(iv) x = 60^\circ + 60^\circ = 120^\circ$$

$$(v) x = 50^\circ + 50^\circ = 100^\circ$$

$$(vi) x = 30^\circ + 60^\circ = 90^\circ$$

2. Find the value of the unknown interior angle  $x$  in the following figures:



**Sol:** Sum of interior opposite angles = Exterior angle

$$(i) \quad x + 50^\circ = 115^\circ$$

$$x = 115^\circ - 50^\circ$$

$$x = 65^\circ$$

$$(ii) \quad x + 70^\circ = 100^\circ$$

$$x = 100^\circ - 70^\circ$$

$$x = 30^\circ$$

$$(iii) \quad x + 90^\circ = 125^\circ$$

$$x = 125^\circ - 90^\circ$$

$$x = 35^\circ$$

$$(iv) \quad x + 60^\circ = 120^\circ$$

$$x = 120^\circ - 60^\circ$$

$$x = 60^\circ$$

$$(v) \quad x + 30^\circ = 80^\circ$$

$$x = 80^\circ - 30^\circ$$

$$x = 50^\circ$$

$$(vi) \quad x + 35^\circ = 75^\circ$$

$$x = 75^\circ - 35^\circ$$

$$x = 40^\circ$$

### ANGLE SUM PROPERTY OF A TRIANGLE

**The total measure of the three angles of a triangle is  $180^\circ$**

**Given:**  $\angle 1, \angle 2, \angle 3$  are angles of  $\triangle ABC$  and  $\angle 4$  is the exterior angle when  $BC$  is extended to  $D$ .

**Proof:**  $\angle 1 + \angle 2 = \angle 4$  (by exterior angle property)

$$\angle 1 + \angle 2 + \angle 3 = \angle 4 + \angle 3 \text{ (adding } \angle 3 \text{ to both the sides)}$$

$$\text{But } \angle 4 + \angle 3 = 180^\circ \text{ (linear pair)}$$

$$\therefore \angle 1 + \angle 2 + \angle 3 = 180^\circ$$

**Example 2:** In the given figure (Fig 6.18) find  $m\angle P$

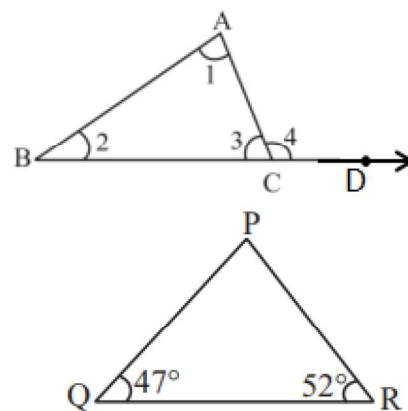
**Solu:** By angle sum property of a triangle,

$$\angle P + 47^\circ + 52^\circ = 180^\circ$$

$$\angle P + 99^\circ = 180^\circ$$

$$\angle P = 180^\circ - 99^\circ$$

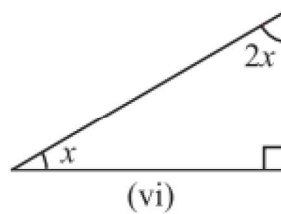
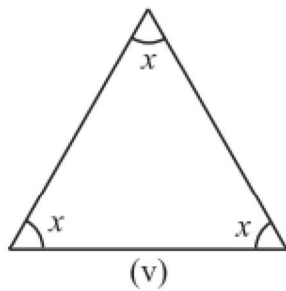
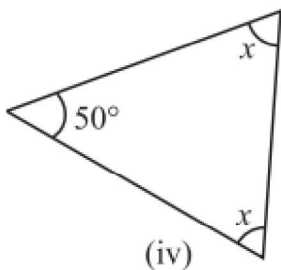
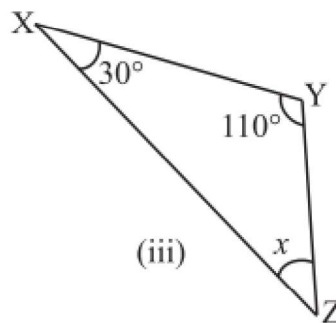
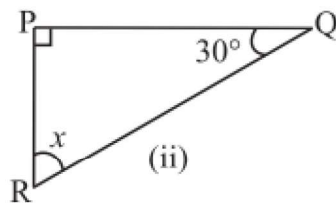
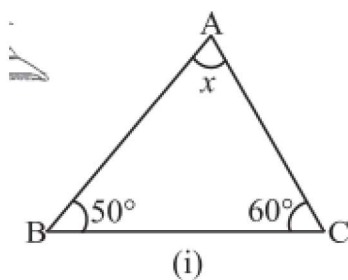
$$m\angle P = 81^\circ$$



## EXERCISE 6.3

1. Find the value of the unknown  $x$  in the following diagrams:





**Sol:** Sum of three angles in a triangle =  $180^\circ$

(i)  $x + 50^\circ + 60^\circ = 180^\circ$

$x + 110^\circ = 180^\circ$

$x = 180^\circ - 110^\circ$

$x = 70^\circ$

(ii)  $x + 30^\circ + 90^\circ = 180^\circ$

$x + 120^\circ = 180^\circ$

$x = 180^\circ - 120^\circ$

$x = 60^\circ$

(iii)  $x + 110^\circ + 30^\circ = 180^\circ$

$x + 140^\circ = 180^\circ$

$x = 180^\circ - 140^\circ$

$x = 40^\circ$

(iv)  $x + x + 50^\circ = 180^\circ$

$2x = 180^\circ - 50^\circ = 130^\circ$

$x = \frac{130^\circ}{2}$

$x = 65^\circ$

(v)  $x + x + x = 180^\circ$

$3x = 180^\circ$

$x = \frac{180^\circ}{3} = 60^\circ$

$x = 70^\circ$

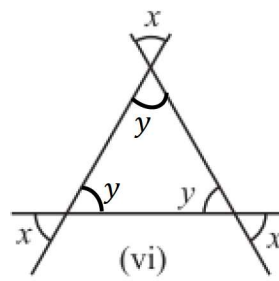
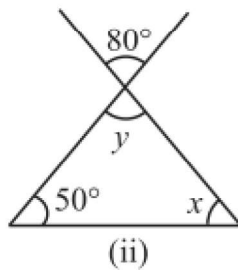
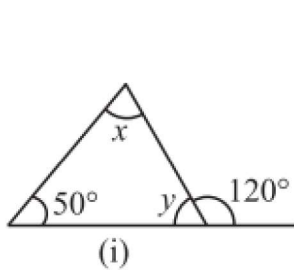
(vi)  $x + 2x + 90^\circ = 180^\circ$

$3x = 180^\circ - 90^\circ$

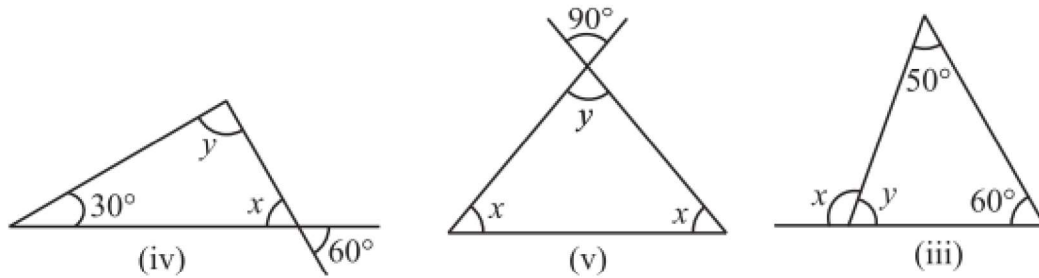
$x = \frac{90^\circ}{3}$

$x = 30^\circ$

2. Find the values of the unknowns  $x$  and  $y$  in the following diagrams:





**Sol:**

(i)  $y + 120^\circ = 180^\circ$  (Linear pair)

$$y = 180^\circ - 120^\circ = 60^\circ$$

$x + 50^\circ = 120^\circ$  (Exterior angle property)

$$x = 120^\circ - 50^\circ = 70^\circ$$

(ii)  $y = 80^\circ$  (Vertically opposite angles)

$x + y + 50^\circ = 180^\circ$  (Angle sum property of triangle)

$$x + 80^\circ + 50^\circ = 180^\circ$$

$$x + 130^\circ = 180^\circ$$

$$x = 180^\circ - 130^\circ$$

$$x = 50^\circ$$

(iii)  $x = 50^\circ + 60^\circ$  (Exterior angle property)

$$x = 110^\circ$$

$x + y = 180^\circ$  (Linear pair)

$$110^\circ + y = 180^\circ$$

$$y = 180^\circ - 110^\circ$$

$$y = 70^\circ$$

(iv)  $x = 60^\circ$  (Vertically opposite angles)

$x + y + 30^\circ = 180^\circ$  (Angle sum property of triangle)

$$60^\circ + y + 30^\circ = 180^\circ$$

$$y + 90^\circ = 180^\circ$$

$$y = 180^\circ - 90^\circ = 90^\circ$$

(v)  $y = 90^\circ$  (Vertically opposite angles)

$x + x + y = 180^\circ$  (Angle sum property of triangle)

$$2x + 90^\circ = 180^\circ$$

$$2x = 180^\circ - 90^\circ$$

$$2x = 90^\circ$$

$$x = \frac{90^\circ}{2} = 45^\circ$$

(vi)  $x = y$  (Vertically opposite angles)

$y + y + y = 180^\circ$  (Angle sum property of triangle)

$$3y = 180^\circ$$

$$y = \frac{180^\circ}{3} = 60^\circ$$

$$x = 60^\circ \text{ and } y = 60^\circ$$

**TRY THESE**

- Two angles of a triangle are  $30^\circ$  and  $80^\circ$ . Find the third angle.

**Sol:** Let third angle =  $x$

$$x + 30^\circ + 80^\circ = 180^\circ \text{ (Angle sum property of triangle)}$$

$$x + 110^\circ = 180^\circ$$

$$x = 180^\circ - 110^\circ = 70^\circ$$

- 2. One of the angles of a triangle is  $80^\circ$  and the other two angles are equal. Find the measure of each of the equal angles.**

**Sol:** Let the other two angles are  $x, x$

$$x + x + 80^\circ = 180^\circ \text{ (Angle sum property of triangle)}$$

$$2x + 80^\circ = 180^\circ$$

$$2x = 180^\circ - 80^\circ = 100^\circ$$

$$2x = 100^\circ$$

$$x = \frac{100^\circ}{2} = 50^\circ$$

- 3. The three angles of a triangle are in the ratio 1:2:1. Find all the angles of the triangle. Classify the triangle in two different ways.**

**Sol:** Let the angles are  $x, 2x, x$

$$x + 2x + x = 180^\circ \text{ (Angle sum property of triangle)}$$

$$4x = 180^\circ$$

$$x = \frac{180^\circ}{4} = 45^\circ$$

The angles are  $45^\circ, 2 \times 45^\circ, 45^\circ$  i.e  $45^\circ, 90^\circ, 45^\circ$

The given triangle is an isosceles triangle and right angled triangle.

### THINK, DISCUSS AND WRITE

- 1. Can you have a triangle with two right angles?**

**Sol:** No

- 2. Can you have a triangle with two obtuse angles?**

**Sol:** No

- 3. Can you have a triangle with two acute angles?**

**Sol:** Yes

- 4. Can you have a triangle with all the three angles greater than  $60^\circ$ ?**

**Sol:** No

- 5. Can you have a triangle with all the three angles equal to  $60^\circ$ ?**

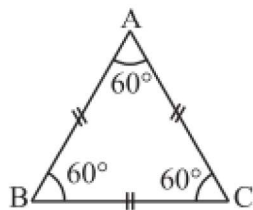
**Sol:** Yes, in equilateral triangle all the three angles equal to  $60^\circ$

- 6. Can you have a triangle with all the three angles less than  $60^\circ$ ?**

**Sol:** No.

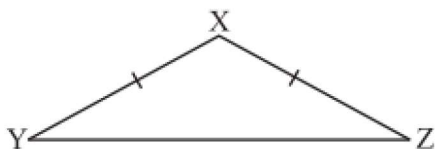
### TWO SPECIAL TRIANGLES: EQUILATERAL AND ISOSCELES

A triangle in which all the three sides are of equal lengths is called an equilateral triangle.



A triangle in which two sides are of equal lengths is called an **isosceles triangle**.

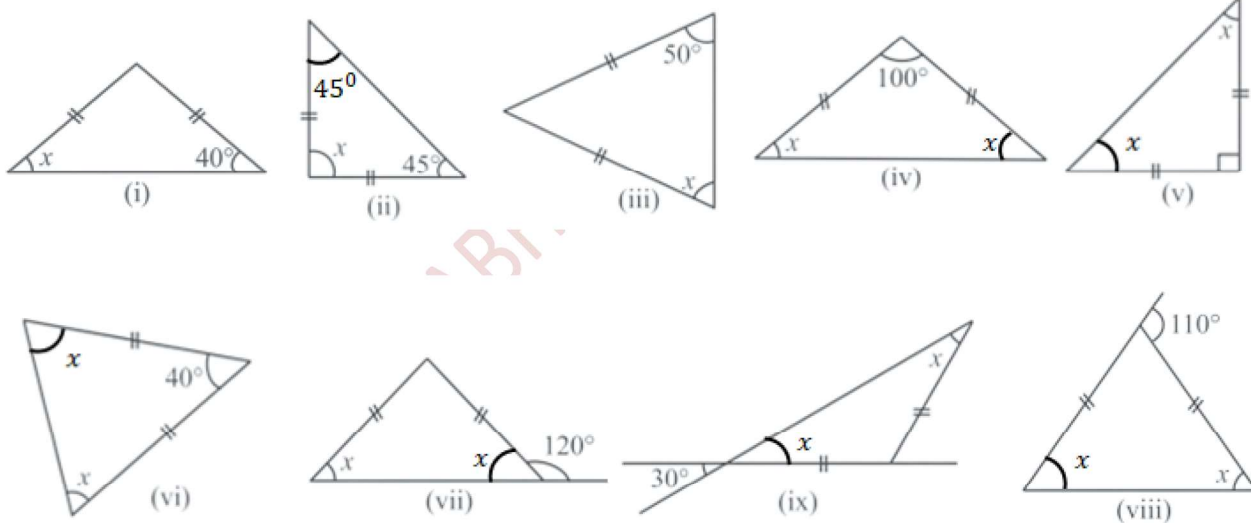
In an isosceles triangle base angles opposite to the equal sides are equal.



$$XY = XZ \Rightarrow \angle Y = \angle Z$$

### TRY THESE

1. Find angle  $x$  in each figure:



**Sol:** Equal sides opposite angles are equal.

$$(i) \quad x = 40^\circ$$

$$(ii) \quad x + 45^\circ + 45^\circ = 180^\circ$$

$$x + 90^\circ = 180^\circ$$

$$x = 180^\circ - 90^\circ = 90^\circ$$

$$(iii) \quad x = 50^\circ$$

$$(iv) \quad x + x + 100^\circ = 180^\circ$$

$$2x = 180^\circ - 100^\circ = 80^\circ$$

$$x = \frac{80^\circ}{2} = 40^\circ$$

$$(v) \quad x + x + 90^\circ = 180^\circ$$

$$2x = 180^\circ - 90^\circ = 90^\circ$$

$$x = \frac{90^\circ}{2} = 45^\circ$$

$$(vi) \quad x + x + 40^\circ = 180^\circ$$

$$2x = 180^\circ - 40^\circ = 140^\circ$$

$$x = \frac{140^\circ}{2} = 70^\circ$$

$$(vii) \quad x + 120^\circ = 180^\circ \text{ (Linear pair)}$$

$$x = 180^\circ - 120^\circ$$

$$x = 60^\circ$$

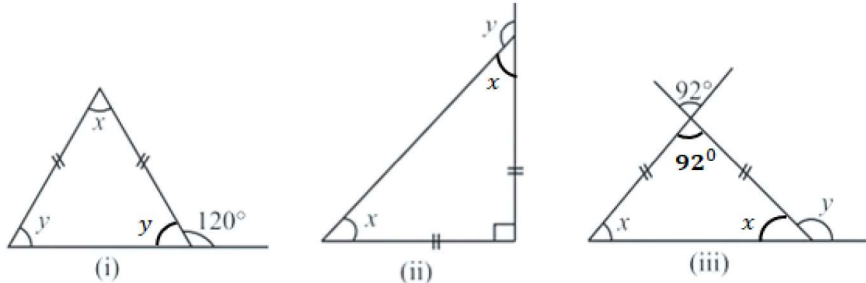
(viii)  $x + x = 110^\circ$  (Exterior angle property)

$$2x = 110^\circ$$

$$x = \frac{110^\circ}{2} = 55^\circ$$

(ix)  $x = 30^\circ$

2. Find angles  $x$  and  $y$  in each figure.



**Sol:**

(i)  $y + 120^\circ = 180^\circ$  (Linear pair)

$$y = 180^\circ - 120^\circ = 60^\circ$$

$x + y = 120^\circ$  (Exterior angle property)

$$x + 60^\circ = 120^\circ$$

$$x = 120^\circ - 60^\circ$$

$$x = 60^\circ$$

(ii)  $x + x + 90^\circ = 180^\circ$  (Angle sum property)

$$2x + 90^\circ = 180^\circ$$

$$2x = 180^\circ - 90^\circ$$

$$2x = 90^\circ$$

$$x = \frac{90^\circ}{2} = 45^\circ$$

$y = x + 90^\circ$  (Exterior angle property)

$$y = 45^\circ + 90^\circ = 135^\circ$$

(iii)  $x + x + 92^\circ = 180^\circ$  (Angle sum property)

$$2x + 92^\circ = 180^\circ$$

$$2x = 180^\circ - 92^\circ$$

$$2x = 88^\circ$$

$$x = \frac{88^\circ}{2} = 44^\circ$$

$x + y = 180^\circ$  (Linear pair)

$$44^\circ + y = 180^\circ$$

$$y = 180^\circ - 44^\circ = 136^\circ$$

### SUM OF THE LENGTHS OF TWO SIDES OF A TRIANGLE

The sum of the lengths of any two sides of a triangle is greater than the length of the third side.

The sum of the lengths of any two sides of a triangle is greater than the third side

**Example 3:** Is there a triangle whose sides have lengths 10.2 cm, 5.8 cm and 4.5 cm?

**Sol:**  $4.5 + 5.8 = 10.3 > 10.2$

$$5.8 + 10.2 = 16 > 4.5$$

$$10.2 + 4.5 = 14.7 > 5.8$$

The sum of the lengths of any two sides would be greater than the length of the third side.

Therefore, the triangle is possible.



**Example 4:** The lengths of two sides of a triangle are 6 cm and 8 cm. Between which two numbers can length of the third side fall?

**Sol:** The third side has to be less than the sum of the two sides.

The third side is thus, less than  $8 + 6 = 14$  cm

The third side has to be greater than the difference of the two sides. The third side is thus, greater than  $8 - 6 = 2$  cm

The length of the third side could be any length greater than 2 and less than 14 cm

## EXERCISE 6.4

1. Is it possible to have a triangle with the following sides?

(i) 2 cm, 3 cm, 5 cm

**Sol:**  $2 \text{ cm} + 3 \text{ cm} = 5 \text{ cm}$

The triangle is not possible

(ii) 3 cm, 6 cm, 7 cm

**Sol:**  $3 + 6 = 9 > 7$

$$3 + 7 = 10 > 6$$

$$6 + 7 = 13 > 3$$

The sum of the lengths of any two sides is greater than the length of the third side.

Therefore, the triangle is possible.

(iii) 6 cm, 3 cm, 2 cm

**Sol:**  $3 + 2 = 5 < 6$

The triangle is not possible

2. Take any point O in the interior of a triangle PQR. Is

(i)  $OP + OQ > PQ$ ?

**Sol:** Yes.

(ii)  $OQ + OR > QR$ ?

**Sol:** Yes.

iii)  $OR + OP > RP$ ?

**Sol:** Yes.

3. AM is a median of a triangle ABC.

Is  $AB + BC + CA > 2AM$ ?

(Consider the sides of triangles  $\triangle ABM$  and  $\triangle AMC$ .)

**Sol:** In  $\triangle ABM$ ,  $AB + BM > AM \rightarrow (1)$

In  $\triangle ACM$ ,  $MC + CA > AM \rightarrow (2)$

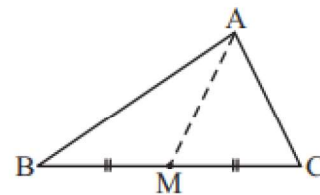
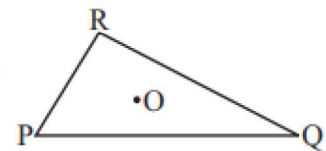
From (1)+(2)

$$AB + BM + MC + CA > AM + AM$$

$$AB + BC + CA > 2AM \quad (BM + MC = BC)$$

Hence, given statement is true.

4. ABCD is a quadrilateral.



**Is  $AB + BC + CD + DA > AC + BD$ ?**

**Sol:** In  $\triangle ABC$ ,  $AB+BC>AC \rightarrow(1)$

In  $\triangle BCD$ ,  $BC+CD>BD \rightarrow(2)$

In  $\triangle DCA$ ,  $CD+DA>AC \rightarrow(3)$

In  $\triangle DAB$ ,  $DA+AB>BD \rightarrow(4)$

Adding (1),(2),(3),(4)

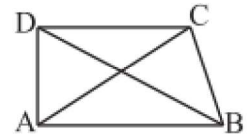
$$AB+BC+ BC+CD+ CD+DA+ DA+AB > AC+ BD+ AC+ BD$$

$$2AB+2BC+2CD+2DA > 2 AC+ 2BD$$

$$2(AB+BC+CD+DA) > 2(AC+BD)$$

$$AB+BC+CD+DA > AC+BD$$

Hence, given statement is true.



**5. ABCD is quadrilateral. Is  $AB + BC + CD + DA < 2(AC + BD)$ ?**

**Sol:** The sum of the lengths of any two sides is greater than the length of the third side.

In  $\triangle AOB$ ,  $OA+OB>AB \rightarrow(1)$

In  $\triangle BOC$ ,  $OB+OC>BC \rightarrow(2)$

In  $\triangle COD$ ,  $OC+OD>CD \rightarrow(3)$

In  $\triangle DOA$ ,  $OD+OA>DA \rightarrow(4)$

From (1)+(2)+(3)+(4)

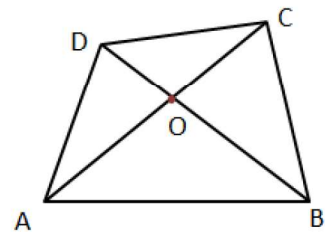
$$OA+OB+ OB+OC+ OC+OD+ OD+OA > AB+BC+CD+DA$$

$$2OA+2OB+2OC+2OD > AB+BC+CD+DA$$

$$2(OA+OB+ OC+OD) > AB+BC+CD+DA$$

$$2(AC+BD) > AB+BC+CD+DA$$

$$AB+BC+CD+DA < 2(AC+BD)$$



**6. The lengths of two sides of a triangle are 12 cm and 15 cm. Between what two measures should the length of the third side fall?**

**Sol:** The third side has to be less than the sum of the two sides.

The third side is thus, less than  $12 + 15 = 27$  cm

The third side has to be greater than the difference of the two sides.

The third side is thus, greater than  $15 - 12 = 3$  cm

The length of the third side could be any length greater than 3cm and less than 27 cm.

### THINK, DISCUSS AND WRITE

1. Is the sum of any two angles of a triangle always greater than the third angle?

**Sol:** No.

### RIGHT-ANGLED TRIANGLES AND PYTHAGORAS PROPERTY

In a right-angled triangle, the square on the hypotenuse = sum of the squares on the legs.

If the Pythagoras property holds, the triangle must be right-angled.

**Example 5:** Determine whether the triangle whose lengths of sides are 3 cm, 4 cm, 5 cm is a right-angled triangle

**Sol:**  $3^2 = 3 \times 3 = 9$ ;  $4^2 = 4 \times 4 = 16$ ;  $5^2 = 5 \times 5 = 25$

$$3^2 + 4^2 = 9 + 16 = 25 = 5^2$$

$$\text{We find } 3^2 + 4^2 = 5^2$$

Therefore, the triangle is right-angled.

**Example 6:**  $\Delta ABC$  is right-angled at C. If AC = 5 cm and BC = 12 cm find the length of AB

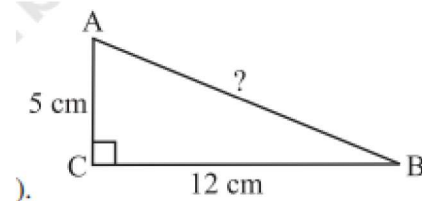
**Sol:** By Pythagoras property,

$$AB^2 = AC^2 + BC^2$$

$$= 5^2 + 12^2 = 25 + 144 = 169 = 13^2$$

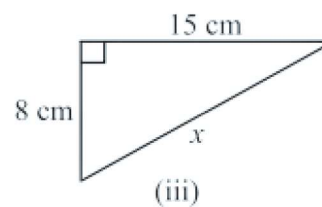
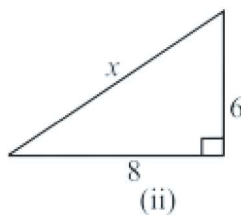
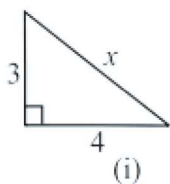
$$\text{So, } AB = 13$$

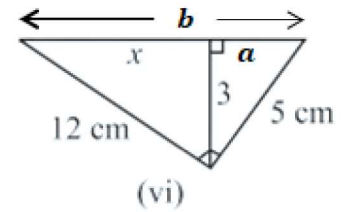
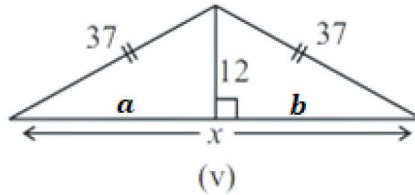
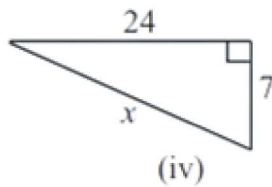
The length of AB is 13 cm



### TRY THESE

Find the unknown length  $x$  in the following figures





**Sol:** By Pythagoras property,

$$(i) \quad x^2 = 3^2 + 4^2 = 9 + 16 = 25 = 5^2$$

$$x = 5$$

$$(ii) \quad x^2 = 6^2 + 8^2 = 36 + 64 = 100 = 10^2$$

$$x = 10$$

$$(iii) \quad x^2 = 8^2 + 15^2 = 64 + 225 = 289 = 17^2$$

$$x = 17$$

$$(iv) \quad x^2 = 7^2 + 24^2 = 49 + 576 = 625 = 25^2$$

$$x = 25$$

$$(v) \quad a^2 = 37^2 - 12^2 = 1369 - 144 = 1225 = 35^2$$

$$a = 35 \text{ also } b = 35$$

$$x = 35 + 35 = 70$$

$$(vi) \quad b^2 = 12^2 + 5^2 = 144 + 25 = 169 = 13^2$$

$$b = 13$$

$$a^2 = 5^2 - 3^2 = 25 - 9 = 16 = 4^2$$

$$a = 4$$

$$x = b - a = 13 - 4 = 9$$

### EXERCISE 6.5

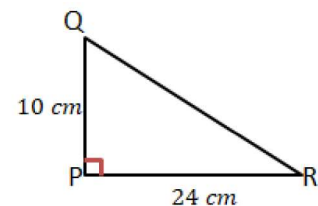
1. PQR is a triangle, right-angled at P. If PQ = 10 cm and PR = 24 cm, find QR.

**Sol:** By Pythagoras property,

$$QR^2 = PQ^2 + PR^2$$

$$= 10^2 + 24^2 = 100 + 576 = 676 = 26^2$$

$$\text{So, } QR = 26 \text{ cm}$$



2. ABC is a triangle, right-angled at C. If AB = 25 cm and AC = 7 cm, find BC.

**Sol:** By Pythagoras property,



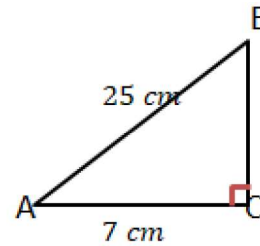
$$AC^2 + BC^2 = AB^2$$

$$7^2 + BC^2 = 25^2$$

$$49 + BC^2 = 625$$

$$BC^2 = 625 - 49 = 576 = 24^2$$

$$BC = 24 \text{ cm}$$



3. A 15 m long ladder reached a window 12 m high from the ground on placing it against a wall at a distance  $a$ . Find the distance of the foot of the ladder from the wall.

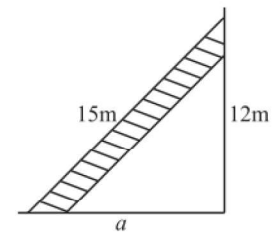
**Sol:** By Pythagoras property,

$$a^2 + 12^2 = 15^2$$

$$a^2 + 144 = 225$$

$$a^2 = 225 - 144 = 81 = 9^2$$

$$a = 9 \text{ cm}$$



4. Which of the following can be the sides of a right triangle? In the case of right-angled triangles, identify the right angles

(i) 2.5 cm, 6.5 cm, 6 cm.

**Sol:**  $(2.5)^2 + 6^2 = 6.25 + 36 = 42.25$

$$(6.5)^2 = 42.25$$

$$(2.5)^2 + 6^2 = (6.5)^2$$

Given sides form a right triangle.

(ii) 2 cm, 2 cm, 5 cm.

**Sol:**  $2^2 + 2^2 = 4 + 4 = 8$

$$5^2 = 25$$

$$2^2 + 2^2 \neq 5^2$$

The given sides are not of a right triangle.

(iii) 1.5 cm, 2cm, 2.5 cm.

**Sol:**  $(1.5)^2 + 2^2 = 2.25 + 4 = 6.25$

$$(2.5)^2 = 6.25$$

$$(1.5)^2 + 2^2 = (2.5)^2$$

Given sides form a right triangle

5. A tree is broken at a height of 5 m from the ground and its top touches the ground at a distance of 12 m from the base of the tree. Find the original height of the tree.

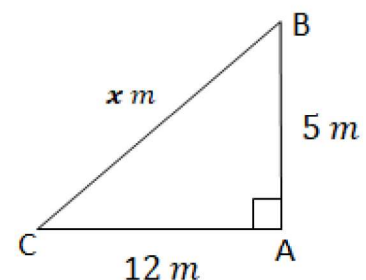
**Sol:** By Pythagoras property,

$$BC^2 = AB^2 + AC^2$$

$$BC^2 = 5^2 + 12^2 = 25 + 144 = 169 = 13^2$$

$$BC = 13 \text{ m}$$

The original height of the tree =  $AB + BC = 5 + 13 = 18 \text{ m}$



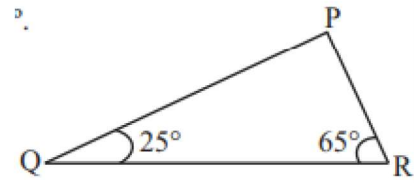
6. Angles Q and R of a  $\Delta PQR$  are  $25^\circ$  and  $65^\circ$ . Write which of the following is true:

$$(i) PQ^2 + QR^2 = RP^2 \quad (ii) PQ^2 + RP^2 = QR^2$$

$$(iii) RP^2 + QR^2 = PQ^2$$

$$\text{Sol: } \angle P = 180^\circ - (25^\circ + 65^\circ) = 180^\circ - 90^\circ = 90^\circ$$

$$(ii) PQ^2 + RP^2 = QR^2 \text{ is correct}$$



7. Find the perimeter of the rectangle whose length is 40 cm and a diagonal is 41 cm.

Sol: By Pythagoras property,

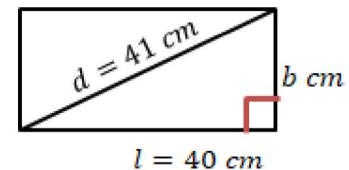
$$l^2 + b^2 = d^2$$

$$40^2 + b^2 = 41^2$$

$$b^2 = 41^2 - 40^2 = 1681 - 1600 = 81 = 9^2$$

$$b = 9 \text{ cm}$$

$$\text{The perimeter of the rectangle} = 2(l + b) = 2(40 + 9) = 2 \times 49 = 98 \text{ cm}$$



8. The diagonals of a rhombus measure 16 cm and 30 cm. Find its perimeter.

Sol: AC = 16 cm and BD = 30 cm

Diagonal of a rhombus perpendicularly bisect each other

$$OA = OC = \frac{AC}{2} = \frac{16}{2} = 8 \text{ cm}$$

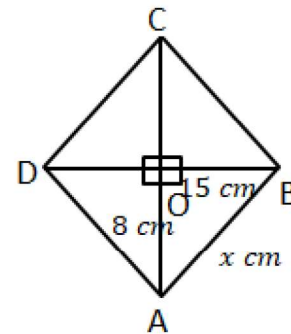
$$OB = OD = \frac{BD}{2} = \frac{30}{2} = 15 \text{ cm}$$

By Pythagoras property

$$AB^2 = OA^2 + OB^2 = 8^2 + 15^2 = 64 + 225 = 289 = 17^2$$

$$AB = 17 \text{ cm}$$

$$\text{Perimeter of the rhombus} = 4 \times AB = 4 \times 17 \text{ cm} = 68 \text{ cm}$$



### THINK, DISCUSS AND WRITE

1. Which is the longest side in the triangle PQR, right-angled at P?

Sol: QR

2. Which is the longest side in the triangle ABC, right-angled at B?

Sol: AC

3. Which is the longest side of a right triangle?

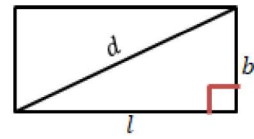
Sol: Opposite side of right angle (Hypotenuse)

4. 'The diagonal of a rectangle produce by itself the same area as produced by its length and breadth' – This is Baudhayan Theorem. Compare it with the Pythagoras property.

**Sol:** From Baudhayan theorem

$$l^2 + b^2 = d^2$$

So, Baudhayan Theorem and Pythagoras theorem are basically same.



### DO THIS

There are many proofs for Pythagoras theorem, using 'dissection' and 'rearrangement' procedure. Try to collect a few of them and draw charts explaining them.

5.  
6.

Please download VI to X class all maths notes from website

<https://sureshmathsmaterial.com/>

