CHAPTER 3

VI-MATHEMATICS-NCERT

3. PLAYING WITH NUMBERS (notes)

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1. **Factor**: A factor of a number is an exact divisor of that number.

NUMBER	FACTORS
1	1
2	1,2
3	1,3
4	1,2,4
5	1,5
6	1,2,3,6
7	1,7

NUMBER	FACTORS
8	1,2,4,8
9	1,3,9
10	1,2,5,10
11	1,11
12	1,2,3,4,6,12
13	1,13
14	1,2,7,14

NUMBER	FACTORS
15	1,3,5,15
16	1,2,4,8,16
17	1,17
18	1,2,3,6,9,18
19	1,19
20	1,2,4,5,10,20
21	1,3,7,21

2. 1 is a factor of every number.
3. Every number is a factor of itself.
4. Every factor is less than or equal to the given number.
5. Number of factors of a given number are finite
6. **Multiple**: A number multiplied by 1,2,3,4,...we get multiples of that number

Number	Multiples
1	1, 2, 3, 4, 5, 6, 7,...
2	2, 4, 6, 8, 10, 12, 14,.....
3	3, 6, 9, 12, 15,18,...
4	4, 8,12,16,20,24,...
5	5,10,15,20,25,30,..
6	6,12,18,24,30,36,...
7	7,14,21,28,35,42,49,56,63,70,.

7. Every multiple of a number is greater than or equal to that number
8. The number of multiples of a given number is infinite.
9. Every number is a multiple of itself.

Perfect number:

A number for which sum of all its factors is equal to twice the number is called a perfect number.

Ex: (i) Factors of 6 are 1,2,3,6

Sum of factors of $6=1+2+3+6=12=2\times6$

$\therefore$ 6 is a perfect number

(ii) Factors of 28 are 1,2,4,7,14,28.

Sum of factors of $28=1+2+4+714+28=56=2\times28$

$\therefore$ 28 is a perfect number

(iii) Next perfect number=496

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Try These

Find the possible factors of 45, 30 and 36.

(i) $45=1\times45$
$=3\times15$
$=5\times9$
Factors of 45 are:1,3,5,9,15,45

(ii) $30=1\times30$
$=2\times15$
$=3\times10$
$=5\times6$
Factors of 30 are:1,2,3,5,6,10,15,30.

(iii) $36=1\times36$
$=2\times18$
$=3\times12$
$=4\times9$
$=6\times6$
Factors of 36 are:1,2,3,4,6,9,12,18,36.

Example 1 : Write all the factors of 68.

Sol: $68 = 1 \times 68$
$= 2 \times 34$
$= 4 \times 17$
The factors of 68 are 1, 2, 4,17,34, 68.

Example 2 : Find the factors of 36.

Sol: $36 = 1 \times 36$
$= 2 \times 18$
$= 3 \times 12$
$= 4 \times 9$
$= 6 \times 6$
The factors of 68 are 1, 2, 3,4,6,9,12,18,36.

Example 3 : Write first five multiples of 6.

Sol: First 5 multiples of 6 are: $6 \times 1, 6 \times 2, 6 \times 3, 6 \times 4, 6 \times 5$

i.e. 6, 12, 18, 24 and 30.

EXERCISE 3.1

1. **Write all the factors of the following numbers :**

(a) 24

Sol:$24 = 1 \times 24$
$= 2 \times 12$
$= 3 \times 8$
$= 4 \times 6$
Factors of 24 are : 1,2,3,4,6,8,12,24

(b) 15

Sol: $15=1\times15=3\times5$
Factors of 15 are: 1,3,5,15.

(c) 21

Sol:$21=1\times21$

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$=3\times7$

Factors of 21 are:1,3,7,21

(d) 27

Sol: $27=1\times27$

$=3\times9$

Factors of 27 are: 1,3,9,27.

(e) 12

Sol: $12=1\times12$

$=2\times6$

$=3\times4$

Factors of 12 are:1,2,3,4,6,12

(f) 20

Sol: $20=1\times20$

$=2\times10$

$=4\times5$

Factors of 20 are 1,2,4,5,10,20.

(g) 18

Sol: $18=1\times18$

$=2\times9$

$=3\times6$

Factors of 18 are 1,2,3,6,9,18.

(h) 23

Sol: $23=1\times23$

Factors of 23 are 1,23.

(i) 36

Sol: $36=1\times36$

$=2\times18$

$=3\times12$

$=6\times6$

Factors of 36 are 1,2,3,6,12,18,36

2. Write first five multiples of :

(a) 5 (b) 8 (c) 9

Sol: (a) First 5 multiples of 5 are:5,10,15,20,25.

(b) First 5 multiples of 8 are: 8,16,24,32,40.

(c) First 5 multiples of 9 are: 9,18,27,36,45.

3. Match the items in column 1 with the items in column 2.

(i) 35 (b) Multiple of 7

(ii) 15 (d) Factor of 30

(iii) 16 (a) Multiple of 8

(iv) 20 (f) Factor of 20

(v) 25 (e) Factor of 50

4. Find all the multiples of 9 upto 100.

Sol: Multiples of 9 are: 9,18,27,36,45,54,63,72,81,90,99

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Prime and Composite Numbers

Number	Factors	No. of factors
1	1	1
2	1,2	2
3	1,3	2
4	1,2,4	3
5	1,5	2
6	1,2,3,6	4
7	1,7	2
8	1,2,4,8	4
9	1,3,9	3
10	1,2,5,10	4

Number	Factors	No. of factors
11	1,11	2
12	1,2,3,4,6,12	6
13	1,13	2
14	1,2,7,14	4
15	1,3,5,15	4
16	1,2,4,8,16	5
17	1,17	2
18	1,2,3,6,9,18	6
19	1,19	2
20	1,2,4,5,10,20	6

The numbers having exactly two factors are **prime numbers**.

Numbers having more than two factors are called **Composite numbers**

1 is neither a prime nor a composite number

Prime numbers up to 100 (25 numbers)
2,3,5,7,11,13,17,19,23,29,31,37,41,43,47,53,59,61,67,71,73,79,83,89,97

Twin primes:

Two prime numbers whose difference is 2 are called twin primes

Ex: 3,5; 5,7; 11,13; 17,19; 71,73;

Try These

Observe that $2 \times 3 + 1 = 7$ is a prime number. Here, 1 has been added to a multiple of 2 to get a prime number. Can you find some more numbers of this type?

(i) $2\times5+1=11$ is a prime number

(ii) $2\times6+1=13$ is a prime number

(iii) $2\times8+1=17$ is a prime number

(iv) $2\times9+1=19$ is a prime number

(v) $2\times11+1=23$ is a prime number

Example 4 : Write all the prime numbers less than 15

Sol: 2,3,5,7,11,13

Even and odd numbers .

The multiples of 2 are called even numbers. The rest of natural numbers are called odd numbers

Even numbers: 2,4,6,8,10,12,14,16,18,20,....

Odd Numbers: 1,3,5,7,9,11,13,15,17,19,.....

1. 2 is the even prime number.
2. Every prime number except 2 is odd.

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EXERCISE 3.2

1. What is the sum of any two (a) Odd numbers? (b) Even numbers?

Sol: (a) (i) 1+3=4 (ii) 3+7=10 (iii) 5+7=12

Sum of two odd numbers is even number.

(b) (i) 2+4=6 (ii) 4+12=16 (iii) 8+12=20

Sum of two even numbers is even number.

2. State whether the following statements are True or False:

(a) The sum of three odd numbers is even. (F)

(b) The sum of two odd numbers and one even number is even. (T)

(c) The product of three odd numbers is odd. (T)

(d) If an even number is divided by 2, the quotient is always odd. (F)

(e) All prime numbers are odd. (F)

(f) Prime numbers do not have any factors. (F)
(g) Sum of two prime numbers is always even. (F)
(h) 2 is the only even prime number. (T)
(i) All even numbers are composite numbers. (F)
(j) The product of two even numbers is always even. (T)

3. The numbers 13 and 31 are prime numbers. Both these numbers have same digits 1 and 3. Find such pairs of prime numbers upto 100

Sol: 17,71; 37,73; 79,97.

4. Write down separately the prime and composite numbers less than 20.

Sol: Prime numbers less than 20: 2,3,5,7,11,13,17,19.

Composite numbers less than 20: 4,6,8,9,10,12,14,15,16,18.

5. What is the greatest prime number between 1 and 10?

Sol: 7

6. Express the following as the sum of two odd primes.
(a) 44 (b) 36 (c) 24 (d) 18

Sol: (a) 44=3+41=7+37=13+31

(b) 36=5+31=7+29=13+23=17+19

(c) 24=5+19=7+17=11+13

(d) 18=5+13=7+11

7. Give three pairs of prime numbers whose difference is 2

Sol: (i) 5,7 (ii) 11,13 (iii) 17,19 (iv) 27,29 (v) 41,43

8. Which of the following numbers are prime?
(a) 23 (b) 51 (c) 37 (d) 26

Sol: (a) 23 (c) 37

9. Write seven consecutive composite numbers less than 100 so that there is no prime number between them.

Sol: 90,91,92,93,94,95,96.

10. Express each of the following numbers as the sum of three odd primes:
(a) 21 (b) 31 (c) 53 (d) 61

Sol: (a) 21=3+5+13=3+7+11

(b) 31=5+7+19=7+11+13

(c) 53=13+17+23=11+19+23

(d) 61=11+13+37=11+19+31

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11. **Write five pairs of prime numbers less than 20 whose sum is divisible by 5.**

Sol: (i) 2,3 (ii) 2,13 (iii) 3,7 (iv) 3,17 (v) 7,13

12. Fill in the blanks :

(a) A number which has only two factors is called a Prime number.
(b) A number which has more than two factors is called a Composite number.
(c) 1 is neither Prime nor Composite.
(d) The smallest prime number is 2.
(e) The smallest composite number is 4.
(f) The smallest even number is 2.

Tests for Divisibility of Numbers

1. **Divisibility by 10**: If a number has 0 in the ones place then it is divisible by 10.
   Ex: 10,30,500,420,6000,... are divisible by 10
2. **Divisibility by 5**: A number which has either 0 or 5 in its ones place is divisible by 5.
   Ex: 15,20,35,545,6020,215,...are divisible by 5.
3. **Divisibility by 2** : A number is divisible by 2 if it has any of the digits 0, 2, 4, 6 or 8 in its ones place.
   Ex: 48,50,64,848,520,362,...
4. **Divisibility by 3**: If the sum of the digits is a multiple of 3, then the number is divisible by 3
   Ex: 36,54,153,642,... are divisible by 3.
5. **Divisibility by 6:** if a number is divisible by 2 and 3 both then it is divisible by 6 also.
6. **Divisibility by 4**: A number with 3 or more digits is divisible by 4 if the number formed by its last two digits (i.e. ones and tens) is divisible by 4.
7. **Divisibility by 8** : A number with 4 or more digits is divisible by 8, if the number formed by the last three digits is divisible by 8.
8. **Divisibility by 9** : If the sum of the digits of a number is divisible by 9, then the number itself is divisible by 9
9. **Divisibility by 11**: Find the difference between the sum of the digits at odd places (from the right) and the sum of the digits at even places (from the right) of the number. If the difference is either 0 or divisible by 11, then the number is divisible by 11.

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EXERCISE 3.3

1. **Using divisibility tests complete the table.**

Number	Divisible by								
	2	3	4	5	6	8	9	10	11
128	Yes	No	Yes	No	No	Yes	No	No	No
990	Yes	Yes	No	Yes	Yes	No	Yes	Yes	Yes
1586	Yes	No	No	No	No	No	No	No	No
275	No	No	No	Yes	No	No	No	No	Yes
6686	Yes	No	No	No	No	No	No	No	No
639210	Yes	Yes	No	Yes	Yes	No	No	Yes	Yes
429714	Yes	Yes	No	No	Yes	No	Yes	No	No
2856	Yes	Yes	Yes	No	Yes	Yes	No	No	No
3060	Yes	Yes	Yes	Yes	Yes	No	Yes	Yes	No
406839	No	Yes	No	No	No	No	No	No	No

2. **Using divisibility tests, determine which of the following numbers are divisible by 4; by 8**

If the last two digit number is divisible by 4 then the number is divisible by 4.
If the last three digit number is divisible by 8 then the number is divisible by 8.

(a) 572

Sol: Number formed by last two digits=72 =4×18 which is divisible by 4
So, 572 is divisible by4
Number formed last three digits=572 is not divisible by 8
So, 572 is not divisible by 8

(b) 726352

Sol: Number formed by last two digits =52 =4×13 which is divisible by 4
So, 726352 is divisible by4
Number formed last three digits =352 =44×8 which is divisible by 8
So, 726352 is divisible by 8

(c) 5500

Sol: Number formed by last two digits =00
So, 5500 is divisible by 4
Number formed last three digits =500 is not divisible by 8
So, 5500 is not divisible by 8

(d) 6000

Sol: Number formed by last two digits =00
So, 6000 is divisible by 4
Number formed last three digits =000 is divisible by 8
So, 6000 is divisible by 8

(e) 12159

Sol: Number formed by last two digits =59 is not divisible by 8
So, 12159 is divisible by 4
Number formed by last three digits =159 is not divisible by 8
So, 12159 is not divisible by 8

(f) 14560

Sol: Number formed by last two digits =60=4×15
So, 14560 is divisible by 4
Number formed by last three digits =560=8×70 is divisible by 8
So, 14560 is divisible by 8

(g) 21084

Sol: Number formed by last two digits =00
So, 5500 is divisible by 4
Number formed last three digits =500 is not divisible by 8
So, 5500 is not divisible by 8

(h) 31795072

Sol: Number formed by last two digits =72(=4×18) is divisible by 4
So, 31795072 is divisible by 4
Number formed last three digits =072=72(=8×9) is divisible by 8
So, 31795072 is divisible by 8

(i) 1700

Sol: Number formed by last two digits =00
So, 1700 is divisible by 4
Number formed last three digits =700 is not divisible by 8

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So, 1700 is not divisible by 8

(j) 2150.

Sol: Number formed by last two digits =50 is not divisible by 4

So, 2150 is not divisible by 4

Number formed last three digits =150 is not divisible by 8

So, 2150 is not divisible by 8

3. Using divisibility tests, determine which of following numbers are divisible by 6:

If a number is divisible by 2 and 3 both then it is divisible by 6 also.

(a) 297144

Sol: Last digit=4 . So, 297144 is divisible by 2

Sum of digits=2+9+7+1+4+4=27 is divisible by3

So, 297144 is divisible by 3

Now 297144 is divisible by 2 and 3 .So, 297144 is divisible by 6

(b) 1258

Sol: Last digit=4 . So, 297144 is divisible by 2

Sum of digits=2+9+7+1+4+4=27 is divisible by3

So, 297144 is divisible by 3

Now 297144 is divisible by 2 and 3 .So, 297144 is divisible by 6

(c) 4335

Sol: Last digit=4 . So, 297144 is divisible by 2

Sum of digits=2+9+7+1+4+4=27 is divisible by3

So, 297144 is divisible by 3

Now 297144 is divisible by 2 and 3 .So, 297144 is divisible by 6

(d) 61233

Sol: Last digit=4 . So, 297144 is divisible by 2

Sum of digits=2+9+7+1+4+4=27 is divisible by3

So, 297144 is divisible by 3

Now 297144 is divisible by 2 and 3 .So, 297144 is divisible by 6

(e) 901352

Sol: Last digit=2 . So, 901352 is divisible by 2

Sum of digits=9+0+1+3+5+2=20 is not divisible by3

So, 901352 is not divisible by 3

Now 901352 is divisible by 2 and not divisible by 3 .So, 901352 is not divisible by 6.

(f) 438750

Sol: Last digit=0 . So, 438750 is divisible by 2

Sum of digits=4+3+8+7+5+0=27 is divisible by3

So, 438750 is divisible by 3

Now 438750 is divisible by 2 and 3 .So, 438750 is divisible by 6

(g) 1790184

Sol: Last digit=4 . So, 1790184 is divisible by 2

Sum of digits=1+7+9+0+1+8+4=30 is divisible by3

So, 1790184 is divisible by 3

Now 1790184 is divisible by 2 and 3 .So, 1790184 is divisible by 6

(h) 12583

Sol: Last digit=3 . So, 297144 is not divisible by 2

Now 297144 is not divisible by 2 .So, 297144 is not divisible by 6

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(i) 639210

Sol: Last digit=0 . So, 639210 is divisible by 2

Sum of digits=6+3+9+2+1+0=21 is divisible by3

So, 639210 is divisible by 3

Now 639210 is divisible by 2 and 3 .So, 639210 is divisible by 6

(j) 17852.

Sol: Last digit=4 . So, 17852 is divisible by 2

Sum of digits=1+7+8+5+2=23 is not divisible by3

So, 17852 is not divisible by 3

Now 17852 is divisible by 2 and not divisible by 3 .So, 17852 is not divisible by 6.

4. **Using divisibility tests, determine which of the following numbers are divisible by 11:**

(If the difference between the sum of the digits at odd places and the sum of the digits at even places of the number is either 0 or divisible by 11 then the number is divisible by 11)

(a) 5445

Sol: Sum of the digits at odd places=5+4=9

Sum of the digits at even places=4+5=9

Difference=9-9=0

$\therefore$ 5445 is divisible by 11

(b) 10824

Sol: Sum of the digits at odd places=4+8+1=13

Sum of the digits at even places=2+0=2

Difference=13-2=11

$\therefore$ 10824 is divisible by 11

(c) 7138965

Sol: Sum of the digits at odd places=5+9+3+7=24

Sum of the digits at even places=6+8+1=15

Difference=24-15=9

$\therefore$ 7138965 is not divisible by 11

(d) 70169308

Sol: Sum of the digits at odd places=8+3+6+0=17

Sum of the digits at even places=0+9+1+7=17

Difference=17-17=0

$\therefore$ 70169308 is divisible by 11

(e) 10000001

Sol: Sum of the digits at odd places=1+0+0+0=1

Sum of the digits at even places=0+0+0+1

Difference=1-1=0

$\therefore$ 10000001 is divisible by 11

(f) 901153

Sol: Sum of the digits at odd places=3+1+0=4

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Sum of the digits at even places=5+1+9=15
Difference=15-4=11
∴ 901153 is divisible by 11

5. Write the smallest digit and the greatest digit in the blank space of each of the following numbers so that the number formed is divisible by 3 :

(a) _ 6724

Sol: Sum of digits=6+7+2+4=19
If we add 2 the number is 21 is divisible by 3
Required smallest digit is 2
If we add 8 the number is 27 is divisible by 3
Required greatest digit=8

(b) 4765 _ 2

Sol: Sum of digits=4+7+6+5+2=24
If we add 0 the number is 24 is divisible by 3
Required smallest digit is 0
If we add 9 the number is 33 is divisible by 3
Required greatest digit=9

6. Write a digit in the blank space of each of the following numbers so that the number formed is divisible by 11

(a) 92 _ 389

Sol: Sum of digits at odd places=9+3+2=14
Sum of digits at even places=8+x+9=17+x
Difference=17+x-14=3+x
3+x=0 or 11or 22...
3+x=11
X=8

(b) 8 _ 9484

Sol: Sum of digits at odd places=4+4+x=8+x
Sum of digits at even places=8+9+8=25
Difference=25-8-x=17-x
17-x=0 or 11or 22...
17-x=11
X=6

Common Factors and Common Multiples.

Try These

Find the common factors of

(a) 8, 20

Sol: Factors of 8: 1,2,4,8

Factors of 20: 1,2,4,5,10,20

Common factors of 8,20 : 1,2,4

(b) 9, 15

Sol: Factors of 9: 1,3,9

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Factors of 15: 1,3,5,15

Common Factors of 9,15 : 1,3

(c) 4,18

Sol: Factors of 4: 1,2,4

Factors of 18: 1,2,3,6,9,18.

Common Factors of4,18 : 1,2

(d) 4,15

Sol: Factors of 4: 1,2,4.

Factors of 15: 1,3,5,15

Common Factors of 9,15 : 1

(e) 4,12,16

Sol: Factors of 4 : 1,2,4.

Factors of 12: 1,2,3,4,6,12.

Factors of 16: 1,2,4,8,16.

Common Factors of 4,12,16 : 1,2,4

Co-prime numbers:

Two numbers having only 1 as a common factor are called co-prime numbers.

Ex: (i) 4,15 (ii) 7,8 (iii) 12,49 (iv) 18,23

Exp 5 : Find the common factors of 75, 60 and 210

Sol: Factors of 75 are 1, 3, 5, 15, 25 and 75.

Factors of 60 are 1, 2, 3, 4, 5, 6, 10, 12, 15, 30 and 60.

Factors of 210 are 1, 2, 3, 5, 6, 7, 10, 14, 15, 21, 30, 35, 42, 70, 105 and 210.

Thus, common factors of 75, 60 and 210 are 1, 3, 5 and 15.

Exp 6 : Find the common multiples of 3, 4 and 9.

Sol : Multiples of 3 are 3, 6, 9, 12, 15, 18, 21, 24, 27, 30, 33, 36, 39, 42, 45, 48, ...

Multiples of 4 are 4, 8, 12, 16, 20, 24, 28, 32, 36, 40, 44, 48,...

Multiples of 9 are 9, 18, 27, 36, 45, 54, 63, 72, 81, ...

Clearly, common multiples of 3, 4 and 9 are 36, 72, 108,...

EXERCISE 3.4

1. **Find the common factors of :**

(a) 20 and 28

Sol: Factors of 20 are 1,2,4,5,10,20.

Factors of 28 are 1,2,4,7,14,28.

Common factors of 20 and 28 are 1,2,4.

(b) 15 and 25

Sol: Factors of 15 are 1,3,5,15.

Factors of 25 are 1,5,25.

Common factors of 15 and 25 are 1,5.

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(c) 35 and 50

Sol: Factors of 35 are 1,5,7,35.

Factors of 50 are 1,2,5,10,25,50.

Common factors of 35 and 50 are 1,5.

(d) 56 and 120

Sol: Factors of 56 are 1,2,4,7,8,14,28,56.

Factors of 120 are 1,2,4,5,6,8,10,12,15,20,30,40,60,120.

Common factors of 56 and 120 are 1,2,4,8.

2. Find the common factors of :

(a) 4, 8 and 12

Sol: Factors of 4 are 1,2,4.

Factors of 8 are 1,2,4,8.

Factors of 12 are 1,2,3,4,6,12.

Common Factors of 4,8,12 are 1, 2, and 4

(b) 5, 15 and 25

Sol: Factors of 5 are 1, 5.

Factors of 15 are 1, 3, 5.

Factors of 25 are 1,5,25.

Common Factors of 5, 15, 25 are 1 and 5.

3. Find first three common multiples of :

(a) 6 and 8

Sol: Multiples of 6 are 6, 12, 18, 24, 30, 36, 42 ,48, 54, 60, 66, 72, 78, 84, ...

Multiples of 8 are 8, 16, 24, 32, 40, 48, 56, 64, 72, 80,.....

First three common multiples of 6 and 8 are 24,48 and 72

(b) 12 and 18

Sol: Multiples of 12 are 12, 24, 36, 48, 60, 72, 84, 96, 108, 120, 132,...

Multiples of 18 are 18, 36, 54, 72, 90, 108, 126, 144,...

First three common multiples of 12 and 18 are 36,72 and 108.

4. Write all the numbers less than 100 which are common multiples of 3 and 4.

Sol: Common multiples of 3 and 4 (multiples of 12) less than 100 are 12,24,36,48,60,72,84,96

5. Which of the following numbers are co-prime?

(a) 18 and 35

Sol: Factors of 18 are 1,2,3,6,9,18

Factors of 35 are 1,5,7,35

Common factor of 18 and 35 is 1

$\therefore$ 18 and 35 are co-primes

(b) 15 and 37

Sol: Factors of 15 are 1,3,5,15

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Factors of 35 are 1,37
Common factor of 15 and 37 is 1
$\therefore$ 15 and 37 are co-primes

(c) 30 and 415

Sol: Factors of 30 are 1,2,3,5,6,10,15,30.
Factors of 415 are 1,5,83,415
Common factor s of 30 and 415 are 1,5
$\therefore$ 30 and 415 are not co-primes

(d) 17 and 68

Sol: Factors of 17 are 1,17.
Factors of 68 are 1,2,4,17,34,68
Common factors of 17 and 68 are 1,17
$\therefore$ 17 and 68 are not co-primes.

(e) 216 and 215

Sol: Factors of 216 are 1,2,3,4,6,8,9,12,18,24,27,36,54,72,108,216
Factors of 215 are 1,5,43,215.
Common factor of 18 and 35 is 1
$\therefore$ 18 and 35 are co-primes

(f) 81 and 16

Sol: Factors of 18 are 1,2,3,6,9,18
Factors of 35 are 1,5,7,35
Common factor of 216 and 215is 1
$\therefore$ 216 and 215are co-primes

6. A number is divisible by both 5 and 12. By which other number will that number be always divisible?

Sol: The number divisible by 5 and 12 is $5\times12=60$ and multiples of 60

The required number is 60.

7. A number is divisible by 12. By what other numbers will that number be divisible?

Sol: If a number divisible by 12 then the number divisible by the factors of 12 also.

The number will also divisible by 1,2,3,4,6 and 12

Prime Factorisation

A number is expressed as a product of prime numbers the factorisation is called prime factorisation.

1. Write the prime factorisations of 16,28,38

Sol: $16=2\times2\times2\times2$.

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$28=2\times2\times7$

$38=2\times19$

2	980
2	490
5	245
7	49
7	7
	1

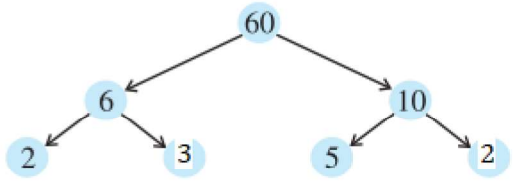
Expl 7 : Find the prime factorisation of 980.

Sol: $980=2\times2\times5\times7\times7$

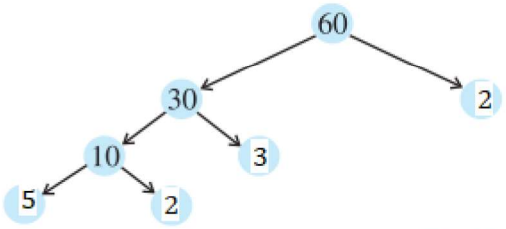
EXERCISE 3.5

1. Here are two different factor trees for 60. Write the missing numbers.

(a)



(b)



2. Which factors are not included in the prime factorisation of a composite number?

Sol: 1 and itself are not included in the prime factorisation of a composite number.

3. Write the greatest 4-digit number and express it in terms of its prime factors.

Sol: The greatest 4-digit number=9999

$9999=3\times3\times11\times101$

3	9999
3	3333
11	1111
	101

4. Write the smallest 5-digit number and express it in the form of its prime factors.

2	10000
2	5000
2	2500
2	1250
5	625
5	125
5	25
	5

Sol: The smallest 5-digit number=10000

$10000=2\times2\times2\times2\times5\times5\times5\times5$

5. Find all the prime factors of 1729 and arrange them in ascending order. Now state the relation, if any; between two consecutive prime factors.

7	1729
13	247
	19

Sol: $1729=7\times13\times19$

Difference between two consecutive prime factors is 6

$13-7=6$ and $19-13=6$

6. The product of three consecutive numbers is always divisible by 6. Verify this statement with the help of some examples.

Sol: Exp 1: 8,9,10 are three consecutive numbers

Product of the numbers=8×9×10=720

720=6×120 is divisible by 6

Exp 2: 13,14,15

Product of the numbers=13×14×15=2730=6×455 is divisible by 6

7. The sum of two consecutive odd numbers is divisible by 4. Verify this statement with the help of some examples.

Sol: (i) Two consecutive odd numbers :7,9

Sum of the numbers=7+9=16=4×4 is divisible by 4.

(ii) 13,15

Sum of the numbers=13+15=28=4×7 is divisible by 4.

8. In which of the following expressions, prime factorisation has been done?

(a) $24 = 2 \times 3 \times 4$

Sol: prime factorisation has not been done. Since 4 is not a prime number

(b) $56 = 7 \times 2 \times 2 \times 2$

Sol: prime factorisation has been done.

(c) $70 = 2 \times 5 \times 7$

Sol: prime factorisation has been done

(d) $54 = 2 \times 3 \times 9$

Sol: prime factorisation has not been done. Since 9 is not a prime number

9. 18 is divisible by both 2 and 3. It is also divisible by $2 \times 3 = 6$. Similarly, a number is divisible by both 4 and 6. Can we say that the number must also be divisible by $4 \times 6 = 24$? If not, give an example to justify your answer

Sol: 12 is divisible by 4 and 6 but 12 is not divisible by 24

10. I am the smallest number, having four different prime factors. Can you find me?

Sol: Required number=2×3×5×7=210

Highest Common Factor (HCF)

The Highest Common Factor (HCF) of two or more given numbers is the highest (or greatest) of their common factors.

Try These

Find the HCF of the following:

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(i) 24 and 36

Sol: Factors of 24 are 1,2,3,4,6,8,12,24.

Factors of 36 are 1,2,3,4,6,9,12,18,36

Common factors of 24 and 36 are 1,2,3,4,6,12

HCF of 24 and 36 =12

(ii) 15, 25 and 30

Sol: Factors of 15:1,3,5,15

Factors of 25:1,5,25

Factors of 30:1,2,3,5,6,10,15,30

Common factors of 15, 25 and 30 are 1,5

HCF of 15, 25 and 30=5

(iii) 8 and 12

Sol: Factors of 8:1,2,4,8

Factors of 12:1,2,3,4,6,12

Common factors of 8 and 12 are 1,2,4

HCF of 8 and 12 =4

(iv) 12, 16 and 28

Sol: Factors of 12:1,2,3,4,6,12

Factors of 16:1,2,4,8,16

Factors of 28:1,2,4,7,14,28

Common factors of 12,16 and 28 are 1,2,4.

HCF of 12,16 and 28 =4

Finding HCF by prime factorisation .

1. **Find HCF of 20,28,36 by prime factorisation.**

Sol:

$20 = \boxed{2} \times \boxed{2} \times 5$

$28 = \boxed{2} \times \boxed{2} \times 7$

$36 = \boxed{2} \times \boxed{2} \times 3 \times 3$

2	20
2	10
5	5
	1

2	28
2	14
7	7
	1

2	36
2	18
3	9
3	3
	1

HCF of 20, 28 and 36 is $2 \times 2 = 4$

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EXERCISE 3.6

1. Find the HCF of the following numbers :

(a) 18, 48

Sol: 18=2×3×3

48=2×2×2×2×3

HCF of 18,48 = 2×3=6

2	18
3	9
	3

2	48
2	24
2	12
2	6
	3

(b) 30, 42

Sol: 30=2×3×5

42=2×3×7

HCF of 30,42=2×3=6

2	30
3	15
	5

2	42
3	21
	7

(c) 18, 60

Sol: 18=2×3×3

60=2×2×3×5

HCF of 18,60=2×3=6

2	18
3	9
	3

2	60
2	30
3	15
	5

(d) 27, 63

Sol: 27=3×3×3

63=3×3×7

HCF of 27,63=3×3=9

3	27
3	9
	3

3	63
3	21
	7

(e) 36, 84

Sol:36=2×2×3×3

84=2×2×3×7

HCF of 36,84=2×2×3=12

2	36
2	18
3	9
	3

2	84
2	42
3	21
	7

(f) 34, 102

Sol:34=2×17

102=2×3×17

HCF of 34,102=2×17=54

2	34
	17

2	102
3	51
	17

(g) 70, 105, 175

Sol: 70=2×5×7

105=3×5×7

175=5×5×7

HCF of 70,105,175=5×7=35

2	70
5	35
	7

3	105
5	35
	7

5	175
5	35
	7

(h) 91, 112, 49

Sol: 91=7×13

112=2×2×2×2×7

49=7×7

HCF of 91,112,49=7

7	91
	13

2	112
2	56
2	28
2	14
	7

7	49
	7

(i) 18, 54, 81

Sol: 18=2×3×3

54=2×3×3×3

81=3×3×3×3

HCF of 18,54,81=3×3=9

2	18
3	9
	3

2	54
3	27
3	9
	3

3	81
3	27
3	9
	3

(j) 12, 45, 75

Sol: 12=2×2×3

45=3×3×5

2	12
2	6
	3

3	45
3	15
	5

3	75
5	25
	5

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$75=3\times5\times5$

HCF of 12,45,75=3

2. What is the HCF of two consecutive (a) numbers? (b) even numbers? (c) odd numbers?

Sol: (a) HCF of two consecutive numbers=1

(b) HCF of two even numbers=2

(c) HCF of two odd numbers=1

3. HCF of co-prime numbers 4 and 15 was found as follows by factorisation : $4 = 2 \times 2$ and $15 = 3 \times 5$ since there is no common prime factor, so HCF of 4 and 15 is 0. Is the answer correct? If not, what is the correct HCF?

Sol: The answer is incorrect.

The HCF of 4 and 15=1

Lowest Common Multiple (LCM)

The Lowest Common Multiple (LCM) of two or more given numbers is the lowest (or smallest or least) of their common multiples.

Exp 8 : Find the LCM of 12 and 18.

Sol: Multiples of 12 are 12, 24, 36, 48, 60, 72, 84, 96, 120, ...

Multiples of 18 are 18, 36, 54,72, 90, 108,.....

Common multiples of 12 and 18 are 36, 72, 108, ...

LCM of 12 and 18=36

Finding LCM by prime factorisation:

The LCM of the two numbers is the product of the prime factors counted the maximum number of times they occur in any of the numbers

Example 9 : Find the LCM of 24 and 90.

Sol: $24=2\times2\times2\times3$

$90=2\times3\times3\times5$

LCM of 24 and 90= $(2 \times 2 \times 2) \times (3 \times 3) \times 5 = 360$

Example 10 : Find the LCM of 40, 48 and 45

Sol: $40 = 2 \times 2 \times 2 \times 5$

$48 = 2 \times 2 \times 2 \times 2 \times 3$

$45 = 3 \times 3 \times 5$

LCM = $(2 \times 2 \times 2 \times 2)\times(3 \times 3) \times 5 = 720$

2	20	25	30
2	10	25	15
3	5	25	15
5	5	25	5
5	1	5	1
	1	1	1

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Example 11 : Find the LCM of 20, 25 and 30

Sol: LCM = $2 \times 2 \times 3 \times 5 \times 5 = 300$

Some Problems on HCF and LCM:

Example 12 : Two tankers contain 850 litres and 680 litres of kerosene oil respectively. Find the maximum capacity of a container which can measure the kerosene oil of both the tankers when used an exact number of times.

2	850
5	425
5	85
17	17
	1

2	680
2	340
2	170
5	85
17	17
	1

Sol:

$850 = 2 \times 5 \times 5 \times 17 = \boxed{2} \times \boxed{5} \times \boxed{17} \times 5$

$680 = 2 \times 2 \times 2 \times 5 \times 17 = \boxed{2} \times \boxed{5} \times \boxed{17} \times 2 \times 2$

The HCF of 850 and 680 = $2 \times 5 \times 17 = 170$.

Maximum capacity of the required container is 170 litres.

Example 13 : In a morning walk, three persons step off together. Their steps measure 80 cm, 85 cm and 90 cm respectively. What is the minimum distance each should walk so that all can cover the same distance in complete steps?

Sol: LCM of 80, 85 and 90 = 12240

The required minimum distance is 12240 cm.

2	12	16	24	36
2	6	8	12	18
2	3	4	6	9
2	3	2	3	9
3	3	1	3	9
3	1	1	1	3
	1	1	1	1

Example 14 : Find the least number which when divided by 12, 16, 24 and 36 leaves a remainder 7 in each case.

Sol: LCM = $2 \times 2 \times 2 \times 2 \times 3 \times 3 = 144$

The required number is 7 more than 144.

The required least number = 144 + 7 = 151.

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EXERCISE 3.7

1. Renu purchases two bags of fertiliser of weights 75 kg and 69 kg. Find the maximum value of weight which can measure the weight of the fertiliser exact number of times.

Sol: we will find the HCF of 75 and 69

$75 = 3 \times 5 \times 5$

$69 = 3 \times 23$

HCF of 75, 69 = 3

The required maximum value of weight = 3 kg.

2. Three boys step off together from the same spot. Their steps measure 63 cm, 70 cm and 77 cm respectively. What is the minimum distance each should cover so that all can cover the distance in complete steps?

7	63, 70, 77
9	9, 10, 11
10	1, 10, 11
11	1, 1, 11
	1, 1, 1

Sol: We will find the LCM of 63, 70, 77

LCM of 63, 70, 77=7×9×10×11=6930

The minimum distance each should cover so that all can cover the distance in complete steps =6930 cm

3. The length, breadth and height of a room are 825 cm, 675 cm and 450 cm respectively. Find the longest tape which can measure the three dimensions of the room exactly.

Sol: We will find HCF of 825,675,450.

825=3×5×5×11

675=3×3×3×5×5

450=2×3×3×5×5

HCF of 825,675,450=3×5×5=75

Required longest tape=75 cm

4. Determine the smallest 3-digit number which is exactly divisible by 6, 8 and 12.

2	6, 8, 12
2	3, 4, 6
2	3, 2, 3
3	3, 1, 3
	1, 1, 1

Sol: We will find LCM of 6,8,12

LCM of 6,8,12=2×2×2×3=24

Multiples of 24 are 24,48,72,96,120,...

The smallest 3-digit number which is exactly divisible by 6, 8 and 12=120

5. Determine the greatest 3-digit number exactly divisible by 8, 10 and 12.

2	8, 10, 12
2	4, 5, 6
2	2, 5, 3
3	1, 5, 3
5	1, 5, 1
	1, 1, 1

Sol: LCM of 8 10 and 12=2×2×2×3×5=120

Multiples of 120 are 120, 240, 360, 480, 600, 720, 840, 960, 1080,..

The greatest 3-digit number exactly divisible by 8, 10 and 12=960

6. The traffic lights at three different road crossings change after every 48 seconds, 72 seconds and 108 seconds respectively. If they change simultaneously at 7 a.m., at what time will they change simultaneously again?

2	48, 72, 108
2	24, 36, 54
2	12, 18, 27
3	6, 9, 27
3	2, 3, 9
	2, 1, 3

Sol: We will find LCM of 48,72,108

LCM of 48,72 and 108=2×2×2×2×3×3×3

=432

The required time=432 sec

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$=7\times60+12=7$ minutes 12 seconds

7. Three tankers contain 403 litres, 434 litres and 465 litres of diesel respectively. Find the maximum capacity of a container that can measure the diesel of the three containers exact number of times.

Sol: We find HCF of 403, 434 and 465.

$403=13\times31$

$434=2\times7\times31$

$465=3\times5\times31$

HCF of 403,434 and $465=31$

Required maximum capacity of container=31 litres.

8. Find the least number which when divided by 6, 15 and 18 leave remainder 5 in each case.

Sol:

LCM of 6,15 and $18=2\times3\times3\times5=90$

Remainder=5

Required number=90+5=95

2	6, 15, 18
3	3, 15, 9
3	1, 5, 3
5	1, 5, 1
	1, 1, 1

9. Find the smallest 4-digit number which is divisible by 18, 24 and 32.

Sol: LCM of 18,24 and $32=2\times2\times2\times2\times2\times3\times3=288$

Multiples of 288 are 288,576,864,1152,..

The smallest 4-digit number which is divisible by 18, 24 and 32=1152

2	18, 24, 32
2	9, 12, 16
2	9, 6, 8
2	9, 3, 4
2	9, 3, 2
3	9, 3, 1
3	3, 1, 1
	1, 1, 1

10. Find the LCM of the following numbers : Observe a common property in the obtained LCMs. Is LCM the product of two numbers in each case?

(a) 9 and 4

Sol: LCM of 9 and $4= 2\times2\times3\times3=36$

Product of 9 and $4=9\times4=36$

2	9,4
2	9, 2
3	9, 1
3	3,1
	1, 1

(b) 12 and 5

Sol: LCM of 12 and $5= 2\times2\times3\times5=60$

Product of 12 and $5=12\times5=60$

2	12,5
2	6, 5
3	3, 5
5	1,5
	1, 1

(c) 6 and 5

Sol: LCM of 6 and $5=2\times3\times5=30$

Product of 6 and $5=6\times5=30$

2	6, 5
3	3, 5
5	1, 5
	1,1

(d) 15 and 4.

Sol: LCM of 15 and $4=2\times2\times3\times5=60$

2	15, 4
2	15, 2
3	15, 1
5	5, 1
	1, 1

Product of 15 and 4=15×4=60

Common property is the LCM of given numbers=Product of given numbers.

(This property holds only the given numbers have no common prime factors)

11. **Find the LCM of the following numbers in which one number is the factor of the other. What do you observe in the results obtained?**

(a) 5, 20

Sol: LCM of 5,20=5×2×2=20

5	5, 20
2	1, 4
2	1, 2
	1,1

(b) 6, 18

Sol: LCM of 6,18=2×3×3=18

2	6, 18
3	3, 9
3	1, 3
	1,1

(c) 12, 48

Sol: LCM of 12,48= 2×2×2×2×3=48

2	12,48
2	6, 24
2	3, 12
2	3, 6
3	3, 3
	1, 1

(d) 9, 45

Sol: LCM of 9,45= 3×3×5=45

3	9, 45
3	3, 15
5	1, 5
	1, 1

We observe that, in two numbers one number is factor of another number then LCM of the numbers =The larger number.

THANK YOU

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