CHAPTER 2

IX-MATHEMATICS-NCERT

2. POLYNOMIALS(notes)

PREPARED BY: BALABHADRA SURESH

1. **Polynomial**: An algebraic expression in which the variables involved have only non-negative integral (whole numbers) powers is called a polynomial.

Polynomials	Not polynomials
$2x$	$4x^{\frac{1}{2}}$
$\frac{1}{3}x-4$	$3x^2+4x^{-1}+5$
x^2-2x-1	$4+\frac{1}{x}$

2. If a polynomial contains only one variable then it is called **polynomial in one variable**.
   Ex: $2x+3$; $5x^2-6x+2$; $5y+6$; $-6y^2+7y-5$
3. In the polynomial x^2+2x, the expressions x^2 and $2x$ are called the terms of the polynomial.
4. Each term of a polynomial has a coefficient. In $-x^3+4x^2+7x-2$
   The coefficient of $x^3=-1$
   The coefficient of $x^2=4$
   The coefficient of $x=7$
   constant term $=-2$
5. 2, –5, 7, etc. are examples of **constant polynomials**.
6. The constant polynomial 0 is called the **zero polynomial**.
7. The degree of the zero polynomial is not defined.
8. If the variable in a polynomial is x, we may denote the polynomial by p(x), or q(x), or r(x), etc
9. The highest power of the variable in a polynomial as the degree of the polynomial.
   Example: i) $3x^2+7x+5$ → degree=2
   ii) $7x^3+5x^2+2x-6$ → degree=3

Types of polynomials according to degree

1. **Constant polynomial**: A polynomial of degree 0 is called constant polynomial.
   Ex: 5,-7,120,…
2. **Linear polynomial:** A polynomial of degree 1 is called a linear polynomial.
   Example: $3x+5, 7x-8, -9x, \ldots$
   The general form a linear polynomial in variable x is $ax+b$ $(a,b\in R, a\neq 0)$.
3. **Quadratic polynomial** : A polynomial of degree 2 is called a quadratic polynomial.
   Example: $x^2-5x+6, 2x^2-5, 7x^2, \ldots\ldots$
   The general form a quadratic polynomial in variable x is ax^2+bx+c $(a,b,c\in R, a\neq 0)$.

4. **Cubic polynomial** : A polynomial of degree 3 is called a cubic polynomial.

Example: $5x^3 - 4x^2 + x - 1, 2x^3 - 3x + 5, -3x^3 - 10,$.......

The general form a cubic polynomial in variable x is $ax^3 + bx^2 + cx + d$ $(a, b, c, d \in R, a \neq 0)$.

10. **The general form of n^{th} degree polynomial in one variable x:**

$p(x) = a_n x^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \cdots\cdots + a_1 x + a_0$ is a polynomial of n^{th} degree ,

where $a_0, a_1, a_2, \ldots\ldots\ldots, a_{n-1}, a_n$ are real coefficients and $a_0 \neq 0$.

EXERCISE-2.1

1. **Which of the following expressions are polynomials in one variable and which are not? State reasons for your answer.**

(i) $\mathbf{4x^2 - 3x + 7}$ → polynomial in one variable x

(ii) $\mathbf{y^2 + \sqrt{2}}$ → polynomial in one variable y

(iii) $\mathbf{3\sqrt{t} + t\sqrt{2}}$ → not a polynomial

(iv) $\mathbf{y + \frac{2}{y}}$ → not a polynomial

(v) $\mathbf{x^{10} + y^3 + t^{50}}$ → polynomial in three variables $x, y$and t

2. **Write the coefficient of x^2 in each of the following**

(i) $\mathbf{2 + x^2 + x}$ → coefficient of $x^2 = 1$

(ii) $\mathbf{2 - x^2 + x^3}$ → coefficient of $x^2 = -1$

(iii) $\mathbf{\frac{\pi}{2}x^2 + x}$ → coefficient of $x^2 = \frac{\pi}{2}$

(iv) $\mathbf{\sqrt{2}x - 1}$ → coefficient of $x^2 = 0$

3. **Give one example each of a binomial of degree 35, and of a monomial of degree 100.**

Sol: A binomial of degree 35 : $x^{35} + x^2$

A monomial of degree 100: $3x^{100}$

4. **Write the degree of each of the following polynomials:**

Polynomial	Degree
(i) $5x^3 + 4x^2 7x$	3
(ii) $4 - y^2$	2
$(iii) 5t - \sqrt{7}$	1
$(iv) 3$	0

5. **Classify the following as linear, quadratic and cubic polynomials:**

Sol: Linear polynomials: $(iv) 1 + x$ $(v) 3t$

Quadratic polynomials: $(i) x^2 + x$ (iii) $y + y^2 + 4$ (vi) r^2

Cubic polynomials: (ii) $x - x^3$ (vii) $7x^3$

Example 2 : Find the value of each of the following polynomials at the indicated value of variables

(i) $p(x) = 5x^2 - 3x + 7$ at $x = 1$

Sol: $p(1) = 5(1)^2 - 3(1) + 7$

$= 5 - 3 + 7$

$= 12 - 3$

$= 9$

(ii) $q(y) = 3y^3 - 4y + \sqrt{11}$ at $y = 2$

Sol: $q(2) = 3(2)^3 - 4(2) + \sqrt{11}$

$= 3 \times 8 - 4 \times 2 + \sqrt{11}$

$= 24 - 8 + \sqrt{11}$

$= 16 + \sqrt{11}$

(iii) $p(t) = 4t^4 + 5t^3 - t^2 + 6$ at $t = a$

Sol: $p(a) = 4a^4 + 5a^3 - a^2 + 6$

Zeroes of a Polynomial

1. A real number 'c' is a zero of a polynomial $p(x)$ if $p(c) = 0$. In this case, 'c' is also called a root of the polynomial equation $p(x) = 0$.
2. Every linear polynomial in one variable has a unique zero, a non-zero constant polynomial has no zero.
3. Every real number is a zero of the **zero polynomial**.

Linear Polynomial	Zero of the polynomial
$x + a$	$-a$
$x - a$	a
$ax + b$	$\frac{-b}{a}$
$ax - b$	$\frac{b}{a}$

Example 3 : Check whether –2 and 2 are zeroes of the polynomial x + 2.

Solu : Let $p(x) = x + 2$

$p(2) = 2 + 2 = 4,$

$p(-2) = -2 + 2 = 0$

∴ –2 is a zero of the polynomial $x + 2$, but 2 is not.

Example 4 : Find a zero of the polynomial p(x) = 2x + 1.

Sol: Let $p(x) = 0$

$2x + 1 = 0$

$2x = -1$

$x = \frac{-1}{2}$

So, $\frac{-1}{2}$ is a zero of the polynomial $2x + 1$

Example 5 : Verify whether 2 and 0 are zeroes of the polynomial $x^2 - 2x$

Sol: $Let\ \ p(x) = x^2 - 2x$

$p(2) = (2)^2 - 2(2) = 4 - 4 = 0$

$p(0) = (0)^2 - 2(0) = 0 - 0 = 0$

Hence, 2 and 0 are both zeroes of the polynomial $x^2 - 2x$.

EXERCISE 2.2

1. Find the value of the polynomial $5x - 4x^2 + 3$ at (i) x = 0 (ii) x = –1 (iii) x = 2

Sol: $Let\ p(x) = 5x - 4x^2 + 3$

(i) $p(0) = 5(0) - 4(0)^2 + 3 = 0 - 0 + 3 = 3$

(ii) $p(-1) = 5(-1) - 4(-1)^2 + 3 = -5 - 4 + 3 = -9 + 3 = -6$

(iii) $p(2) = 5(2) - 4(2)^2 + 3 = 10 - 16 + 3 = 13 - 16 = -3$

2. Find p(0), p(1) and p(2) for each of the following polynomials:

(*i*) $p(y) = y^2 - y + 1$

Sol: $p(0) = (0)^2 - 0 + 1 = 0 - 0 + 1 = 1$

$p(1) = (1)^2 - 1 + 1 = 1 - 1 + 1 = 1$

$p(2) = (2)^2 - 2 + 1 = 4 - 2 + 1 = 5 - 2 = 3$

(*ii*) $p(t) = 2 + t + 2t^2 - t^3$

Sol: $p(0) = 2 + 0 + 2 \times (0)^2 - (0)^3 = 2 + 0 + 0 - 0 = 2$

$p(1) = 2 + 1 + 2 \times 1^2 - 1^3 = 2 + 1 + 2 - 1 = 4$

$p(2) = 2 + 2 + 2 \times 2^2 - 2^3 = 4 + 8 - 8 = 4$

(*iii*) $p(x) = x^3$

Sol: $p(0) = 0^3 = 0$

$p(1) = 1^3 = 1$

$p(2) = 2^3 = 8$

(*iv*) $p(x) = (x - 1)(x + 1)$

Sol: $p(0) = (0 - 1)(0 + 1) = (-1) \times 1 = -1$

$p(1) = (1 - 1)(1 + 1) = 0 \times 2 = 0$

$p(2) = (2 - 1)(2 + 1) = 1 \times 3 = 3$

3. Verify whether the following are zeroes of the polynomial, indicated against them

(*i*) $p(x) = 3x + 1;\ x = -\frac{1}{3}$

Sol: $p\left(-\frac{1}{3}\right) = 3 \times \left(-\frac{1}{3}\right) + 1 = -1 + 1 = 0$

$p\left(-\frac{1}{3}\right) = 0$

$So, \left(-\frac{1}{3}\right)\ is\ a\ zero\ of\ the\ polynomial\ 3x + 1$

(ii) $p(x) = 5x - \pi;\ x = \frac{4}{5}$

Sol: $p(\frac{4}{5}) = 5 \times \left(\frac{4}{5}\right) - \pi = 4 - \pi$

$p\left(\frac{4}{5}\right) \neq 0$

So $\frac{4}{5}$ is not a zero of the polynomial $5x - \pi$.

(iii) $p(x) = x^2 - 1;\ x = 1, -1$

Sol: $p(1) = 1^2 - 1 = 1 - 1 = 0$

$p(-1) = (-1)^2 - 1 = 1 - 1 = 0$

$p(1) = 0\ and\ p(-1) = 0$

So, $1, -1$ *are the zeroes of the polynomial* $x^2 - 1$.

(iv) $p(x) = (x + 1)(x - 2), x = -1, 2$

sol: $p(-1) = (-1 + 1)(-1 - 2) = 0 \times (-3) = 0$

$p(2) = (2 + 1)(2 - 2) = 3 \times 0 = 0$

$So, -1,2$ *are the zeroes of the polynomial* $(x + 1)(x - 2)$

(v) $p(x) = x^2;\ x = 0$

Sol: $p(0) = 0^2 = 0$

So, 0 is a zero of the polynomial x^2

(vi) $p(x) = lx + m, x = -\frac{m}{l}$

Sol: $p(x) = lx + m$

$p\left(-\frac{m}{l}\right) = l\left(-\frac{m}{l}\right) + m = -m + m = 0$

(vii) $p(x) = 3x^2 - 1, x = -\frac{1}{\sqrt{3}}, \frac{2}{\sqrt{3}}$

Sol: $p\left(-\frac{1}{\sqrt{3}}\right) = 3\left(-\frac{1}{\sqrt{3}}\right)^2 - 1 = 3 \times \frac{1}{3} - 1 = 1 - 1 = 0$

$p\left(\frac{2}{\sqrt{3}}\right) = 3\left(\frac{2}{\sqrt{3}}\right)^2 - 1 = 3 \times \frac{4}{3} - 1 = 4 - 1 = 3$

$-\frac{1}{\sqrt{3}}$ is the zero of the polynomial $3x^2 - 1$, but $\frac{2}{\sqrt{3}}$ is not.

(viii) $p(x) = 2x + 1, x = \frac{1}{2}$

Sol: $p\left(\frac{1}{2}\right) = 2\left(\frac{1}{2}\right) + 1\ \ 2 + 1 = 3$

$\frac{1}{2}$ is not a zero of the polynomial $2x + 1$

Find the zero of the polynomial in each of the following cases

$(i)\ p(x) = x + 5$

Sol: $Let\ p(x) = 0$

$x + 5 = 0$

$x = -5$

$\therefore\ -5$ is the zero of the polynomial $x + 5$

$(ii)\ p(x) = x - 5$

Sol: $Let\ p(x) = 0$

$x - 5 = 0$

$x = 5$

$\therefore\ 5\ is\ the\ zero\ of\ the\ polynomial\ x - 5$

$(iii)\ p(x) = 2x + 5$

Sol: $Let\ p(x) = 0$

$2x + 5 = 0$

$2x = -5 \Rightarrow x = \frac{-5}{2}$

$\therefore\ \frac{-5}{2}\ is\ the\ zero\ of\ the\ polynomial\ 2x + 5$

$(vi)\ p(x) = 3x - 2$

Sol: $Let\ p(x) = 0$

$3x - 2 = 0$

$3x = 2 \Rightarrow x = \frac{2}{3}$

$\therefore\ \frac{2}{3}\ is\ the\ zero\ of\ the\ polynomial\ 3x - 2$

$(v)\ p(x) = 3x$

Sol: $Let\ p(x) = 0$

$3x = 0$

$x = 0$

$\therefore\ 0\ is\ the\ zero\ of\ the\ polynomial\ 3x$

$(vi)\ p(x) = ax\ , a \neq 0$

Sol: $Let\ p(x) = 0$

$ax = 0$

$x = 0$

$\therefore\ 0\ is\ the\ zero\ of\ the\ polynomial\ ax$

$(vii)\ p(x) = cx + d, c \neq 0, c, d\ are\ real\ numbers.$

Sol: $Let\ p(x) = 0$

$cx + d = 0$

$cx = -d \Rightarrow x = \frac{-d}{c}$

$\therefore \frac{-d}{c}$ *is the zero of the polynomial* $cx + d$

Remainder Theorem: Let $p(x)$ be any polynomial of degree greater than or equal to one and let 'a' be any real number. If $p(x)$ is divided by the linear polynomial $(x - a)$, then the remainder is $p(a)$.

Factor Theorem : $p(x)$ is a polynomial of degree n ≥ 1 and 'a' is any real number

(i) *If* $p(a) = 0$ *then* $(x - a)$ *is a factor of* $p(x)$. and

(ii) *If* $(x - a)$ *is a factor of polynomial* $p(x)$ *then* $p(a) = 0$.

***Example 6* : *Examine whether* $x + 2$ *is a factor of* $x^3 + 3x^2 + 5x + 6$ *and of* $2x + 4$.**

Sol: Let $p(x) = x^3 + 3x^2 + 5x + 6$

$p(-2) = (-2)^3 + 3(-2)^2 + 5(-2) + 6$

$= -8 + 12 - 10 + 6 = 18 - 18 = 0$

$\therefore x + 2$ *is a factor of* $x^3 + 3x^2 + 5x + 6$

since $2x + 4 = 2(x + 2)$ *but* 2 *is not a factor of* $p(x)$

So, $2x + 4$ *is not a factor of* $p(x)$

Example 7 : *Find the value of* k, *if* $x - 1$ *is a factor of* $4x^3 + 3x^2 - 4x + k$

Sol: *Let* $p(x) = 4x^3 + 3x^2 - 4x + k$

If $x - 1$ *is a factor of* $p(x)$ *then* $p(1) = 0$

$4(1)^3 + 3(1)^2 - 4(1) + k = 0$

$4 + 3 - 4 + k = 0$

$3 + k = 0$

$k = -3$

Factorisation of the polynomial $ax^2 + bx + c$ ***by splitting the middle term.***

Let its factors be $(px + q)$ and $(rx + s)$.

$ax^2 + bx + c = (\mathrm{px} + \mathrm{q})(\mathrm{rx} + \mathrm{s})$

$ax^2 + bx + c = prx^2 + (ps + qr)x + qs$

$ps \times qr = a \times c$ *and* $ps + qr = b$

Example 8 : Factorise $6x^2 + 17x + 5$ by splitting the middle term, and by using the Factor Theorem

Sol: $6x^2 + 17x + 5 = 6x^2 + 2x + 15x + 5$

$= 2x(3x + 1) + 5(3x + 1)$

$= (3x + 1)(2x + 5)$

Example 9 : *Factorise* $y^2 - 5y + 6$ *by using the Factor Theorem.*

Sol: $p(y) = y^2 - 5y + 6$

$p(1) = (1)^2 - 5(1) + 6 = 1 - 5 + 6 = 7 - 5 = 2$

$p(2) = (2)^2 - 5(2) + 6 = 4 - 10 + 6 = 10 - 10 = 0$

$(y - 2)$ *is a factor of* $p(y)$

$p(3) = (3)^2 - 5(3) + 6 = 9 - 15 + 6 = 15 - 15 = 0$

$(y - 3)$ *is a factor of* $p(y)$

$\therefore y^2 - 5y + 6 = (y - 2)(y - 3)$

Example 10 **:** ***Factorise*** $\boldsymbol{x^3 - 23x^2 + 142x - 120}$.

Sol: $p(x) = x^3 - 23x^2 + 142x - 120$

$p(1) = (1)^3 - 23(1)^2 + 142(1) - 120 = 1 - 23 + 142 - 120 = 143 - 143 = 0$

$\therefore (x - 1)$ *is a factor of* $p(x)$

$x^3 - 23x^2 + 142x - 120 = x^3 - x^2 - 22x^2 + 22x + 120x - 120$

$= x^2(x - 1) - 22x(x - 1) + 120(x - 1)$

$= (x - 1)(x^2 - 22x + 120)$

$(x^2 - 22x + 120) = (x^2 - 12x - 10x + 120)$

$= x(x - 12) - 10(x - 12)$

$= (x - 12)(x - 10)$

$x^3 - 23x^2 + 142x - 120 = (x - 1)(x - 12)(x - 10)$

(OR)

By using Horner's synthetic method

$p(x) = x^3 - 23x^2 + 142x - 120$

$p(1) = (1)^3 - 23(1)^2 + 142(1) - 120$

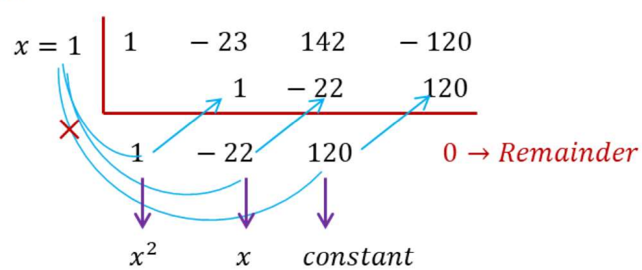
$= 1 - 23 + 142 - 120$

$= 143 - 143 = 0$

$\therefore (x - 1)$ *is a factor of* $p(x)$

Using Horner's synthetic division method

Divide $p(x)$ *by* $(x - 1)$



Quotient $= (x^2 - 22x + 120)$

$= (x^2 - 12x - 10x + 120)$

$= x(x - 12) - 10(x - 12)$

$= (x - 12)(x - 10)$

$x^3 - 23x^2 + 142x - 120 = (x - 1)(x - 12)(x - 10)$

EXERCISE 2.3

1. **Determine which of the following polynomials has (x + 1) a factor**

(*i*) $\mathbf{x^3 + x^2 + x + 1}$

Sol: $p(x) = x^3 + x^2 + x + 1$

$p(-1) = (-1)^3 + (-1)^2 + (-1) + 1$

$= -1 + 1 - 1 + 1 = 2 - 2 = 0$

$(x + 1)$ is a factor of $x^3 + x^2 + x + 1$

(ii) $x^4 + x^3 + x^2 + x + 1$

Sol: $p(x) = x^4 + x^3 + x^2 + x + 1$

$p(-1) = (-1)^4 + (-1)^3 + (-1)^2 + (-1) + 1$

$= 1 - 1 + 1 - 1 + 1 = 3 - 2 = 1$

$(x + 1)$ is not a factor of $x^4 + x^3 + x^2 + x + 1$

(iii) $x^4 + 3x^3 + 3x^2 + x + 1$

Sol: $p(x) = x^4 + 3x^3 + 3x^2 + x + 1$

$p(-1) = (-1)^4 + 3 \times (-1)^3 + 3 \times (-1)^2 + (-1) + 1$

$= 1 - 3 + 3 - 1 + 1 = 1$

$(x + 1)$ is not a factor of $x^4 + 3x^3 + 3x^2 + x + 1$

(iv) $x^3 - x^2 - (2 + \sqrt{2})x + \sqrt{2}$

Sol: $p(x) = x^3 - x^2 - (2 + \sqrt{2})x + \sqrt{2}$

$p(-1) = (-1)^3 - (-1)^2 - (2 + \sqrt{2}) \times (-1) + \sqrt{2}$

$= -1 - 1 + 2 - \sqrt{2} + \sqrt{2} = 0$

$(x + 1)$ is a factor of $x^3 - x^2 - (2 + \sqrt{2})x + \sqrt{2}$

2. Use the Factor Theorem to determine whether g(x) is a factor of p(x) in each of the following cases:

(i) $p(x) = 2x^3 + x^2 - 2x - 1, g(x) = x + 1$

Sol: $p(x) = 2x^3 + x^2 - 2x - 1$

$p(-1) = 2(-1)^3 + (-1)^2 - 2(-1) - 1$

$= -2 + 1 + 2 - 1 = 0$

$\therefore g(x)$ is a factor of $p(x)$

(ii) $p(x) = x^3 + 3x^2 + 3x + 1, g(x) = x + 2$

Sol: $p(x) = x^3 + 3x^2 + 3x + 1$

$p(-2) = (-2)^3 + 3(-2)^2 + 3(-2) + 1$

$= -8 + 12 - 6 + 1$

$= 14 - 14 = 0$

$\therefore\ g(x)\ is\ a\ factor\ of\ p(x)$

(iii) $\boldsymbol{p(x) = x^3 - 4x^2 + x + 6, g(x) = x - 3}$

Sol: $p(x) = x^3 - 4x^2 + x + 6$

$p(3) = 3^3 - 4 \times 3^2 + 2 + 6$

$= 27 - 36 + 8$

$= 36 - 36 = 0$

$\therefore\ g(x)\ is\ a\ factor\ of p(x)$

3. Find the value of k, if x – 1 is a factor of p(x) in each of the following cases:

$(\boldsymbol{i})$ $\boldsymbol{p(x) = x^2 + x + k}$

If $x - 1$ is a factor of p(x)then p(1) = 0

$1^2 + 1 + k = 0$

$2 + k = 0$

$k = -2$

$(\boldsymbol{ii})$ $\boldsymbol{p(x) = 2x^2 + kx + 2}$

If $x - 1$ is a factor of p(x)then p(1) = 0

$2 \times 1^2 + k \times 1 + 2 = 0$

$2 + k + 2 = 0$

$k + 4 = 0$

$k = -4$

$(\boldsymbol{iii})$ $\boldsymbol{p(x) = kx^2 - \sqrt{2}\,x + 1}$

If $x - 1$ is a factor of p(x)then p(1) = 0

$k \times (1)^2 - \sqrt{2} \times 1 + 1 = 0$

$k - \sqrt{2} + 1 = 0$

$k = \sqrt{2} - 1$

(iv) $\boldsymbol{p(x) = kx^2 - 3x + k}$

If $x - 1$ is a factor of p(x)then p(1) = 0

$k(1)^2 - 3(1) + k = 0$

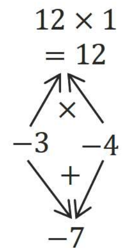
$k - 3 + k = 0$

$2k - 3 = 0$

$k = \frac{3}{2}$

4. Factorise :

(i) **$12x^2 - 7x + 1$**

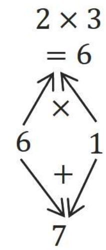


Sol: $12x^2 - 7x + 1 = 12x^2 - 3x - 4x + 1$

$= 3x(4x - 1) - 1(4x - 1)$

$= (4x - 1)(3x - 1)$

(ii) **$2x^2 + 7x + 3$**

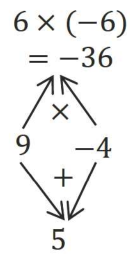


Sol: $2x^2 + 7x + 3 = 2x^2 + 6x + x + 3$

$= 2x(x + 3) + 1(x + 3)$

$= (x + 3)(2x + 1)$

(iii) **$6x^2 + 5x - 6$**

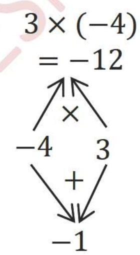


Sol: $6x^2 + 5x - 6 = 6x^2 + 9x - 4x - 6$

$= 3x(2x + 3) - 2(2x + 3)$

$= (2x + 3)(3x - 2)$

(iv) **$3x^2 - x - 4$**



Sol: $3x^2 - x - 4 = 3x^2 - 4x + 3x - 4$

$= x(3x - 4) + 1(3x - 4)$

$= (3x - 4)(x + 1)$

5. Factorise :

(i) **$x^3 - 2x^2 - x + 2$**

Sol: $Let\ p(x) = x^3 - 2x^2 - x + 2$

$p(x) = x^3 - 2x^2 - x + 2$

$p(1) = 1^3 - 2 \times 1^2 - 1 + 2$

$= 1 - 2 - 1 + 2 = 0$

So $(x - 1)$ is a factor of $p(x)$

$x^3 - 2x^2 - x + 2$

$= x^3 - x^2 - x^2 + x - 2x + 2$

$= x^2(x - 1) - x(x - 1) - 2(x - 1)$

$= (x - 1)(x^2 - x - 2)$

$= (x - 1)(x + 1)(x - 2)$

$x^2 - x - 2$ $= x^2 - 2x + x - 2$ $= x(x - 2) + 1(x - 2)$	$-1 \times 2 = -2$ $-1 + 2 = 1$

(ii) **$x^3 - 3x^2 - 9x - 5$**

Sol: $p(x) = x^3 - 3x^2 - 9x - 5$

$p(-1) = (-1)^3 - 3 \times (-1)^2 - 9 \times (-1) - 5$

$= -1 - 3 + 9 - 5$

$= 9 - 9 = 0$

So $(x + 1)$ is a factor of $p(x)$

$x^3 - 3x^2 - 9x - 5$

$= x^3 + x^2 - 4x^2 - 4x - 5x - 5$

$= x^2(x+1) - 4x(x+1) - 5(x+1)$

$= (x+1)(x^2 - 4x - 5)$

$= (x+1)(x+1)(x-5)$

$= (x+1)^2(x-5)$

$x^2 - 4x - 5$ $= x^2 - 5x + x - 5$ $= x(x-5) + 1(x-5)$	$-5 \times 1 = -5$ $-5 + 1 = -4$

(iii) $x^3 + 13x^2 + 32x + 20$

Sol: $p(x) = x^3 + 13x^2 + 32x + 20$

$p(-1) = (-1)^3 + 13 \times (-1)^2 + 32 \times (-1) + 20$

$= -1 + 13 - 32 + 20$

$= 33 - 33 = 0$

So $(x+1)$ is a factor of $p(x)$

$x^3 + 13x^2 + 32x + 20$

$= x^3 + x^2 + 12x^2 + 12x + 20x + 20$

$= x^2(x+1) + 12x(x+1) + 20(x+1)$

$= (x+1)(x^2 + 12x + 20)$

$= (x+1)(x+2)(x+10)$

$x^2 + 12x + 20$ $= x^2 + 2x + 10x + 20$ $= x(x+2) + 10(x+2)$	$2 \times 10 = 20$ $2 + 10 = 12$

(iv) $2y^3 + y^2 - 2y - 1$

Sol: $p(y) = y^3 + y^2 - y - 1$

$p(-1) = (-1)^3 + (-1)^2 - (-1) - 1$

$= -1 + 1 + 1 - 1$

$= 2 - 2 = 0$

So $(y+1)$ is a factor of $p(y)$

$y^3 + y^2 - y - 1 = y^3 + y^2 - y - 1$

$= y^2(y+1) - 1(y+1)$

$= (y+1)(y^2 - 1)$

$= (y+1)(y+1)(y-1)$

Algebraic Identities

(i) $(x + y)^2 \equiv x^2 + 2xy + y^2$

(ii) $(x - y)^2 \equiv x^2 - 2xy + y^2$

(iii) $(x + y)(x - y) \equiv x^2 - y^2$

(iv) $(x + a)(x + b) \equiv x^2 + (a + b)x + ab.$

Example 11 : Find the following products using appropriate identities:

(i) $(x + 3)(x + 3)$

Sol: $(x + y)^2 = x^2 + 2xy + y^2$

$(x + 3)(x + 3) = (x + 3)^2 = x^2 + 2 \times x \times 3 + 3^2$

$= x^2 + 6x + 9$

(ii) $(x - 3)(x + 5)$

Sol: $(x + a)(x + b) = x^2 + (a + b)x + ab$

$(x - 3)(x + 5) = x^2 + (-3 + 5)x + (-3) \times 5$

$= x^2 + 2x - 15$

Example 12 : Evaluate 105 × 106 without multiplying directly

Sol: $(x + a)(x + b) \equiv x^2 + (a + b)x + ab$

$105 \times 106 = (100 + 5)(100 + 6)$

$= (100)^2 + (5 + 6) \times 100 + 5 \times 6$

$= 10000 + 1100 + 30 = 11130$

Example 13 : Factorise:

(i) $49a^2 + 70ab + 25b^2$

Sol: $x^2 + 2xy + y^2 = (x + y)^2$

$49a^2 + 70ab + 25b^2 = (7a)^2 + 2 \times 7a \times 5b + (5b)^2$

$= (7a + 5b)^2 = (7a + 5b)(7a + 5b)$

(ii) $\frac{25}{4}x^2 - \frac{y^2}{9}$

Sol: $x^2 - y^2 = (x + y)(x - y)$

$$\frac{25}{4}x^2 - \frac{y^2}{9} = \left(\frac{5}{2}x\right)^2 - \left(\frac{y}{3}\right)^2 = \left(\frac{5}{2}x + \frac{y}{3}\right)\left(\frac{5}{2}x - \frac{y}{3}\right)$$

Identity V : $(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$

Sol: $(x + y + z)^2 = [(x + y) + z]^2 = (x + y)^2 + 2(x + y)z + z^2$

$= x^2 + 2xy + y^2 + 2xz + 2yz + z^2$

$= x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$

Example 14 : Write $(3a + 4b + 5c)^2$ in expanded form

Sol: $(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$

$(3a + 4b + 5c)^2 = (3a)^2 + (4b)^2 + (5c)^2 + 2(3a)(4b) + 2(4b)(5c) + 2(3a)(5c)$

$= 9a^2 + 16b^2 + 25c^2 + 24ab + 40bc + 30ac$

Example 16 : Factorise $4x^2 + y^2 + z^2 - 4xy - 2yz + 4xz$

Sol: $x^2 + y^2 + z^2 + 2xy + 2yz + 2zx = (x + y + z)^2$

$4x^2 + y^2 + z^2 - 4xy - 2yz + 4xz$

$= (2x)^2 + (-y)^2 + (z)^2 + 2(2x)(-y) + 2(-y)(z) + 2(2x)(z)$

$= (2x - y + z)^2 = (2x - y + z)(2x - y + z)$

Identity VI : $(x + y)^3 = x^3 + 3x^2y + 3xy^2 + y^3 = x^3 + y^3 + 3xy(x + y)$

Sol: $(x + y)^3 = (x+y)(x + y)^2 = (x+y)(x^2 + 2xy + y^2)$

$= x^3 + 2x^2y + xy^2 + x^2y + 2xy^2 + y^3 = x^3 + y^3 + 3x^2y + 3xy^2 = x^3 + y^3 + 3xy(x + y)$

Identity VII : $(x - y)^3 = x^3 - 3x^2y + 3xy^2 - y^3 = x^3 - y^3 - 3xy(x - y)$

Sol: $(x - y)^3 = (x-y)(x - y)^2 = (x-y)(x^2 - 2xy + y^2)$

$= x^3 - 2x^2y + xy^2 - x^2y + 2xy^2 - y^3 = x^3 - y^3 - 3x^2y + 3xy^2 = x^3 - y^3 - 3xy(x - y)$

Example 17 : Write the following cubes in the expanded form:

$(i)(3a + 4b)^3$

Sol: $(x + y)^3 = x^3 + y^3 + 3xy(x + y)$

$(3a + 4b)^3 = (3a)^3 + (4b)^3 + 3(3a)(4b)(3a + 4b)$

$= 27a^3 + 64b^4 + 36ab(3a + 4b) = 27a^3 + 64b^4 + 108a^2b + 144ab^2$

$(ii)\ (5p - 3q)^3$

Sol: $(x - y)^3 = x^3 - y^3 - 3xy(x - y)$

$(5p - 3q)^3 = (5p)^3 - (3q)^3 - 3(5p)(3q)(5p - 3q)$

$= 125p^3 - 27q^3 - 45pq(5p - 3q)$

$= 125p^3 - 27q^3 - 225p^2q + 135pq^2$

Example 18 : Evaluate each of the following using suitable identities:

$(i)\ (104)^3$

Sol: $(x + y)^3 = x^3 + y^3 + 3xy(x + y)$

$(104)^3 = (100 + 4)^3 = (100)^3 + (4)^3 + 3(100)(4)(100 + 4)$

$= 1000000 + 64 + 1200 \times 104$

$= 1000000 + 64 + 124800 = 1124864$

$(ii)(999)^3$

Sol: $(x - y)^3 = x^3 - y^3 - 3xy(x - y)$

$(999)^3 = (1000 - 1)^3 = (100)^3 - (1)^3 - 3(100)(1)(1000 - 1)$

$= 1000000 - 1 - 300 \times 999$

$= 1000000 - 1 - 2997000 = 997002999$

Example 19 : ***Factorise*** $8x^3 + 27y^3 + 36x^2y + 54xy^2$

Sol: $x^3 + y^3 + 3x^2y + 3xy^2 = (x + y)^3$

$8x^3 + 27y^3 + 36x^2y + 54xy^2 = (2x)^3 + (3y)^3 + 3(2x)^2(3y) + 3(2x)(3y)^2 = (2x + 3y)^3$

Identity VIII : $x^3 + y^3 + z^3 - 3xyz = (x + y + z)(x^2 + y^2 + z^2 - xy - yz - xz)$

Example 20 : Factorise : $8x^3 + y^3 + 27z^3 - 18xyz$

Sol: $x^3 + y^3 + z^3 - 3xyz = (x + y + z)(x^2 + y^2 + z^2 - xy - yz - xz)$

$8x^3 + y^3 + 27z^3 - 18xyz = (2x)^3 + (y)^3 + (3z)^3 - 3(2x)(y)(3z)$

$= (2x + y + 3z)[(2x)^2 + (y)^2 + (3z)^2 - (2x)(y) - (y)(3z) - (2x)(3z)]$

$= (2x + y + 3z)(4x^2 + y^2 + 9z^2 - 2xy - 3yz - 6xz)$

EXERCISE 2.4

1. Use suitable identities to find the following products

(i) $(x+4)(x+10)$

Sol: $(x + a)(x + b) \equiv x^2 + (a + b)x + ab$; $a = 4, b = 10$

$(x+4)(x+10) = x^2 + (4+10)x + 4 \times 10 = x^2 + 14x + 40$

(ii) $(x+8)(x-10)$

Sol: $(x + a)(x + b) \equiv x^2 + (a + b)x + ab; a = 8, b = -10$

$(x+8)(x-10) = x^2 + (8-10)x + 8 \times (-10) = x^2 - 2x - 80$

(iii) $(3x + 4)(3x - 5)$

Sol: $(x + a)(x + b) \equiv x^2 + (a + b)x + ab;\ x = 3x, a = 4, b = -5$

$(3x + 4)(3x - 5) = (3x)^2 + (4-5)(3x) + 4 \times (-5)$

$= 9x^2 - 3x - 20$

(iv) $\left(y^2 + \frac{3}{2}\right)\left(y^2 - \frac{3}{2}\right)$

Sol: $(a+b)(a-b) = a^2 - b^2;\ a = y^2, b = \frac{3}{2}$

$\left(y^2 + \frac{3}{2}\right)\left(y^2 - \frac{3}{2}\right) = (y^2)^2 - \left(\frac{3}{2}\right)^2 = y^4 - \frac{9}{4}$

(v) $(3 - 2x)(3 + 2x)$

Sol: $(a+b)(a-b) = a^2 - b^2; a = 3, b = 2x$

$(3 - 2x)(3 + 2x) = (3)^2 - (2x)^2 = 9 - 4x^2$

2. Evaluate the following products without multiplying directly

(i) 103×107

Sol: $(x + a)(x + b) \equiv x^2 + (a + b)x + ab; x = 100, a = 3, b = 7$

$103 \times 107 = (100+3)(100+7) = (100)^2 + (3 + 7)(100) + 3 \times 7$

$= 10000 + 1000 + 21 = 11021$

(ii) 95×96

Sol: $(x + a)(x + b) \equiv x^2 + (a + b)x + ab; x = 90, a = 5, b = 6$

$95 \times 96 = (90+5)(90+6) = (90)^2 + (5 + 6)(90) + 5 \times 6$

$= 8100 + 990 + 30 = 9120$

(iii) 104×96

Sol: $(a+b)(a-b) = a^2 - b^2; a = 100, b = 4$

$104 \times 96 = (100+4)(100-4) = (100)^2 - (4)^2 = 10000 - 16 = 9984$

3. Factorise the following using appropriate identities

(i) $9x^2 + 6xy + y^2$

Sol: $a^2 + 2ab + b^2 = (a+b)^2$; $a = 3x, b = y$

$9x^2 + 6xy + y^2 = (3x)^2 + 2(3x)(y) + (y)^2 = (3x+y)^2$

(ii) $4y^2 - 4y + 1$

Sol: $a^2 - 2ab + b^2 = (a-b)^2$; $a = 2y, b = 1$

$4y^2 - 4y + 1 = (2y)^2 + 2(2y)(1) + (1)^2 = (2y+1)^2$

(iii) $x^2 - \frac{y^2}{100}$

Sol: $a^2 - b^2 = (a+b)(a-b)$; $a = x, b = \frac{y}{10}$

$x^2 - \frac{y^2}{100} = x^2 - \left(\frac{y}{10}\right)^2 = \left(x + \frac{y}{10}\right)\left(x - \frac{y}{10}\right)$

4. Expand each of the following, using suitable identities:

(i) $(x + 2y + 4z)^2$

Sol: $(a+b+c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ac$

$(x + 2y + 4z)^2 = (x)^2 + (2y)^2 + (4z)^2 + 2(x)(2y) + 2(2y)(4z) + 2(x)(4z)$

$= x^2 + 4y^2 + 16z^2 + 4xy + 16yz + 8xz$

(ii) $(2x - y + z)^2$

Sol: $(a+b+c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ac$

$(\mathbf{2x - y + z})^2 = (2x)^2 + (-y)^2 + (z)^2 + 2(2x)(-y) + 2(-y)(z) + 2(2x)(z)$

$= 4x^2 + y^2 + z^2 - 4xy - 2yz + 4xz$

(iii) $(-2x + 3y + 2z)^2$

Sol: $(a+b+c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ac$

$(-2x + 3y + 2z)^2 = (-2x)^2 + (3y)^2 + (2z)^2 + 2(-2x)(3y) + 2(3y)(2z) + 2(-2x)(2z)$

$= 4x^2 + 9y^2 + 4z^2 - 12xy + 12yz - 8xz$

(iv) $(3a - 7b - c)^2$

Sol: $(x+y+z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2xz$

$(3a - 7b - c)^2 = (3a)^2 + (-7b)^2 + (-c)^2 + 2(3a)(-7b) + 2(-7b)(-c) + 2(3a)(-c)$

$= 9a^2 + 49b^2 + c^2 - 42ab + 14bc - 6ac$

(v) $(-2x + 5y - 3z)^2$

Sol: $(a+b+c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ac$

$(-2x + 5y - 3z)^2 = (-2x)^2 + (5y)^2 + (-3z)^2 + 2(-2x)(5y) + 2(5y)(-3z) + 2(-2x)(-3z)$

$= 4x^2 + 25y^2 + 9z^2 - 20xy - 30yz + 12xz$

(vi) $\left[\frac{1}{4}a - \frac{1}{2}b + 1\right]^2$

Sol: $(x+y+z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2xz$

$\left(\frac{1}{4}a - \frac{1}{2}b + 1\right)^2 = \left(\frac{1}{4}a\right)^2 + \left(-\frac{1}{2}b\right)^2 + (1)^2 + 2\left(\frac{1}{4}a\right)\left(-\frac{1}{2}b\right) + 2\left(-\frac{1}{2}b\right)(1) + 2\left(\frac{1}{4}a\right)(1)$

$$=\frac{1}{16}a^2+\frac{1}{4}b^2+1-\frac{1}{4}ab-b+\frac{1}{2}a=\frac{a^2}{16}+\frac{b^2}{4}+1-\frac{ab}{4}-b+\frac{a}{2}$$

5. Factorise:

(i) $4x^2+9y^2+16z^2+12xy-24yz-16xz$

Sol: $a^2+b^2+c^2+2ab+2bc+2ac=(a+b+c)^2$

$4x^2+9y^2+16z^2+12xy-24yz-16xz$

$=(2x)^2+(3y)^2+(-4z)^2+2(2x)(3y)+2(3y)(-4z)+2(2x)(-4z)$

$=(2x+3y-4z)^2$

(ii) $2x^2+y^2+8z^2-2\sqrt{2}xy+4\sqrt{2}yz-8xz$

Sol: $a^2+b^2+c^2+2ab+2bc+2ac=(a+b+c)^2$

$2x^2+y^2+8z^2-2\sqrt{2}xy+4\sqrt{2}yz-8xz$

$=\left(-\sqrt{2}x\right)^2+(y)^2+\left(2\sqrt{2}z\right)^2+2\left(-\sqrt{2}x\right)(y)+2(y)\left(2\sqrt{2}z\right)+2\left(-\sqrt{2}x\right)\left(2\sqrt{2}z\right)$

$=\left(-\sqrt{2}x+y2\sqrt{2}z\right)^2$

6. Write the following cubes in expanded form

(i) $(2x+1)^3$

Sol: $(a+b)^3=a^3+b^3+3ab(a+b)$

$(2x+1)^3=(2x)^3+(1)^3+3(2x)(1)(2x+1)$

$=8x^3+1+6x(2x+1)$

$=8x^3+1+12x^2+6x$

$=8x^3+12x^2+6x+1$

(ii) $(2a-3b)^3$

Sol: $(x+y)^3=x^3+y^3+3xy(x+y)$

$(2a-3b)^3=(2a)^3-(3b)^3-3(2a)(3b)(2a-3b)$

$=8a^3-27b^3-18ab(2a-3b)$

$=8a^3-27b^3-36a^2b+54ab^2$

(iii) $\left[\frac{3}{2}x+1\right]^3$

Sol: $(a+b)^3=a^3+b^3+3ab(a+b)$

$\left[\frac{3}{2}x+1\right]^3=\left(\frac{3}{2}x\right)^3+(1)^3+3\left(\frac{3}{2}x\right)(1)\left(\frac{3}{2}x+1\right)$

$=\frac{27}{8}x^3+1+\frac{9}{2}x\left(\frac{3}{2}x+1\right)$

$=\frac{9}{4}x^2+1+\frac{9}{4}x^2+\frac{9}{2}x$

(iv) $\left[x-\frac{2}{3}y\right]^3$

Sol: $(a-b)^3 = a^3 - b^3 - 3ab\,(a-b)$

$$\left[x-\frac{2}{3}y\right]^3 = (x)^3 - \left(\frac{2}{3}y\right)^3 - 3(x)\left(\frac{2}{3}y\right)\left(x-\frac{2}{3}y\right)$$

$$= x^3 - \frac{8}{27}y^3 - 2xy\left(x-\frac{2}{3}y\right)$$

$$= x^3 - \frac{8}{27}y^3 - 2x^2y + \frac{4}{3}xy^2$$

7. Evaluate the following using suitable identities:

(i) $\mathbf{(99)^3}$

Sol: $(x-y)^3 \equiv x^3 - y^3 - 3xy\,(x-y)$

$(99)^3 = (100-1)^3$

$= 100^3 - 1^3 - 3(100)(1)[100-1]$

$= 1000000 - 1 - 300(99)$

$= 1000000 - 1 - 29700$

$= 9{,}70{,}299$

$(\boldsymbol{ii})$ $\mathbf{(102)^3}$

Sol: $(x+y)^3 \equiv x^3 + y^3 + 3xy\,(x+y)$

$(102)^3 = (100+2)^3$

$= (100)^3 + (2)^3 + 3(100)(2)[100+2]$

$= 1000000 + 8 + 600(102)$

$= 1000000 + 8 + 61200$

$= 10{,}61{,}208$

$(\boldsymbol{iii})$ $\mathbf{(998)^3}$

Sol: $(x-y)^3 \equiv x^3 - y^3 - 3xy\,(x-y)$

$(998)^3 = (1000-2)^3$

$= (1000)^3 - (2)^3 - 3(1000)(2)[1000-2]$

$= 1000000000 - 8 - 6000(998)$

$= 1000000000 - 8 - 5988000$

$= 99{,}40{,}11{,}992$

8. **Factorise each of the following**:

$(\boldsymbol{i})$ $\mathbf{8a^3 + b^3 + 12a^2b + 6ab^2}$

Sol: : $x^3 + 3x^2y + 3xy^2 + y^3 = (x+y)^3$

$8a^3 + b^3 + 12a^2b + 6ab^2$

$= (2a)^3 + (b)^3 + 3(2a)^2b + 3(2a)(b)^2$

$= (2a + b)^3$

(ii) $\mathbf{8a^3 - b^3 - 12a^2b + 6ab^2}$

Sol: $x^3 - y^3 - 3x^2y + 3xy^2 \equiv (x - y)^3$

$8a^3 - b^3 - 12a^2b + 6ab^2$

$= (2a)^3 - (b)^3 - 3(2a)^2b + 3(2a)(b)^2$

$= (2a - b)^3$

(iii) $\mathbf{27 - 125a^3 - 135a + 225a^2}$

Sol: $x^3 - y^3 - 3x^2y + 3xy^2 = (x - y)^3$

$27 - 125a^3 - 135a + 225a^2$

$= (3)^3 - (5a)^3 - 3 \times 3^2 \times 5a + 3 \times 3 \times (5a)^2$

$= (3 - 5a)^3$

(vi) $\mathbf{64a^3 - 27b^3 - 144a^2b + 108ab^2}$

Sol: $x^3 - y^3 - 3x^2y + 3xy^2 = (x - y)^3$

$64a^3 - 27b^3 - 144a^2b + 108ab^2$

$= (4a)^3 - (3b)^3 - 3(4a)^2(3b) + 3(4a)(3b)^2$

$= (4a - 3b)^3$

(v) $\mathbf{27p^3 - \frac{1}{216} - \frac{9}{2}p^2 + \frac{1}{4}p}$

Sol: $x^3 - y^3 - 3x^2y + 3xy^2 = (x - y)^3$

$$27p^3 - \frac{1}{216} - \frac{9}{2}p^2 + \frac{1}{4}p = (3p)^3 - \left(\frac{1}{6}\right)^3 - 3(3p)^2\left(\frac{1}{6}\right) + 3(3p)\left(\frac{1}{6}\right)^2$$

$$= \left(3p - \frac{1}{6}\right)^3$$

9. Verify (i) $\mathbf{x^3 + y^3 = (x + y)(x^2 - xy + y^2)}$ (ii) $\mathbf{x^3 - y^3 = (x - y)(x^2 + xy + y^2)}$

Sol: (i) $\text{R.H.S} = (x + y)(x^2 - xy + y^2) = x(x^2 - xy + y^2) + y(x^2 - xy + y^2)$

$= x^3 - x^2y + xy^2 + x^2y - xy^2 + y^3 = x^3 + y^3 = \text{L.H.S}$

(i) $\text{R.H.S} = (x - y)(x^2 + xy + y^2) = x(x^2 + xy + y^2) - y(x^2 + xy + y^2)$

$= x^3 + x^2y + xy^2 - x^2y - xy^2 - y^3 = x^3 - y^3 = \text{L.H.S}$

10. (i)Factorise $\mathbf{27y^3 + 125z^3}$

Sol: (i) $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

$27y^3 + 125z^3 = (3y)^3 + (5z)^3 = (3y + 5z)[(3y)^2 - (3y)(5z) + (5z)^2]$

$= (3y + 5z)(9y^2 - 15yz + 25z^2)$

(ii) ***Factorise*** **: $64m^3 - 343n^3$**

Sol: $x^3 - y^3 = (x - y)(x^2 + xy + y^2)$

$64m^3 - 343n^3 = (4m)^3 - (7n)^3 = (4m - 7n)[(4m)^2 + (4m)(7n) + (7n)^2]$

$= (4m - 7n)(16m^2 + 28mn + 49n^2)$

11. Factorise : $27x^{33} + y^3 + z^3 - 9xyz$

Sol: $x^3 + y^3 + z^3 - 3xyz \equiv (x + y + z)(x^2 + y^2 + z^2 - xy - yz - xz)$

$27x^3 + y^3 + z^3 - 9xyz = (3x)^3 + y^3 + z^3 - 3(3x)(y)(z)$

$= (3x + y + z)[(3x)^2 + y^2 + z^2 - (3x)(y) - (y)(z) - (z)(3x)]$

$= (3x + y + z)(9x^2 + y^2 + z^2 - 3xy - yz - 3xz)$

12. Verify that $x^3 + y^3 + z^3 - 3xyz = \frac{1}{2}(x + y + z)[(x - y)^2 + (y - z)^2 + (z - x)^2]$

Sol: $x^3 + y^3 + z^3 - 3xyz$

$= (x + y + z)(x^2 + y^2 + z^2 - xy - yz - xz)$

$= (x + y + z)\frac{1}{2}(2x^2 + 2y^2 + 2z^2 - 2xy - 2yz - 2xz)$

$= (x + y + z)\frac{1}{2}[(x^2 - 2xy + y^2) + (y^2 - 2yz + z^2) + (z^2 - 2xz + x^2)]$

$= (x + y + z)\frac{1}{2}[(x - y)^2 + (y - z)^2 + (z - x)^2]$

13. If $x + y + z = 0$, show that $x^3 + y^3 + z^3 = 3xyz$.

Sol: We know that

$x^3 + y^3 + z^3 - 3xyz = (x + y + z)(x^2 + y^2 + z^2 - xy - yz - xz)$

If $x + y + z = 0$ then

$x^3 + y^3 + z^3 - 3xyz = (0)(x^2 + y^2 + z^2 - xy - yz - xz)$

$x^3 + y^3 + z^3 - 3xyz = 0$

$x^3 + y^3 + z^3 = 3xyz$

14. Without actually calculating the cubes, find the value of each of the following:

(i) **$(-12)^3 + (7)^3 + (5)^3$**

Sol: Let $x = -12, y = 7, z = 5$

$x + y + z = -12 + 7 + 5 = 0$

We know that if $x + y + z = 0$ then $x^3 + y^3 + z^3 = 3xyz$

$\Rightarrow (-12)^3 + (7)^3 + (5)^3 = 3(-12)(7)(5)$

$\Rightarrow (-12)^3 + (7)^3 + (5)^3 = -1260$

(ii) $(28)^3 + (-15)^3 + (-13)^3$

Sol: $Let\ x = 28, y = -15, z = -13$

$x + y + z = 28 - 15 - 13 = 0$

We know that if $x + y + z = 0$ then $x^3 + y^3 + z^3 = 3xyz$

$\Rightarrow (28)^3 + (-15)^3 + (-13)^3 = 3(28)(-15)(-13)$

$\Rightarrow (28)^3 + (-15)^3 + (-13)^3 = 16380$

15. Give possible expressions for the length and breadth of each of the following rectangles, in which their areas are given:

(i)**Area**: $25a^2 - 35a + 12$

Sol: $25a^2 - 35a + 12 = 25a^2 - 15a - 20a + 12$

$= 5a(5a - 3) - 4(5a - 3)$

$= (5a - 3)(5a - 4)$

Length=$5a - 3$ and Breadth= $5a - 4$

(ii)**Area**: $35y^2 + 13y - 12$

Sol: $35y^2 + 13y - 12 = 35y^2 + 28y - 15y - 12$

$= 7y(5y + 4) - 3(5y + 4)$

$= (5y + 4)(7y - 3)$

Length=$5y + 4$ and Breadth= $7y - 3$

16. What are the possible expressions for the dimensions of the cuboids whose volumes are given below?

(i)**Volume** : $3x^2 - 12x$

Sol: $3x^2 - 12x = 3x(x - 4)$

$length = 3, \quad breadth = x, \quad height = x - 4$

(ii) $12ky^2 + 8ky - 20k$

Sol: $12ky^2 + 8ky - 20k =$

$= 4k(3y^2 + 2y - 5)$

$= 4k(3y^2 - 3y + 5y - 5)$

$= 4k[3y(y-1) + 5(y-1)]$

$= 4k(3y+5)(y-1) = l \times b \times h$

$length = 4k, \quad breadth = (3y+5), \quad height = (y-1)$