CHAPTER 11

IX-MATHEMATICS-NCERT-2023-24

11. SURFACE AREAS AND VOLUMES (Notes)

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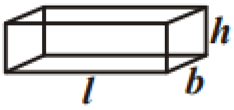
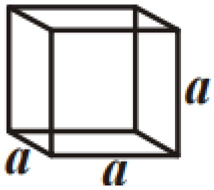
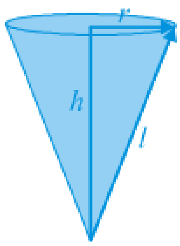
1.

S.N	Figure	Name of the solid and Nomenclature	Lateral/Curved surface area L.S.A/C.S.A(unit²)	Total surface are T.S.A(unit²)	Volume (unit³)
1		**Cuboid** $l = length$ b = breadth h = height	$2h(l+b)$	$2(lb+bh+lh)$	lbh
2		**Cube** $a = side$	$4a^2$	$6a^2$	a^3
3		**Regular circular** **Cylinder** $r = radius$ h = height	$2\pi rh$	$2\pi r(h+r)$	$\pi r^2 h$

Surface Area of a Right Circular Cone



Base radius $= r$, height $= h$, slant height $= l$

$l^2 = h^2 + r^2 \Rightarrow l = \sqrt{h^2 + r^2}$

Curved Surface Area of a Cone $= \pi rl$

Total Surface Area of a Cone $= \pi rl + \pi r^2 = \pi r(l + r)$

Example1: Find the curved surface area of a right circular cone whose slant height is 10 cm and base radius is 7 cm

Sol: $l = 10\ cm, \quad r = 7\ cm$

$$\text{Curved Surface Area of a Cone} = \pi rl = \frac{22}{7} \times 7 \times 10 = 220\ cm^2$$

Example 2 : The height of a cone is 16 cm and its base radius is 12 cm. Find the curved surface area and the total surface area of the cone (Use p = 3.14).

Solution : Here, h = 16 cm and r = 12 cm.

$$l = \sqrt{h^2 + r^2} = \sqrt{16^2 + 12^2} = \sqrt{256 + 144} = \sqrt{400} = 20\ cm$$

$$\text{Curved Surface Area} = \pi rl = 3.14 \times 12 \times 20 = 753.6\ cm^2$$

Total Surface Area of a Cone $= \pi r(l + r) = 3.14 \times 12 \times (20 + 12)$

$= 3.14 \times 12 \times 32 = 1205.76\ cm^2$

Example 3 : A corn cob (see Fig. 11.5), shaped somewhat like a cone, has the radius of its broadest end as 2.1 cm and length (height) as 20 cm. If each 1 cm2 of the surface of the cob carries an average of four grains, find how many grains you would find on the entire cob.

Sol: $r = 2.1\ cm, h = 20\ cm$

$$l = \sqrt{h^2 + r^2} = \sqrt{2.1^2 + 20^2} = \sqrt{4.41 + 400} = \sqrt{404.41} = 20.11\ cm$$

The curved surface area of the corn cob $= \pi r l = \frac{22}{7} \times 2.1 \times 20.11$

$= 22 \times 0.3 \times 20.11 = 132.726\ cm^2$

Number of grains of corn on 1 cm^2= 4

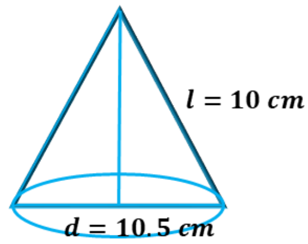
Number of grains on the entire curved surface of the cob $= 132.726 \times 4 = 530.904 \approx 531$

So, there would be approximately 531 grains of corn on the cob.

EXERCISE 11.1

1. Diameter of the base of a cone is 10.5 cm and its slant height is 10 cm. Find its curved surface area.

Sol: Diameter$(d) = 10.5\ cm$



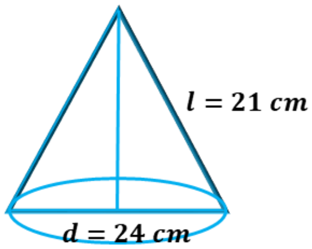
Radius$(r) = \frac{10.5}{2} cm$

Slant height$(l) = 10\ cm$

Curved Surface Area of Cone $= \pi r l = \frac{22}{7} \times \frac{10.5}{2} \times 10 = 165\ cm^2$.

2. Find the total surface area of a cone, if its slant height is 21 m and diameter of its base is 24 m.

Sol: Slant height$(l) = 21\ cm$



Diameter$(d) = 24\ cm$

Radius$(r) = \frac{24}{2} = 12cm$

Total Surface Area of a Cone $= \pi r(l + r) = \frac{22}{7} \times 12 \times (21 + 12)$

$= \frac{22}{7} \times 12 \times 33$

$= \frac{8712}{7} = 1244.57\ m^2$

3. Curved surface area of a cone is 308 cm^2 and its slant height is 14 cm. Find (i) radius of the base and (ii) total surface area of the cone.

Sol: Slant height$(l) =$ 14 cm

Curved surface area of a cone $=$ 308 cm^2

$\pi r l = 308$

$\frac{22}{\cancel{7}} \times r \times \cancel{14}^2 = 308$

$r = \frac{308}{22 \times 2} = 7\ cm$

Total Surface Area of Cone $= \pi r(l + r) = \frac{22}{\cancel{7}} \times \cancel{7} \times (14 + 7) = 22 \times 21 = 462\ cm^2$

4. A conical tent is 10 m high and the radius of its base is 24 m. Find (i) slant height of the tent. (ii) cost of the canvas required to make the tent, if the cost of 1 m^2 canvas is ₹ 70.

Sol: Height$(h) = 10\ m$, radius$(r) = 24\ m$

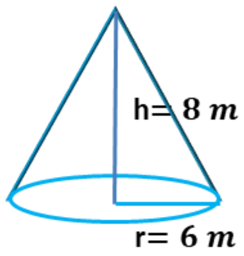
(i)Slant height of the tent $= l = \sqrt{h^2 + r^2} = \sqrt{10^2 + 24^2} = \sqrt{100 + 576} = \sqrt{676} = 26\ m$

(ii) Curved surface area of the cone $= \pi r l = \frac{22}{7} \times 24 \times 26 = \frac{13728}{7}\ m^2$

The cost of 1 m^2 canvas = ₹ 70

The cost of the required canvas = ₹ 70 × $\frac{13728}{7}$ = ₹137280

5. What length of tarpaulin 3 m wide will be required to make conical tent of height 8 m and base radius 6 m? Assume that the extra length of material that will be required for stitching margins and wastage in cutting is approximately 20 cm (Use p = 3.14).



Sol: Radius of cone$(r) = 6\ m$

Height$(h) = 8\ m$

Slant height of the tent $= l = \sqrt{h^2 + r^2} = \sqrt{8^2 + 6^2} = \sqrt{64 + 36}$
$= \sqrt{100} = 10m$

Curved surface area $= \pi r l = 3.14 \times 6m \times 10m = 188.4\ m^2$

Width of the tarpaulin is 3m and area of the tarpaulin=188.4 m^2

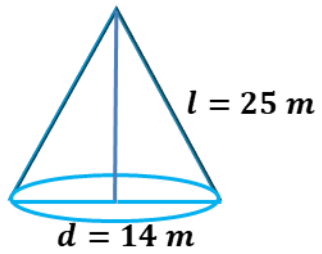
Length of tarpaulin $= \frac{188.4}{3} = 62.8\ m$

The extra length of material=20 cm=0.2 m

Required length of tarpaulin=62.8 m+0.2 m=63 m

6. The slant height and base diameter of a conical tomb are 25 m and 14 m respectively. Find the cost of white-washing its curved surface at the rate of ₹ 210 per 100 m^2 .

Sol: Slant height of cone$(l) = 25\ m$



Diameter$(d) = 14\ m$

Radius$(r) = 7\ m$

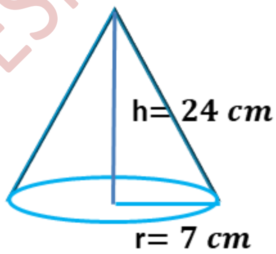
Curved surface area $= \pi r l = \frac{22}{7} \times 7m \times 25m = 550\ m^2$

Cost of white washing per 100 m^2=₹210

Total cost for white washing the tomb $= ₹\ \frac{210}{100} \times 550 = ₹21 \times 55 = ₹1155$

7. A joker's cap is in the form of a right circular cone of base radius 7 cm and height 24 cm. Find the area of the sheet required to make 10 such caps.

Sol: Radius of the cap (cone) =r=7 cm



Height of the cap (cone) =h=24 cm

Slant height of the cap $= l = \sqrt{r^2 + h^2}$

$= \sqrt{7^2 + 24^2} = \sqrt{49 + 576} = \sqrt{625} = 25$ cm

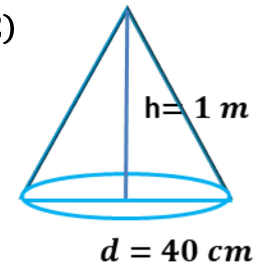
L. S. A of cap $= \pi rl$

$= \frac{22}{7} \times 7\text{cm} \times 25\text{cm} = 22 \times 25\text{cm}^2 = 550\text{cm}^2$

Area of the sheet required to make 1cap $= 550\ \text{cm}^2$.

Area of the sheet required to make 10 such caps $= 10 \times 550\ \text{cm}^2 = 5500\ \text{cm}^2$

8. A bus stop is barricaded from the remaining part of the road, by using 50 hollow cones made of recycled cardboard. Each cone has a base diameter of 40 cm and height 1 m. If the outer side of each of the cones is to be painted and the cost of painting is ₹ 12 per m^2 , what will be the cost of painting all these cones? (Use p = 3.14 and take $\sqrt{1.04}$= 1.02)



Sol: Diameter$(d) = 40\ cm = 0.4\ m$

Radius$(r) = 0.2\ m$

Height$(h) = 1$ m

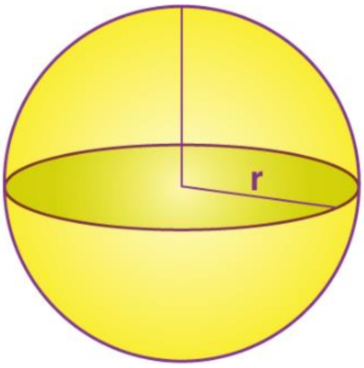
Slant height $= l = \sqrt{r^2 + h^2} = \sqrt{(0.2)^2 + 1^2} = \sqrt{0.04 + 1} = \sqrt{1.04} = 1.02\ m$

Curved surface area $= \pi r l = 3.14 \times 0.2 \times 1.02 = 0.64056\, m^2$

Curved surface area of 50 cones $= 50 \times 0.64056 = 32.028\, m^2$

Cost of painting per 1 m^2 = ₹ 12

Cost of painting for all cones = ₹ 12 × 32.028 = ₹384.336



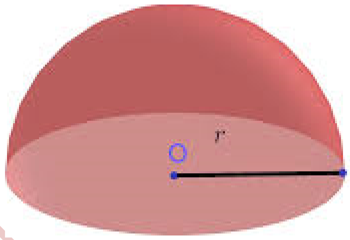
Surface Area of a Sphere

Surface Area of a Sphere $= 4\pi r^2$

Hemi sphere

Curved Surface Area of a Hemisphere $= 2\pi r^2$

Total Surface Area of a Hemisphere $= 3\pi r^2$



Example 4 : Find the surface area of a sphere of radius 7 cm.

Sol: $r = 7\, cm$

Surface Area of a Sphere $= 4\pi r^2 = 4 \times \frac{22}{7} \times 7 \times 7 = 616\, cm^2$

Example 5 : Find (i) the curved surface area and (ii) the total surface area of a hemisphere of radius 21 cm.

Sol: $r = 21\, cm$

(i) Curved Surface Area of a Hemisphere $= 2\pi r^2 = 2 \times \frac{22}{7} \times 21 \times 21 = 2772\, cm^2$

(ii) Total Surface Area of a Hemisphere $= 3\pi r^2 = 3 \times \frac{22}{7} \times 21 \times 21 = 4158\, cm^2$

Example 6 : The hollow sphere, in which the circus motorcyclist performs his stunts, has a diameter of 7 m. Find the area available to the motorcyclist for riding.

Solution : Diameter of the sphere(d) = 7 m

Radius$(r) = \frac{7}{2}\, m$

Surface Area of a Sphere $= 4\pi r^2 = 4^{\cancel{2}} \times \frac{22}{\cancel{7}} \times \frac{\cancel{7}}{\cancel{2}} \times \frac{7}{\cancel{2}} = 22 \times 7 = 154\, m^2$

Example 7 : A hemispherical dome of a building needs to be painted (see Fig. 11.9). If the circumference of the base of the dome is 17.6 m, find the cost of painting it, given the cost of painting is ₹ 5 per 100 cm^2 .

Sol: circumference of the dome = 17.6 m

$2\pi r = 17.6$

$2 \times \frac{22}{7} \times r = 17.6$

$$r = \frac{17.6 \times 7}{2 \times 22} = 0.4 \times 7 = 2.8\, m$$

The curved surface area of the dome $= 2\pi r^2 = 2 \times \frac{22}{7} \times 2.8 \times 2.8 = 49.28\, m^2$

Cost of painting per 100 cm² = ₹ 5.

Cost of painting per 1m² = ₹ 5×100=₹500

Cost of painting the whole dome=₹500×49.28=₹24640

EXERCISE 11.2

1. Find the surface area of a sphere of radius:
(i) 10.5 cm

Sol: Radius$(r) = 10.5\, m$

Surface Area of the Sphere $= 4\pi r^2 = 4 \times \frac{22}{\cancel{7}} \times \cancel{10.5}^{1.5} \times 10.5$

$= 88 \times 1.5 \times 10.5 = 1386\, m^2$

(ii) 5.6 cm

Sol: Radius$(r) = 5.6\, cm$

Surface Area of the Sphere $= 4\pi r^2 = 4 \times \frac{22}{\cancel{7}} \times \cancel{5.6}^{0.8} \times 5.6$

$= 88 \times 0.8 \times 5.6 = 394.24\, cm^2$

(iii) 14 cm

Sol: Radius$(r) = 14\, cm$

Surface Area of the Sphere $= 4\pi r^2 = 4 \times \frac{22}{\cancel{7}} \times \cancel{14}^2 \times 14$

$= 88 \times 2 \times 14 = 2464\, cm^2$

2. Find the surface area of a sphere of diameter:
(i) 14 cm

Sol: Diameter$(d) = 14$ cm, Radius$(r) = 7cm$

Surface Area of the Sphere $= 4\pi r^2 = 4 \times \frac{22}{\cancel{7}} \times \cancel{7} \times 7$

$= 88 \times 7 = 616\, cm^2$

(ii) 21 cm

Sol: Diameter$(d) = 21$ cm, Radius$(r) = \frac{21}{2} cm$

Surface Area of the Sphere $= 4\pi r^2 = 4 \times \frac{22}{\cancel{7}} \times \frac{\cancel{21}^{3}}{\cancel{2}} \times \frac{21}{\cancel{2}}$

$= 22 \times 63 = 1386\ cm^2$

(iii) 3.5m

Sol: Diameter$(d) = 3.5 = \frac{35}{10} = \frac{7}{2}$cm, Radius$(r) = \frac{7}{4} cm$

Surface Area of the Sphere $= 4\pi r^2 = 4 \times \frac{\cancel{22}^{11}}{\cancel{7}} \times \frac{\cancel{7}}{4} \times \frac{7}{4_2}$

$= \frac{11 \times 7}{2} = \frac{77}{2} = 38.5\ cm^2$

3. **Find the total surface area of a hemisphere of radius 10 cm. (Use π = 3.14)**

Sol: Radius$(r) = 10\ cm$

Total surface area of a hemisphere $= 3\pi r^2 = 3 \times 3.14 \times 10 \times 10 = 942\ cm^2$

4. **The radius of a spherical balloon increases from 7 cm to 14 cm as air is being pumped into it. Find the ratio of surface areas of the balloon in the two cases.**

Sol: Radius of balloon before pumping $(r) = 7\ cm$

Radius of balloon after pumping (R) $= 14\ cm$

Ratio of the surface areas of the balloon $= \frac{4\pi r^2}{4\pi R^2} = \frac{r^2}{R^2} = \frac{7 \times 7}{14 \times 14} = \frac{1}{4} = 1:4$

5. **A hemispherical bowl made of brass has inner diameter 10.5 cm. Find the cost of tin-plating it on the inside at the rate of ₹ 16 per 100 cm² .**

Sol: Inner diameter(d) $= 10.5\ cm$

Inner radius$(r) = \frac{10.5}{2} = 5.25\ cm$

CSA of hemispherical bowl $= 2\pi r^2 = 2 \times \frac{22}{\cancel{7}} \times \cancel{5.25}^{7.5} \times 5.25$

$= 44 \times 0.75 \times 5.25 = 173.25\ cm^2$

The cost of tinplating the bowl per 100 cm²= ₹16 per

Total cost of tinplating to the boul $= ₹16 \times \frac{173.25}{100} = ₹\frac{2772}{100} = ₹27.72$

6. **Find the radius of a sphere whose surface area is 154 cm² .**

Sol: The surface area of the sphere $= 154\ \text{cm}^2$

$$4\pi r^2 = 154\text{cm}^2$$

$$4 \times \frac{22}{7} \times r^2 = 154\text{cm}^2$$

$$r^2 = \frac{154 \times 7\text{cm}^2}{4 \times 22}$$

$$r^2 = \frac{49}{4}\text{cm}^2$$

$$r = \frac{7}{2} = 3.5\ cm$$

Radius of the sphere=3.5 cm

7. The diameter of the moon is approximately one fourth of the diameter of the earth. Find the ratio of their surface areas.

Sol: Radius of moon $= r$ and radius of earth $= R$

The diameter of the moon = one fourth of the diameter of the earth'

$$2r = \frac{1}{4} \times 2R$$

$$\frac{r}{R} = \frac{1}{4}$$

The ratio of their surface areas $= \frac{4\pi r^2}{4\pi R^2} = \left(\frac{r}{R}\right)^2 = \left(\frac{1}{4}\right)^2 = \frac{1}{16} = 1:16$

8. A hemispherical bowl is made of steel, 0.25 cm thick. The inner radius of the bowl is 5 cm. Find the outer curved surface area of the bowl.

Sol: Inner radius of the bowl$(r) = 5\ cm$

Thickness of steel$(t) = 0.25\ cm$

Outer radius of the bowl$(R) = 5 + 0.25 = 5.25cm$

Outer CSA of the hemispherical bowl $= 2\pi R^2 = 2 \times \frac{22}{\cancel{7}} \times \cancel{5.25}^{0.75} \times 5.25$

$$= 44 \times 0.75 \times 5.25 = 173.25\ cm^2$$

The outer curved surface area of the sphere $= 173.25\ cm^2$

9. A right circular cylinder just encloses a sphere of radius r (see Fig. 11.10). Find (i) surface area of the sphere, (ii) curved surface area of the cylinder, (iii) ratio of the areas obtained in (i) and (ii).

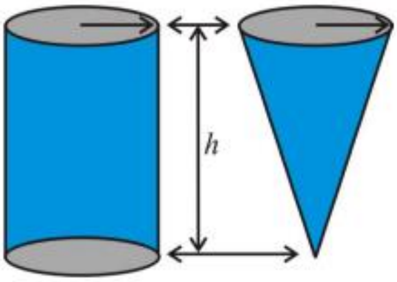
Sol: Radius of the cylinder $=$ Radius of the spher $= r$

Height of the cylinder $= 2r$

(i)surface area of the sphere $= 4\pi r^2$

(ii)curved surface area of the cylinder $= 2\pi rh = 2\pi r \times 2r = 4\pi r^2$

(iii)ratio of the areas obtained in (i)and (ii) $= \frac{4\pi r^2}{4\pi r^2} = 1:1$



Volume of a Right Circular Cone

$$\boxed{\text{Volume of a Cone } = \frac{1}{3}\pi r^2 h}$$

Example 8 : The height and the slant height of a cone are 21 cm and 28 cm respectively. Find the volume of the cone.

Sol: $h = 21\ cm, l = 28\ cm$

$$l^2 = h^2 + r^2 \Rightarrow r = \sqrt{l^2 - h^2} = \sqrt{28^2 - 21^2} = \sqrt{(28+21)(28-21)} = \sqrt{49 \times 7} = 7\sqrt{7}\ cm$$

$$\text{Volume of a Cone } = \frac{1}{3}\pi r^2 h = \frac{1}{\cancel{3}} \times \frac{22}{\cancel{7}} \times 7\sqrt{7} \times 7\sqrt{7} \times \cancel{21}^{7} = 22 \times 49 \times 7 = 7546\ cm^3$$

Example 9 : Monica has a piece of canvas whose area is 551 m2 . She uses it to have a conical tent made, with a base radius of 7 m. Assuming that all the stitching margins and the wastage incurred while cutting, amounts to approximately 1 m2 , find the volume of the tent that can be made with it.

Solution: Radius(r) = 7 m

The area of the canvas = 551 m^2

Area of the canvas lost in wastage = 1 m^2

The area of canvas available for making the tent=551-1=550 m^2

Curved surface area of tent = 550 m^2

$$\pi r l = 550$$

$$\frac{22}{7} \times 7 \times l = 550$$

$$l = \frac{550 \times 7}{22 \times 7} = 25\ m$$

$$l^2 = h^2 + r^2 \Rightarrow h = \sqrt{l^2 - r^2} = \sqrt{25^2 - 7^2} = \sqrt{625 - 49} = \sqrt{576} = 24\ m$$

$$\text{The volume of a Cone } = \frac{1}{3}\pi r^2 h = \frac{1}{\cancel{3}} \times \frac{22}{\cancel{7}} \times \cancel{7} \times 7 \times \cancel{24}^{8} = 22 \times 7 \times 8 = 1232\ m^3$$

EXERCISE 11.3

1. Find the volume of the right circular cone with
(i) radius 6 cm, height 7 cm

Sol: The volume of Cone $=\frac{1}{3}\pi r^2 h=\frac{1}{\cancel{3}}\times\frac{22}{\cancel{7}}\times\cancel{6}^2\times 6\times\cancel{7}=22\times 12=264\ cm^3$

(ii) radius 3.5 cm, height 12 cm

Sol: The volume of Cone $=\frac{1}{3}\pi r^2 h=\frac{1}{\cancel{3}}\times\frac{22}{\cancel{7}}\times\cancel{3.5}^{0.5}\times 3.5\times\cancel{12}^4=22\times 0.5\times 3.5\times 4=154\ cm^3$

2. Find the capacity in litres of a conical vessel with

(i) Radius 7 cm, slant height 25 cm

$1000\ cm^3 = 1\ l$

Sol: $r=7\ cm, l=25\ cm$

$$h=\sqrt{l^2-r^2}=\sqrt{25^2-7^2}=\sqrt{625-49}=\sqrt{576}=24\ cm$$

The capacity of the conical vessel $=\frac{1}{3}\pi r^2 h$

$$=\frac{1}{\cancel{3}}\times\frac{22}{\cancel{7}}\times\cancel{7}\times 7\times\cancel{24}^8$$

$$=1232 cm^3$$

$$=1.232\ litres$$

$1000\ m^3 = 1000\ l = 1\ kiololitre$

(ii) Height 12 cm, slant height 13 cm

Sol: $h=12\ cm, l=13\ cm$

$$r=\sqrt{l^2-h^2}=\sqrt{13^2-12^2}=\sqrt{169-144}=\sqrt{25}=5\ cm$$

The capacity of the conical vessel $=\frac{1}{3}\pi r^2 h$

$$=\frac{1}{\cancel{3}}\times\frac{22}{7}\times 5\times 5\times\cancel{12}^4$$

$$=\frac{2200}{7}cm^3$$

$$=\frac{2200}{7\times 1000}\ l$$

$$=\frac{11}{35}\ litres$$

3. The height of a cone is 15 cm. If its volume is 1570 cm³ , find the radius of the base. (Use p = 3.14)

Sol: Height$(h)=15\ cm$

Volume of the cone $=1570\ \mathrm{cm}^3$

$$\frac{1}{3}\pi r^2 h=1570\mathrm{cm}^3$$

$$\frac{1}{3}\times\frac{22}{7}\times r^2\times 15=1570$$

$$r^2=\frac{1570\times 7\times 3}{22\times 15}=100$$

$$r=10\ cm$$

The radius of the base=10 cm

4. If the volume of a right circular cone of height 9 cm is 48πcm³ , find the diameter of its base.

Sol: Height$(h)=9\ cm$

The volume of the cone=48πcm³

$$\frac{1}{3}\pi r^2 h = 48\pi$$

$$\frac{1}{3}\times\pi\times r^2\times 9 = 48\pi$$

$$r^2 = \frac{48\times 3}{9} = 16$$

$$r = 4\ cm$$

The diameter of circular cone base=2×4=8 cm

5. **A conical pit of top diameter 3.5 m is 12 m deep. What is its capacity in kilolitres?**

Sol: Diameter (d) = 3.5 m

$$\text{Radius}(r) = \frac{3.5}{2} = 1.75\ m$$

$$\text{Depth}(h) = 12\ m$$

$$\text{Volume of the conical } pit = \frac{1}{3}\pi r^2 h = \frac{1}{\cancel{3}}\times\frac{22}{\cancel{7}}\times\cancel{1.75}^{0.25}\times 1.75\times\cancel{12}^{4}$$

$$= 22\times 0.25\times 1.75\times 4 = 38.5\ m^3 = 38.5\text{kilolitre.}$$

6. **The volume of a right circular cone is 9856 cm3 . If the diameter of the base is 28 cm, find (i) height of the cone (ii) slant height of the cone (iii)curved surface area of the cone**

Sol: (i) Diameter of the cone$(d) = 28cm$

$$\text{Radius}(r) = 14\ cm$$

$$\text{Volume of the cone} = 9856\ \text{cm}^3$$

$$\frac{1}{3}\pi r^2 h = 9856$$

$$\frac{1}{3}\times\frac{22}{\cancel{7}}\times\cancel{14}^2\times 14\times h = 9856$$

$$h = \frac{9856\times 3}{22\times 14} = 48\ cm$$

Height of the cone=48 cm

$$\text{(ii)Slant height of the cone} = l = \sqrt{r^2+h^2} = \sqrt{14^2+48^2} = \sqrt{196+2304} = \sqrt{2500} = 50\ cm$$

$$\text{(iii) CSA of the cone} = \pi r l = \frac{22}{\cancel{7}}\times\cancel{14}^2\times 50 = 2200\ cm^2$$

7. **A right triangle ABC with sides 5 cm, 12 cm and 13 cm is revolved about the side 12 cm. Find the volume of the solid so obtained.**

Sol: Radius of the cone$(r) = 5\ cm$

Height of the cone (h)=12 cm

Volume of the cone $= \frac{1}{3}\pi r^2 h = \frac{1}{\cancel{3}} \times \pi \times 5 \times 5 \times \cancel{12}^4 = 100\ \pi\ cm^3$

8. If the triangle ABC in the Question 7 above is revolved about the side 5 cm, then find the volume of the solid so obtained. Find also the ratio of the volumes of the two solids obtained in Questions 7 and 8.

Sol: Radius of the cone$(r) = 12\ cm$

Height of the cone (h)=5 cm

Volume of the cone $= \frac{1}{3}\pi r^2 h = \frac{1}{\cancel{3}} \times \pi \times 12 \times \cancel{12}^4 \times 5 = 240\ \pi\ cm^3$

The ratio of the volumes of the two solids obtained $= \frac{100\pi}{240\pi} = \frac{5}{12} = 5:12$

A heap of wheat is in the form of a cone whose diameter is 10.5 m and height is 3 m. Find its volume. The heap is to be covered by canvas to protect it from rain. Find the area of the canvas required.

Sol: Diameter = 10.5 m

Radius(r) $= \frac{10.5}{2} = 5.25$ m

Height$(h) = 3m$

Slant height of the cone $= l = \sqrt{r^2 + h^2} = \sqrt{5.25^2 + 3^2} = \sqrt{27.5625 + 9} = \sqrt{36.5625} = 6.05\ m$

Volume of the conical heap $= \frac{1}{3}\pi r^2 h = \frac{1}{\cancel{3}} \times \frac{22}{\cancel{7}} \times \cancel{5.25}^{0.75} \times 5.25 \times \cancel{3} = 22 \times 0.75 \times 5.25$

$= 86.625\ m^3$

CSA of cone $= \pi r l = \frac{22}{\cancel{7}} \times \cancel{5.25}^{0.75} \times 6.05 = 22 \times 0.75 \times 6.05 = 99.825\ m^2$

The area of the canvas required=99.825 m^2.

Volume of a Sphere

Volume of sphere $= \frac{4}{3}\pi r^3$

Volume of hemisphere $= \frac{4}{3}\pi r^3$

Example 10 : Find the volume of a sphere of radius 11.2 cm.

Sol: Volume of sphere $= \frac{4}{3}\pi r^3 = \frac{4}{3}\times\frac{22}{7}\times 11.2\times 11.2\times 11.2 = 5887.32\ cm^3$

Example 11 : A shot-putt is a metallic sphere of radius 4.9 cm. If the density of the metal is 7.8 g per cm³ , find the mass of the shot-putt.

Sol: Volume of sphere $= \frac{4}{3}\pi r^3 = \frac{4}{3}\times\frac{22}{\cancel{7}}\times \cancel{4.9}^{0.7}\times 4.9\times 4.9\ cm^3 = 493\ cm^3$

Mass of 1 cm³ of metal= 7.8 g.

Mass of the shot-putt = 7.8 × 493 g=3845.44 g=3.85 kg

Example 12 : A hemispherical bowl has a radius of 3.5 cm. What would be the volume of water it would contain?

Sol: Volume of hemispherical bowl $= \frac{2}{3}\pi r^3 = \frac{2}{3}\times\frac{22}{\cancel{7}}\times \cancel{3.5}^{0.5}\times 3.5\times 3.5 = 89.8\ cm^3$

EXERCISE 11.4

1. Find the volume of a sphere whose radius is

(i) 7 cm

Volume of sphere $= \frac{4}{3}\pi r^3 = \frac{4}{3}\times\frac{22}{\cancel{7}}\times \cancel{7}\times 7\times 7 = \frac{4312}{3} = 1437\frac{1}{3}\ cm^3$

(ii) 0.63 m

Volume of sphere $= \frac{4}{3}\pi r^3 = \frac{4}{\cancel{3}}\times\frac{22}{\cancel{7}}\times \cancel{0.63}^{\cancel{0.09}^{0.03}}\times 0.63\times 0.63 = 1.0478\ m^3 = 1.05\ m^3$

2. Find the amount of water displaced by a solid spherical ball of diameter

(i) 28 cm

Sol: Diameter$(d) = 28\ cm$

Radius$(r) = \frac{28\ cm}{2} = 14\ cm$

Volume of sphere $= \frac{4}{3}\pi r^3$

$= \frac{4}{3}\times\frac{22}{\cancel{7}}\times \cancel{14}^2\times 14\times 14$

$= \frac{34496}{3}cm^3 = 11498\frac{2}{3}cm^3$

The amount of water displaced by the solid $= 11498\frac{2}{3}cm^3$

(ii) 0.21 m

Sol: Diameter$(d) = 0.21\ cm$

Radius$(r) = \frac{0.21\ cm}{2} = 0.105\ cm$

Volume of sphere $= \frac{4}{3}\pi r^3$

$= \frac{4}{\cancel{3}}\times\frac{22}{\cancel{7}}\times \cancel{0.105}^{\cancel{0.015}^{0.005}}\times 0.105\times 10.105$

$= 0.004851 cm^3$

The amount of water displaced by the solid $== 0.004851 cm^3$

3. The diameter of a metallic ball is 4.2 cm. What is the mass of the ball, if the density of the metal is 8.9 g per cm³?

Sol: Diameter$(d) = 4.2\ cm$

Radius$(r) = \frac{4.2\ cm}{2} = 2.1\ cm$

Volume of metallic ball $= \frac{4}{3}\pi r^3 = \frac{4}{\cancel{3}} \times \frac{22}{\cancel{7}} \times \cancel{2.1}^{\cancel{0.3}^{0.1}} \times 2.1 \times 2.1 = 38.808\ cm^3$

The density of the metal per 1 cm³=8.9 g

The mass of the ball $= 38.808 \times 8.9\ g = 345.39\ g$

4. The diameter of the moon is approximately one-fourth of the diameter of the earth. What fraction of the volume of the earth is the volume of the moon?

Sol: Let radius of the earth $= R$, and radius of moon $= r$

The diameter of the moon = one-fourth of the diameter of the earth.

$2r = \frac{1}{4} \times 2R \Rightarrow \frac{r}{R} = \frac{1}{4}$

$$\frac{\text{Volume of the moon}}{\text{Volume of the earth}} = \frac{\frac{4}{3}\pi r^3}{\frac{4}{3}\pi R^3} = \frac{r^3}{R^3} = \left(\frac{r}{R}\right)^3 = \left(\frac{1}{4}\right)^3 = \frac{1}{64}$$

Required fraction $= \frac{1}{64}$

5. How many litres of milk can a hemispherical bowl of diameter 10.5 cm hold?

Sol: Diameter$(d) = 10.5\ cm$

Radius$(r) = \frac{10.5}{2} = 5.25\ cm$

Volume of hemispherical bowl $= \frac{2}{3}\pi r^3$

$= \frac{2}{\cancel{3}} \times \frac{22}{\cancel{7}} \times \cancel{5.25}^{\cancel{0.75}^{0.25}} \times 5.25 \times 5.25$

$= 303.1875\ cm^3$

$= \frac{303.1875}{1000}\ l = 0.3031875\ l = 0.303\ l\ (approx)$

0.303 litres of milk can be held in the bowl.

6. A hemispherical tank is made up of an iron sheet 1 cm thick. If the inner radius is 1 m, then find the volume of the iron used to make the tank.

Sol: Inner radius of tank$(r) = 1\ m$

Thickness of iron sheet $= 1\ cm = 0.01\ m$

Outer radius of tank$(R) = 1m + 0.01m = 1.01m$

Volume of the iron $= \frac{2}{3}\pi R^3 - \frac{2}{3}\pi r^3 = \frac{2}{3}\pi(R^3 - r^3) = \frac{2}{3} \times \frac{22}{7} \times (1.01^3 - 1^3)$

$= \frac{44}{21} \times (1.030301 - 1) = \frac{44}{21} \times 0.030301 = \frac{1.33}{21} = 0.063\ m^3(approx)$

7. Find the volume of a sphere whose surface area is 154 cm^2 .

Sol: Surface area of the sphere $= 154\ cm^2$

$4\pi r^2 = 154$

$4 \times \frac{22}{7} \times r^2 = 154$

$r^2 = \frac{154 \times 7}{4 \times 22} = \frac{49}{4}$

$r = \frac{7}{2}$

Volume of the sphere $= \frac{4}{3}\pi r^3 = \frac{4}{3} \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times \frac{7}{2} = \frac{539}{3} = 179\frac{2}{3}\ cm^3$

8. A dome of a building is in the form of a hemisphere. From inside, it was white-washed at the cost of ₹ 4989.60. If the cost of white-washing is ₹ 20 per square metre, find the (i) inside surface area of the dome, (ii) volume of the air inside the dome.

Sol: Let inside radius of the dome $= r\ m$

Inside surface area of the dome $= 2\pi r^2 m^2$

The cost of white-washing per 1 m^2= ₹ 20

Total cost of white wash=₹ 20×$2\pi r^2$

$\therefore$ ₹ $20 \times 2\pi r^2 =$ ₹ 4989.60

$20 \times 2 \times \frac{22}{7} \times r^2 = 4989.60$

$r^2 = \frac{4989.60 \times 7}{20 \times 2 \times 22} = 39.69 = (6.3)^2$

$r = 6.3\ m$

(i)Inside surface area of the dome $= 2\pi r^2 = 2 \times \frac{22}{\cancel{7}} \times \cancel{6.3}^{0.9} \times 6.3 = 249.48\ m^2$

(ii)Volume of the air inside the dome $= \frac{2}{3}\pi r^3 = \frac{2}{\cancel{3}} \times \frac{22}{\cancel{7}} \times \cancel{6.3}^{\cancel{0.9}^{0.3}} \times 6.3 \times 6.3 = 523.9\ m^3$

9. Twenty seven solid iron spheres, each of radius r and surface area S are melted to form a sphere with surface area S^1. Find the (i) radius r^1 of the new sphere, (ii) ratio of S and S^1

Sol: Volume of solid iron sphere $= \frac{4}{3}\pi r^3$

Volume of new solid iron sphere $= \frac{4}{3}\pi (r^1)^3$

Volume of new solid iron sphere $= 27 \times$ Volume of solid iron sphere

$\frac{4}{3}\pi (r^1)^3 = 27 \times \frac{4}{3}\pi r^3 = \frac{4}{3}\pi (3r)^3$

$r^1 = 3r$

(i) Radius of the new sphere $r^1 = 3r$

(ii)Ratio of S and $S^1 = 4\pi r^2 : 4\pi (r^1)^2 = r^2 : (r^1)^2 = r^2 : (3r)^2 = r^2 : 9r^2 = 1:9$

10. A capsule of medicine is in the shape of a sphere of diameter 3.5 mm. How much medicine (in mm^3) is needed to fill this capsule?

Sol: Diameter of capsule$(d) = 3.5\ mm$

Radius of capsule$(r) = \frac{3.5}{2} = 1.75\ mm$

Volume of the capsule $= \frac{4}{3}\pi r^3 = \frac{4}{3} \times \frac{22}{7} \times 1.75 \times 1.75 \times 1.75 = 22.46\ mm^3$

$\therefore$ $22.46\ mm^3$ medicine is needed to fill this capsule.