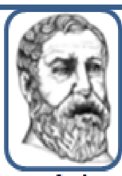
CHAPTER 10

IX-MATHEMATICS-NCERT(2023-24)

10. HERON'S FORMULA

PREPARED BY: BALABHADRA SURESH

https://sureshmathsmaterial.com/

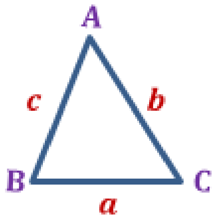

Heron of Alexandria

1. **Heron's formula:**

Area of triangle $= \sqrt{s(s-a)(s-b)(s-c)}$

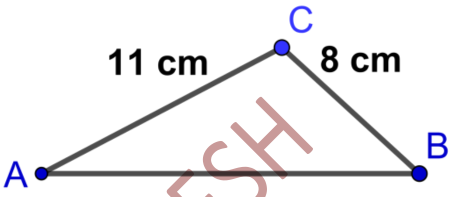
Where a, b, c are sides of the triangle and $s = \dfrac{a+b+c}{2}$

s=semi-perimeter i.e., half the perimeter of the triangle



Example 1 : Find the area of a triangle, two sides of which are 8 cm and 11 cm and the perimeter is 32 cm

Sol: a = 8 cm and b = 11 cm.



Perimeter of the triangle = 32 cm

$a + b + c = 32$

$8 + 11 + c = 32$

$c = 32 - 19 = 13\ cm$

2s =32 cm, i.e., s = 16 cm,

s – a =(16 – 8) cm = 8 cm,

s – b =(16 – 11) cm = 5 cm,

s – c =(16 – 13) cm = 3 cm.

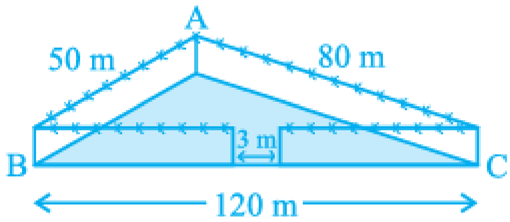
Area of triangle $= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{16 \times 8 \times 5 \times 3}$

$= \sqrt{2 \times 8 \times 8 \times 5 \times 3}$

$= 8\sqrt{2 \times 5 \times 3} = 8\sqrt{30}\ cm^2$

Example 2 : A triangular park ABC has sides 120m, 80m and 50m (see Fig. 10.4). A gardener Dhania has to put a fence all around it and also plant grass inside. How much area does she need to plant? Find the cost of fencing it with barbed wire at the rate of `20 per metre leaving a space 3m wide for a gate on one side.



Solution: $a = 120\ m, b = 80\ m, c = 50\ m$

$s = \dfrac{a+b+c}{2} = \dfrac{50 + 80 + 120}{2} = \dfrac{250}{2} = 125\ m$

$s = 125\ m$

$s - a = (125 - 120)\ m = 5\ m,$

$s - b = (125 - 80)\ m = 45\ m,$

$s - c = (125 - 50)\ m = 75\ m$

Area of the park $= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{125 \times 5 \times 45 \times 75}$

$= \sqrt{25 \times 5 \times 5 \times 3 \times 15 \times 25 \times 3}$

$= 25 \times 5 \times 3 \times \sqrt{15} = 375\sqrt{15}m^2$

Perimeter of the park = AB + BC + CA = 250 m

Length of the wire needed for fencing = 250 m – 3 m=247 m

The cost of fencing = ₹20 × 247 = ₹4940.

Example 3 : The sides of a triangular plot are in the ratio of 3 : 5 : 7 and its perimeter is 300 m. Find its area.

Sol: The ratio of sides of triangle=3:5:7

Let the sides $a = 3x, b = 5x, c = 7x$

Perimeter of the triangle=300m

$3x + 5x + 7x = 300$

$15x = 300$

$x = \frac{300}{15} = 20$

$a = 3 \times 20 = 60m, b = 5 \times 20 = 100m, c = 7 \times 20 = 140m$

$s = \frac{60 + 100 + 140}{2} = \frac{300}{2} = 150m$

Area of the plot $= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{150 \times (150 - 60) \times (150 - 100) \times (150 - 140)}\ m^2$

$= \sqrt{150 \times 90 \times 50 \times 10}\ m^2$

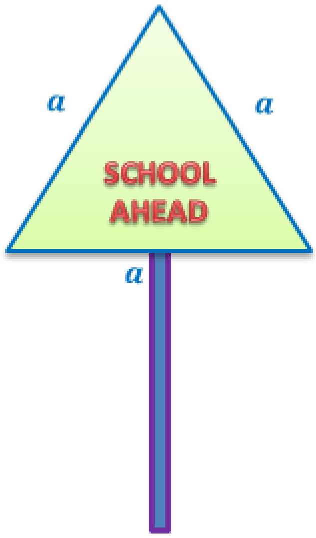
$= \sqrt{3 \times 50 \times 3 \times 3 \times 10 \times 50 \times 10}\ m^2$

$= 3 \times 10 \times 50 \times \sqrt{3}\ m^2$

$= 1500\sqrt{3}\ m^2$

EXERCISE 10.1

1. A traffic signal board, indicating 'SCHOOL AHEAD', is an equilateral triangle with side 'a'. Find the area of the signal board, using Heron's formula. If its perimeter is 180 cm, what will be the area of the signal board?



Sol: $2s = 3a$

$$s = \frac{3a}{2}$$

$$s - a = \frac{3a}{2} - a = \frac{3a - 2a}{2} = \frac{a}{2}$$

Area of the signal board $= \sqrt{s(s-a)(s-a)(s-a)}$

$$= \sqrt{\frac{3a}{2} \times \frac{a}{2} \times \frac{a}{2} \times \frac{a}{2}} = \frac{\sqrt{3} \times a \times a}{2 \times 2} = \frac{\sqrt{3}}{4}a^2$$

If perimeter of the board=180 cm

$$3a = 180\ cm$$

$$a = \frac{180}{3} = 60\ cm$$

Area of the signal board $= \frac{\sqrt{3}}{4}a^2 = \frac{\sqrt{3}}{4} \times 60 \times 60 = 900\sqrt{3}\ cm^2$

Method 2:

Let a=2x

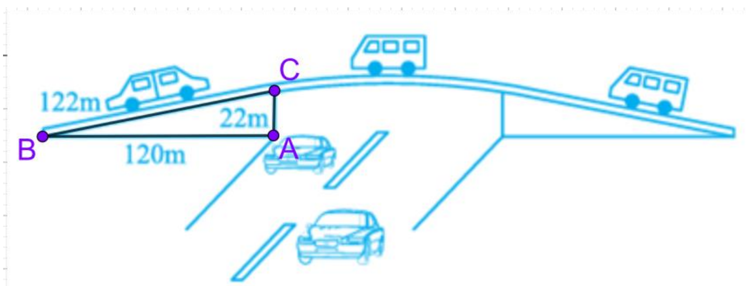
$$2s = 6x \Rightarrow s = \frac{6x}{2} = 3x$$

$$s - a = 3x - 2x = x$$

Area of the signal board $= \sqrt{s(s-a)(s-a)(s-a)}$

$$= \sqrt{3x \times x \times x \times x} = \sqrt{3x^4} = \sqrt{3}x^2 = \sqrt{3} \times \left(\frac{a}{2}\right)^2 = \frac{\sqrt{3}}{4}a^2$$

2. The triangular side walls of a flyover have been used for advertisements. The sides of the walls are 122 m, 22 m and 120 m (see Fig. 10.6). The advertisements yield earnings of ₹ 5000 per m² per year. A company hired one of its walls for 3 months. How much rent did it pay?



Sol: $a = 122$ m, b $= 22$ m , c $= 120$ m

$$s = \frac{a+b+c}{2} = \frac{122+22+120}{2} = \frac{264}{2} = 132\ m$$

Area of the triangle $= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{132(132-122)(132-22)(132-120)}$

$= \sqrt{132 \times 10 \times 110 \times 12}$

$= \sqrt{11 \times 12 \times 10 \times 11 \times 10 \times 12}$

$= 10 \times 11 \times 12 = 1320\ m^2$

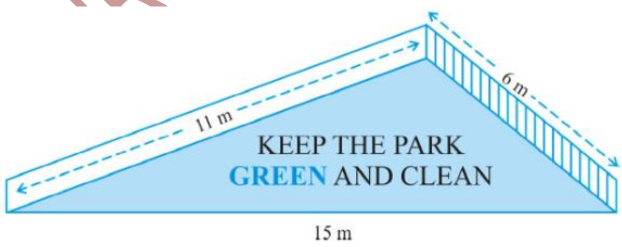
Rent for 1 m^2 area per year=₹ 5000

Rent for 1 m^2 area per 3 months $= \frac{₹\ 5000}{12} \times 3 = ₹1250$

Rent for 1320 m^2 area per 3 months $= ₹1250 \times 1320 = ₹16,50,000$

3. There is a slide in a park. One of its side walls has been painted in some colour with a message "KEEP THE PARK GREEN AND CLEAN" (see Fig. 10.7). If the sides of the wall are 15 m, 11 m and 6 m, find the area painted in colour.

Sol: $a = 15$ m, b = 11 m , c = 6 m



$$s = \frac{a+b+c}{2} = \frac{15+11+6}{2} = \frac{32}{2} = 16\ m$$

Area of the triangle $= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{16(16-15)(16-11)(16-6)}$

$= \sqrt{16 \times 1 \times 5 \times 10}$

$= \sqrt{4 \times 4 \times 5 \times 5 \times 2}$

$= 20\sqrt{2}\ m^2$

$\therefore$ The area painted in colour= $20\sqrt{2}\ m^2$

4. Find the area of a triangle two sides of which are 18cm and 10cm and the perimeter is 42cm.

Sol: a = 18 cm and b = 10 cm.

Perimeter of the triangle = 42 cm

$a + b + c = 42$

$18 + 10 + c = 42$

$c = 42 - 28 = 14\ cm$

2s =42 cm, i.e., s = 21 cm,

Area of triangle $= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{21(21-18)(21-10)(21-14)}$

$= \sqrt{21 \times 3 \times 11 \times 7}$

$= \sqrt{3 \times 7 \times 3 \times 11 \times 7} = 3 \times 7 \times \sqrt{11}\ cm^2 = 21\sqrt{11}\ cm^2$

5. Sides of a triangle are in the ratio of 12 : 17 : 25 and its perimeter is 540cm. Find its area.

Sol: The ratio of sides=12 : 17 : 25

Let the sides are $a = 12x, b = 17x, c = 25x$

Perimeter = 540cm

$12x + 17x + 25x = 540$

$54x = 540$

$x = 10$

$a = 12 \times 10 = 120cm$, $b = 17 \times 10 = 170\ cm, c = 25 \times 10 = 250\ cm$

2s=540cm

s=270 cm

Area of triangle $= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{270(270-120)(270-170)(270-250)}$

$= \sqrt{270 \times 150 \times 100 \times 20}$

$= \sqrt{3 \times 3 \times 3 \times 10 \times 3 \times 5 \times 5 \times 2 \times 10 \times 10 \times 2 \times 10}$

$= 3 \times 3 \times 2 \times 5 \times 10 \times 10 = 9000\ cm^2$

6. An isosceles triangle has perimeter 30 cm and each of the equal sides is 12 cm. Find the area of the triangle.

Sol: $a = 12\ cm$, $b = 12\ cm$

Perimeter= 30 cm

$a + b + c = 30$

$12 + 12 + c = 30$

$24 + c = 30$

$c = 30 - 24 = 6\ cm$

$2s = 30\ cm \Rightarrow s = 15\ cm$

Area of triangle $= \sqrt{s(s-a)(s-b)(s-c)}$

$$= \sqrt{15(15-12)(15-12)(15-6)}$$

$$= \sqrt{15 \times 3 \times 3 \times 9} = 9\sqrt{15}\ cm^2$$

***Proof of Heron's formula*:**

$$2s = P = a + b + c$$

From ΔABD

$$x^2 + h^2 = c^2$$

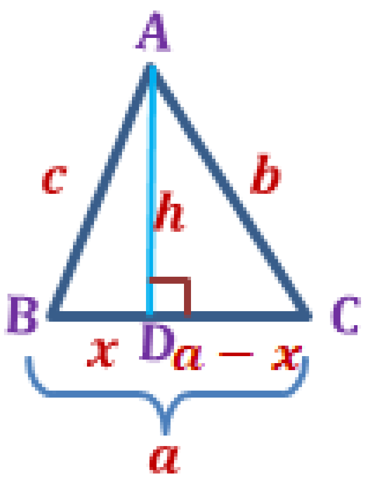
$$h^2 = c^2 - x^2 \rightarrow (1)$$

From ΔADC

$$(a-x)^2 + h^2 = b^2$$

$$h^2 = b^2 - (a-x)^2 \rightarrow (2)$$

From (1) and (2)



$$c^2 - x^2 = b^2 - (a-x)^2$$

$$c^2 - x^2 = b^2 - a^2 + 2ax - x^2$$

$$c^2 = b^2 - a^2 + 2ax$$

$$x = \frac{a^2 + c^2 - b^2}{2a}$$

Substitute x value in (1)

$$h^2 = c^2 - x^2 = (c+x)(c-x)$$

$$= \left(c + \frac{a^2+c^2-b^2}{2a}\right)\left(c - \frac{a^2+c^2-b^2}{2a}\right)$$

$$= \left(\frac{2ac + a^2 + c^2 - b^2}{2a}\right)\left(\frac{2ac - a^2 - c^2 + b^2}{2a}\right)$$

$$= \left(\frac{(a+c)^2 - b^2}{2a}\right)\left(\frac{b^2 - (a-c)^2}{2a}\right)$$

$$= \left[\frac{(a+c+b)(a+c-b)}{2a}\right]\left[\frac{(b+a-c)(b-a+c)}{2a}\right]$$

$$= \left[\frac{2s(2s-2b)}{2a}\right]\left[\frac{(2s-2c)(2s-2a)}{2a}\right]$$

$$= \left[\frac{16s(s-a)(s-b)(s-c)}{4a^2}\right]$$

$$= \frac{4s(s-a)(s-b)(s-c)}{a^2}$$

$$h = \sqrt{\frac{4s(s-a)(s-b)(s-c)}{a^2}}$$

$$h = \frac{2\sqrt{s(s-a)(s-b)(s-c)}}{a}$$

Area of the triangle $= \frac{1}{2} \times base \times height$

$$= \frac{1}{2} \times a \times \frac{2\sqrt{s(s-a)(s-b)(s-c)}}{a}$$

$$= \sqrt{s(s-a)(s-b)(s-c)}$$

Heron (10 C.E. – 75 C.E.)

For VI to X all chapters notes visit my website

https://sureshmathsmaterial.com/