CHAPTER

9

IX-MATHEMATICS-NCERT-202324

9.CIRCLES (Notes)

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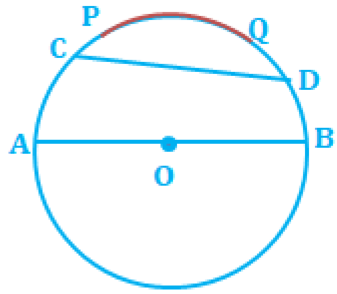
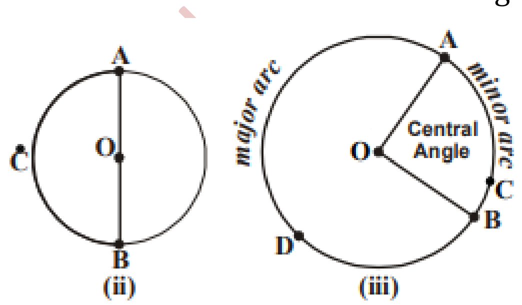
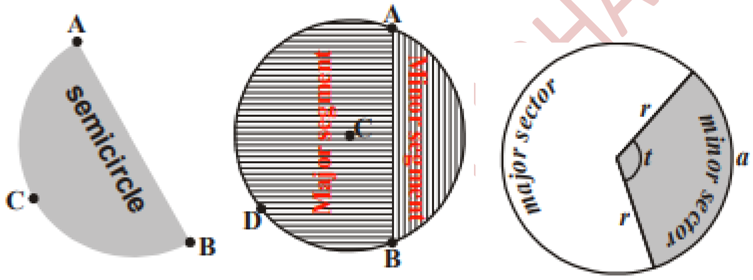
https://sureshmathsmaterial.com

1. **Circle**: A circle is a collection of all the points in a plane which are at a fixed distance from a fixed point on the plane. The fixed point 'O' is called the centre of the circle and the fixed distance OA, is called the radius of the circle.
2. A line segment joining any two points on the circle that passes through the centre is called the diameter (AB)
3. A line segment joining any two points on the circle is called a chord (CD)
4. The part of the circle between any two points on it is called an arc.
5. If the end points of an arc become the end points of a diameter then such an arc is called a semi-circular arc or a semicircle.
6. If the arc is smaller than a semicircle, then the arc is called a minor arc and if the arc is longer than a semicircle, then the arc is called a major arc
7. The region between the chord and the minor arc is called the minor segment and the region between the chord and the major arc is called the major segment.
8. The area enclosed by an arc and the two radii joining the centre to the end points of an arc is called a sector. One is minor sector and another is major sector.

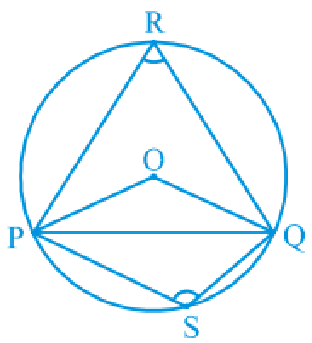




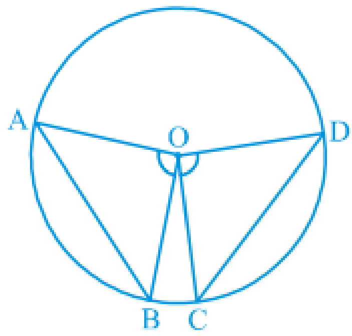


Angle Subtended by a Chord at a Point

1. In adjacent figure ∠ PRQ is called the angle subtended by the line segment PQ at the point R and ∠ POQ is the angle subtended by the chord PQ at the centre O
2. ∠ PRQ and ∠ PSQ are respectively the angles subtended by PQ at points R and S on the major and minor arcs PQ.



Theorem 9.1 : Equal chords of a circle subtend equal angles at the centre.



Proof: AB and CD are two equal chords of a circle with centre O.

In ΔAOB and ΔCOD

OA = OC (Radii of a circle)

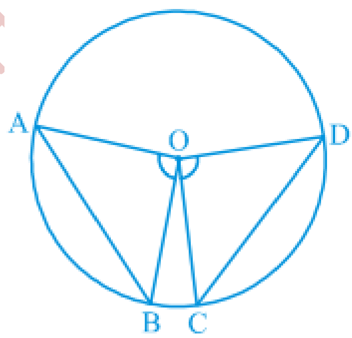
OB = OD (Radii of a circle)

AB = CD (Given)

Δ AOB ≅ Δ COD (By SSS congruence rule)

∠AOB = ∠COD (By CPCT)(Corresponding parts of congruent triangles)

Theorem 9.2 : If the angles subtended by the chords of a circle at the centre are equal, then the chords are equal.



Proof: The angles subtended by the chords AB and CD of a circle at the centre O are ∠AOB and ∠COD respectively.

Given ∠AOB = ∠COD

In ΔAOB and ΔCOD

OA = OC (Radii of a circle)

OB = OD (Radii of a circle)

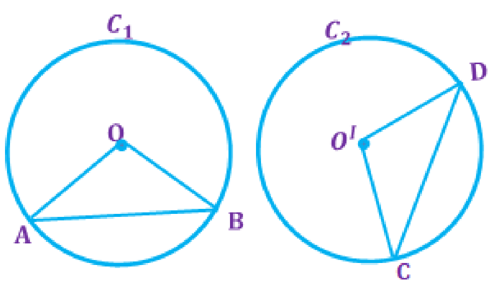
∠AOB = ∠COD (Given)

Δ AOB ≅ Δ COD (By SAS congruence rule)

AB=CD (By CPCT)

EXERCISE 9.1

1. **Recall that two circles are congruent if they have the same radii. Prove that equal chords of congruent circles subtend equal angles at their centres.**



Sol: C_1, C_2 are two congruent circles.

In ΔAOB and ΔCO^ID

$OA = O^IC$ (Radii ofcongruent circles)

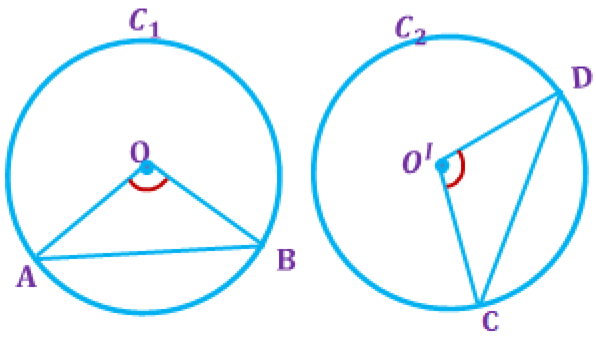
$OB = O^ID$ (Radii ofcongruent circles)

AB = CD (Given)

$\Delta AOB \cong \Delta CO^ID$(By SSS congruence rule)

$\angle AOB = \angle CO^ID$ (By CPCT)

2. Prove that if chords of congruent circles subtend equal angles at their centres, then the chords are equal.



Sol: C_1, C_2 are two congruent circles

$OA = O^IC$ (Radii ofcongruent circles)

$OB = O^ID$ (Radii ofcongruent circles)

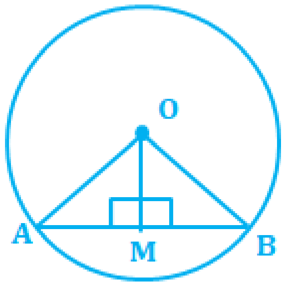
$\angle AOB = \angle CO^ID$ (Given)

$\Delta AOB \cong \Delta CO^ID$(By SAS congruence rule)

$AB = CD$ (By CPCT)

Perpendicular from the Centre to a Chord

Theorem 9.3 : The perpendicular from the centre of a circle to a chord bisects the chord.



Proof: AB is a chord for the circle with centre O and $OM \perp AB$.

Joining O to A and B

In ΔAMO and ΔBMO

$OA = OB$ (Radii of a circle)

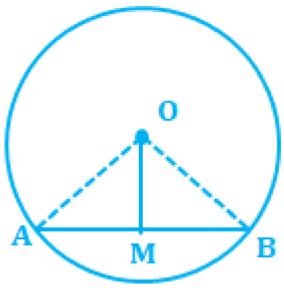
$OM = OM$ (Common))

$\angle AMO = \angle BMO = 90^0$ ($OP \perp AB$)

$\Delta\ AMO \cong \Delta\ BMO$ (By RHS congruence rule)

AM=BM (By CPCT)

Theorem 9.4 : The line drawn through the centre of a circle to bisect a chord is perpendicular to the chord.



Proof: Let AB be a chord of a circle with centre O and O is joined to the mid-point M of AB. Join OA and OB.

In ΔAMO and ΔBMO

$OA = OB$ (Radii of a circle)

$OM = OM$ (Common))

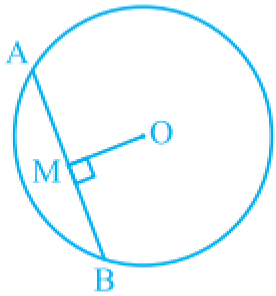
$AM = BM$ (M is midpoint of AB)

$\Delta\ AMO \cong \Delta\ BMO$ (By SSS congruence rule)

$\angle AMO = \angle BMO$ (By CPCT)

But ∠AMO and ∠BMO are linear pair angles.

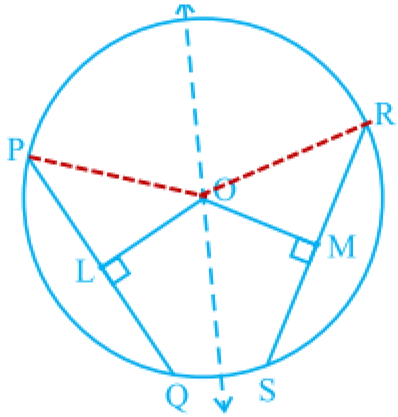
So, ∠AMO = ∠BMO=90^0



Equal Chords and their Distances from the Centre

The length of the perpendicular from a point to a line is the distance of the line from the point.

Theorem 9.5 : Equal chords of a circle (or of congruent circles) are equidistant from the centre (or centres).



Proof:PQ and RS are two equal chords of circle with center O.

OL and OM are perpendiculars to PQ and RS respectively.

We know that the perpendicular from the centre of a circle to a chord bisects the chord.

$PL = QL = \frac{PQ}{2}$ and $SM = RM = \frac{RS}{2}$

But PQ=RS (Given)

⇒PL=RM→(1)

In ΔPOL and ΔROM

∠OLP=∠OMR=90^0

OP=OR (Radii of same circle)

PL=RM (from(1))

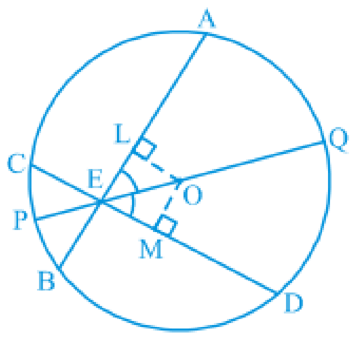
ΔPOL ≅ ΔROM (by RHS rule)

OL=OM(by CPCT)

Hence proved.

Theorem 9.6 : Chords equidistant from the centre of a circle are equal in length.

Example 1 : If two intersecting chords of a circle make equal angles with the diameter passing through their point of intersection, prove that the chords are equal.



Sol: AB and CD are two chords of a circle, with centre O intersecting at a point E.

PQ is a diameter through E, such that ∠ AEQ = ∠ DEQ.

Draw OL ⊥ AB and OM ⊥ CD.

$\angle LEO = \angle MEO \rightarrow (1)$

In Δ OLE and ΔOME

$\angle LEO = \angle MEO$ (from(1))

$\angle OLE = \angle OME$ $(=90^0)$

EO = EO (Common)

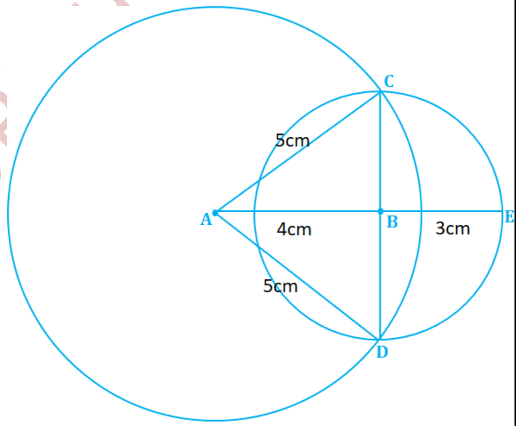
Δ OLE $\cong$ Δ OME (AAS congruency rule)

$\Rightarrow$ OL = OM (by CPCT)

$\Rightarrow$ AB = CD

EXERCISE 9.2

1. Two circles of radii 5 cm and 3 cm intersect at two points and the distance between their centres is 4 cm. Find the length of the common chord.



Sol: Let A,B are the centers of the circles.

CD is the common chord.

AC=AD=5 cm; BE=3 cm

AB⊥CD

From Pythagoras theorem

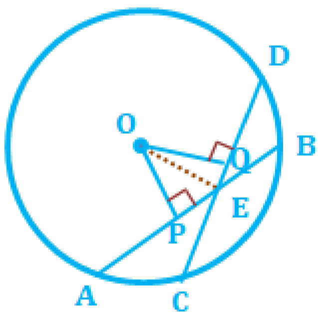
$BC^2 = AC^2 - AB^2 = 5^2 - 4^2 = 25 - 16 = 9$

$BC = 3\ cm$

CD=3+3=6 cm

Length of the chord=6 cm

2. If two equal chords of a circle intersect within the circle, prove that the segments of one chord are equal to corresponding segments of the other chord.



Sol: AB,CD are two chords and AB=CD.

Let AB,CD intersect at E.

Now we prove AE=DE and CE=BE

OP and OQ are perpendiculars to AB and CD from O.

We know that the perpendicular from the centre of a circle to a chord bisects the chord.

AP=PB =CQ=QD→(1)

$In\ \Delta OPE\ and\ \Delta OQE$

$\angle OPE = \angle OQE = 90^0$

$OP = OQ$ (Distance from centre to equal chords)

$OE = OE\ (common)$

$\Delta OPE \cong \Delta OQE(\ RHS\ congruence\ rule)$

$PE = QE\ (By\ CPCT) \rightarrow (2)$

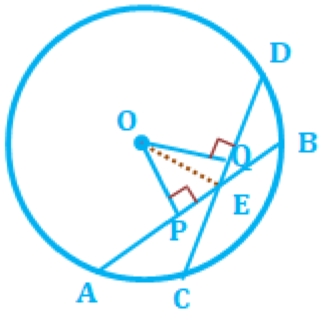
$Now\ AE = AP + PE = DQ + QE\ [\ From\ (1) and\ (2)]$

$\therefore$ AE = DE

$Given\ AB = CD$

$AB - AE = CD - DE\ (\because AE = DE)$
$\therefore$ BE=CE
Hence proved.

3. If two equal chords of a circle intersect within the circle, prove that the line joining the point of intersection to the centre makes equal angles with the chords.



Sol: AB,CD are two chords and AB=CD.

Let AB,CD intersect at E.

Now we show that ∠OEA=∠OED

Let OP and OQ are perpendiculars to AB and CD from O.

$In\ \Delta OPE\ and\ \Delta OQE$

$\angle OPE = \angle OQE = 90^0$

$OP = OQ$ (Distance from centre to equal chords)

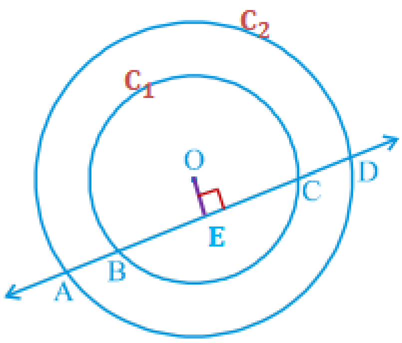
$OE = OE\ (common)$

$\Delta OPE \cong \Delta OQE(\ RHS\ congruence\ rule)$

$\angle \mathrm{OEP} = \angle \mathrm{OEQ}\ (By\ CPCT)$

$\Rightarrow \angle \mathrm{OEA} = \angle \mathrm{OED}$

4. If a line intersects two concentric circles (circles with the same centre) with centre O at A, B, C and D, prove that AB = CD (see Fig. 9.12).



Sol: $Let\ C_1$ and C_2 are two concentric circles with centre O.

$A\ line\ intersects\ \ C_1\ and\ C_2\ at\ B, C\ and\ A, D.$

Let OE⊥AD

We know that the perpendicular from the centre of a circle to a chord bisects the chord.

$In\ circle\ C_2: AE = ED \longrightarrow (1)$

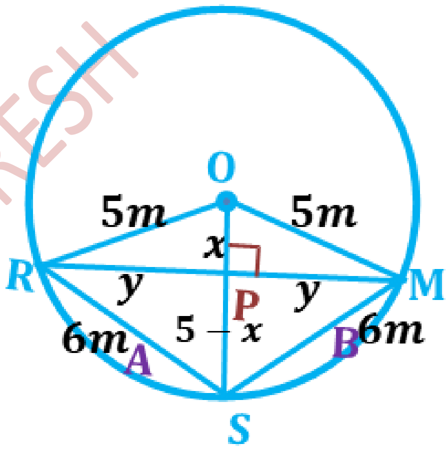
$In\ circle\ C_1 : BE = EC \longrightarrow (2)$

From (1)-(2)

$AE - BE = ED - EC$

AB=CD

5. Three girls Reshma, Salma and Mandip are playing a game by standing on a circle of radius 5m drawn in a park. Reshma throws a ball to Salma, Salma to Mandip, Mandip to Reshma. If the distance between Reshma and Salma and between Salma and Mandip is 6m each, what is the distance between Reshma and Mandip?



Sol: Join OR,OS,OM,RS,MS. OS intersect RM at P

In ΔORS and ΔOMS

OR=OM (Radii)

RS=MS (Given)

OS=OS (Common)

ΔORS ≅ ΔOMS (By SSS congruence rule)

∠ROS=∠MOS (By CPCT)

∴∠ROP=∠MOP →(1)

In ΔROP and ΔMOP

RO=MO (Radii)

∠ROP=∠MOP (From (1))

OP=OP (Common)

ΔROP ≅ ΔMOP (By SAS congruence rule)

RP=PM and ∠RPO=∠MPO (By CPCT)→(2)

∠RPO=∠MPO=90^0

So, OP bisects the chord RM and OP⊥RM

Let RP = PM = y ; $OP = x \Rightarrow$ PS = $5 - x$

In ΔOPR

$x^2 + y^2 = 5^2$(From Pythagoras theorem)

$y^2 = 25 - x^2 \rightarrow (3)$

In ΔRPS

$(5 - x)^2 + y^2 = 6^2$

$y^2 = 36 - (5 - x)^2$

$y^2 = 36 - (25 - 10x + x^2)$

$y^2 = 36 - 25 + 10x - x^2$

$y^2 = 11 + 10x - x^2 \rightarrow (4)$

From (3)and (4)

$11 + 10x - x^2 = 25 - x^2$

$10x = 25 - 11 = 14$

$x = \frac{14}{10} = 1.4\ cm$

From (3)

$y^2 = 25 - x^2 = 25 - (1.4)^2 = 25 - 1.96 = 23.04$

$y = \sqrt{23.04} = 4.8\ cm$

The distance between Reshma and Mandip $= 2y = 2 \times 4.8\ cm = 9.6\ cm$

6. A circular park of radius 20m is situated in a colony. Three boys Ankur, Syed and David are sitting at equal distance on its boundary each having a toy telephone in his hands to talk each other. Find the length of the string of each phone.

Sol: Ankur (A), Syed(S) and David(D)

Let AS = SD = DA = $2x$

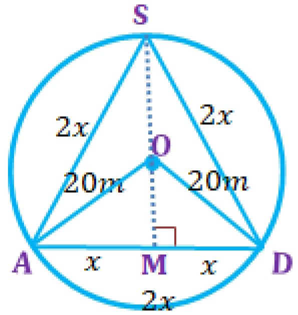
Let OM⊥AD

ΔAMO ≅ ΔDMO (By RHS congruence rule)

AM = MD (By CPCT)

$\therefore$ AM = MD = x

From ΔAMS



$SM^2 = (2x)^2 - x^2 = 4x^2 - x^2 = 3x^2$

$SM = \sqrt{3x^2} = \sqrt{3}x$

$OM = SM - OS = \sqrt{3}x - 20$

From ΔAMO

$OM^2 + AM^2 = OA^2$

$OM^2 = OA^2 - AM^2$

$OM^2 = 20^2 - x^2 = 400 - x^2$

$\left(\sqrt{3}x - 20\right)^2 = 400 - x^2$

$3x^2 - 40\sqrt{3}x + 400 = 400 - x^2$

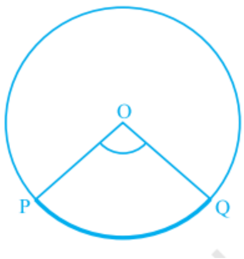
$3x^2 + x^2 = 40\sqrt{3}x$

$4x^2 = 40\sqrt{3}x$

$x = 10\sqrt{3}\, m$

The length of the string of each phone $= 2x = 2 \times 10\sqrt{3} = 20\sqrt{3}\, m$

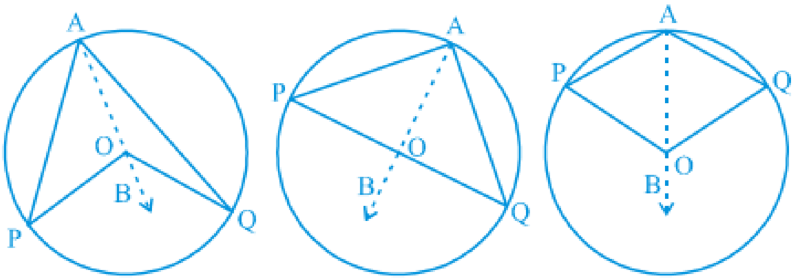
Angle Subtended by an Arc of a Circle



1. If two chords of a circle are equal, then their corresponding arcs are congruent and conversely, if two arcs are congruent, then their corresponding chords are equal.

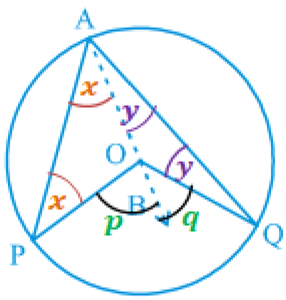
2. Congruent arcs (or equal arcs) of a circle subtend equal angles at the centre.

Theorem 9.7 : The angle subtended by an arc at the centre is double the angle subtended by it at any point on the remaining part of the circle.



(i) (ii) (iii)

Given: : Let O be the centre of the circle. PQ is an arc subtending $\angle POQ$ at the centre. Let A be a point on the remaining part of the circle.



Proof: Draw ray $\overrightarrow{OA}$

In ΔAOP, $OA = OP$ (radii of the same circle)

$\Rightarrow \angle OAP = \angle OPA = x$(say)(Angles opposite to equal sides are equal)

simlarly In ΔAOQ, $OA = OQ$

$\Rightarrow \angle OAQ = \angle OQA = y$(say)

$Let\ \angle POB = p$ and $\angle QOB = q$

$p = x + x$(exterior angle is equal to sum of the opposite interior angles)

$p = 2x$

Similarly $q = 2y$

$p + q = 2x + 2y$

$p + q = 2(x + y)$

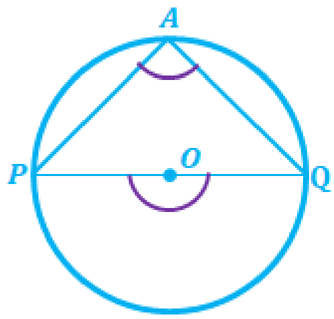
$\angle POQ = 2\angle PAQ$

For the case (iii), where PQ is the major arc

Reflex of $\angle POQ = 2\angle PAQ$

Theorem 9.8 : Angles in the same segment of a circle are equal.

Example: Angle in a semicircle is a right angle.



Sol: PQ is a diameter and 'O' is the centre of the circle.

$\therefore \angle POQ = 180o$ [Angle on a straight line]

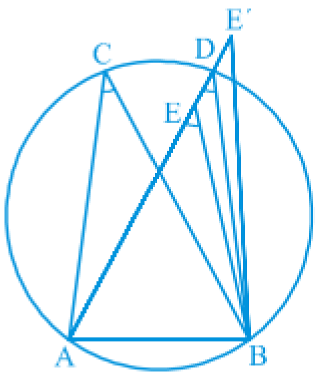
$\angle POQ = 2\angle PAQ$ [Angle subtended by an arc at the centre is twice the angle subtended by it at any other point on circle]

$$\therefore \angle PAQ = \frac{\angle POQ}{2} = \frac{180^0}{2} = 90^0$$

Theorem 9.9 : If a line segment joining two points subtends equal angles at two other points lying on the same side of the line containing the line segment, the four points lie on a circle (i.e. they are concyclic).

Proof: AB is a line segment, which subtends equal angles at two points C and D. That is $\angle ACB = \angle ADB$.

R.T.P : A, B, C and D are concyclic.(they lie on the same circle)



let us draw a circle through the points A, C and B. Suppose it does not pass through the point D. Then it will intersect AD (or extended AD) at a point, say E (or E′).

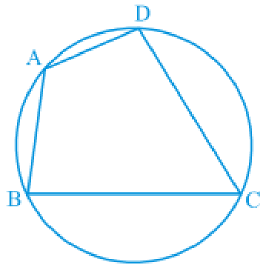
If points A, C, E and B lie on a circle,

$\angle ACB = \angle AEB$

But it is given that $\angle ACB = \angle ADB$.

$\therefore \angle$ AEB = $\angle$ ADB.

This is not possible unless E coincides with D.



Cyclic Quadrilateral: A quadrilateral ABCD is called cyclic if all the four vertices A, B, C, D of it lie on a circle.

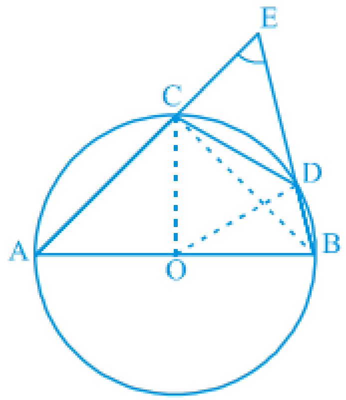
Theorem 9.10 : The sum of either pair of opposite angles of a cyclic quadrilateral is 180º.(OR)

If ABCD is a Cyclic quadrilateral then $\angle A+\angle C=180^0$ and $\angle B+\angle D=180^0$

Theorem 9.11 : If the sum of a pair of opposite angles of a quadrilateral is 180º, the quadrilateral is cyclic.(OR)

If in quadrilateral $\angle A+\angle C=180^0$ and $\angle B+\angle D=180^0$ then ABCD is a Cyclic quadrilateral.

Example 2 : In Fig. 9.19, AB is a diameter of the circle, CD is a chord equal to the radius of the circle. AC and BD when extended intersect at a point E. Prove that $\angle$ AEB = 60°.



Solution : Join OC, OD and BC.

In ΔODC , OC=OD=DC

$\therefore$ ΔODC is an equilateral.

$\Rightarrow \angle COD = 60°$

$\angle CBD = \frac{1}{2}\angle COD$ (*By* Angle subtended theorem)

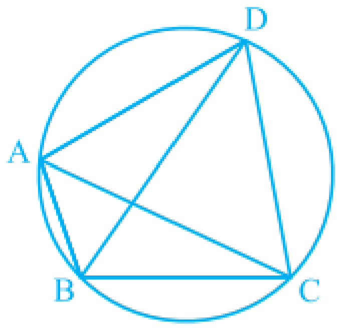
$\angle CBD = \frac{1}{2} \times 60^0 = 30^0 \Rightarrow \angle CBE = 30^0$

$\angle ACB = 90°$ (angle subtended by semi – circle is 90^0)

$\angle BCE = 180° – \angle ACB = 180° – 90° = 90°$

$\angle CEB = 90° – \angle CBE = 90° - 30° = 60°$, i.e., $\angle AEB = 60°$

Example 3 : In Fig 9.20, ABCD is a cyclic quadrilateral in which AC and BD are its diagonals. If $\angle$ DBC = 55° and $\angle$ BAC = 45°, find $\angle$ BCD.



Solution: $\angle CAD = \angle DBC = 55°$ (Angles in the same segment)

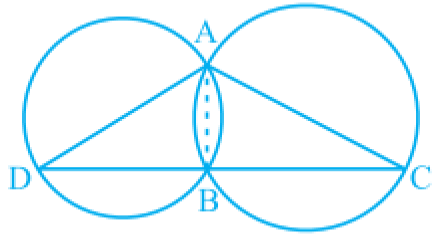
$\therefore$ $\angle DAB = \angle CAD + \angle BAC = 55° + 45° = 100°$

But $\angle DAB + \angle BCD = 180°$ (Opposite angles of a cyclic quadrilateral)

So, $\angle BCD = 180° – 100° = 80°$

Example 4 : Two circles intersect at two points A and B. AD and AC are diameters to the two circles (see Fig. 9.21). Prove that B lies on the line segment DC.

Solution : Join AB.



$\angle ABD = 90°$ (Angle in a semicircle)

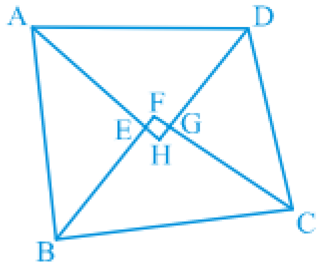
$\angle ABC = 90°$ (Angle in a semicircle)

So, $\angle ABD + \angle ABC = 90° + 90° = 180°$

Therefore, DBC is a line. That is B lies on the line segment DC.

Example 5: Prove that the quadrilateral formed (if possible) by the internal angle bisectors of any quadrilateral is cyclic.

Solution : In Fig. 9.22, ABCD is a quadrilateral in which the angle bisectors AH, BF, CF and DH of internal angles A, B, C and D respectively form a quadrilateral EFGH.



Now, $\angle FEH = \angle AEB = 180° - \angle EAB - \angle EBA$

$= 180° - \frac{1}{2}(\angle A + \angle B)$

$\angle FGH = \angle CGD = 180° - \angle GCD - \angle GDC$

$= 180° - \frac{1}{2}(\angle C + \angle D)$

Therefore, $\angle FEH + \angle FGH = 180° - \frac{1}{2}(\angle A + \angle B) + 180° - \frac{1}{2}(\angle C + \angle D)$

$= 360° - \frac{1}{2}(\angle A + \angle B + \angle C + \angle D) = 360° - \frac{1}{2}(360°) = 360° - 180° = 180°$

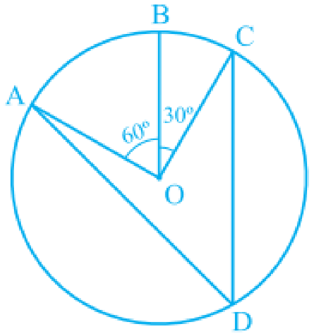
In EFGH the pair of opposite angles are supplementary.

So, the quadrilateral EFGH is cyclic.

EXERCISE 9.3

1. **In Fig. 9.23, A,B and C are three points on a circle with centre O such that $\angle BOC = 30°$ and $\angle AOB = 60°$. If D is a point on the circle other than the arc ABC, find $\angle ADC$.**

Sol: $\angle AOC = \angle AOB + \angle BOC = 60^0 + 30^0 = 90^0$.



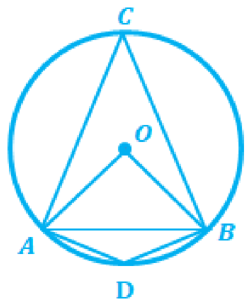
The angle subtended by an arc at the centre is double the angle subtended by it at any point on the remaining part of the circle.

$2 \times \angle ADC = \angle AOC$

$2 \times \angle ADC = 90^0$

$\angle ADC = \frac{90^0}{2} = 45^0$

2. **A chord of a circle is equal to the radius of the circle. Find the angle subtended by the chord at a point on the minor arc and also at a point on the major arc.**



Sol: O is the center of the circle and AB is a chord with length of radius.

OA=OB=AB (Radius)

ΔABO becomes an equilateral triangle.

$\therefore \angle AOB = 60^0$

Let C be a point on the major arc and D be a point on the minor arc.

The angle subtended by an arc at the centre is double the angle subtended by it at any point on the remaining part of the circle.

$2 \times \angle ACB = \angle AOB$

$2 \times \angle ACB = 60^0$

$\angle ACB = \frac{60^0}{2} = 30^0$

Since A, B. C and D lie on the same circle. ADBC is a cyclic quadrilateral.

$\angle ACB + \angle ADB = 180^0$ (*In a cyclic quadrilateral opposite angles are supplementary*)

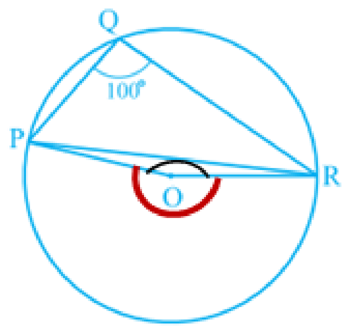
$30^0 + \angle ADB = 180^0$

$\angle ADB = 180^0 - 30^0 = 150^0$

Required angles are $150^0, 30^0$.

3. **In Fig. 9.24, ∠ PQR = 100°, where P, Q and R are points on a circle with centre O. Find ∠ OPR.**

Sol: We know that, the angle subtended by an arc at the center is double the angle subtended by it at any point on the remaining part of the circle.



Reflex $\angle POR = 2 \times \angle PQR$

Reflex$\angle POR = 2 \times 100^0 = 200^0$

$\angle POR = 360^0 - \text{Reflex}\angle POR = 360^0 - 200^0 = 160^0$

In ΔPOR, OP=OR (Radii of the circle)

$\angle OPR = \angle ORP = x$ (Angles of opposite to equal sides are equal)

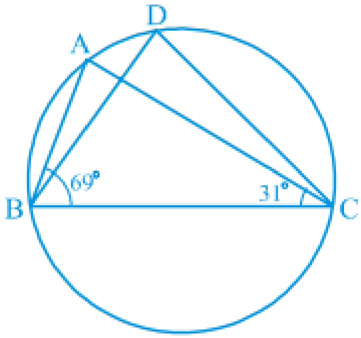
$\angle OPR + \angle ORP + \angle POR = 180^0$ (*Angle sum property of triangle*)

$x + x + 160^0 = 180^0$

$2x = 180^0 - 160^0 = 20^0$

$x = \frac{20^0}{2} = 10^0 \Rightarrow \angle OPR = 10^0$

4. **In Fig. 9.25, ∠ ABC = 69°, ∠ ACB = 31°, find ∠ BDC.**



Sol: In ΔABC, the sum of all angles = 180º

$\angle BAC + \angle ABC + \angle ACB = 180^0$

$\angle BAC + 69^0 + 31^0 = 180^0$

$\angle BAC + 100^0 = 180^0$

$\angle BAC = 180^0 - 100^0$

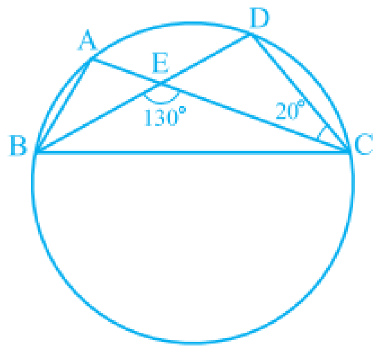
$\angle BAC = 80^0$

We know that angles in the same segment of a circle are equal.

$\angle BDC = \angle BAC$

$\angle BDC = 80^0$

5. **In Fig. 9.26, A, B, C and D are four points on a circle. AC and BD intersect at a point E such that ∠ BEC = 130° and ∠ ECD = 20°. Find ∠ BAC.**



Sol: $\angle BEC + \angle CED = 180^0 (\ Linear\ pair)$

$130^0 + \angle CED = 180^0$

$\angle CED = 180^0 - 130^0 = 50^0$

In ΔDEC, the sum of all angles = 180^0

$\angle CDE + \angle CED + \angle ECD = 180^0$

$\angle CDE + 50^0 + 20^0 = 180^0$

$\angle CDE + 70^0 = 180^0$

$\angle CDE = 180^0 - 70^0 = 110^0$
$\angle CDB = 110^0$

We know that angles in the same segment of a circle are equal.

$\angle BAC = \angle CDB$
$\angle BAC = 110^0$

6. **ABCD is a cyclic quadrilateral whose diagonals intersect at a point E. If ∠ DBC = 70°, ∠ BAC is 30°, find ∠ BCD. Further, if AB = BC, find ∠ ECD.**

Sol: We know that angles in the same segment of a circle are equal.

$\angle CAD = \angle CBD = 70^0$

$\angle BAC = \angle BDC = 30^0$

In ΔABC, AB=BC

$\angle BAC = \angle BCA = 30^0$(*Angles opposite to equal sides are equal*)

$\angle BAD = \angle BAC + \angle CAD = 30^0 + 70^0 = 100^0$

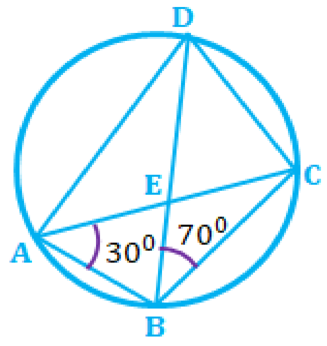
ABCD is a cyclic quadrilateral

$\angle BAD + \angle BCD = 180^0$

$100^0 + \angle BCD = 180^0$

$\angle BCD = 180^0 - 100^0 = 80^0$

$\angle ECD = \angle BCD - \angle BCA = 80^0 - 30^0 = 50^0$.

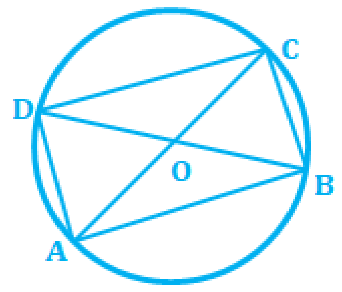


7. **If diagonals of a cyclic quadrilateral are diameters of the circle through the vertices of the quadrilateral, prove that it is a rectangle.**

Sol: ABCD is a cyclic quadrilateral.

AC and BD are diameters.

We know that, the angle subtended by an arc at the centre is double the angle subtended by it at any point on the remaining part of the circle.



$2 \times \angle ADC = \angle AOC$

$2 \times \angle ADC = 180^0$

$\angle ADC = \dfrac{180^0}{2} = 90^0$

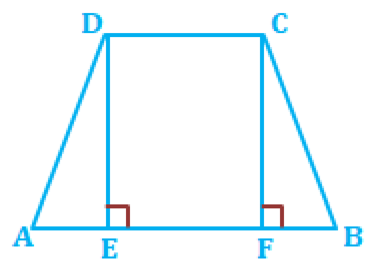
In ABCD one angle is 90^0 and diagonals are equal.

So, ABCD is a rectangle.

8. **If the non-parallel sides of a trapezium are equal, prove that it is cyclic.**

Sol: ABCD is a trapezium with AB || DC and AD=BC.

Draw DE⊥AB and CF⊥AB



In ΔADE and ΔBCF

AD=BC (Given)

$\angle AED = \angle BFC = 90^0$

DE=CF (Distance between parallel lines)

$\Delta ADE \cong \Delta BCF$ (By RHS congruence rule)

$\angle DAE = \angle CBF$ (By CPCT)

$\angle DAB = \angle CBA \rightarrow (1)$

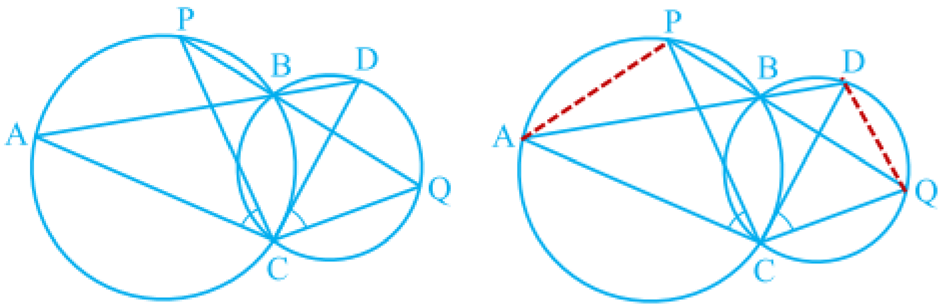
$\angle ADC + \angle DAB = 180^0$(co- interior angles are supplementary)

$\angle ADC + \angle CBA = 180^0$

Opposite angles are supplementary.

Hence, ABCD is a cyclic quadrilateral.

9. **Two circles intersect at two points B and C. Through B, two line segments ABD and PBQ are drawn to intersect the circles at A, D and P, Q respectively (see Fig. 9.27). Prove that ∠ACP = ∠ QCD.**



Sol: Join AP and DQ

For chord AP

$\angle ABP = \angle ACP($ *Angles in the sqme segment are equal*$) \rightarrow (1)$

For chord DQ

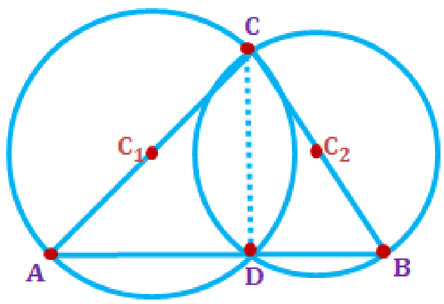
$\angle QBD = \angle QCD$ (*Angles in the sqme segment are equal*$) \rightarrow (2)$

$\angle ABP = \angle QBD$ (*vertically opposite angles*$) \rightarrow (3)$

From (1) ,(2) and (3)

$\angle ACP = \angle QCD$

10. **If circles are drawn taking two sides of a triangle as diameters, prove that the point of intersection of these circles lie on the third side.**



Sol: Two circles are drawn on the sides AC and BC of the ΔABC

The circles intersected at D.

Join CD.

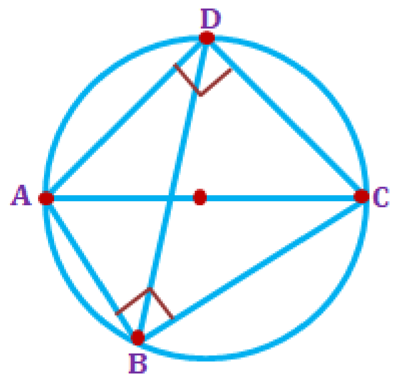
$\angle ADC = \angle BDC = 90^0$(*Angle in semi circle*)

$\angle ADC + \angle BDC = 90^0 + 90^0$

$\angle ADC + \angle BDC = 180^0$

ΔADB is a straight angle

So, D lies on AB.



11. **ABC and ADC are two right triangles with common hypotenuse AC. Prove that ∠ CAD = ∠ CBD.**

Sol: Draw a circle with diameter AC.

ABCD is a quadrilateral.

$\angle B + \angle D = 90^0 + 90^0 = 180^0$

In quadrilateral ABCD , opposite angles are supplementary

ABCD is a cyclic quadrilateral.

CD is a chord.

∠ CAD = ∠ CBD(Angles in the same segment in a circle)

12. **Prove that a cyclic parallelogram is a rectangle.**

Sol: Let ABCD is a cyclic parallelogram.

$\angle A + \angle C = 180^0$ (*opposite angles are supplementary in cyclic quadrilateral*)

But $\angle A = \angle C$ (*In a parallelogramopposite angle are equal*)

$\therefore \angle A = \angle C = 90^0$

ABCD is a parallelogram and one interior angle is 90^0

So, ABCD is a rectangle.