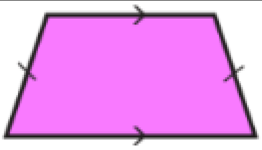
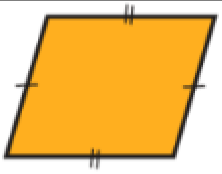
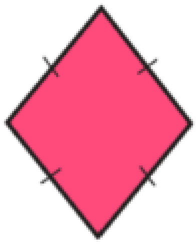
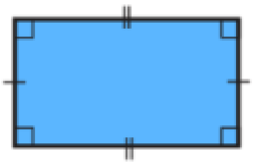
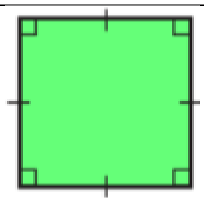
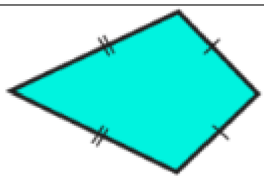
CHAPTER 8

IX-MATHEMATICS-NCERT-2023-24

8. QUADRILATERALS (NOTES)

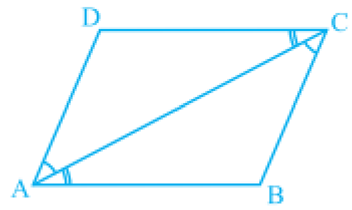
PREPARED BY: BALABHADRA SURESH

1. A quadrilateral has four sides, four angles and four vertices.

Quadrilateral	Figure	Properties
Trapezium A quadrilateral with a pair of parallel sides.		1. One pair of parallel lines
Parallelogram: A quadrilateral with each pair of opposite sides parallel		1. Opposite sides are equal. 2. Opposite angles are equal. 3. Diagonals not equal and bisect one another. 4. Adjacent angles are supplementary
Rhombus: A parallelogram with sides of equal length.		1. All sides are equal. 2. Opposite angles are equal 3. Diagonals are not equal and perpendicularly bisect one another. 4. Adjacent angles are supplementary
Rectangle: A parallelogram with a right angle		1. Opposite sides are equal 2. All angles are right angles. 3. Diagonals are equal and bisect one another.
Square: A rectangle with sides of equal length.		1. All sides are equal. 2. All angles are right angles. 3. Diagonals are equal and perpendicularly bisect one another.
Kite: A quadrilateral with exactly two pairs of equal consecutive sides		1. The diagonals are perpendicular to one another. 2. Diagonals bisect each other.

Theorem 8.1 : A diagonal of a parallelogram divides it into two congruent triangles.

Proof : Let ABCD be a parallelogram and AC be a diagonal



In Δ ABC and Δ CDA,

BC || AD and AC is a transversal.

∠ BCA = ∠ DAC (Pair of alternate angles)

AB || DC and AC is a transversal.

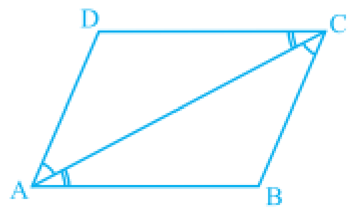
∠ BAC = ∠ DCA (Pair of alternate angles)

AC = CA (Common)

Δ ABC ≅ Δ CDA (ASA rule)

Diagonal AC divides parallelogram ABCD into two congruent triangles ABC and CDA.

Theorem 8.2 : In a parallelogram, opposite sides are equal.

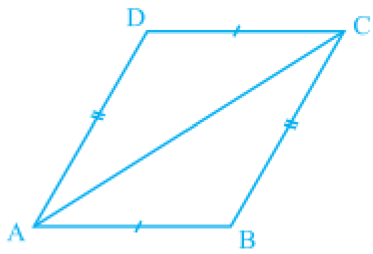


Proof: Let ABCD be a parallelogram and AC be a diagonal.

Δ ABC ≅ Δ CDA (ASA rule)

So, AB = DC and BC = AD (CPCT)

Theorem 8.3 : If each pair of opposite sides of a quadrilateral is equal, then it is a parallelogram.



Proof: ABCD be a quadrilateral and AB = DC and BC = AD

In Δ ABC and Δ CDA

AB = DC (given)

BC = AD(given)

AC=AC (common)

Δ ABC ≅ Δ CDA (SSS congruence rule)

∠ BAC = ∠ DCA(CPCT)

Alternate interior angles are equal⟹ AB || CD

Similarly BC || DA

Each pair of opposite sides are parallel .

ABCD is a parallelogram.

Theorem 8.4 : In a parallelogram, opposite angles are equal.

Proof: ABCD is a parallelogram.

$AB \parallel CD$ and AC is transversal

$x = p$(Alternate interior angles)

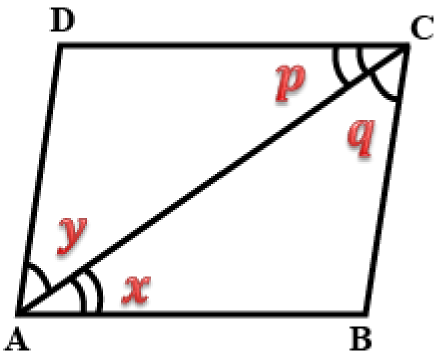
$BC \parallel AD$ and AC is transversal

$y = q$ (Alternate interior angles)

$x + y = p + q$

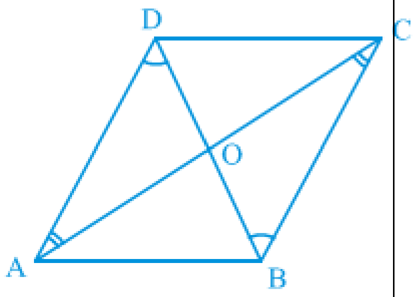
$\angle BAD = \angle BCD \Rightarrow \angle A = \angle C$

Similarly $\angle B = \angle D$



Theorem 8.5 : If in a quadrilateral, each pair of opposite angles is equal, then it is a parallelogram.

Proof: In quadrilateral ABCD , $\angle A = \angle C$ and $\angle B = \angle D$



$\angle A + \angle B + \angle C + \angle D = 360^0$(Sum of angles in quadrilateral)

$\angle A + \angle D + \angle A + \angle D = 360^0$

$2(\angle A + \angle D) = 360^0$

$\angle A + \angle D = 180^0$

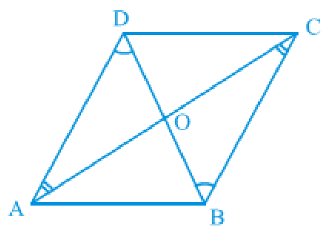
Co interior angles are supplementary

$\Rightarrow AB \parallel DC$

Similarly $BC \parallel AD$

$\therefore$ ABCD is a parallelogram.

Theorem 8.6 : The diagonals of a parallelogram bisect each other.



Proof: In parallelogram ABCD diagonals AC,BD intersect at O

$\Delta\, AOD = \Delta\, COB$ (ASA rule)

$AO = CO$ and $OD = OB$ (CPCT)

$\Rightarrow O$ is mid point of AC and BD

$\Rightarrow AC$ and BD are bisect each other

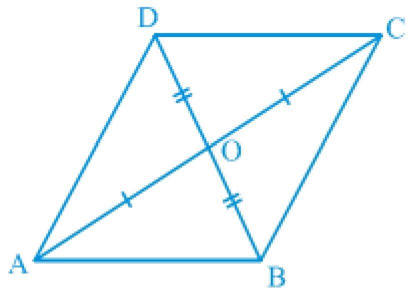
Theorem 8.7 : If the diagonals of a quadrilateral bisect each other, then it is a parallelogram

Proof: ABCD is a quadrilateral. The diagonals AC and BD bisect at O

In Δ AOB , Δ COD

OA = OC and OB = OD (given)

$\angle$AOB=$\angle$COD (vertically opposite angles)



Δ AOB $\cong$ Δ COD (SAS congruence rule)

$\therefore \angle$ ABO = $\angle$ CDO (By CPCT)

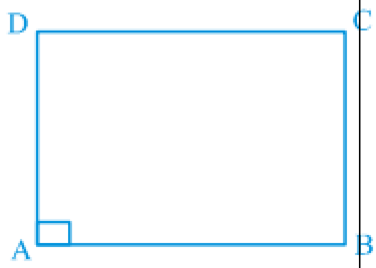
Alternate interior angles are equal

$\therefore$ AB || CD

Similarly BC || AD

Therefore ABCD is a parallelogram.

Example 1 : Show that each angle of a rectangle is a right angle.



Sol: Rectangle is a parallelogram in which one angle is a right angle.

ABCD is a rectangle. Let one angle is $\angle A = 90^0$

We have, AD || BC and AB is a transversal.

$\angle A + \angle B = 180°$ (Interior angles on the same side of the transversal)

$90^0 + \angle B = 180°$

$\angle B = 180° - 90^0 = 90^0$

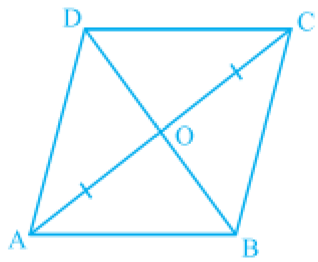
$\angle C = \angle A$ and $\angle D = \angle B$(Opposite angles of the parallelogram)

$\angle C = 90^0$ and $\angle D = 90^0$

Therefore, each of the angles of a rectangle is a right angle.

Example 2 : Show that the diagonals of a rhombus are perpendicular to each other.

Sol: Let ABCD is a rhombus.



AB = BC = CD = DA (All sides are equal in rhombus)

In Δ AOD and Δ COD

OA = OC (Diagonals of a parallelogram bisect each other)

OD = OD (Common)

AD = CD (given)

Δ AOD $\cong$ Δ COD (SSS congruence rule)

$\angle AOD = \angle COD$ (CPCT)

But, $\angle AOD + \angle COD = 180°$ (Linear pair)

$2\angle AOD = 180°$

$\angle AOD = 90°$

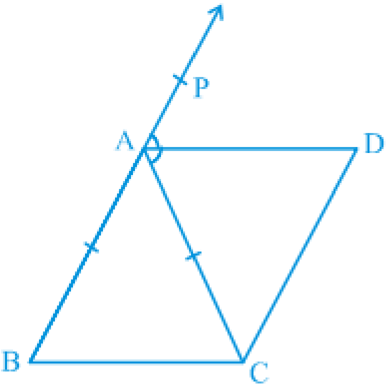
So, the diagonals of a rhombus are perpendicular to each other.

Example 3 : ABC is an isosceles triangle in which AB = AC. AD bisects exterior angle PAC and CD || AB (see Fig. 8.8). Show that (i) ∠ DAC = ∠ BCA and (ii) ABCD is a parallelogram.

Sol: (i) Δ ABC is isosceles in which AB = AC (Given)

So, ∠ ABC = ∠ ACB (Angles opposite to equal sides)

Also, ∠ PAC = ∠ ABC + ∠ ACB (Exterior angle of a triangle)



or, ∠ PAC = 2∠ ACB →(1)

Now, AD bisects ∠ PAC.

So, ∠ PAC = 2∠ DAC →(2)

2∠ DAC = 2∠ ACB [From (1) and (2)]

∠ DAC = ∠ BCA

(ii) ∠ DAC = ∠ ACB i.e alternate interior angles are equal.

⇒ BC ||AD

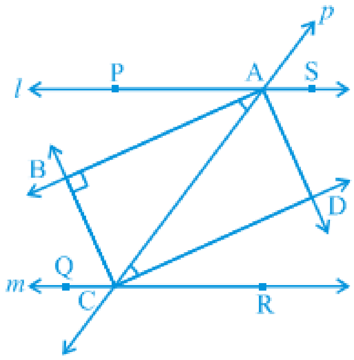
Also, BA || CD (*Given*)

Now, both pairs of opposite sides of quadrilateral ABCD are parallel.

So, ABCD is a parallelogram.

Example 4 : Two parallel lines l and m are intersected by a transversal p (see Fig. 8.9). Show that the quadrilateral formed by the bisectors of interior angles is a rectangle.

Sol: PS || QR and transversal p intersects them at points A and C .



The bisectors of∠PAC and ∠ACQ intersect at B and bisectors of ∠ ACR and ∠SAC intersect at D.

Now, ∠ PAC = ∠ ACR (Alternate angles as l || m and p is a transversal)

$$\frac{1}{2}\angle PAC = \frac{1}{2}\angle ACR$$

i.e., ∠ BAC = ∠ ACD ⇒ alternate interior angles are equal.

$AB \parallel DC$

Similarly, BC || AD (Considering ∠ ACB and ∠ CAD)

Therefore, quadrilateral ABCD is a parallelogram.

∠ PAC + ∠ CAS = 180° (Linear pair)

$\frac{1}{2}\angle PAC + \frac{1}{2}\angle CAS = \frac{1}{2}\times 180° = 90°$

∠ BAC + ∠ CAD = 90°

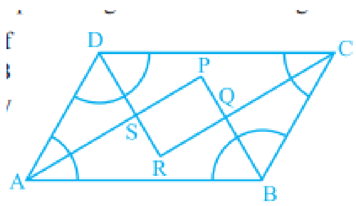
∠ BAD = 90°

So, ABCD is a parallelogram in which one angle is 90°.

Therefore, ABCD is a rectangle.

Example 5 : Show that the bisectors of angles of a parallelogram form a rectangle.

Sol : Let P, Q, R and S be the points of intersection of the bisectors of



∠ A and ∠ B, ∠ B and ∠ C, ∠ C and ∠ D, and ∠ D and ∠ A of parallelogram ABCD In Δ ASD , DS bisects ∠ D and AS bisects ∠ A

$\therefore \angle DAS + \angle ADS = \frac{1}{2}\angle A + \frac{1}{2}\angle D = \frac{1}{2}(\angle A + \angle D)$

$= \frac{1}{2}\times 180^0$ (∠ A and ∠ D are adjacent angles of parallelogram ABCD)

$= 90^0$

Also, ∠DAS + ∠ ADS + ∠DSA = 180° (Angle sum property of a triangle)

90° + ∠ DSA = 180°

∠ DSA = 90°

∠ PSR = 90° (Being vertically opposite to ∠ DSA)

Similarly ∠ SPQ = 90° ,∠ PQR = 90° and ∠ SRQ = 90°

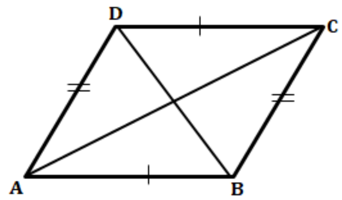
So, PQRS is a quadrilateral in which all angles are right angles.

So, PQRS is a rectangle.

EXERCISE 8.1

1. If the diagonals of a parallelogram are equal, then show that it is a rectangle.

Sol: ABCD is a parallelogram and diagonals AC=BD



$\Delta\, ABC \cong \Delta\, DCB$ (SSS $congruence$ $rule$)

$\angle ABC = \angle DCB$ ($CPCT$)

$\angle ABC + \angle DCB = 180^0$ ($co - interior$ $angles$ are $supplementary$)

$\angle ABC + \angle ABC = 180^0$

$2\angle ABC = 180^0$

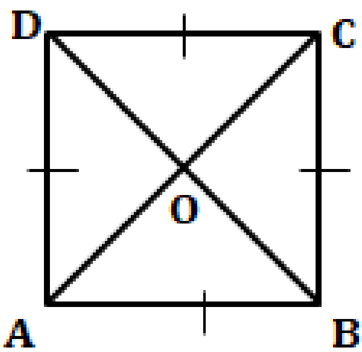
$\angle ABC = 90^0$

ABCD is a parallelogram and one of the angles is 90^0

ABCD is a rectangle.

2. How that the diagonals of a square are equal and bisect each other at right angles.

Sol: Let ABCD is square.



$\Delta\, ABC \cong \Delta\, DCB(\,SAS\ rule)$

$AC = BD\ (By\ CPCT) \Rightarrow Diagonals\ are\ equal$

$\Delta\, AOB \cong \Delta COD\ (ASA\ congruence\ rule)$

$\therefore AO = CO\ and\ OB = OD\ (by\ CPCT)$

$\Rightarrow Diogonals\ are\ bisect\ each\ other$

$\Delta\, AOB \cong \Delta COB\ (SSS\ congruence\ rule)$

$\angle AOB = \angle COB\ (by\ CPCT)$

$But\ \angle AOB + \angle COB = 180^0\ (Linear\ pair)$

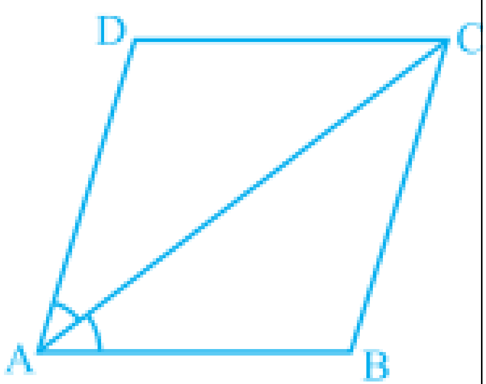
$\angle AOB + \angle AOB = 180^0$

$2\angle AOB = 180^0$

$\angle AOB = 90^0$

Diagonals are bisect each other at right angles.

3. Diagonal AC of a parallelogram ABCD bisects ∠ A Show that (i) it bisects ∠ C also, (ii) ABCD is a rhombus.



Sol: (i) ∠BAC=∠DAC (AC bisects ∠A)→(1)

∠BAC=∠DCA (Alternate interior angles)→(2)

∠DAC =∠BCA (Alternate interior angles)→(3)

From (1),(2),(3)

∠DCA=∠BCA

Hence AC bisects ∠ C also.

(ii) In ΔBAC

∠BAC=∠BCA (From (1),(2),(3))

AB=BC (opposite sides of equal angles are equal) →(4)

But AB=DC and BC=AD (Opposite sides of parallelogram)→(5)

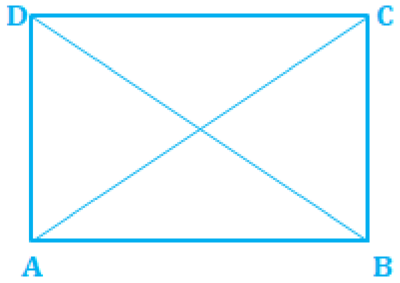
From (4) ,(5)

AB =BC= CD= DA

Hence, ABCD is a rhombus.

4. ABCD is a rectangle in which diagonal AC bisects ∠ A as well as ∠ C. Show that: (i) ABCD is a square (ii) diagonal BD bisects ∠ B as well as ∠ D.

Sol: ABCD is a rectangle



Diagonal AC bisects ∠ A as well as ∠ C

$\angle BAC = \angle DAC = \frac{1}{2}\angle A$ and $\angle BCA = \angle DCA = \frac{1}{2}\angle C$

But $\angle A = \angle C \Rightarrow \frac{1}{2}\angle A = \frac{1}{2}\angle C$

$\Rightarrow \angle BAC = \angle BCA$

$\Rightarrow AB = BC$ (Sides opposite to equal angles are equal)

But $AB = DC$ and $BC = DA$ (opposite sides of a rectangle are equal)

$\therefore AB = BC = CD = DA$

∴ ABCD is a square.

(ii) In ΔBCD , BC=CD (ABCD is a square)

$\angle CBD = \angle CDB$ (Angles opposite to equal sides are equal)

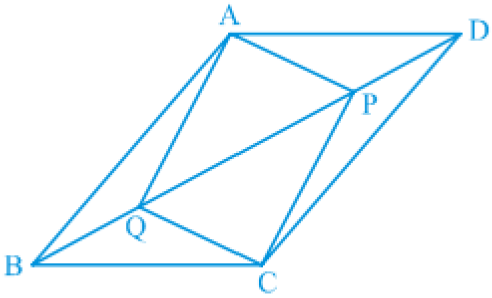
But $\angle CBD = \angle ADB$ and $\angle CDB = \angle ABD$ (alternate interior angles)

$\therefore \angle CBD = \angle ABD$ and $\angle CDB = \angle ADB$

BD bisects ∠D and ∠B.

5. In parallelogram ABCD, two points P and Q are taken on diagonal BD such that DP = BQ (see Fig. 8.12). Show that:
(i) Δ APD ≅ Δ CQB (ii) AP = CQ (iii) Δ AQB ≅Δ CPD (iv) AQ = CP (v) APCQ is a parallelogram.

Sol: (i) In ΔAPD and ΔCQB



∠ADP=∠CBQ (Alternate interior angles)

AD=BC (Opposite sides of a parallelogram are equal)

DP=BQ (Given)

Δ APD ≅ Δ CQB (SAS congruence rule)

(ii) Δ APD ≅ Δ CQB (From (i))

∴AP=CQ (CPCT)

(iii) In ΔAQB and ΔCPD

∠ABQ=∠CDP (Alternate interior angles)

AB=CD (Opposites of a parallelogram are equal)

BQ=DP (Given)

ΔAQB ≅ ΔCPD (SAS congruence rule)

(iv) ΔAQB ≅ ΔCPD (From (iii))

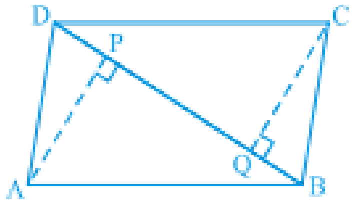
∴AQ=CP (CPCT)

(v) In quadrilateral APCQ

AP=CQ and AQ=CP

Hence APCQ is a parallelogram.

6. ABCD is a parallelogram and AP and CQ are perpendiculars from vertices A and C on diagonal BD (see Fig. 8.13). Show that (i) Δ APB ≅ Δ CQD (ii) AP = CQ



Sol: (i) In ΔAPB and ΔCQD

∠APB=∠CQD=90^0

AB=CD (opposite sides of parallelogram are equal)

∠ABD=∠CDQ (AB||CD , alternate interior angles)

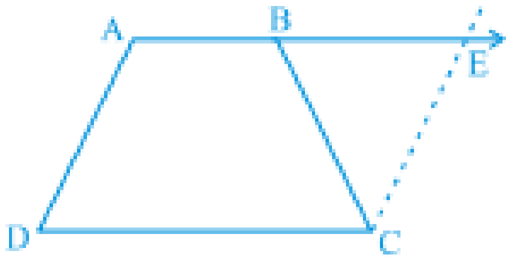
∴ΔAPB ≅ ΔCQD (AAS congruence rule)

(ii) Δ APB ≅ Δ CQD (from (i))

∴AP=CQ (CPCT)

7. ABCD is a trapezium in which AB || CD and AD = BC (see Fig. 8.14). Show that (i) ∠ A = ∠ B (ii) ∠ C = ∠ D (iii) Δ ABC ≅ Δ BAD (iv) diagonal AC = diagonal BD

[Hint: Extend AB and draw a line through C parallel to DA intersecting AB produced at E.]



Sol: Draw AE ||DC and CE||DA

ADCE is a parallelogram

AD=CE (opposite sides of ADCE)

AD = BC (given)

∴BC=CE

∠CEB=∠CBE (equal sides opposite angles are equal)

$\angle A+\angle CEB=180^0$(co-interior angles are supplementary)

$\angle A+\angle CBE=180^0(\angle CEB=\angle CBE)\rightarrow(1)$

$\angle B+\angle CBE=180^0$ (Linear pair)$\rightarrow(2)$

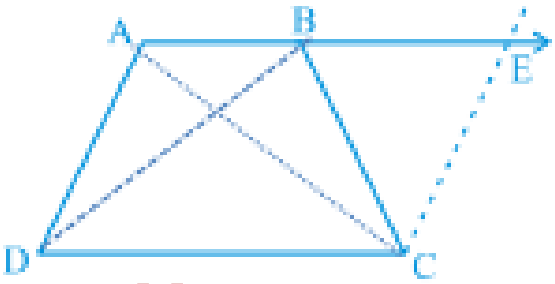
From (1) and (2)

$\angle A=\angle B$

(ii) $\angle A+\angle D=180^0$(co-interior angles)$\rightarrow(3)$

$\angle B+\angle C=180^0$(co-interior angles)$\rightarrow(4)$

From (3) and (4)



$\angle B+\angle C=\angle A+\angle D$

But $\angle A=\angle B$

$\therefore \angle C=\angle D$

(iii) InΔABC and ΔBAD,

BC=AD (given)

AB=BA (common side)

$\angle B=\angle A$ (From (i))

$\Delta ABC\cong\Delta BAD$(SAS congruence rule)

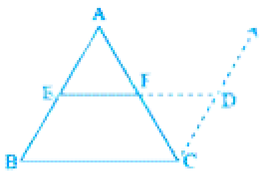
(iv) $\Delta ABC\cong\Delta BAD$ (from (iii))

$\therefore$ AC=BD (by CPCT)

The Mid-point Theorem

Theorem 8.8 : The line segment joining the mid-points of two sides of a triangle is parallel to the third side.

Proof: In ΔABC, E and F are mid-points of AB and AC respectively and draw CD || BA.



In ΔAEF , ΔCDF

$\angle AEF = \angle CDF$ (*Alternate interior angles*)

$\angle EAF = \angle FCD$ (*Alternate interior angles*)

$AF = FC$ (*F is mid point of AC*)

$\Delta AEF \cong \Delta CDF$ (*ASA rule*)

$EF = DF$ *and* $BE = AE = DC$ (*CPCT*)

∴ BCDE is a parallelogram. So, EF || BC

Theorem 8.9 : The line drawn through the mid-point of one side of a triangle, parallel to another side bisects the third side.

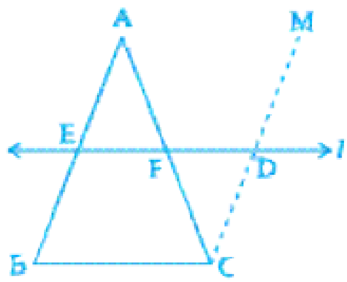
Proof: In ΔABC , E is midpoint of AB.

Draw a line l passing through E and parallel to BC . The line intersects AC at F.

Construct CD || BA

EB || DC and ED || BC

⇒EBCD is a parallelogram.



BE=DC (opposite sides of parallelogram)

But BE=AE (E is midpoint of AB)

∴ AE=CD →(1)

In ΔAFE and ΔCFD

∠EAF = ∠DCF (BA || CD and AC is transversal, alternate interior angles)

∠AEF = ∠CDF (BA || CD and ED is transversal, alternate interior angles)

AE=CD (from (1))

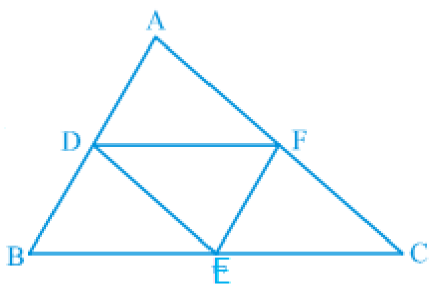
ΔAFE ≅ ΔCFD (ASA congruence rule)

∴ AF = CF (CPCT)

⇒ l bisects AC.

Example 6 : In ΔABC, D, E and F are respectively the mid-points of sides AB, BC and CA (see Fig. 8.18). Show that Δ ABC is divided into four congruent triangles by joining D, E and F.

Solution: We know that the line segment joining the mid-points of two sides of a triangle is parallel to the third side.



DE || AC , DF || BC and EF || AB

Therefore ADEF, BDFE and DFCE are all parallelograms.

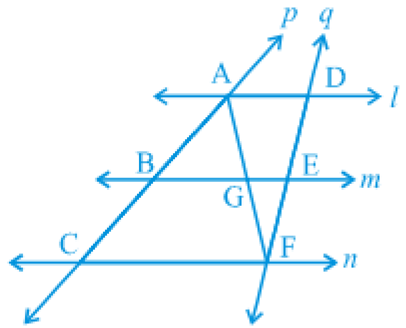
Now DE is a diagonal of the parallelogram BDFE,

Therefore, ΔBDE ≅ Δ FED

Similarly ΔDAF ≅ ΔFED and ΔEFC ≅ ΔFED

So, all the four triangles are congruent

Example 7 : l, m and n are three parallel lines intersected by transversals p and q such that l, m and n cut off equal intercepts AB and BC on p (see Fig. 8.19). Show that l, m and n cut off equal intercepts DE and EF on q also.



Sol: Let us join A to F intersecting m at G.

The trapezium ACFD is divided into two triangles; namely ΔACF and ΔAFD.

In ΔACF, it is given that B is the mid-point of AC (AB = BC)

and BG || CF (since m || n).

So, G is the mid-point of AF .

Now, in ΔAFD, we can apply the same argument as G is the mid-point of AF, GE || AD, so E is the mid-point of DF,

i.e., DE = EF

$\Rightarrow l, m \text{ } and \text{ } n$ cut off equal intercepts on q also.

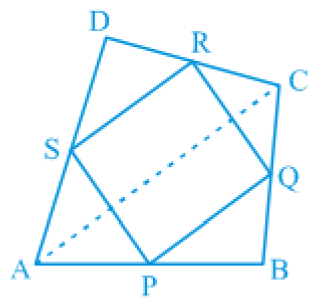
EXERCISE 8.2

1. **ABCD is a quadrilateral in which P, Q, R and S are mid-points of the sides AB, BC, CD and DA. AC is a diagonal. Show that :**

   (i) SR || AC and SR $= \frac{1}{2}$ AC

   (ii) PQ = SR

   (iii) PQRS is a parallelogram.



Sol: We know that the line segment joining the mid-points of two sides of a triangle is parallel to the third side and also half of it.

(i) In ΔABC , P and Q are midpoints of AB and BC.

PQ || AC and PQ $= \frac{1}{2}$ AC$\rightarrow$(1)

(ii) In ΔADC , S and R are midpoints of DA and DC.

PQ || AC and PQ $= \frac{1}{2}$ AC$\rightarrow$(2)

From (1) and (2)

PQ = SR

(iii) From (1) and (2)

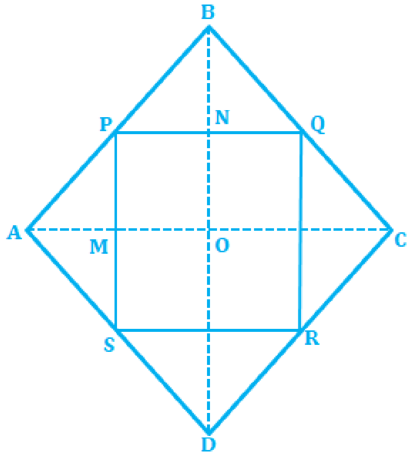
SR || AC and PQ || AC

$\Rightarrow$ PQ ||SR also PQ = SR

$\therefore$ PQRS is a parallelogram.

2. **ABCD is a rhombus and P, Q, R and S are the mid-points of the sides AB, BC, CD and DA respectively. Show that the quadrilateral PQRS is a rectangle.**

Sol: We know that the line segment joining the mid-points of two sides of a triangle is parallel to the third side and also half of it.



In ΔABC , P and Q are midpoints of AB and BC.

$PQ \parallel AC$ and $PQ = \frac{1}{2}$ AC → (1)

In ΔADC , S and R are midpoints of AD and DC.

$SR \parallel AC$ and $SR = \frac{1}{2}$ AC → (2)

From (1)*and* (2) : $PQ \parallel SR$ and $PQ = SR$

Similarly : $PS \parallel QR$ and $PS = QR$

∴ PQRS is a parallelogram.

$MO \parallel PN$ and $PM \parallel NO$

PMON also a parallelogram.

∠MPN=∠MON (opposite angles in a parallelogram)

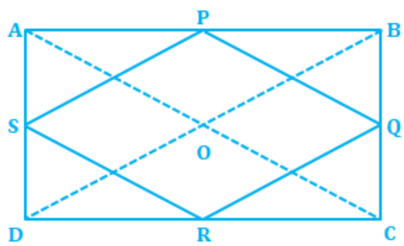
But ∠MON=90^0 (Diagonals of a rhombus perpendicular to each other)

∴ ∠MPN=90^0

In parallelogram PQRS one angle is 90^0

So, PQRS is a rectangle.

3. ABCD is a rectangle and P, Q, R and S are mid-points of the sides AB, BC, CD and DA respectively. Show that the quadrilateral PQRS is a rhombus.



Sol: We know that the line segment joining the mid-points of two sides of a triangle is parallel to the third side and also half of it.

In ΔABC , P and Q are midpoints of AB and BC.

$PQ \parallel AC$ and $PQ = \frac{1}{2}$ AC → (1)

In ΔADC , S and R are midpoints of AD and DC.

$SR \parallel AC$ and $SR = \frac{1}{2}$ AC → (2)

From (1)*and* (2) : $PQ \parallel SR$ and $PQ = SR = \frac{1}{2}AC$

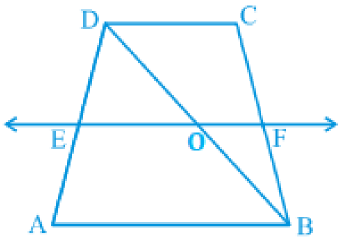
Similarly : $PS \parallel QR$ and $PS = QR = \frac{1}{2}$ BD

$Also, AC = BD$ ($Diagonals\ of\ a\ rectangle\ AC, BD\ are\ equal$)

$\therefore PQ=QR=RS=SP$

So, PQRS is a rhombus.

4. ABCD is a trapezium in which AB || DC, BD is a diagonal and E is the mid-point of AD. A line is drawn through E parallel to AB intersecting BC at F (see Fig. 8.21). Show that F is the mid-point of BC.



Sol: We know that the line drawn through the mid-point of one side of a triangle, parallel to another side bisects the third side.

$In\ \Delta ABD, EO \parallel AB\ and\ E\ is\ mid\ point\ of\ AD$

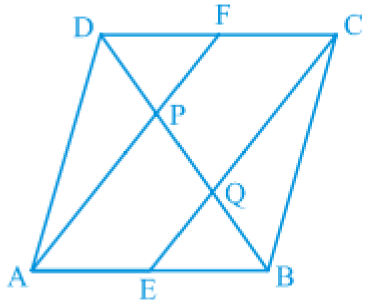
$\Rightarrow O\ is\ mid\ point\ of\ BD$

$In\ \Delta CBD, OF \parallel CD\ and\ O\ is\ mid\ point\ of\ BD$

$\Rightarrow F\ is\ mid\ point\ of\ BC$

5. In a parallelogram ABCD, E and F are the mid-points of sides AB and CD respectively (see Fig. 8.22). Show that the line segments AF and EC trisect the diagonal BD.

Sol: E and F are the mid-points of sides AB and CD



$DF = FC = \frac{1}{2}DC\ and\ AE = EB = \frac{1}{2}AB$

$AB \parallel DC\ \ and\ AB = CD$ ($ABCD\ is\ a\ parallelogram$)

$\Rightarrow AE \parallel FC\ \ and\ \frac{1}{2}AB = \frac{1}{2}CD$

$\Rightarrow AE \parallel FC\ \ and\ AE = FC$

$\therefore AEFC\ is\ a\ parallelogram$.

$\Rightarrow AF \parallel EC$

$In\ \Delta ABP, EQ \parallel AP\ and\ E\ is\ midpoint\ of\ AB.$

$\Rightarrow Q\ is\ midpoint\ of\ BP$

$\Rightarrow BQ = QP \rightarrow (1)$

$In\ \Delta DQC, FP \parallel CQ\ and\ F\ is\ midpoint\ of\ DC.$

$\Rightarrow P\ is\ midpoint\ of\ DQ$

$\Rightarrow QP = PD \rightarrow (2)$

From (1) and (2)

BQ = QP = PD

∴ The line segments AF and EC trisect the diagonal BD.

6. ABC is a triangle right angled at C. A line through the mid-point M of hypotenuse AB and parallel to BC intersects AC at D. Show that (i) D is the mid-point of AC (ii) MD ⊥ AC

(iii) CM = MA $= \frac{1}{2}$ **AB**

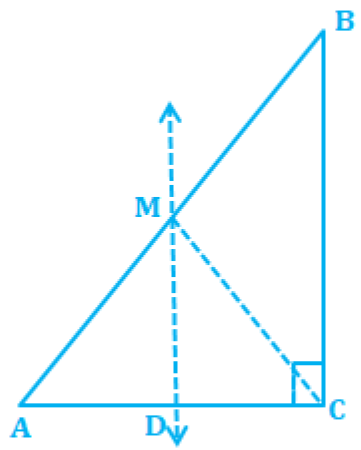
Sol: We know that the line drawn through the mid-point of one side of a triangle, parallel to another side bisects the third side.

(i) $In\ \Delta ABC, MD \parallel BC\ and\ M\ is\ midpoint\ of\ AB$

$\Rightarrow D\ is\ midpoint\ of\ AC.$

(ii) $MD\ \parallel BC\ and\ AC\ is\ transversal$

$\angle MDC + \angle BCD = 180^0 (\ Co - interior\ angles\ are\ supplementary)$



$\angle MDC + 90^0 = 180^0$

$\angle MDC = 180^0 - 90^0 = 90^0$

$\therefore MD \perp AC$

(iii) $In\ \Delta AMD\ and\ \Delta CMD$

AD=DC (D is midpoint of AC)

$\angle ADM = \angle CDM (= 90^0)$

$MD = MD\ (Common)$

$\Delta AMD \cong \Delta CMD\ (SAS\ congruence\ rule)$

$AM = CM\ (By\ CPCT)$

$But\ AM = \frac{1}{2} AB\ (\ M\ is\ mid\ point\ of\ AB)$

$\therefore CM = MA = \frac{1}{2} AB$