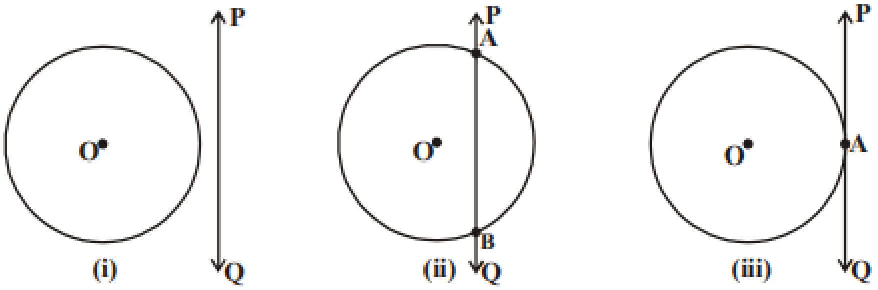
X CLASS

TANGENTS AND SECANTS TO A CIRCLE(NOTES)

Prepared by: BALABHADRA SURESH

1.



(i) The line PQ and the circle have no common point. In this case PQ is a nonintersecting line with respect to the circle.

(ii) The line PQ intersects the circle at two points A and B. It forms a chord AB on the circle with two common points. In this case the line PQ is a secant of the circle.

(iii) There is only one point A, common to the line PQ and the circle. This line is called a tangent to the circle.

The common point of the tangent and the circle is called the point of contact and the tangent is said to touch the circle at the common point.

2. We can draw infinite tangents to the circle.
3. We can draw two tangents to the circle from a point away from it.
4. The tangent at any point of a circle is perpendicular to the radius through the point of contact.

EXERCISE - 9.1

1. Fill in the blanks

(i) A tangent to a circle intersects it in ....one........... point (s).

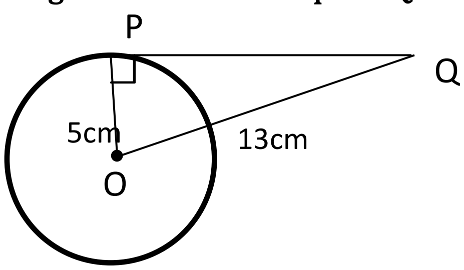
(ii) A line intersecting a circle in two points is called a secant

(iii) The number of tangents drawn at the end points of the diameter is two

(iv) The common point of a tangent to a circle and the circle is called point of contact

(v) We can draw infinite tangents to a given circle.

2. **A tangent PQ at a point P of a circle of radius 5 cm meets a line through the centre O at a point Q so that OQ = 13 cm. Find length of PQ.**



Sol: Radius OP = 5cm and OQ= 13cm

We know that angle between radius and tangent = 90^0

$In\ \Delta OPQ, \angle P = 90^0$

$PQ^2 + OP^2 = OQ^2$

$PQ^2 + 5^2 = 13^2$

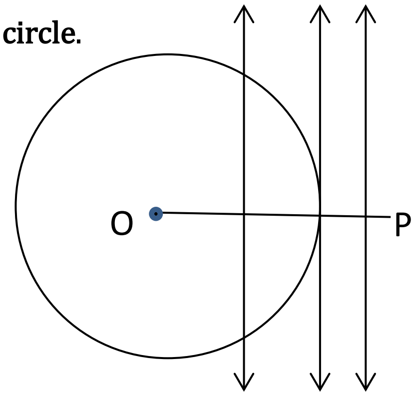
$PQ^2 + 25 = 169$

$PQ^2 = 169 - 25 = 144 = 12^2$

$\therefore PQ = 12\ cm$

3. **Draw a circle and two lines parallel to a given line such that one is a tangent and the other, a secant to the circle.**

Sol:

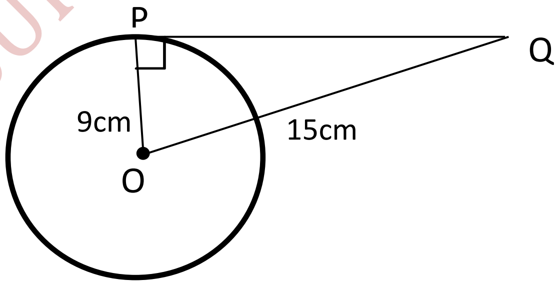


4. **Calculate the length of tangent from a point 15 cm. away from the centre of a circle of radius 9 cm.**

Sol: Radius OP = 9cm and OQ= 15cm

We know that angle between radius and tangent = 90^0

In $\Delta OPQ, \angle P = 90^0$



$PQ^2 + OP^2 = OQ^2$

$PQ^2 + 9^2 = 15^2$

$PQ^2 + 81 = 225$

$PQ^2 = 225 - 81 = 144 = 12^2$

$\therefore PQ = 12$ cm

(OR)

Radius(r)= 9 cm

Distance between centre to point (d) =15 cm

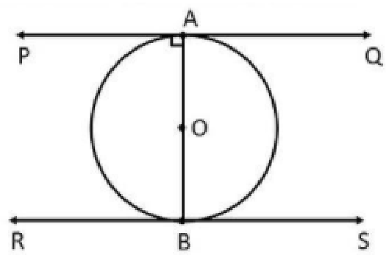
The length of tangent=$\sqrt{d^2 - r^2} = \sqrt{15^2 - 9^2} = \sqrt{225 - 81} = \sqrt{144} = 12\ cm$

5. Prove that the tangents to a circle at the end points of a diameter are parallel.

Sol: **Given**: A circle with centre O and AB is diameter

PQ , RS are tangents at A,B.

To prove: $PQ \parallel RS$



Proof:

Tangent is perpendicular to the radius at the point of contact

$OA \perp PQ$ and $OB \perp RS$

$\angle OAP = 90^0$ and $\angle OBS = 90^0$

$\Rightarrow \angle BAP = 90^0$ and $\angle ABS = 90^0$

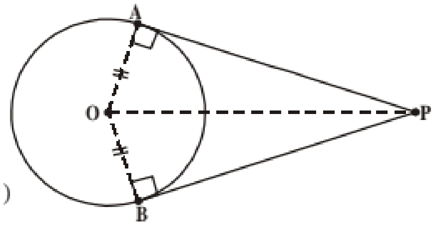
$\Rightarrow \angle BAP = \angle ABS$

⇒ Alternate interior angles are equal

⇒ $PQ \parallel RS$

Theorem-9.2 : The lengths of tangents drawn from an external point to a circle are equal.

Sol: **Given** : A circle with centre O , PA and PB are two tangents to the circle from P.



To prove : PA=PB

Proof : Join OA,OB and OP.

In ΔOAP and ΔOBP

$\angle OAP = \angle OBP = 90^0$ (Angle between radii and tangents)

OA =OB (radii of same circle)

OP=OP (common)

$\Delta OAP \cong \Delta OBP$ (R.H.S congruency axiom)

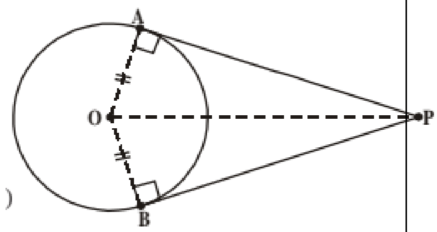
$\therefore$ PA = PB (CPCT)

Hence proved.

Try This

"Prove the lengths of tangents drawn from an external point to a circle are equal" use Pythagoras theorem.

Sol: **Given** : A circle with centre O , PA and PB are two tangents to the circle from P.



To prove : PA=PB

Proof : Join OA,OB and OP.

In ΔOAP and ΔOBP

$\angle OAP = \angle OBP = 90^0$ (Angle between radii and tangents)

From ΔOAP

$OP^2 = OA^2 + PA^2 \rightarrow (1)$ (Pythagoras theorem)

From ΔOBP

$OP^2 = OB^2 + PB^2 \rightarrow (2)$ (Pythagoras theorem)

From (1) and (2)

$OA^2 + PA^2 = OB^2 + PB^2$

$\Rightarrow PA^2 = PB^2$ (OA = OB = radius)

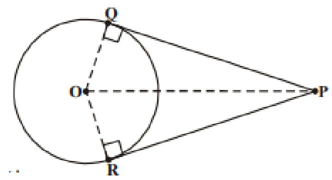
$\Rightarrow PA = PB$

Prob1:Prove The centre of a circle lies on the bisector of the angle between two tangents drawn from a point outside it.

Sol: Let PQ and PR be two tangents drawn from a point P outside of the circle with centre O.

Join OQ , OR and OP

In ΔOQP and ΔORP



$\angle OQP = \angle ORP = 90^0$(Angle between radii and tangents)

OP=OP (common)

OQ =OR (radii of same circle)

$\Delta OQP \cong \Delta ORP$(R. H. S rule)

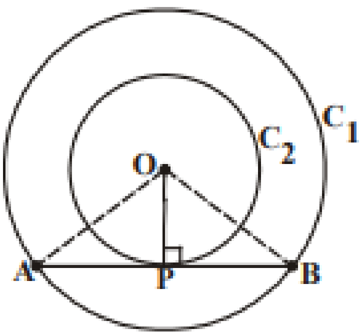
$\angle OPQ = \angle OPR$(CPCT)

Therefore, OP is the angle bisector of $\angle QPR$.

Hence, the centre lies on the bisector of the angle between the two tangents

Prob2: Prove in two concentric circles, such that a chord of the bigger circle, that touches the smaller circle is bisected at the point of contact with the smaller circle.

Sol: C_1 and C_2 are two concentric circles with centre O.



AB is chord of larger circle touching the smaller circle at P

Now we prove that $AP = BP$

Join OA,OB and OP

In ΔAPO and ΔBPO

$\angle APO = \angle BPO = 90^0$(Angle between radii and tangents)

$OA = OB$(radii of same circle)

OP=OP (common)

$\Delta APO \cong \Delta BPO$(R. H. S rule)

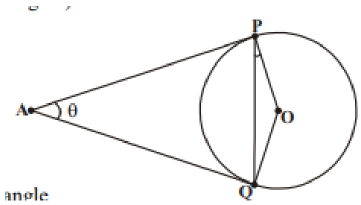
$\therefore AP = BP$(CPCT)

Prob3: If two tangents AP and AQ are drawn to a circle with centre O from an external point A then prove $\angle PAQ = 2\angle OPQ = 2\angle OQP$

Sol: Join OP,OQ and PQ . Let $\angle PAQ = \theta$

We know that the lengths of tangents drawn from an external point to a circle are equal.

AP=AQ, So ΔAPQ is an isosceles triangle



$\Rightarrow \angle APQ = \angle AQP = \alpha(say)$

$\theta + \alpha + \alpha = 180^0(Angle\ sum\ property)$

$2\alpha = 180^0 - \theta$

$$\Rightarrow \alpha = \frac{180^0 - \theta}{2} = 90^0 - \frac{1}{2}\theta$$

$\angle APO = \angle AQO = 90^0$(Angle between radii and tangents)

$$\angle OPQ = \angle APO - \angle APQ = 90^0 - \alpha = 90^0 - \left(90^0 - \frac{1}{2}\theta\right) = \frac{1}{2}\theta = \frac{1}{2}\angle PAQ$$

$$\angle OPQ = \frac{1}{2}\angle PAQ \Rightarrow \angle PAQ = 2\angle OPQ$$

Similarly $\angle PAQ = 2\angle OQP$

Prob4: If a circle touches all the four sides of a quadrilateral ABCD at points PQRS. Then AB+CD = BC + DA.

Sol: We know that the lengths of tangents drawn from an external point to a circle are equal.

A is external point and AP,AS are tangents then

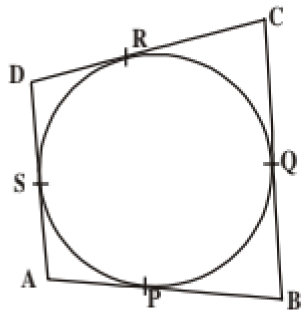
AP = AS -------(1)

Similarly

BP = BQ-------(2)

CR = CQ------(3)

DR = DS ------(4)



(1)+(2)+(3)+(4) we get

AP+BP+CR+DR = AS+BQ+CQ+DS

(AP+BP)+(CR+DR) = (BQ+CQ)+ (AS+DS)

AB + CD = BC + DA

Example-1. Draw a pair of tangents to a circle of radius 5cm which are inclined to each other at an angle 60°.

Sol: Let 'O' is the centre of the circle. Radius OA=OB= 5cm and

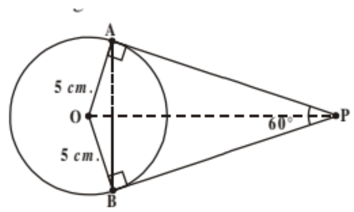
$\angle APB = 60^0$

$\Delta OAP \cong \Delta OBP \ (R.H.S\ congruency)$

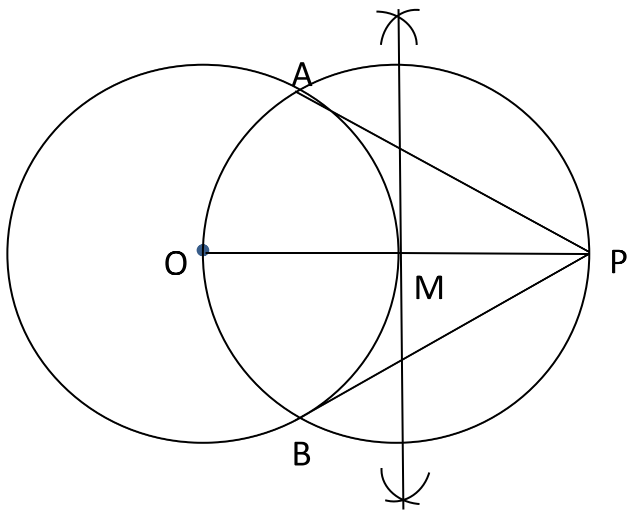
$\angle OPA = \angle OPB = \frac{1}{2} \times \angle APB = \frac{1}{2} \times 60^0 = 30^0$

Now $In\ \Delta OAP, \sin 30^0 = \frac{OA}{OP} = \frac{5}{OP}$

$\frac{1}{2} = \frac{5}{OP} \Rightarrow OP = 5 \times 2 = 10cm$



Rough diagram



Steps of construction:

1. Draw a circle with centre O and radius 5cm.
2. Take a point P such that OP= 10cm. Join OP.
3. Draw a perpendicular bisector to OP to meet at M.

4. Take M as centre and PM=OM as radius draw a circle.
5. Let the circle intersects the given circle at A and B . Join PA and PB
6. PA and PB are required tangents.

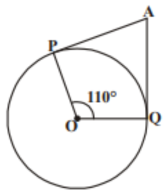
Exercise - 9.2

(i) The angle between a tangent to a circle and the radius drawn at the point of contact is 90^0

(ii) From a point Q, the length of the tangent to a circle is 24 cm. and the distance of Q from the centre is 25 cm. The radius of the circle is 7cm.

Sol: Radius $= \sqrt{d^2 - l^2} = \sqrt{25^2 - 24^2} = \sqrt{625 - 576} = \sqrt{49} = 7cm$

(iii) If AP and AQ are the two tangents a circle with centre O so that $\angle$ POQ =110^0 , then $\angle$PAQ is equal to 70^0



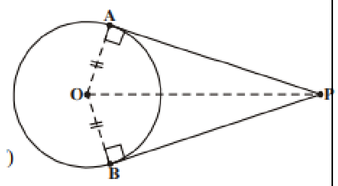
Sol: $\angle APO = \angle AQO = 90^0 (Angle\ between\ radii\ and\ tangents)$

$\angle APO + \angle AQO + \angle POQ + \angle PAQ = 360^0$(sum of angles in quadrilateral)

$90^0 + 90^0 + 100^0 + \angle PAQ = 360^0$

$\angle PAQ = 360^0 - 280^0 = 70^0$

(iv) If tangents PA and PB from a point P to a circle with centre O are inclined to each other at angle of 80°, then $\angle$POA is equal to



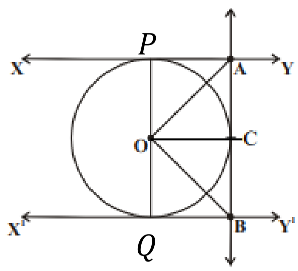
Sol: $\angle APB + \angle AOB = 180^0$

$80^0 + \angle AOB = 180^0$

$\angle AOB = 180^0 - 80^0 = 100^0$

$\angle POA = \angle POB = \dfrac{100^0}{2} = 50^0$

(v) In the figure XY and X^1Y^1 are two parallel tangents to a circle with centre O and another tangent AB with point of contact C intersecting XY at A and X^1Y^1 at B then $\angle$AOB =



Sol: $In\ \Delta OPA\ and\ \Delta OCA$

$OP = OC(Radii)$

$AP = AC(Tangents\ from\ A)$

$OA = OA(\ common)$

$\Delta OPA\ \cong\ \Delta OCA(S.S.S\ Rule)$

$\angle POA = \angle COA(CPCT) \rightarrow (1)$

Similarly

$\angle QOB = \angle COB \rightarrow (2)$

$\angle POA + \angle COA + \angle QOB + \angle COB = 180^0$(Sum of angles on strait line)

$\angle COA + \angle COA + \angle COB + \angle COB = 180^0\ \left(from\ (1),(2)\right)$

$2(\angle COA + \angle COB) = 180^0$

$\angle AOB = \frac{180^0}{2} = 90^0$

2. **Two concentric circles are radii 5 cm and 3cm are drawn. Find the length of the chord of the larger circle which touches the smaller circle**

Sol: Let O is the centre of circles and AB is chord of larger circle touches smaller circle at P.

Radii of concentric circle OP=3 cm , OA=5 cm

$In\ \Delta APO\ , \angle P = 90^0 (Angle\ between\ radii\ and\ tangents)$

$AP^2 + OP^2 = OA^2$

$AP^2 + 3^2 = 5^2$

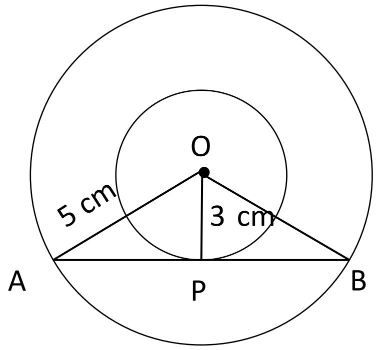
$AP^2 + 9 = 25$

$AP^2 = 25 - 9 = 16 = 4^2$

$AP = 4\ cm$

Similarly BP=4 cm

AB=4+4=8 cm



3. **Prove that the parallelogram circumscribing a circle is a rhombus.**

Sol: Let ABCD is a parallelogram circumscribing a circle.

Let P,Q,R,S be points of contact

We know that the lengths of tangents drawn from an external point to a circle are equal.

A is external point and AP,AS are tangents then

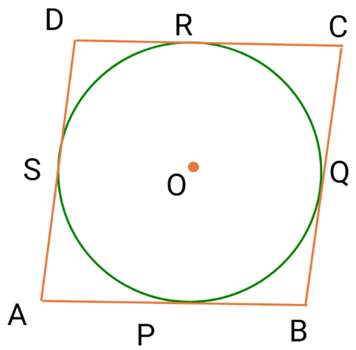
AP = AS ------- (1)

Similarly

BP = BQ------- (2)

CR = CQ------ (3)

DR = DS ------ (4)



(1)+ (2) +(3)+(4) we get

AP+BP+CR+DR = AS+BQ+CQ+DS

(AP+BP)+ (CR+DR) = (BQ+CQ)+ (AS+DS)

AB + CD = BC + DA

AB + AB = BC + BC (opposite sides of a parallelogram are equal AB=CD and BC=DA)

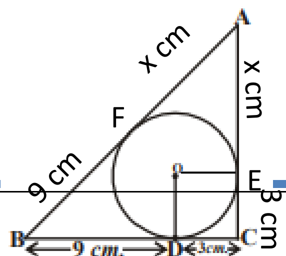
2 AB = 2 BC ⇒ AB = BC

∴ AB = BC = CD = DA . Hence ABCD is a rhombus

4. A triangle ABC is drawn to circumscribe a circle of radius 3 cm. such that the segments BD and DC into which BC is divided by the point of contact D are of length 9 cm. and 3 cm. respectively (See adjacent figure). Find the sides AB and AC.

Sol: Radius OD=OE=3 cm

Tangents from an external point to circle are equal



CD=CE= 3 cm and BD=BF= 9 cm

AF=AE=x (say)

$AB = (x+9)cm \ \ and \ \ AC = (x+3)cm$

Now in quadrilateral CDOE

OD=OE=CD=CE=3 cm and $\angle CDO = 90^0$

∴ CDOE is a square

$\Rightarrow \angle DCE = 90^0$

In ΔACB , $\angle C = 90^0$

$AB^2 = BC^2 + AC^2$(Pythagoras theorem)

$(x+9)^2 = 12^2 + (x+3)^2$

$x^2 + 18x + 81 = 144 + x^2 + 6x + 9$

$18x - 6x = 153 - 81$

$12x = 72$

$x = \frac{72}{12} = 6$

$AB = (x+9)cm = (6+9)cm = 15\ cm$

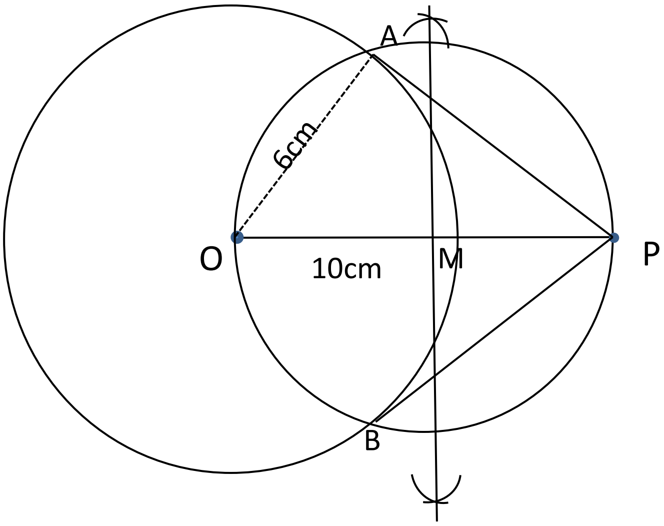
$AC = (x+3)cm = (6+3)cm = 9\ cm$

5. **Draw a circle of radius 6cm. From a point 10 cm away from its centre, construct the pair of tangents to the circle and measure their lengths. Verify by using Pythagoras Theorem.**

Sol:

Steps of construction :

1. Draw a circle with centre O and radius 6cm.
2. Take a point P such that OP= 10cm. Join OP.
3. Draw a perpendicular bisector to OP to meet at M.
4. Take M as centre and PM=OM as radius draw a circle.
5. Let the circle intersects the given circle at A and B . Join PA and PB
6. PA and PB are required tangents and their lengths are 8cm



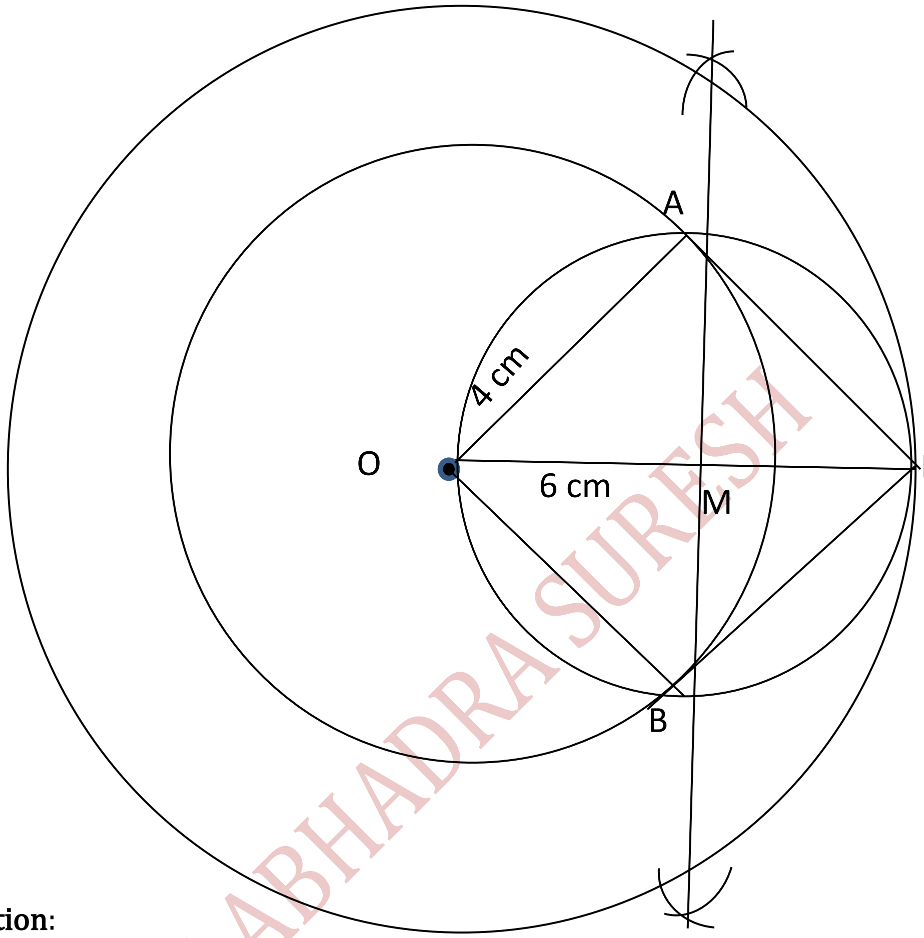
Verification:

OA= 6cm , OP = 10cm

$$length\ of\ tangent = AP = \sqrt{OP^2 - OA^2} = \sqrt{10^2 - 6^2} = \sqrt{100-36} = \sqrt{64} \quad = 8cm$$

6. **Construct a tangent to a circle of radius 4cm from a point on the concentric circle of radius 6cm and measure its length. Also verify the measurement by actual calculation.**

Sol:



Steps of construction:

1. Draw two concentric circles with centre O and radius 4cm and 6cm
2. Take a point P on larger circle and Join OP.
3. Draw a perpendicular bisector to OP to meet at M.
4. Take M as centre and PM=OM as radius draw a circle.
5. Let the circle intersects the given circle at A and B . Join PA and PB
6. PA and PB are required tangents.
7. Length of tangent = 4.5 cm

Verification:

OA= 4 cm , OP = 6cm

$$length\ of\ tangent = AP = \sqrt{OP^2 - OA^2} = \sqrt{6^2 - 4^2} = \sqrt{36-16} = \sqrt{20} \quad = 4.47cm$$

8. In a right triangle ABC, a circle with a side AB as diameter is drawn to intersect the hypotenuse AC in P. Prove that the tangent to the circle at P bisects the side BC.

Sol: $In\ \Delta ABC\ , \angle B = 90^0 \Rightarrow QB\ is\ tangent$

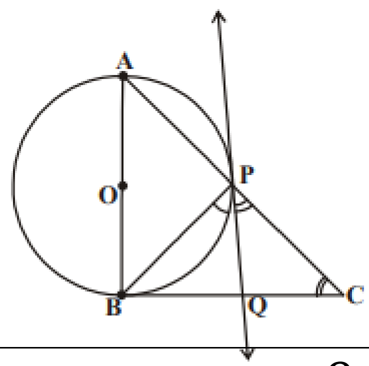
QP is also tangent.

We know that the lengths of tangents drawn from an external point to a circle are equal.

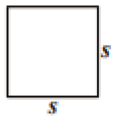
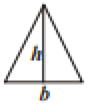
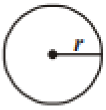
$\therefore QB = QP \rightarrow (1)$

From the figure $\angle QCP = \angle QPC$

$\Rightarrow QP = QC \rightarrow (2)$

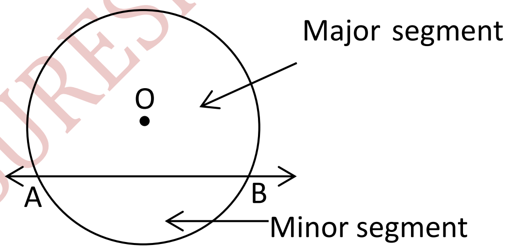


From (1) and (2)
$QB = QC$
$\Rightarrow PQ \text{ bisects } BC$
$\Rightarrow \text{The tangent to the circle at } P \text{ bisects the side } BC$

S.No.	Figure	Dimentions	Area
1.		length $= l$ breadth $= b$	$A = lb$
2.		Side $=$ s	$A = s^2$
3.		base $= b$ height $= h$	$A = \frac{1}{2} bh$
4.		radius $= r$	$A = \pi r^2$

Segment of a circle formed by a secant

A secant is a line which intersects the circle at two distinct points.



Do This

1. Find the area of sector, whose radius is 7 cm. with the given angle:

i. **60^0**

Sol: Radius(r) =7 cm ; Angle $(x^0)= 60^0$

$$Area\ of\ sector = \frac{x^0}{360^0} \times \pi r^2$$

$$= \frac{60^0}{360^0} \times \frac{22}{7} \times 7 \times 7$$

$$= \frac{1}{6} \times 22 \times 7 = \frac{77}{3}\ cm^2$$

ii. **30^0**

Sol: Radius(r) =7 cm ; Angle $(x^0)= 30^0$

$$Area\ of\ sector = \frac{x^0}{360^0} \times \pi r^2$$

$$= \frac{30^0}{360^0} \times \frac{22}{7} \times 7 \times 7$$

$$= \frac{1}{12} \times 22 \times 7 = \frac{77}{6}\ cm^2$$

iii. **72^0**

Sol: Radius(r) =7 cm
Angle $(x^0)= 72^0$

$$Area\ of\ sector = \frac{x^0}{360^0} \times \pi r^2$$

$$= \frac{72^0}{360^0} \times \frac{22}{7} \times 7 \times 7$$

$$= \frac{1}{5} \times 22 \times 7$$
$$= \frac{154}{5}\ cm^2$$

iv 90^0

Sol: Radius(r) =7 cm

Angle (x^0)= 90^0

$$Area\ of\ sector = \frac{x^0}{360^0} \times \pi r^2$$
$$= \frac{90^0}{360^0} \times \frac{22}{7} \times 7 \times 7$$
$$= \frac{1}{4} \times 22 \times 7$$
$$= \frac{77}{2}\ cm^2$$

v. 120^0

Sol: Radius(r) =7 cm

Angle (θ)= 120^0

$$Area\ of\ sector = \frac{x^0}{360^0} \times \pi r^2$$
$$= \frac{120^0}{360^0} \times \frac{22}{7} \times 7 \times 7$$
$$= \frac{1}{3} \times 22 \times 7$$
$$= \frac{154}{3}\ cm^2$$

2. **The length of the minute hand of a clock is 14 cm. Find the area swept by the minute hand in 10 minutes.**

Sol: Radius(r) =14 cm

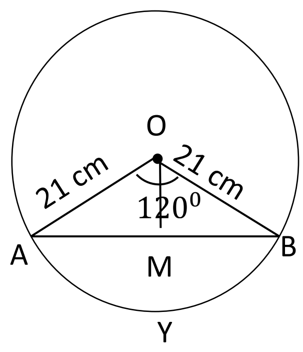
Swept by minute hand in 1 minute=$\frac{360^0}{60} = 6^0$

Angle $(\theta) = 10 \times 6^0 = 60^0$

$$Area\ of\ sector = \frac{x^0}{360^0} \times \pi r^2$$
$$= \frac{60}{360^0} \times \frac{22}{7} \times 14 \times 14$$
$$= \frac{1}{6} \times 22 \times 2 \times 14 = \frac{22 \times 14}{3} = \frac{308}{3}\ cm^2$$

The area swept by the minute hand in 10 minutes $= \frac{308}{3}\ cm^2$

Example-1. Find the area of the segment AYB showing in the adjacent figure. If radius of the circle is 21 cm and ∠ AOB = 120⁰ (Use $\pi = \frac{22}{7}$ and $\sqrt{3} = 1.732$)



Sol: Radius of the circle(r) =OA=OB=21 cm

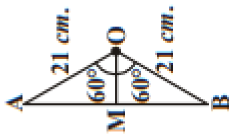
Let $OM \perp AB$

$\angle AMO = \angle BMO = 90^0$

$\Delta AMO \cong \Delta BMO (R.H.S\ rule)$

$\angle AOM = \angle BOM = \frac{120^0}{2} = 60^0$

From ΔAPO



$\sin 60^0 = \frac{AM}{OA} \Rightarrow \frac{\sqrt{3}}{2} = \frac{AM}{21} \Rightarrow AM = \frac{21 \times \sqrt{3}}{2} cm$

$\cos 60^0 = \frac{OM}{OA} \Rightarrow \frac{1}{2} = \frac{OM}{21} \Rightarrow OM = \frac{21}{2}\ cm$

$AB = 2 \times AM = 2 \times \frac{21 \times \sqrt{3}}{2} cm = 21\sqrt{3} cm$

Area of the sector $OAYB = \frac{x^0}{360^0} \times \pi r^2$

$= \frac{120^0}{360^0} \times \frac{22}{7} \times 21 \times 21$

$= 462\ cm^2$

Area of triangle $OAB = \frac{1}{2} \times AB \times OM$

$= \frac{1}{2} \times 21\sqrt{3} \times \frac{21}{2}$

$= \frac{441\sqrt{3}}{4} cm^2$

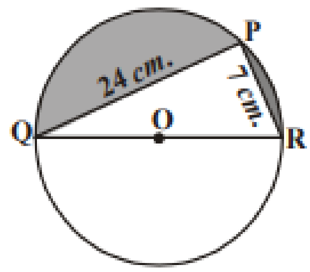
$= \frac{441 \times 1.732}{4} = 441 \times 0.433 = 190.953\ cm^2$

Area of the segment AYB= $Area\ of\ the\ sector\ OAYB - Area\ of\ triangle\ OAB$

$= 462 - 190.953 = 271.074\ cm^2$

Example-2. Find the area of the segments shaded in figure, if PQ = 24 cm., PR = 7 cm. and QR is the diameter of the circle with centre O (Take $\pi = \frac{22}{7}$)

Sol: Area of the segments shaded = Area of semicircle OQPR - Area of triangle PQR



$\angle QPR = 90°$ (Angle in a semicircle)

$QR^2 = PQ^2 + PR^2$(pythagoras theorem)

$= 24^2 + 7^2$

$= 576 + 49$

$= 625$

$QR = \sqrt{625} = 25\ cm$

Radius of the circle$(r) = \frac{25}{2} cm$

Area of semicircle OQPR $= \frac{1}{2} \times \pi r^2$

$= \frac{1}{2} \times \frac{22}{7} \times \frac{25}{2} \times \frac{25}{2}$

$= \frac{6875}{28} = 245.53\ cm^2$

Area of triangle QPR $= \frac{1}{2} \times QP \times PR$

$= \frac{1}{2} \times 24 \times 7$

$= 84\ cm^2$

Area of the shaded segments = 245.53 - 84 = 161.53 cm ^{2}

Example-3. A round table top has six equal designs as shown in the figure. If the radius of the table top is 14 cm., find the cost of making the designs with paint at the rate of ₹5 per cm 2 . (use $\sqrt{3} =$ 1.732)

Sol: We know that the radius of circumscribing circle of a regular hexagon is equal to the length of its side.

$\therefore$ Each side of regular hexagon = 14 cm.

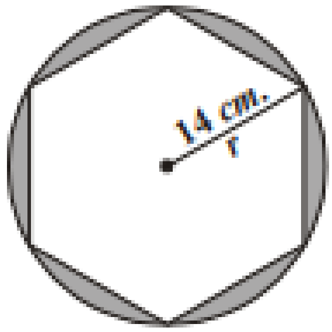
Therefore, Area of six design segments = Area of circle - Area of the regular hexagon.

Area of circle $= \pi r^2$

$= \frac{22}{7} \times 14 \times 14$

$= 22 \times 2 \times 14$

$= 616\ cm^2$



Area of regular hexagon $= 6 \times \frac{\sqrt{3}}{4} \times a^2$

$= 6 \times \frac{\sqrt{3}}{4} \times 14 \times 14$

$= 294 \times 1.732$

$= 509.21\ cm^2$

$Area\ of\ six\ design = 616 - 509.21 = 106.79\ cm^2$

Cost of making the designs with paint for 1 cm^2= ₹5

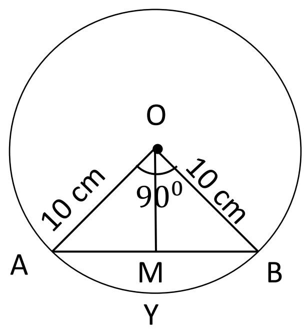
Total cost = ₹5 × 106.79 = ₹533.95

EXERCISE - 9.3

1. A chord of a circle of radius 10 cm. subtends a right angle at the centre. Find the area of the corresponding: (use π = 3.14) i. Minor segment ii. Major segment.

Sol: Area of the sector OAYB $= \frac{x^0}{360^0} \times \pi r^2$

$= \frac{90^0}{360^0} \times 3.14 \times 10 \times 10$



$= \frac{1}{4} \times 314 = 78.5 cm^2$

Area of triangle OAB $= \frac{1}{2} \times OA \times OB$

$= \frac{1}{2} \times 10 \times 10 = 50 cm^2$

Area of circle $= \pi r^2$

$= 3.14 \times 10 \times 10$

$= 314\ cm^2$

i) Area of minor segment

$= Area\ of\ the\ sector\ OAYB - Area\ of\ triangle\ OAB$

$= 78.5 - 50 = 28.5\ cm^2$

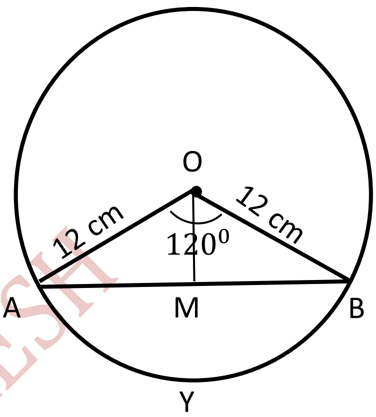
ii) Area of major segment

$= Area\ of\ circle - Area\ of\ minor\ segment$

$= 314 - 28.5 = 285.5\ cm^2$

2. A chord of a circle of radius 12 cm. subtends an angle of 120° at the centre. Find the area of the corresponding minor segment of the circle (use $\pi = 3.14$ and $\sqrt{3} = 1.732$).

Sol: Radius of the circle(r) =OA=OB=21 cm



Let OM ⊥ AB

$\angle AMO = \angle BMO = 90^0$

$\Delta AMO \cong \Delta BMO (R.H.S\ rule)$

$\angle AOM = \angle BOM = \frac{120^0}{2} = 60^0$

From ΔAMO

$\sin 60^0 = \frac{AM}{OA} \Rightarrow \frac{\sqrt{3}}{2} = \frac{AM}{12} \Rightarrow AM = \frac{12 \times \sqrt{3}}{2} = 6\sqrt{3}\ cm$

$\cos 60^0 = \frac{OM}{OA} \Rightarrow \frac{1}{2} = \frac{OM}{12} \Rightarrow OM = \frac{12}{2} = 6\ cm$

$AB = 2 \times AM = 2 \times 6\sqrt{3}cm = 12\sqrt{3}cm$

Area of the sector OAYB $= \frac{x^0}{360^0} \times \pi r^2$

$= \frac{120^0}{360^0} \times 3.14 \times 12 \times 12 = 150.72\ cm^2$

Area of triangle OAB $= \frac{1}{2} \times AB \times OM$

$= \frac{1}{2} \times 12\sqrt{3} \times 6$

$= 36\sqrt{3}cm^2$

$= 36 \times 1.732 = 62.352\ cm^2$

Area of the segment AYB= Area of the sector OAYB − Area of triangle OAB

$= 150.72 - 62.352 = 88.368\ cm^2$

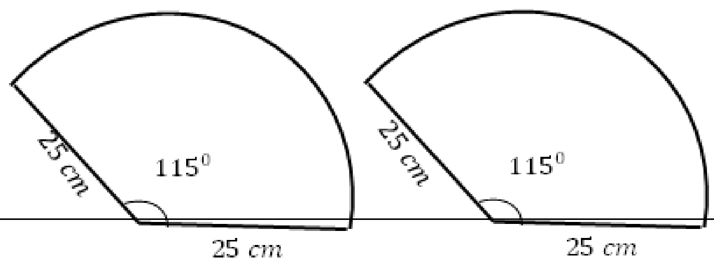
3. A car has two wipers which do not overlap. Each wiper has a blade of length 25 cm. sweeping through an angle of 115°. Find the total area cleaned at each sweep of the blades. (use $\pi = \frac{22}{7}$)

Sol:

Length of wiper blade(r) =25 cm

Angle made by blade $(x) = 115^0$

Area cleaned each swipe of blade



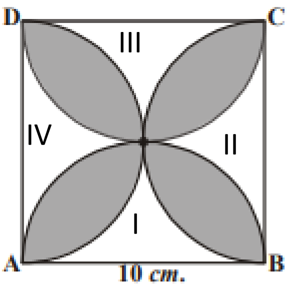
$= \frac{x^0}{360^0} \times \pi r^2$

$= \frac{115^0}{360^0} \times \frac{22}{7} \times 25 \times 25$

$= 627.48\ cm^2$

The total area cleaned at each sweep of the blades$=2 \times 627.48 = 1254.96\ cm^2$

4. Find the area of the shaded region in figure, where ABCD is a square of side 10 cm. and semicircles are drawn with each side of the square as diameter (use $\pi = 3.14$).



Sol: Side of square(s) =10cm

Radius of semi-circle (r) $=\frac{10}{2} = 5cm$

Area of I + Area of III = Area of ABCD − 2 × Area of semi circle

$= s^2 - 2 \times \frac{1}{2}\pi r^2$

$= 10 \times 10 - 3.14 \times 5 \times 5$

$= 100 - 78.5$

$= 21.5\ cm^2$

Similarly

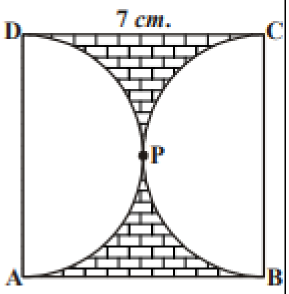
Area of II + Area of IV $= 21.5\ cm^2$

∴ Area of unshaded region $= 21.5\ cm^2 + 21.5\ cm^2 = 43cm^2$

Area of the shaded region in figure = Area of ABCD − Area of unshaded region

$= 100 - 43 = 57\ cm^2$

5. **Find the area of the shaded region in figure, if ABCD is a square of side 7 cm. and APD and BPC are semicircles. (use $= \frac{22}{7}$)**



Sol: Side of square(s) =7cm

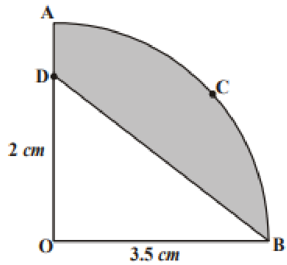
Radius of semi-circle (r) $=\frac{7}{2} cm$

Area of the shaded region = Area of ABCD − 2 × Area of semicircle

$= s^2 - 2 \times \frac{1}{2}\pi r^2$

$= 7 \times 7 - \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} = 49 - 38.5 = 10.5\ cm^2$

6. **In figure, OACB is a quadrant of a circle with centre O and radius 3.5 cm. If OD = 2 cm., find the area of the shaded region(use $\pi = \frac{22}{7}$)**



Sol: Radius of the sector(r)=3.5 $= \frac{35}{10} = \frac{7}{2} cm$

Angle of sector$(x) = 90^0$

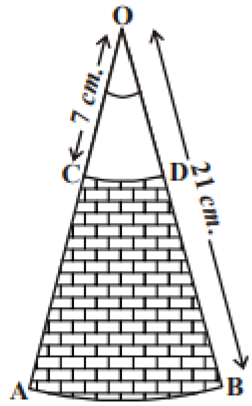
Area of shaded region = Area of sector AOBC − Area of ΔBOD

$= \frac{x^0}{360^0} \times \pi r^2 - \frac{1}{2} \times \text{OB} \times \text{OD}$

$= \frac{90^0}{360^0} \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} - \frac{1}{2} \times \frac{7}{2} \times 2$

$= \frac{77}{8} - \frac{7}{2} = 9.625 - 3.5 = 6.125\ cm^2$

7. **AB and CD are respectively arcs of two concentric circles of radii 21cm. and 7cm. with centre O (See figure). If ∠AOB = 30°, find the area of the shaded region**



Sol: Radii of sectors OAB and OCD are 21 cm and 7 cm, ∠AOB = 30°

Area of the sector $= \frac{x^0}{360^0} \times \pi r^2$

Area of shaded region=Area of sector OAB-Area of sector OCD

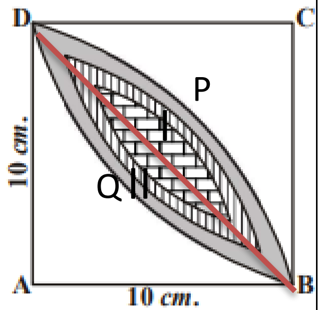
$= \left(\frac{30^0}{360^0} \times \frac{22}{7} \times 21 \times 21\right) - \left(\frac{30^0}{360^0} \times \frac{22}{7} \times 7 \times 7\right)$

$= \frac{1}{12} \times \frac{22}{7}(21 \times 21 - 7 \times 7)$

$= \frac{11}{42}(441 - 49)$

$= \frac{11 \times 392}{42} = \frac{11 \times 56}{6} = \frac{11 \times 28}{3} = \frac{308}{3} = 102.67\ cm^2$

8. **Calculate the area of the designed region in figure, common between the two quadrants of the circles of radius 10 cm. each. (use π = 3.14)**



Sol: $Are\ of\ I = Area\ of\ sector ABPD - Area\ of\ \Delta ABD$

$= \frac{x^0}{360^0} \times \pi r^2 - \frac{1}{2} \times AB \times AD$

$= \frac{90^0}{360^0} \times 3.14 \times 10 \times 10 - \frac{1}{2} \times 10 \times 10$

$= \frac{314}{4} - 50$

$= 78.5 - 50$

$= 28.5\ cm^2$

Similarly Area of II=$28.5 cm^2$

∴ Area of shaded region=$28.5\ cm^2 + 28.5\ cm^2 = 57 cm^2$