CHAPTER
6

X CLASS-2023-24
PROGRESSIONS (NOTES)

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Carl Fredrich Gauss

1. The Indian mathematician Aryabhata (470 AD) was the first to give formula for the sum of squares and cubes of natural number in his famous work Aryabhatiyam written around 499 A.D
2. Carl Fredrich Gauss (1777-1855) is a great German Mathematician find the formula for sum of first n terms in AP
3. Aryabhata also gave the formula for finding the sum of n terms of an Arithmetic Progression starting with p^{th} term.

Arithmetic Progressions

(i) An arithmetic progression is a list of numbers in which each term is obtained by adding a fixed number to the preceding term except the first term.

(ii) This fixed number is called the common difference of the AP.

(iii) The first term is denoted by $a_1, second\ term = a_2, third\ term = a_3, \dots \dots ..$

(iv) Generally first term denoted by $'a'$ and common difference by $'d'$.

(v) General form of AP:

$a, a + d, a + 2d, a + 3d \dots \dots$

(vi) $n^{th}\ term\ of\ AP\ a_n = a + (n - 1)d$

Example1: For the AP $\frac{1}{4}, \frac{-1}{4}, \frac{-3}{4}, \frac{-5}{4}, \dots \dots$ write the first term a and the common difference d.

And find the 7^{th} term

Sol: $a_1 = \frac{1}{4}, a_2 = \frac{-1}{4}, a_3 = \frac{-3}{4}, a_4 = \frac{-5}{4}$

First term $= a = a_1 = \frac{1}{4}$

Common difference $= d = a_2 - a_1 = \frac{-1}{4} - \frac{1}{4} = \frac{-1-1}{4} = \frac{-2}{4} = \frac{-1}{2}$

$7^{th}\ term = a + 6d = \frac{1}{4} + 6 \times \left(\frac{-1}{2}\right)$

$= \frac{1}{4} - 3 = \frac{1-12}{4} = \frac{-11}{4}$

Example-2. Which of the following forms an AP? If they form AP then write next two terms?

(i) 4, 10, 16, 22, . . .

Sol: $a_1 = 4, a_2 = 10, a_3 = 16, a_4 = 22$

$a_2 - a_1 = 10 - 4 = 6$

$a_3 - a_2 = 16 - 10 = 6$

$a_4 - a_3 = 22 - 16 = 6$

$a_2 - a_1 = a_3 - a_2 = a_4 - a_3 = \cdots$

i.e., $a_{k+1} - a_k$ is same every time

So, the given list of numbers forms an AP with the common difference $d = 6$

The next two terms are: $22 + 6 = 28$ and $28 + 6 = 34$

(ii) 1, – 1, – 3, – 5, . . .

Sol: $a_1 = 1, a_2 = -1, a_3 = -3, a_4 = -5, ..$

$a_2 - a_1 = -1 - 1 = -2$

$a_3 - a_2 = -3 - (-1) = -3 + 1 = -2$

$a_4 - a_3 = -5 - (-3) = -5 + 3 = -2$

$a_2 - a_1 = a_3 - a_2 = a_4 - a_3 = \cdots$

i.e., $a_{k+1} - a_k$ is same every time

So, the given list of numbers forms an AP with the common difference $d = -2$

The next two terms are: $-5 - 2 = -7$ *and* $-7 - 2 = -9$

(iii) – 2, 2, – 2, 2, – 2, . . .

Sol: $a_1 = -2, a_2 = 2, a_3 = -2, a_4 = 2, a_5 = -2$

$a_2 - a_1 = 2 - (-2) = 2 + 2 = 4$

$a_3 - a_2 = -2 - 2 = -4$

$a_2 - a_1 \neq a_3 - a_2$

So, the given list of numbers does not form an AP.

(iv) 1, 1, 1, 2, 2, 2, 3, 3, 3, . . .

Sol: $a_1 = 1, a_2 = 1, a_3 1, = a_4 = 2,$

$a_2 - a_1 = 1 - 1 = 0$

$a_3 - a_2 = 1 - 1 = 0$

$a_4 - a_3 = 2 - 1 = 1$

$a_3 - a_2 \neq a_4 - a_3$

So, the given list of numbers does not form an AP.

(v) $\boldsymbol{x, 2x, 3x, 4x}$ .....

Sol: $a_1 = x, a_2 = 2x, a_3 = 3x, a_4 = 4x,$

$a_2 - a_1 = 2x - x = x$

$a_3 - a_2 = 3x - 2x = x$

$a_4 - a_3 = 4x - 3x = x$

$a_2 - a_1 = a_3 - a_2 = a_4 - a_3 = \cdots$

i.e., $a_{k+1} - a_k$ is same every time

So, the given list of numbers forms an AP with the common difference $d = x$

The next two terms are: $4x + x = 5x \text{ and } 5x + x = 6x$

Exercise - 6.1

1. In which of the following situations, does the list of numbers involved make an arithmetic progression, and why?

(i) The taxi fare after each km when the fare is ₹20 for the first km and rises by ₹8 for each additional km.

Sol: Taxi fare for first km= ₹20

Taxi fare for second km= ₹20+₹8=₹28

Taxi fare for third km=₹28+₹8=₹36

∴ The taxi fares are ₹20, ₹28, ₹36, ₹42,……

It is an arithmetic progression with common difference=8

(ii) The amount of air present in a cylinder when a vacuum pump removes $\frac{1}{4}$ of the air remaining in the cylinder at a time.

Sol: let the amount of air present in cylinder= x

(If a vacuum pump removes $\frac{1}{4}$ of the air then the remaining air is $\frac{3}{4}$ of it)

When vacuum pump use first time remaining air $= \frac{3}{4} \times x = \frac{3x}{4}$

Vacuum pump use second time remaining air $= \frac{3}{4} \times \frac{3x}{4} = \frac{9x}{16}$

Vacuum pump use third time remaining air $= \frac{3}{4} \times \frac{9x}{16} = \frac{27x}{64}$

List of air present in cylinder is $x, \frac{3x}{4}, \frac{9x}{16}, \frac{27x}{64}, \ldots$

$$a_2 - a_1 = \frac{3x}{4} - x = \frac{3x - 4x}{4} = \frac{-x}{4}$$

$$a_3 - a_2 = \frac{9x}{16} - \frac{3x}{4} = \frac{9x - 12x}{16} = \frac{-3x}{16}$$

$$a_2 - a_1 \neq a_3 - a_2$$

So, the given list of numbers does not form an AP.

(iii) The cost of digging a well, after every metre of digging, when it costs ₹150 for the first metre and rises by ₹50 for each subsequent metre.

Sol: The cost of digging for first metre=₹150

The cost of digging for second metre=₹150+₹50=₹200

The cost of digging for third metre=₹200+₹50=₹250

The cost of digging for fourth metre=₹250+₹50=₹300

The costs are ₹150, ₹200, ₹250, ₹300,….

It is an arithmetic progression with common difference=₹50

(iv) The amount of money in the account every year, when ₹10000 is deposited at compound interest at 8 % per annum.

Sol: $P = ₹10000\,, R = 8\,\%,$

$$A = P\left(1 + \frac{R}{100}\right)^n$$

$$First\ year\ ammount = 10000\left(1 + \frac{8}{100}\right)^1 = 10000 \times \frac{108}{100} = ₹10800$$

$$Second\ year\ ammount = 10000\left(1 + \frac{8}{100}\right)^2 = 10000 \times \frac{108}{100} \times \frac{108}{100} = ₹11664$$

$$Third\ year\ ammount = 10000\left(1 + \frac{8}{100}\right)^1 = 10000 \times \frac{108}{100} \times \frac{108}{100} \times \frac{108}{100} = ₹12597.12$$

The amounts are ₹10000, ₹10800, ₹11664, ₹12597.12, …

$a_2 - a_1 = ₹10800 - ₹10000 = ₹800$

$a_3 - a_2 = ₹11664 - ₹10800 = ₹864$

$a_2 - a_1 \neq a_3 - a_2$

The given situations does not form an AP

2. Write first four terms of the AP, when the first term $\boldsymbol{a}$ and the common difference $\boldsymbol{d}$ are given as follows:

(i) $\boldsymbol{a = 10, d = 10}$

Sol: $a_1 = a = 10$

$a_2 = a + d = 10 + 10 = 20$

$a_3 = a + 2d = 10 + 2 \times 10 = 10 + 20 = 30$

$a_4 = a + 3d = 10 + 3 \times 10 = 10 + 30 = 40$

The first four terms of AP are 10,20,30,40

(ii) $\boldsymbol{a = -2, d = 0}$

Sol: $a_1 = a = -2$

$a_2 = a + d = -2 + 0 = -2$

$a_3 = a + 2d = -2 + 2 \times 0 = -2 + 0 = -2$

$a_4 = a + 3d = -2 + 3 \times 0 = -2 + 0 = -2$

The first four terms of AP are $-2, -2, -2, -2, ..$

(iii) **a = 4, d = – 3**

Sol: $a_1 = a = 4$

$a_2 = a + d = 4 + (-3) = 4 - 3 = 1$

$a_3 = a + 2d = 4 + 2 \times (-3) = 4 - 6 = -2$

$a_4 = a + 3d = 4 + 3 \times (-3) = 4 - 9 = -5$

The first four terms of AP are $4, 1, -2, -5$

(iv) $\boldsymbol{a = -1, d = \frac{1}{2}}$

Sol: $a_1 = a = -1$

$$a_2 = a + d = -1 + \frac{1}{2} = \frac{-2+1}{2} = \frac{-1}{2}$$

$$a_3 = a + 2d = -1 + 2 \times \left(\frac{1}{2}\right) = -1 + 1 = 0$$

$$a_4 = a + 3d = -1 + 3 \times \left(\frac{1}{2}\right) = -1 + \frac{3}{2} = \frac{-2+3}{2} = \frac{1}{2}$$

$The\ first\ four\ terms\ of\ AP\ are\ -1, \frac{-1}{2}, 0, \frac{1}{2}$

(v) $\boldsymbol{a = -1.25, d = -0.25}$

Sol: $a_1 = a = -1.25$

$a_2 = a + d = -1.25 + (-0.25) = -1.25 - 0.25 = -1.5$

$a_3 = a + 2d = -1.25 + 2 \times (-0.25) = -1.25 - 0.50 = -1.75$

$a_4 = a + 3d = -1.25 + 3 \times (-0.25) = -1.25 - 0.75 = -2$

The first four terms of AP are $-1.25, -1.5, -1.75, -2$

3. For the following APs, write the first term and the common difference:

(i) 3, 1, – 1, – 3, . .

Sol: First term=a=3

Common difference=$d = a_2 - a_1 = 1 - 3 = -2$

(ii) – 5, – 1, 3, 7, . . .

Sol: first term=a=3

Common difference=$d = a_2 - a_1 = 1 - 3 = -2$

$(\boldsymbol{iii})\ \boldsymbol{\frac{1}{3}, \frac{5}{3}, \frac{9}{3}, \frac{13}{3}, \ldots..}$

Sol: first term=a=$\frac{1}{3}$

$$\text{Common difference} = d = a_2 - a_1 = \frac{5}{3} - \frac{1}{3} = \frac{5-1}{3} = \frac{4}{3}$$

(iv) 0.6, 1.7, 2.8, 3.9, . . .

Sol: First term=a=0.6

Common difference=$d = a_2 - a_1 = 1.7 - 0.6 = 1.1$

4. **Which of the following are APs ? If they form an AP, find the common difference d and write three more terms.**

(i) 2, 4, 8, 16, . . .

Sol: $a_1 = 2, a_2 = 4, a_3 = 8, a_4 = 16$

$a_2 - a_1 = 4 - 2 = 2$

$a_3 - a_2 = 8 - 4 = 4$

$a_2 - a_1 \neq a_3 - a_2$

So, the given list of numbers does not form an AP

(ii) $\mathbf{2, \frac{5}{2}, 3, \frac{7}{2} \ldots}$

Sol: $a_1 = 2, a_2 = \frac{5}{2} a_3 = 3, a_4 = \frac{7}{2}$

$$a_2 - a_1 = \frac{5}{2} - 2 = \frac{5-4}{2} = \frac{1}{2}$$

$$a_3 - a_2 = 3 - \frac{5}{2} = \frac{6-5}{2} = \frac{1}{2}$$

$$a_4 - a_3 = \frac{7}{2} - 3 = \frac{7-6}{3} = \frac{1}{2}$$

$a_2 - a_1 = a_3 - a_2 = a_4 - a_3 = \cdots$

i.e., $a_{k+1} - a_k$ is same every time

So, the given list of numbers forms an AP with the common difference $d = \frac{1}{2}$

The next three terms are: $\frac{7}{2} + \frac{1}{2} = \frac{8}{2}, \frac{8}{2} + \frac{1}{2} = \frac{9}{2}, \frac{9}{2} + \frac{1}{2} = \frac{10}{2} \Rightarrow 4, \frac{9}{2}, 5$

(iii) $\mathbf{-1.2, -3.2, -5.2, -7.2, \ldots}$

Sol: $a_1 = -1.2, a_2 = -3.2, a_3 = -5.2, a_4 = -7.2$

$a_2 - a_1 = -3.2 - (-1.2) = -3.2 + 1.2 = -2$

$a_3 - a_2 = -5.2 - (-3.2) = -5.2 + 3.2 = -2$

$a_4 - a_3 = -7.2 - (-5.2) = -7.2 + 5.2 = -2$

$a_2 - a_1 = a_3 - a_2 = a_4 - a_3 = \cdots$

i.e., $a_{k+1} - a_k$ is same every time

So, the given list of numbers forms an AP with the common difference $d = -2$

The next three terms are: $-7.2 - 2 = -9.2, -9.2 - 2 = -11.2, -11.2 - 2 = -13.2$

$\Rightarrow -9.2, -11.2, -13.2$

(iv) $\mathbf{-10, -6, -2, 2, \ldots}$

Sol: $a_1 = -10, a_2 = -6, a_3 = -2, a_4 = 2$

$a_2 - a_1 = -6 - (-10) = -6 + 10 = 4$

$a_3 - a_2 = -2 - (-6) = -2 + 6 = 4$

$a_4 - a_3 = 2 - (-2) = 2 + 2 = 4$

$a_2 - a_1 = a_3 - a_2 = a_4 - a_3 = \cdots$

i.e., $a_{k+1} - a_k$ is same every time

So, the given list of numbers forms an AP with the common difference $d = 4$

The next three terms are: $-2+4=6, 6+4=10, 10+4=14$

$\Rightarrow 6,10,14$

(v) $\mathbf{3, 3+\sqrt{2}, 3+2\sqrt{2}, 3+3\sqrt{2}, \ldots\ldots}$

Sol: $a_1 = 3, a_2 = 3+\sqrt{2}, a_3 = 3+2\sqrt{2}, a_4 = 3+3\sqrt{2}$

$a_2 - a_1 = 3+\sqrt{2} - 3 = \sqrt{2}$

$a_3 - a_2 = 3+2\sqrt{2} - (3+\sqrt{2}) = 3+2\sqrt{2} - 3 - \sqrt{2} = \sqrt{2}$

$a_4 - a_3 = 3+3\sqrt{2} - (3+2\sqrt{2}) = 3+3\sqrt{2} - 3 + 2\sqrt{2} = \sqrt{2}$

$a_2 - a_1 = a_3 - a_2 = a_4 - a_3 = \cdots$

i.e., $a_{k+1} - a_k$ is same every time

So, the given list of numbers forms an AP with the common difference $d = \sqrt{2}$

The next three terms are:−

$(3+3\sqrt{2}) + \sqrt{2} = 3+4\sqrt{2}; (3+4\sqrt{2}) + \sqrt{2} = 3+5\sqrt{2}; (3+5\sqrt{2}) + \sqrt{2} = 3+6\sqrt{2}$

(vi) **0.2, 0.22, 0.222, 0.2222, . . .**

Sol: $a_1 = 0.2, a_2 = 0.22, a_3 = 0.222, a_4 = 0.2222$

$a_2 - a_1 = 0.22 - 0.2 = 0.02$

$a_3 - a_2 = 0.222 - 0.22 = 0.002$

$a_2 - a_1 \neq a_3 - a_2$

So, the given list of numbers does not form an AP

(vii) $\mathbf{0, -4, -8, -12, \ldots.}$

Sol: $a_1 = 0, a_2 = -4, a_3 = -8, a_4 = -12$

$a_2 - a_1 = -4 - 0 = -4$

$a_3 - a_2 = -8 - (-4) = -8 + 4 = -4$

$a_4 - a_3 = -12 - (-8) = -12 + 8 = -4$

$a_2 - a_1 = a_3 - a_2 = a_4 - a_3 = \cdots$

i.e., $a_{k+1} - a_k$ is same every time

So, the given list of numbers forms an AP with the common difference $d = -4$

The next three terms are:−

$-12 - 4 = -16; \quad -16 - 4 = -20; \quad -20 - 4 = -24$

(viii) $\mathbf{-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \ldots.}$

Sol: $a_2 - a_1 = -\frac{1}{2} - \left(-\frac{1}{2}\right) = -\frac{1}{2} + \frac{1}{2} = 0$

$a_3 - a_2 = -\frac{1}{2} - \left(-\frac{1}{2}\right) = -\frac{1}{2} + \frac{1}{2} = 0$

$a_4 - a_3 = -\frac{1}{2} - \left(-\frac{1}{2}\right) = -\frac{1}{2} + \frac{1}{2} = 0$

$a_2 - a_1 = a_3 - a_2 = a_4 - a_3 = \cdots$

i.e., $a_{k+1} - a_k$ is same every time

So, the given list of numbers forms an AP with the common difference $d = 0$

The next three terms are:$- -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}$

(ix) 1,3,9,27,.......

Sol: $a_2 - a_1 = 3 - 1 = 2$

$a_3 - a_2 = 9 - 3 = 6$

$a_2 - a_1 \neq a_3 - a_2$

So, the given list of numbers does not form an AP

(x) $\boldsymbol{a, 2a, 3a, 4a, \ldots \ldots}$

Sol: $a_2 - a_1 = 2a - a = a$

$a_3 - a_2 = 3a - 2a = a$

$a_4 - a_3 = 4a - 3a = a$

$a_2 - a_1 = a_3 - a_2 = a_4 - a_3 = \cdots$

i.e., $a_{k+1} - a_k$ is same every time

So, the given list of numbers forms an AP with the common difference $d = a$

The next three terms are: $5a, 6a, 7a$

(xi) $\boldsymbol{a, a^2, a^3, a^4, \ldots}$

Sol: $a_2 - a_1 = a^2 - a = a(a-1)$

$a_3 - a_2 = a^3 - a^2 = a^2(a-1)$

$a_2 - a_1 \neq a_3 - a_2$

So, the given list of numbers does not form an AP

(xii) $\boldsymbol{\sqrt{2}, \sqrt{8}, \sqrt{18}, \sqrt{32}, \ldots \ldots}$

Sol: $\sqrt{2}, 2\sqrt{2}, 3\sqrt{2}, 4\sqrt{2}, \ldots \ldots ..$

$\sqrt{8} = \sqrt{4 \times 2} = \sqrt{4} \times \sqrt{2} = 2\sqrt{2}$ $\sqrt{18} = \sqrt{9 \times 2} = \sqrt{9} \times \sqrt{2} = 3\sqrt{2}$ $\sqrt{32} = \sqrt{16 \times 2} = \sqrt{16} \times \sqrt{2} = 4\sqrt{2}$

$a_2 - a_1 = 2\sqrt{2} - \sqrt{2} = \sqrt{2}$

$a_3 - a_2 = 3\sqrt{2} - 2\sqrt{2} = \sqrt{2}$

$a_4 - a_3 = 4\sqrt{2} - 3\sqrt{2} = \sqrt{2}$

$a_2 - a_1 = a_3 - a_2 = a_4 - a_3 = \cdots$

i.e., $a_{k+1} - a_k$ is same every time

So, the given list of numbers forms an AP with the common difference $d = \sqrt{2}$

The next three terms are: $5\sqrt{2}, 6\sqrt{2}, 7\sqrt{2}$

$\Rightarrow \sqrt{50}, \sqrt{72}, \sqrt{98}$

(xiii) $\boldsymbol{\sqrt{3}, \sqrt{6}, \sqrt{9}, \sqrt{12}, \ldots \ldots}$

Sol: $a_2 - a_1 = \sqrt{6} - \sqrt{3}$

$a_3 - a_2 = \sqrt{9} - \sqrt{6} = 3 - \sqrt{6}$

$a_2 - a_1 \neq a_3 - a_2$

So, the given list of numbers does not form an AP.

n^{th} TERM(GENERAL TERM) OF AN ARITHMETIC PROGRESSION

If first term is a and common difference is d then the n^{th} term of an AP is

$$a_n = a + (n-1)d$$

Example-3. Find the 10th term of the AP : 5, 1, –3, –7 . . .

Sol: Given AP is 5, 1, –3, –7 . . .

$a = 5$

$d = a_2 - a_1 = 1 - 5 = -4$

The 10th term $= a_{10} = a + 9d$

$= 5 + 9 \times (-4)$

$= 5 - 36 = -31$

Example-4. Which term of the AP : 21, 18, 15, . . . is – 81? Is there any term 0? Give reason for your answer

Sol: First term=a=21

Common difference=$d = a_2 - a_1 = 18 - 21 = -3$

$Let\ a_n = -81$

$\Rightarrow a + (n-1)d = -81$

$\Rightarrow 21 + (n-1) \times (-3) = -81$

$\Rightarrow (n-1) \times (-3) = -81 - 21 = -102$

$\Rightarrow n - 1 = \frac{-102}{-3} = 34$

$\Rightarrow n = 34 + 1 = 35$

$\therefore$ -81 is the 35^{th} term of the given AP.

$Let\ a_n = 0$

$\Rightarrow a + (n-1)d = 0$

$\Rightarrow 21 + (n-1) \times (-3) = 0$

$\Rightarrow (n-1) \times (-3) = -21$

$\Rightarrow n - 1 = \frac{-21}{-3} = 7$

$\Rightarrow n = 7 + 1 = 8$

$\therefore$ The 8^{th} term of the given AP is 0.

Example-5. Determine the AP whose 3^{rd} term is 5 and the 7^{th} term is 9.

Sol: 3^{rd} term of AP=5$\Rightarrow a + 2d = 5 \rightarrow (1)$

7^{th} term of AP=9 $\Rightarrow a + 6d = 9 \rightarrow (2)$

$(2)-(1) \Rightarrow \; a+6d=9$

$$\begin{array}{r} a+2d=5 \\ \underline{(-)\;(-)\quad(-)} \\ 4d=4 \end{array}$$

$d=1$

Substitute d=1 value in (1)

$a+2\times 1=5$

$a=5-2$

$a=3$

Hence, the required AP is 3,4,5,6,……

Example-6. Check whether 301 is a term of the list of numbers 5, 11, 17, 23, .

Sol: Given list of numbers 5, 11, 17, 23, ….

$a_2-a_1=11-5=6$

$a_3-a_2=17-11=6$

$a_4-a_3=23-17=6$

$a_2-a_1=a_3-a_2=a_4-a_3=\cdots$

i.e., $a_{k+1}-a_k$ is same every time

So, the given list of numbers forms an AP with the common difference $d=6$ and $a=5$

$Let\ a_n=301$

$\Rightarrow a+(n-1)d=301$

$\Rightarrow 5+(n-1)\times(6)=301$

$\Rightarrow (n-1)\times(6)=301-5=296$

$\Rightarrow n-1=\frac{296}{6}=\frac{148}{3}$

$\Rightarrow n=\frac{148}{3}+1=\frac{153}{3}$ it is not a positive integer

So, 301 is not a term of the given list of numbers.

Example-7. How many two-digit numbers are divisible by 3?

Sol: The list of two-digit numbers divisible by 3 is : 12, 15, 18, . . . , 99

Clearly it is an AP. $a=12\ \ and\ d=15-12=3$

$Let\ a_n=99 \Rightarrow a+(n-1)d=99$

$12+(n-1)\times 3=99$

$(n-1)\times 3=99-12=87$

$n-1=\frac{87}{3}=27$

$n=29+1=30$

So, there are 30 two-digit numbers divisible by 3.

Example-8. Find the 11th term from the last of the AP series given below :

AP : 10, 7, 4, . . ., – 62.

Sol: Given AP is 10,7,4,…

$a = 10, d = 7 - 10 = -3$

$Let\ a_n = -62 \Rightarrow a + (n-1)d = -62$

$10 + (n-1) \times (-3) = -62$

$(n-1) \times (-3) = -62 - 10 = -72$

$n - 1 = \frac{-72}{-3} = 24$

$n = 24 + 1 = 25$

So, there are 25 terms in the given AP.

The 11th term from the last $= (25 - 10)^{th}\ term$

$= 15^{th}\ term = a + 14d$

$= 10 + 14 \times (-3)$

$= 10 - 42 = -32$

The 11th term from the last of the AP is -32.

Example-9. A sum of 1000 is invested at 8% simple interest per year. Calculate the interest at the end of each year. Do these interests form an AP? If so, find the interest at the end of 30 years.

Sol: simple interest$(I) = \frac{P \times T \times R}{100}$

Here P=1000, R= 8%

The interest at the end of 1^{st} year $= \frac{1000 \times 1 \times 8}{100} =$ ₹ 80

The interest at the end of 2^{nd} year $= \frac{1000 \times 2 \times 8}{100} =$ ₹ 160

The interest at the end of 3^{rd} year $= \frac{1000 \times 3 \times 8}{100} =$ ₹ 240

The interests are 80,160,240,…..

The interests form an AP with $a = 80, d = 80$

The interest at the end of 30 years=$a_{30} = a + 29d$

$= 80 + 29 \times 80 =$ ₹2400

Example-10: In a flower bed, there are 23 rose plants in the first row, 21 in the second, 19 in the third, and so on. There are 5 rose plants in the last row. How many rows are there in the flower bed?

Sol: The number of rose plants in the 1st,2nd ,3rd ,….rows are

23,21,19,…..,5 Clearly it is an AP

$a = 23, d = 21 - 23 = -2$

$Let\ a_n = 5 \Rightarrow a + (n-1)d = 5$

$23 + (n-1) \times (-2) = 5$

$(n-1) \times (-2) = 5 - 23 = -18$

$n - 1 = \frac{-18}{-2} = 9$

$n = 9 + 1 = 10$

So, there are 10 rows in the flower bed.

Exercise - 6.2

1. Fill in the blanks in the following table, given that a is the first term, d the common difference and a_n the n^th term of the AP:

(i) $a = 7, d = 3, n = 8, a_n = ?$

Sol: $a_n = a + (n-1)d$

$= 7 + (8-1) \times 3$

$= 7 + 7 \times 3$

$= 7 + 21 = 28$

(ii) $a = -18, d = ?, n = 10, a_n = 0$

Sol: $a_n = 0$

$a + (n-1)d = 0$

$-18 + (10-1)d = 0$

$9d = 18$

$d = \frac{18}{9} = 2$

(iii) $a = ?, d = -3, n = 18, a_n = -5$

Sol: $a_n = -5$

$a + (n-1)d = -5$

$a + (18-1) \times (-3) = -5$

$a + 17 \times (-3) = -5$

$a - 51 = -5$

$a = -5 + 51 = 46$

(iv) $a = -18.9, d = 2.5, n = ?, a_n = 3.6$

Sol: $a_n = 3.6$

$a + (n-1)d = 3.6$

$-18.9 + (n-1) \times (2.5) = 3.6$

$(n-1) \times (2.5) = 3.6 + 18.9$

$n - 1 = \frac{22.5}{2.5} = 9$

$n = 9 + 1 = 10$

(v) $\boldsymbol{a = 3.5, d = 0, n = 105, a_n = ?}$

Sol: $a_n = a + (n-1)d$

$= 3.5 + (105 - 1) \times 0 = 3.5$

2. (i) Find the 30th term of the A.P 10,7,4,……..

Sol: Given A.P is 10,7,4,……..

$a = 10, d = 7 - 10 = -3$

30^{th} term of the A. P $= a + 29d$

$= 10 + 29 \times (-3)$

$= 10 - 87 = -77$

(ii) Find the 11th term of the A.P : $-3, \frac{-1}{2}, 2, \ldots..$

Sol: Given A.P is $-3, \frac{-1}{2}, 2, \ldots..$

$$a = -3, \quad d = a_2 - a_1 = \frac{-1}{2} - (-3) = \frac{-1}{2} + 3 = \frac{-1+6}{2} = \frac{5}{2}$$

11th term of the A.P$= a + 10d$

$= -3 + 10 \times \left(\frac{5}{2}\right)$

$= -3 + 25 = 22$

3. Find the respective terms for the following APs.

(i) $\boldsymbol{a_1 = 2, a_3 = 26 \ find \ a_2}$

Sol: $a_1 = a = 2$

$a_3 = a + 2d = 26$

$\Rightarrow 2 + 2d = 26$

$\Rightarrow 2d = 26 - 2$

$\Rightarrow d = \frac{24}{2} = 12$

Now $a_2 = a + d = 2 + 12 = 14$

Short cut:
$$a_2 = \frac{a_1 + a_3}{2} = \frac{2 + 26}{2} = \frac{28}{2} = 14$$

(ii) $\boldsymbol{a_2 = 13, a_4 = 3 \ find \ a_1, a_3}$

Sol: $a_2 = a + d = 13 \rightarrow (1)$

$a_4 = a + 3d = 3 \rightarrow (2)$

(2)-(1)$\Rightarrow$ $a + 3d = 3$

$$\begin{array}{r} a + d = 13 \\ (-) \ (-) \ \ (-) \\ \hline 2d = -10 \\ \hline \end{array}$$

$d = \frac{-10}{2} = -5$

Substitute $d = -5$ in (1)

Shortcut:
$$a_3 = \frac{a_2 + a_4}{2} = \frac{13 + 3}{2} = \frac{16}{2} = 8$$
$$a_1 = 2a_2 - a_3 = 26 - 8 = 18$$

$a - 5 = 13$

$a = 13 + 5 = 18$

Now $a_1 = a = 18$

$a_3 = a + 2d = 13 + 2(-5) = 18 - 10 = 8$

(iii) $\boldsymbol{a_1 = 5;\ a_4 = 9\frac{1}{2}\ find\ a_2, a_3}$

Sol: $a_1 = a = 5$

$a_4 = a + 3d = \frac{19}{2}$

$5 + 3d = \frac{19}{2}$

$3d = \frac{19}{2} - 5 = \frac{19 - 10}{2} = \frac{9}{2}$

$d = \frac{9}{2 \times 3} = \frac{3}{2}$

$a_2 = a + d = 5 + \frac{3}{2} = \frac{13}{2}$

$a_3 = a + 2d = 5 + 2 \times \frac{3}{2} = 5 + 3 = 8$

(iv) $\boldsymbol{a_1 = -4; a_6 = 6\ find\ a_2, a_3, a_4, a_5}$

Sol: $a_1 = -4 \Rightarrow a = -4$

$a_6 = 6 \Rightarrow a + 5d = 6$

$-4 + 5d = 6$

$5d = 6 + 4 = 10$

$d = \frac{10}{5} = 2$

$a_2 = a + d = -4 + 2 = -2$

$a_3 = a + 2d = -4 + 2 \times 2 = -4 + 4 = 0$

$a_4 = a + 3d = -4 + 3 \times 2 = -4 + 6 = 2$

$a_5 = a + 4d = -4 + 4 \times 2 = -4 + 8 = 4$

(v) $\boldsymbol{a_2 = 38; a_6 = -22\ find\ a_1, a_3, a_4, a_5}$

Sol: $a_2 = 38 \Rightarrow a + d = 38 \rightarrow (1)$

$a_6 = -22 \Rightarrow a + 5d = -22 \rightarrow (2)$

$(2) - (1) \Rightarrow a + 5d = -22$

$$\begin{array}{r} a + d \;\;=\;\; 38 \\ (-)\;(-)\quad\;(-) \\ \hline 4d = -60 \\ \hline \end{array}$$

$d = \frac{-60}{4} = -15$

Substitute $d = -15$ in (1)

$a - 15 = 38$

$a = 38 + 15 = 53$

$a_1 = a = 53$

$a_3 = a + 2d = 53 + 2(-15) = 53 - 30 = 23$

$a_3 = a + 2d = 53 + 3(-15) = 53 - 45 = 8$

$a_3 = a + 2d = 53 + 2(-15) = 53 - 60 = -7$

4. Which term of the AP : 3, 8, 13, 18, . . . ,is 78?

Sol: given A.P: 3, 8, 13, 18, . .

$a = 3; d = 8 - 3 = 5$

$let\ a_n = 78$

$a + (n-1)d = 78$

$3 + (n-1) \times 5 = 78$

$(n-1) \times 5 = 78 - 3 = 75$

$$n - 1 = \frac{75}{5} = 15$$

$n = 15 + 1 = 16$

∴ 78 is the 16^{th} term of A.P

5. Find the number of terms in each of the following APs :

(i) 7, 13, 19, . . . , 205

Sol: $a = 7, d = 13 - 7 = 6$

$let\ a_n = 205$

$a + (n-1)d = 205$

$7 + (n-1) \times 6 = 205$

$(n-1) \times 6 = 205 - 7 = 198$

$$n - 1 = \frac{198}{6} = 33$$

$n = 33 + 1 = 34$

The number of terms in given A.P are 34.

(ii) $\mathbf{18, 15\frac{1}{2}, 13, \ldots\ldots, -47}$

Sol: $a = 18,$

$$d = \frac{31}{2} - 18 = \frac{31 - 36}{2} = \frac{-5}{2}$$

$let\ a_n = -47$

$a + (n-1)d = -47$

$$18 + (n-1) \times \left(\frac{-5}{2}\right) = -47$$

$(n-1) \times \left(\frac{-5}{2}\right) = -47 - 18$

$(n-1) \times \left(\frac{-5}{2}\right) = -65$

$n - 1 = -65 \times \frac{-2}{5} = 26$

$n = 26 + 1 = 27$

The number of terms in given A.P are 27.

6. Check whether, –150 is a term of the AP : 11, 8, 5, 2 . . .

Sol: $a = 11, d = 8 - 11 = -3$

$let\ a_n = -150$

$a + (n-1)d = -150$

$11 + (n-1) \times (-3) = -150$

$(n-1) \times (-3) = -150 - 11 = -161$

$n - 1 = \frac{-161}{-3} = \frac{161}{3}$ *it is not a natural number*

∴ –150 is not a term of given AP

7. Find the 31st term of an AP whose 11th term is 38 and the 16th term is 73.

Sol: 11th term is 38⇒ $a + 10d = 38 \rightarrow (1)$

16th term is 73⇒ $a + 15d = 73 \rightarrow (2)$

$(2) - (1) \Rightarrow a + 15d = 73$

$a + 10d = 38$

$5d = 35 \Longrightarrow d = \frac{35}{5} = 7$

Substitute d=7 in (1)

$a + 10 \times 7 = 38$

$a = 38 - 70 = -32$

31st term $= a + 30d$

$= -32 + 30 \times 7$

$= -32 + 210$

$= 178$

8. If the 3rd and the 9th terms of an AP are 4 and – 8 respectively, which term of this AP is zero?

Sol: 3rd term of an A.P=4⇒ $a + 2d = 4 \rightarrow (1)$

9th term of an A.P= $-8 \Rightarrow a + 8d = -8 \rightarrow (2)$

$(2) - (1) \Rightarrow a + 8d = -8$

$$\begin{array}{ccc} a + & 2d = & 4 \\ (-) & (-) & (-) \end{array}$$

$$\overline{6d = -12}$$

$$d = \frac{-12}{6} = -2$$

Substitute d=-2 in (1) we get

$a + 2 \times (-2) = 4$

$a - 4 = 4$

$a = 4 + 4 = 8$

$let\ a_n = 0$

$a + (n-1)d = 0$

$8 + (n-1) \times (-2) = 0$

$(n-1) \times (-2) = 0 - 8$

$n - 1 = \frac{-8}{-2} = 4$

$n = 4 + 1 = 5$

∴ The 5^{th} term of A.P is '0'

9. The 17^{th} term of an AP exceeds its 10^{th} term by 7. Find the common difference.

Sol: 17^{th} term of an AP=10^{th} term+7

$a + 16d = a + 9d + 7$

$a + 16d - a - 9d = 7$

$7d = 7 \Longrightarrow d = 1$

The common difference=1

10. Two APs have the same common difference. The difference between their 100th terms is 100, what is the difference between their 1000th terms?

Sol: Let the first A.P is $a, a+d, a+2d, a+3d, \ldots ..$

The second A.P is $b, b+d, b+2d, b+3d, \ldots ....$

The difference between their 100th terms $= 100$

$a_{100} - b_{100} = 100$

$(a + 99d) - (b + 99d) = 100$

$a + 99d - b - 99d = 100$

$a - b = 100 \rightarrow (1)$

The difference between their 1000th terms $= a_{1000} - b_{1000}$

$= (a + 999d) - (b + 999d)$

$= a + 99d - b - 99d$

$= a - b$

$= 100 \quad (from\ (1))$

11. How many three-digit numbers are divisible by 7?

Sol: The three-digit numbers are divisible by 7 are

$105, 112, 119, \ldots\ldots, 994$

$a = 105, d = 7$

$let\ a_n = 994$

$a + (n-1)d = 994$

$105 + (n-1) \times 7 = 994$

$(n-1) \times 7 = 994 - 105 = 889$

$n - 1 = \frac{889}{7} = 127$

$n = 127 + 1 = 128$

∴ 128 three digit numbers are divisible by 7

12. **How many multiples of 4 lie between 10 and 250?**

Sol: Multiples of 4 lie between 10 and 250 are

$12, 16, 20, \ldots\ldots, 248$

$a = 12, d = 4$

$let\ a_n = 248$

$a + (n-1)d = 248$

$12 + (n-1) \times 4 = 248$

$(n-1) \times 4 = 248 - 12 = 236$

$n - 1 = \frac{236}{4} = 59$

$n = 59 + 1 = 60$

∴ 60 multiples of 4 lie between 10 and 250.

13. **For what value of n, are the n^{th} terms of two APs: 63, 65, 67, . .. and 3, 10, 17, . . . equal?**

Sol: First A.P : 63,65,67,….

$a = 63, d = 2$

$a_n = a + (n-1)d$

$= 63 + (n-1) \times 2$

$= 63 + 2n - 2$

$= 2n + 61$

Second A.P: 3,10,17,…..

$a = 3, d = 7$

$a_n = a + (n-1)d$

$= 3 + (n-1) \times 7$

$= 3 + 7n - 7$

$= 7n - 4$

If n^{th} terms of two A.Ps are equal then

$7n - 4 = 2n + 61$

$7n - 2n = 61 + 4$

$5n = 65$

$n = \frac{65}{5} = 13$

$\therefore$ 13th terms of the two A.Ps are equal.

14. Determine the AP whose third term is 16 and the 7th term exceeds the 5th term by 12.

Sol: Third term of AP=16⇒ $a + 2d = 16 \rightarrow (1)$

7th term=5th term + 12

$a + 6d = a + 4d + 12$

$a + 6d - a - 4d = 12$

$2d = 12$

$d = 6$

Substitute $d = 6$ in (1) we get

$a + 2 \times 6 = 16$

$a = 16 - 12 = 4$

The required AP is $a, a + d, a + 2d, a + 3d, \ldots.$

⇒ 4,10,16,22, … ….

15. Find the 20th term from the end of the AP : 3, 8, 13, . . ., 253

Sol: $a = 3, d = 8 - 3 = 5$

$let \quad a_n = l = 253$

$a + (n - 1)d = 253$

$3 + (n - 1) \times 5 = 253$

$(n - 1) \times 5 = 253 - 3 = 250$

$$n - 1 = \frac{250}{5} = 50$$

$n = 50 + 1 = 51$

The 20th term from the end of the AP= (51-20)+1=32th term from first

$= a + 31d = 3 + 31 \times 5 = 3 + 155$=158

(OR)

$a = 3, \qquad d = 8 - 3 = 5$

$a_n = l = 253$

$\mathrm{n^{th}}$term from the end of the AP $= l - (n - 1)d$

$\mathrm{20^{th}}$term from the end of the AP $= 253 - 19 \times 5 = 253 - 95 = 158$

16. The sum of the 4th and 8th terms of an AP is 24 and the sum of the 6th and 10th terms is 44. Find the first three terms of the AP.

Sol: 4th term +8th term of an AP =24

$\Rightarrow a + 3d + a + 7d = 24$

$\Rightarrow 2a + 10d = 24$

$\Rightarrow a + 5d = 12 \rightarrow (1)$

6th term +10th term of an AP =44

$\Rightarrow a + 5d + a + 9d = 44$

$\Rightarrow 2a + 14d = 44$

$\Rightarrow a + 7d = 22 \rightarrow (2)$

$(2) - (1) \Rightarrow a + 7d = 22$

$$\begin{array}{r} a + 5d = 12 \\ (-)\ (-)\quad (-) \\ \hline 2d = 10 \\ \hline \end{array}$$

$d = 5$

Substitute d=5 in (1) we get

$a + 5 \times 5 = 12$

$a = 12 - 25$

$a = -13$

$\therefore$ The first three terms of AP are $a, a + d, a + 2d$

$\Rightarrow -13, -13 + 5, -13 + 10$

$\Rightarrow -13, -8, -3$

17. Subba Rao started work in 1995 at an annual salary of ₹5000 and received an increment of ₹ 200 each year. In which year did his income reach ₹ 7000?

Sol: subbarao salary in 1995=₹5000, Increment =₹200

Salary in 1996=5000+200=₹5200

Salary in 1997=5200+200=₹5400

Salary in 1998=5400+200=₹5600

The salaries are ₹5000, ₹5200, ₹5400, ₹5600,…….forms an AP

$a = 5000, d = 200$

Let $a_n = 7000$

$a + (n - 1) \times 200 = 7000$

$5000 + (n - 1) \times 200 = 7000$

$(n - 1) \times 200 = 7000 - 5000 = 2000$

$n - 1 = \frac{2000}{200} = 10$

$n = 10 + 1$

$n = 11$

$\therefore$ In 11th year subbarao income reached 7000

SUM OF FIRST n TERMS IN ARITHMETIC PROGRESSION:

i. If first term of an AP is $\boldsymbol{a}$ and common difference is $\boldsymbol{d}$ then

Sum of first n terms $= S_n = \frac{n}{2}[2a + (n-1)d]$

ii. If first term is $\boldsymbol{a}$, last term is $\boldsymbol{l}$ and number of terms is $\boldsymbol{n}$ then

$S_n = \frac{n}{2}(a + l)$

iii. $a_n = S_n - S_{n-1}$

Do This

Find the sum of indicated number of terms in each of the following A.P.s

(i) 16, 11, 6 .....; 23 terms

Sol: $a = 16, d = 11 - 16 = -5, n = 23$

$S_n = \frac{n}{2}[2a + (n-1)d]$

$= \frac{23}{2}[2 \times 16 + (23 - 1) \times (-5)]$

$= \frac{23}{2}[32 + 22(-5)]$

$= \frac{23}{2}[32 - 110]$

$= \frac{23}{2}(-78)$

$= 23 \times (-39)$

$= -897$

(ii) –0.5, –1.0, –1.5, ...; 10 terms

Sol: $a = -0.5, d = -1.0 + 0.5 = -0.5, n = 10$

$S_n = \frac{n}{2}[2a + (n-1)d]$

$S_{10} = \frac{10}{2}[2 \times (-0.5) + (10 - 1) \times (-0.5)]$

$= 5[-1 + 9 \times (-0.5)]$

$= 5[-1 - 4.5]$

$= 5 \times (-5.5)$

$= -27.5$

(iii) $\boldsymbol{-1, \frac{1}{4}, \frac{3}{2}, \ldots \ldots, 10\ terms}$

$Sol: a = -1, d = \frac{1}{4} + 1 = \frac{5}{2}, n = 10$

$S_n = \frac{n}{2}[2a + (n-1)d]$

$S_{10} = \frac{10}{2}\left[2 \times (-1) + (10 - 1) \times \frac{5}{2}\right]$

$$= 5\left[-2 + \frac{45}{2}\right]$$

$$= 5 \times \frac{41}{2}$$

$$= \frac{205}{2}$$

Example-11. If the sum of the first 14 terms of an AP is 1050 and its first term is 10, find the 20th term

Sol: $a = 10, n = 14,$

$S_{14} = 1050$

$\frac{n}{2}[2a + (n-1)d] = 1050$

$\frac{14}{2}[2 \times 10 + (14-1)d] = 1050$

$7[20 + 13d] = 1050$

$20 + 13d = \frac{1050}{7} = 150$

$13d = 150 - 20$

$13d = 130$

$d = 10$

20^{th} term$= a + 19d$

$= 10 + 19 \times 10 = 10 + 190 = 200$

Example-12. How many terms of the AP : 24, 21, 18, . . . must be taken so that their sum is 78?

Sol: $a = 24, d = 21 - 24 = -3$

$Let\ S_n = 78$

$\frac{n}{2}[2a + (n-1)d] = 78$

$n[2 \times 24 + (n-1)(-3)] = 2 \times 78$

$n[48 - 3n + 3] = 156$

$n[-3n + 51] = 156$

$-3n^2 + 51n - 156 = 0$

$3n^2 - 51n + 156 = 0$

$n^2 - 17n + 52 = 0$

$(n-4)(n-13) = 0$

$n - 4 = 0 \quad or \quad n - 13 = 0$

$n = 4\ or\ 13$

Example-13. (i) Find the sum of the first 1000 positive integers.

Sol: The first 1000 positive integers are 1,2,3,4,…..,1000

$a = 1, d = 1, n = 1000$

$$S_n = \frac{n}{2}[a + l]$$

$$S_{1000} = \frac{1000}{2}[1 + 1000] = 500 \times 1001 = 500500$$

(ii) Find the sum of the first n positive integers

Sol: $a = 1, d = 1, n = n$

$$S_n = \frac{n}{2}[a + l] = \frac{n}{2}(1 + n) = \frac{n(n + 1)}{2}$$

$$The\ sum\ of\ the\ first\ n\ positive\ integers = \frac{n(n + 1)}{2}$$

Example-14. Find the sum of first 24 terms of the list of numbers whose nth term is given by

$\boldsymbol{a_n = 3 + 2n}$

Sol: $a_n = 3 + 2n$

$a_1 = 3 + 2 \times 1 = 3 + 2 = 5$

$a_2 = 3 + 2 \times 2 = 3 + 4 = 7$

$a_3 = 3 + 2 \times 3 = 3 + 6 = 9$

List of numbers are 5,7,9,...... clearly it is an AP

$a = 5, d = 7 - 5 = 2, n = 24$

$$S_{24} = \frac{24}{2}[2 \times 5 + (24 - 1) \times 2]$$

$$S_{24} = \frac{24}{2}[10 + (24 - 1) \times 2]$$

$= 12[10 + 23 \times 2]$

$= 12 \times 56$

$= 672$

Example-15. A manufacturer of TV sets produced 600 sets in the third year and 700 sets in the seventh year. Assuming that the production increases uniformly by a fixed number every year, find : (i) the production in the 1st year (ii) the production in the 10th year (iii) the total production in first 7 years.

Sol: $a_3 = 600, a_7 = 700$

$a_7 = 700 \Rightarrow a + 6d = 700 \rightarrow (1)$

$a_3 = 600 \Rightarrow a + 2d = 600 \rightarrow (2)$

$4d = 100$

$$d = \frac{100}{4} = 25$$

Substitute d=25 in (2)

$a + 2 \times 25 = 600$

$a + 50 = 600$

$a = 600 - 50 = 550$

(i) The production in the 1st year=550

(ii) The production in the 10th year=$a + 9d$

$= 550 + 9 \times 25$

$= 550 + 225$

$= 775$

(iii) The total production in first 7 years=S_7

$= \frac{7}{2}[2 \times 550 + (7-1) \times 25]$

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$= \frac{7}{2}[1100 + 6 \times 25]$

$= \frac{7}{2}[1250] = 7 \times 625 = 4375$

EXERCISE - 6.3

1. Find the sum of the following APs:

(i) 2, 7, 12, . . ., to 10 terms.

Sol: $a = 2, d = 7 - 2 = 5, n = 10$

$S_n = \frac{n}{2}[2a + (n-1)d]$

$S_{10} = \frac{10}{2}[2 \times 2 + (10-1) \times 5]$

$= 5[4 + 45]$

$= 5 \times 50$

$= 250$

(ii) –37, –33, –29, . . ., to 12 terms.

Sol: $a = -37, d = -33 + 37 = 4, n = 12$

$S_n = \frac{n}{2}[2a + (n-1)d]$

$S_{12} = \frac{12}{2}[2 \times (-37) + (12-1) \times 4]$

$= 6[-74 + 44]$

$= 6 \times (-30)$

$= -180$

(iii) 0.6, 1.7, 2.8, . . ., to 100 terms

Sol: $a = 0.6, d = 1.7 - 0.6 = 1.1, n = 100$

$S_n = \frac{n}{2}[2a + (n-1)d]$

$$S_{100} = \frac{100}{2}[2 \times 0.6 + (100 - 1) \times 1.1]$$

$$= 50[1.2 + 99 \times 1.1]$$

$$= 50[1.2 + 108.9]$$

$$= 50 \times 110.1$$

$$= 5505$$

(iv) $\frac{1}{15}, \frac{1}{12}, \frac{1}{10}, \ldots\ldots, \boldsymbol{to\ 11\ terms}$

Sol: $a = \frac{1}{15}, d = \frac{1}{12} - \frac{1}{15} = \frac{5-4}{60} = \frac{1}{60}, n = 11$

$$S_n = \frac{n}{2}[2n + (n-1)d]$$

$$S_{11} = \frac{11}{2}\left[2\left(\frac{1}{15}\right) + (11-1)\left(\frac{1}{60}\right)\right]$$

$$= \frac{11}{2}\left[\frac{2}{15} + 10 \times \frac{1}{60}\right]$$

$$= \frac{11}{2}\left[\frac{2}{15} + \frac{1}{6}\right]$$

$$= \frac{11}{2}\left[\frac{8+10}{60}\right]$$

$$= \frac{11}{2} \times \frac{18}{60}$$

$$= \frac{33}{20} = 1\frac{13}{20}$$

2. Find the sums given below

(i) $7 + 10\frac{1}{2} + 14 + \cdots + 84$

Sol: $a = 7, d = 10\frac{1}{2} - 7 = 3\frac{1}{2} = \frac{7}{2}, l = 84$

$$l = a_n = 84$$

$$a + (n-1)d = 84$$

$$7 + (n-1)\left(\frac{7}{2}\right) = 84$$

$$(n-1)\left(\frac{7}{2}\right) = 84 - 7$$

$$n - 1 = 77 \times \frac{2}{7} = 22$$

$$n = 22 + 1 = 23$$

$$S_n = \frac{n}{2}(a + l)$$

$$S_{23} = \frac{23}{2}(7 + 84)$$

$= \frac{23}{2} \times 91$

$= \frac{2093}{2} = 1046\frac{1}{2}$

(ii) 34 + 32 + 30 + . . . + 10

Sol: $a = 34, d = 32 - 34 = -2$

$l = a_n = 10$

$34 + (n-1)(-2) = 10$

$(n-1)(-2) = 10 - 34$

$(n-1)(-2) = -24$

$n - 1 = \frac{-24}{-2} = 12$

$n = 12 + 1 = 13$

$S_n = \frac{n}{2}(a + l)$

$S_{13} = \frac{13}{2}(34 + 10)$

$= \frac{13}{2} \times 44$

$= 13 \times 22$

$= 286$

(iii) –5 + (–8) + (–11) + . . . + (–230)

Sol: $a = -5, d = -8 + 5 = -3$

$l = a_n = -230$

$-5 + (n-1)(-3) = -230$

$(n-1)(-3) = -230 + 5$

$(n-1)(-3) = -225$

$n - 1 = \frac{-225}{-3} = 75$

$n = 75 + 1 = 76$

$S_n = \frac{n}{2}(a + l)$

$S_{76} = \frac{76}{2}[-5 + (-230)]$

$= 38 \times (-235)$

$= -8930$

3. **In an AP**:

(i) $\boldsymbol{Given\ a = 5, d = 3, a_n = 50, find\ n\ and\ S_n}$.

Sol: $a_n = 50$

$a+(n-1)d=50$

$5+(n-1)\times 3=50$

$(n-1)\times 3=50-5$

$(n-1)\times 3=45$

$n-1=\frac{45}{3}=15$

$n=15+1$

$n=16$

$S_n=\frac{n}{2}[2a+(n-1)d]$

$S_{16}=\frac{16}{2}[2\times 5+(16-1)\times 3]$

$=8[10+15\times 3]$

$=8[10+45]$

$=8\times 55=440$

(ii) $\boldsymbol{Given\ a\ =\ 7, a_{13}\ =\ 35, find\ d\ and\ S_{13}.}$

Sol: $a_{13}\ =\ 35$

$a+12d=35$

$7+12d=35$

$12d=35-7$

$12d=28$

$d=\frac{28}{12}=\frac{7}{3}$

$S_n=\frac{n}{2}[2a+(n-1)d]$

$S_{13}=\frac{13}{2}\left[2\times 7+(13-1)\times\frac{7}{3}\right]$

$=\frac{13}{2}\left[14+12\times\frac{7}{3}\right]$

$=\frac{13}{2}[14+28]$

$=\frac{13}{2}\times 42=13\times 21=273$

(iii) $\boldsymbol{Given\ a_{12}\ =\ 37, d\ =\ 3, find\ a\ and\ S_{12}.}$

Sol: $a_{12}\ =\ 37$

$a+11d=37$

$a+11\times 3=37$

$a+33=37$

$a=37-33=4$

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$S_{12} = \frac{12}{2}[2 \times 4 + (12-1) \times 3]$$

$$= 6[8 + 33]$$

$$= 6 \times 41 = 246$$

(iv) ***Given*** $\boldsymbol{a_3 = 15, S_{10} = 125,}$ ***find*** $\boldsymbol{d}$ ***and*** $\boldsymbol{a_{10}}$

Sol: $a_3 = 15$

$$a + 2d = 15 \Rightarrow a = 15 - 2d \rightarrow (1)$$

$$S_{10} = 125$$

$$\frac{10}{2}[2a + (10-1)d] = 125$$

$$[2(15 - 2d) + 9d] = \frac{125}{5}$$

$$30 - 4d + 9d = 25$$

$$5d = 25 - 30$$

$$d = \frac{-5}{5} = -1$$

Substitute $d = -1$ in (1)

$$a = 15 - 2 \times (-1) = 15 + 2 = 17$$

$$a_n = a + 9d$$

$$= 17 + 9 \times (-1)$$

$$= 17 - 9 = 8$$

(v) ***Given*** $\boldsymbol{a = 2, d = 8, S_n = 90,}$ ***find*** $\boldsymbol{n}$ ***and*** $\boldsymbol{a_n}$.

Sol: $S_n = 90$

$$\frac{n}{2}[2a + (n-1)d] = 90$$

$$\frac{n}{2}[2 \times 2 + (n-1) \times 8] = 90$$

$$n[4 + 8n - 8] = 90 \times 2$$

$$4n + 8n^2 - 8n - 180 = 0$$

$$8n^2 - 4n - 180 = 0$$

$$2n^2 - n - 45 = 0$$

$$2n^2 - 10n + 9n - 45 = 0$$

$$2n(n-5) + 9(n-5) = 0$$

$$(n-5)(2n+9) = 0$$

$$n - 5 = 0 \text{ or } 2n + 9 = 0$$

$n = 5 \ or \ n = \frac{-9}{2}$

$\therefore n = 5$ (n is a natural number)

$a_n = a_5 = a + 4d$

$= 2 + 4 \times 8 = 2 + 32 = 34$

(vi) Given $a_n = 4, d = 2, S_n = -14$, find n and a.

Sol: $a_n = 4$

$a + (n-1)d = 4$

$a + (n-1) \times 2 = 4$

$a + 2n - 2 = 4$

$a = 4 - 2n + 2$

$a = 6 - 2n \rightarrow (1)$

$S_n = -14$

$\frac{n}{2}[a + a_n] = -14$

$n[6 - 2n + 4] = -14 \times 2$

$n[10 - 2n] = -28$

$10n - 2n^2 + 28 = 0$

$-2n^2 + 10n + 28 = 0$

$n^2 - 5n - 14 = 0$

$(n-7)(n+2) = 0$

$n - 7 = 0 \ or \ n + 2 = 0$

$n = 7 \ or \ n = -2$

$\therefore n = 7$ (n is a natural number)

From (1)

$a = 6 - 2 \times 7 = 6 - 14 = -8$

(vii) Given $l = 28, S = 144$, and there are total 9 terms. Find a.

Sol: $l = a_n = 28, S = 144, n = 9$

$S = 144$

$\frac{n}{2}[a + l] = 144$

$\frac{9}{2}[a + 28] = 144$

$a + 28 = \frac{144 \times 2}{9}$

$a = 32 - 28 = 4$

4. The first and the last terms of an AP are 17 and 350 respectively. If the common difference is 9,

how many terms are there and what is their sum?

Sol: $a = 17\ , d = 9\ and\ \ l = a_n = 350$

$a_n = 350$

$a + (n-1)d = 350$

$17 + (n-1) \times 9 = 350$

$(n-1) \times 9 = 350 - 17$

$n - 1 = \frac{333}{9} = 37$

$n = 37 + 1 = 38$

$S_n = \frac{n}{2}(a + l)$

$= \frac{38}{2}(17 + 350)$

$= 19 \times 367 = 6973$

There are 38 terms and their sum is 6973.

5. Find the sum of first 51 terms of an AP whose second and third terms are 14 and 18 respectively.

Sol: $a_2 = 14 \Rightarrow a + d = 14 \rightarrow (1)$

$a_3 = 18 \Rightarrow a + 2d = 18 \rightarrow (2)$

$(2) - (1) \Rightarrow a + 2d = 18$

$$\begin{array}{rrl} & a + d & = 14 \\ (-) & (-) & (-) \\ \hline & d & = 4 \\ \hline \end{array}$$

Substitute d=4 in (1)

$a + 4 = 14$

$a = 14 - 4 = 10$

$S_n = \frac{n}{2}[2a + (n-1)d]$

$S_{51} = \frac{51}{2}[2 \times 10 + (51 - 1) \times 4]$

$= \frac{51}{2}[20 + 50 \times 4]$

$= \frac{51}{2} \times 220$

$= 51 \times 110$

$= 5610$

6. If the sum of first 7 terms of an AP is 49 and that of 17 terms is 289, find the sum of first n terms.

Sol: $S_n = \frac{n}{2}[2a + (n-1)d]$

$S_7 = 49 \Rightarrow \frac{7}{2}[2a + (7-1)d] = 49$

$\Rightarrow [2a + 6d] = \frac{2 \times 49}{7}$

$\Rightarrow 2a + 6d = 14$

$\Rightarrow a + 3d = 7 \rightarrow (1)$

$S_{17} = 289 \Rightarrow \frac{17}{2}[2a + (17-1)d] = 289$

$\Rightarrow [2a + 16d] = \frac{2\times289}{17}$

$\Rightarrow 2a + 16d = 34$

$\Rightarrow a + 8d = 17 \rightarrow (2)$

Shortcut:
$S_7 = 49 = 7^2$
$S_{17} = 289 = 17^2$
$\therefore S_n = n^2$

$(2) - (1) \Rightarrow a + 8d = 17$

$$\begin{array}{r} a + 3d = \ \ 7 \\ (-)\ (-) \quad (-) \\ \hline 5d = 10 \\ \hline d = 2 \end{array}$$

Substitute d=2 in (1)

$a + 3 \times 2 = 7 \Rightarrow a + 6 = 7 \Rightarrow a = 1$

$S_n = \frac{n}{2}[2a + (n-1)d]$

$= \frac{n}{2}[2 \times 1 + (n-1)2]$

$= \frac{n}{2}[2 + 2n - 2]$

$= \frac{n}{2} \times 2n = n^2$

7. Show that $a_1, a_2, \ldots, a_n, \ldots$ form an AP where a_n is defined as below

$(i)\ a_n = 3 + 4n$

Sol: $a_n = 3 + 4n$

$a_1 = 3 + 4 \times 1 = 3 + 4 = 7$

$a_2 = 3 + 4 \times 2 = 3 + 8 = 11$

$a_3 = 3 + 4 \times 3 = 3 + 12 = 15$

$a_4 = 3 + 4 \times 4 = 3 + 16 = 19$

The list of terms are 7,11, 15,19,.....

$a_2 - a_1 = 11 - 7 = 4$

$a_3 - a_2 = 15 - 11 = 4$

$a_4 - a_3 = 19 - 15 = 4$

$a_2 - a_1 = a_3 - a_2 = a_4 - a_3 = \cdots$

i.e., $a_{k+1} - a_k$ is same every time

So, the given list of numbers forms an AP. $a = 7, d = 4$

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$S_{15} = \frac{15}{2}[2 \times 7 + (15-1) \times 4]$$

$$= \frac{15}{2}[14 + 56]$$

$$= \frac{15}{2} \times 70 = 15 \times 35 = 525$$

(ii) $a_n = 9 - 5n$

Sol: $a_n = 9 - 5n$

$a_1 = 9 - 5 \times 1 = 9 - 5 = 4$

$a_2 = 9 - 5 \times 2 = 9 - 10 = -1$

$a_3 = 9 - 5 \times 3 = 9 - 15 = -6$

$a_4 = 9 - 5 \times 4 = 9 - 20 = -11$

The list of terms is $4, -1, -6, -11, \ldots \ldots$

$a_2 - a_1 = -1 - 4 = -5$

$a_3 - a_2 = -6 + 1 = -5$

$a_4 - a_3 = -11 + 6 = -5$

$a_2 - a_1 = a_3 - a_2 = a_4 - a_3 = \cdots$

i.e., $a_{k+1} - a_k$ is same every time

So, the given list of numbers forms an AP. $a = 4, d = -5$

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$S_{15} = \frac{15}{2}[2 \times 4 + (15-1) \times (-5)]$$

$$= \frac{15}{2}[8 - 70]$$

$$= \frac{15}{2} \times (-62) = 15 \times (-31) = -465$$

8. If the sum of the first n terms of an AP is $4n - n^2$, what is the first term? What is the sum of first two terms? What is the second term? Similarly, find the 3rd, the 10th and the nth terms

Sol: $S_n = 4n - n^2$

$S_1 = 4 \times 1 - 1^2 = 4 - 1 = 3$

$S_2 = 4 \times 2 - 2^2 = 8 - 4 = 4$

$S_3 = 4 \times 3 - 3^2 = 12 - 9 = 3$

$S_4 = 4 \times 4 - 4^2 = 16 - 16 = 0$

$a_1 = S_1 = 3$

$a_2 = S_2 - S_1 = 4 - 3 = 1$

$a_3 = S_3 - S_2 = 3 - 4 = -1$

$\therefore a = 3, d = a_2 - a_1 = 1 - 3 = -2$

$a_{10} = a + 9d = 3 + 9 \times (-2) = 3 - 18 = -15$

$a_n = a + (n-1)d = 3 + (n-1) \times (-2) = 3 - 2n + 2 = 5 - 2n$

9. Find the sum of the first 40 positive integers divisible by 6.

Sol: The first 40 positive integers divisible by 6 are

$6 \times 1, 6 \times 2, 6 \times 3, \dots \dots \dots., 6 \times 40$

$\Rightarrow 6, 12, 18, \dots \dots, 240$

$a = 6, d = 6, n = 40, l = 240$

$S_n = \frac{n}{2}[a + l]$

$S_{40} = \frac{40}{2}[6 + 240]$

$= 20 \times 246 = 4920$

Shortcut:

$S_{40} = 6 + 12 + 18 + \cdots .. +240$

$= 6(1 + 2 + 3 + 4 + \cdots + 40)$

$= 6 \times \frac{40 \times 41}{2} = 3 \times 1640 = 4920$

10. A sum of 700 is to be used to give seven cash prizes to students of a school for their overall academic performance. If each prize is 20 less than its preceding prize, find the value of each of the prizes.

Sol: Let the prizes be $x, x + 20, x + 40, x + 60, x + 80, x + 100, x + 120$

$a = x, d = 20, l = x + 120$

$S_7 = 700$

$\frac{7}{2}[x + x + 120] = 700$

$2x + 120 = \frac{700 \times 2}{7} = 200$

$2x = 200 - 120 = 80$

$x = 40$

The prizes are ₹40, ₹60, ₹80, ₹100, ₹120, ₹140, ₹160.

11. In a school, students thought of planting trees in and around the school to reduce air pollution. It was decided that the number of trees, that each section of each class will plant, will be the same as the class, in which they are studying, e.g., a section of Class I will plant 1 tree, a section of Class II will plant 2 trees and so on till Class XII. There are three sections of each class. How many trees will be planted by the students?

Sol: Trees planted by each class are

$3 \times 1, 3 \times 2, 3 \times 3, \dots \dots \dots \dots., 3 \times 12$

$\Rightarrow 3, 6, 9, \dots \dots \dots., 36$ it is an AP

$a = 3, d = 3, n = 12, l = 36$

$S_n = \frac{n}{2}[a + l]$

$S_{12} = \frac{12}{2}[3 + 36] = 6 \times 39 = 234$

Total plants=234

12. A spiral is made up of successive semicircles, with centres alternately at A and B, starting with centre at A, of radii 0.5 cm, 1.0 cm, 1.5 cm, 2.0 cm, . . . as shown in Fig. 5.4. What is the total length of such a spiral made up of thirteen consecutive semicircles? (Take $\pi = \frac{22}{7}$)

Sol: The radii are 0.5 cm, 1.0 cm, 1.5 cm, 2.0 cm,……. these terms are in AP

$a = 0.5, d = 0.5, n = 13$

$l_1 = \pi \times r = \pi \times 0.5 = \pi \times \frac{1}{2} = \frac{\pi}{2}$

$l_2 = \pi \times 1 = \pi, \qquad l_3 = \pi \times 1.5 = \pi \times \frac{3}{2} = \frac{3\pi}{2}, \ldots \ldots$

Total length of spiral$= l_1 + l_2 + l_3 + \cdots . + l_{13}$

$= \frac{\pi}{2} + \pi + \frac{3\pi}{2} + \cdots \ldots .13\ terms$

$= \frac{\pi}{2}[1 + 2 + 3 + \cdots 13\ terms]$

$= \frac{\pi}{2}\left[\frac{13(13+1)}{2}\right] = \frac{\pi}{2}\left(\frac{13 \times 14}{2}\right) = \frac{\pi}{2} \times 91 = \frac{22}{7} \times \frac{1}{2} \times 91 = 11 \times 13 = 143\ cm$

13. 200 logs are stacked in the following manner: 20 logs in the bottom row, 19 in the next row, 18 in the row next to it and so on. In how may rows are the 200 logs placed and how many logs are in the top row?

Sol: The logs in rows are 20,19,18,……is an AP

$a = 20, d = -1$

$S_n = 200$

$\frac{n}{2}[2a + (n - 1)d] = 200$

$\frac{n}{2}[2 \times 20 + (n - 1) \times (-1)] = 200$

$n[40 - n + 1] = 200 \times 2$

$41n - n^2 - 400 = 0$

$-n^2 + 41n - 400 = 0$

$n^2 - 41n + 400 = 0$

$(n - 16)(n - 25) = 0$

$n - 16 = 0\ \ or\ \ n - 25 = 0$

$n = 16\ or\ n = 25$

$\therefore n = 16$ (n cannot be 25)

The number of logs in the top row=16.

14. In a bucket and ball race, a bucket is placed at the starting point, which is 5 m from the first ball, and the other balls are placed 3 m apart in a straight line. There are ten balls in the line. A competitor starts from the bucket, picks up the nearest ball, runs back with it, drops it in the bucket, runs back to pick up the next ball, runs to the bucket to drop it in, and she continues in the same way until all the balls are in the bucket. What is the total distance the competitor has to run?

Sol: The distance of first ball (from bucket)=5m

The distance of second ball=5+3=8m

The distance of third ball= 8+3=11 m

The distance of fourth ball= 11+3=14 m

……….

The distance of fourth ball= 11+3=14 m

The distance covered the competitor for 1st ,2nd ,3rd, …. Balls are

$2 \times 5m, 2 \times 8m, 2 \times 11m, \dots \dots (10\ terms)$

$10m, 16m, 22m, \dots \dots \dots . (10\ terms)$ *clearly these terms are in AP*

$a = 10, d = 6, n = 10$

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$= \frac{10}{2}[2 \times 10 + (10-1) \times 6]$$

$$= 5[20 + 54]$$

$$= 5 \times 74 = 370m$$

GEOMETRIC PROGRESSIONS

1. A Geometric Progression is a list of numbers in which each term is obtained by multiplying preceding term with a fixed number. This fixed number is called common ratio 'r'
2. The first term is $'a'$ and common ratio is $'r'$ then the G.P is $a, ar, ar^2, ar^3, \dots ..$
3. $a_1 = a, a_2 = ar, a_3 = ar^2, a_4 = ar^3, \dots.$
4. $r = \dfrac{a_2}{a_1} = \dfrac{a_3}{a_2} = ..$
5. $a_n = ar^{n-1}$

Do This

Find which of the following are not G.P.

1. 6, 12, 24, 48, …..

Sol: $a_1 = 6, a_2 = 12, a_3 = 24, a_4 = 48 \dots ..$

$$\frac{a_2}{a_1} = \frac{12}{6} = 2$$

$$\frac{a_3}{a_2} = \frac{24}{12} = 2$$

$$\frac{a_4}{a_3} = \frac{48}{24} = 2$$

$$\frac{a_2}{a_1} = \frac{a_3}{a_2} = \frac{a_4}{a_3} = \cdots.$$

∴ Given list of terms form a G.P

2. 1, 4, 9, 16, ...

Sol: $a_1 = 1, a_2 = 4, a_3 = 9, a_4 = 16 \ldots ..$

$$\frac{a_2}{a_1} = \frac{4}{1} = 4$$

$$\frac{a_3}{a_2} = \frac{9}{4} = 2.25$$

$$\frac{a_2}{a_1} \neq \frac{a_3}{a_2}$$

∴ Given list of terms not form a G.P

3. 1, –1, 1, –1, ...

Sol: $a_1 = 1, a_2 = -1, a_3 = 1, a_4 = -1 \ldots ..$

$$\frac{a_2}{a_1} = \frac{-1}{1} = -1$$

$$\frac{a_3}{a_2} = \frac{1}{-1} = -1$$

$$\frac{a_4}{a_3} = \frac{-1}{1} = -1$$

$$\frac{a_2}{a_1} = \frac{a_3}{a_2} = \frac{a_4}{a_3} = \cdots.$$

∴ Given list of terms form a G.P

4. –4, –20, –100, –500, ...

Sol: $a_1 = -4, a_2 = -20, a_3 = -100, a_4 = -500 \ldots ..$

$$\frac{a_2}{a_1} = \frac{-20}{-4} = 5$$

$$\frac{a_3}{a_2} = \frac{-100}{-20} = 5$$

$$\frac{a_4}{a_3} = \frac{-500}{-100} = 5$$

$$\frac{a_2}{a_1} = \frac{a_3}{a_2} = \frac{a_4}{a_3} = \cdots.$$

∴ Given list of terms form a G.P

Example-16. Write the GP. if the first term $a = 3$, and the common ratio r = 2.

Sol: The required G.P : $a, ar, ar^2, ar^3, \dots..$

$\Rightarrow 3, 3\times 2, 3\times 2^2, 3\times 2^3, \dots.$

$\Rightarrow 3, 6, 12, 24, \dots \dots.$

Example-17. Write GP. if $a = 256, r = \frac{-1}{2}$

Sol: The required G.P : $a, ar, ar^2, ar^3, \dots..$

$$= 256, 256\times\left(\frac{-1}{2}\right), 256\times\left(\frac{-1}{2}\right)^2, 256\times\left(\frac{-1}{2}\right)^3, \dots.$$

$$= 256, 256\times\left(\frac{-1}{2}\right), 256\times\frac{1}{4}, 256\times\left(\frac{-1}{8}\right), \dots$$

$$= 256, -128, 64, -32, \dots..$$

Example-18. Find the common ratio of the GP $25, -5, 1, \frac{-1}{5}, \dots$

$Sol:$ $r = \frac{a_2}{a_1} = \frac{-5}{25} = \frac{-1}{5}$

Example-19. Which of the following list of numbers form GP. ?

(i) 3, 6, 12, ...

$Sol: \frac{a_2}{a_1} = \frac{6}{3} = 2$

$\frac{a_3}{a_2} = \frac{12}{6} = 2$

$\frac{a_2}{a_1} = \frac{a_3}{a_2} = \cdots.$

∴ Given list of terms form a G.P

(ii) 64, –32, 16,.....

$Sol: \frac{a_2}{a_1} = \frac{-32}{64} = \frac{-1}{2}$

$\frac{a_3}{a_2} = \frac{16}{-32} = \frac{-1}{2}$

$\frac{a_2}{a_1} = \frac{a_3}{a_2} = \cdots.$

∴ Given list of terms form a G.P

(iii) $\frac{1}{64}, \frac{1}{32}, \frac{1}{8}, \dots$

$Sol: \frac{a_2}{a_1} = \frac{\frac{1}{32}}{\frac{1}{64}} = \frac{1}{32}\times\frac{64}{1} = 2$

$\frac{a_3}{a_2} = \frac{\frac{1}{8}}{\frac{1}{32}} = \frac{1}{8}\times\frac{32}{1} = 4$

$\frac{a_2}{a_1} \neq \frac{a_3}{a_2}$

∴ Given list of terms does not form G.P.

Exercise - 6.4

1. In which of the following situations, does the list of numbers involved in form a GP.?

(i) Salary of Sharmila, when her salary is ₹ 5,00,000 for the first year and expected to receive yearly increase of 10% .

Sol: First year salary of sharmila $= a_1 = ₹5,00,000$

Second year salary of Sharmila $= a_2 = ₹5,00,000 \times \frac{110}{100} = ₹5,50,000$

Third year salary of Sharmila $= a_3 = ₹5,50,000 \times \frac{110}{100} = ₹6,05,000$

Fourth year salary of Sharmila $= a_4 = ₹6,05,000 \times \frac{110}{100} = ₹6,65,500$

$$\frac{a_2}{a_1} = \frac{₹5,50,000}{₹5,00,000} = \frac{11}{10}$$

$$\frac{a_3}{a_2} = \frac{₹6,05,000}{₹5,50,000} = \frac{11}{10}$$

$$\frac{a_4}{a_3} = \frac{₹6,65,500}{₹6,05,000} = \frac{11}{10}$$

$$\frac{a_2}{a_1} = \frac{a_3}{a_2} = \frac{a_4}{a_3} = \cdots.$$

∴ Given list of terms form a G.P

(ii) Number of bricks needed to make each step, if the stair case has total 30 steps. Bottom step needs 100 bricks and each successive step needs 2 brick less than the previous step.

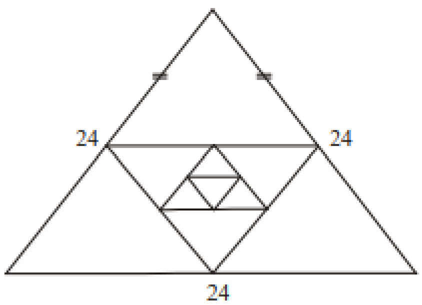
Sol: The bricks needed to make 30 steps are

$100, 100-2, 100-4, 100-6, \dots \dots$

$= 100, 98, 96, 94, \dots \dots$

Clearly this is not G.P. This is an A.P.

(iii) Perimeter of the each triangle, when the mid points of sides of an equilateral triangle whose side is 24 cm are joined to form another triangle, whose mid points in turn are joined to form still another triangle and the process continues indefinitely.



Sol: Perimeter of first triangle= $a_1 = 3 \times 24\ cm = 72\ cm$

Perimeter of second triangle= $a_2 = 3 \times 12\ cm = 36\ cm$

Perimeter of third triangle= $a_3 = 3 \times 6\ cm = 18\ cm$

Perimeter of fourth triangle= $a_4 = 3 \times 3\ cm = 9\ cm$

$$\frac{a_2}{a_1} = \frac{36}{72} = \frac{1}{2}$$

$$\frac{a_3}{a_2}=\frac{18}{36}=\frac{1}{2}$$

$$\frac{a_4}{a_3}=\frac{9}{18}=\frac{1}{2}$$

$$\frac{a_2}{a_1}=\frac{a_3}{a_2}=\frac{a_4}{a_3}=\cdots.$$

∴ Given list of terms form a G.P

2. Write three terms of the GP when the first term '*a*' and the common ratio '*r*' are given?

(i) $\boldsymbol{a = 4; r = 3}$

Sol: GP: $a, ar. ar^2, \ldots$

$\Rightarrow 4,4\times 3,4\times 3^2,\ldots$

$\Rightarrow 4,12,36, \ldots.$

(ii) $\boldsymbol{a = \sqrt{5}\,;\, r = \frac{1}{5}}$

Sol: GP: $a, ar. ar^2, \ldots$

$$\Rightarrow \sqrt{5}, \sqrt{5}\times\frac{1}{5}, \sqrt{5}\times\left(\frac{1}{5}\right)^2, \ldots..$$

$$\Rightarrow \sqrt{5}, \frac{\sqrt{5}}{5}, \frac{\sqrt{5}}{25}, \ldots$$

(iii) $\boldsymbol{a = 81; r = \frac{-1}{3}}$

Sol: GP: $a, ar. ar^2, \ldots$

$$\Rightarrow 81, 81\times\left(\frac{-1}{3}\right), 81\times\left(\frac{-1}{3}\right)^2, \ldots.$$

$$\Rightarrow 81, -\frac{81}{3}, \frac{81}{9}, ..$$

$\Rightarrow 81, -27, 9, \ldots$

(iv) $\boldsymbol{a = \frac{1}{64}\,;\, r = 2}$

Sol: GP: $a, ar. ar^2, \ldots$

$$\Rightarrow \frac{1}{64}, \frac{1}{64}\times 2, \frac{1}{64}\times 2^2, ..$$

$$\Rightarrow \frac{1}{64}, \frac{1}{32}, \frac{1}{16}, \ldots$$

3. Which of the following are GP? If they are GP. Write three more terms?

(i) 4, 8, 16 .....

$$Sol:\ \frac{a_2}{a_1}=\frac{8}{4}=2$$

$$\frac{a_3}{a_2}=\frac{16}{8}=2$$

$$\frac{a_2}{a_1} = \frac{a_3}{a_2} = \cdots.$$

$\therefore$ Given list of terms form a G.P

$a_4 = ar^3 = 4 \times 2^3 = 4 \times 8 = 32$

$a_5 = ar^4 = 4 \times 2^4 = 4 \times 16 = 64$

$a_6 = ar^5 = 4 \times 2^5 = 4 \times 32 = 128$

Next three terms are 32,64,128

Shortcut:
Next three terms are
$16 \times 2, 16 \times 2^2, 16 \times 2^3$
$= 32,64,128$

(ii) $\boldsymbol{\frac{1}{3}, \frac{-1}{6}, \frac{1}{12}, \ldots.}$

Sol: $\frac{a_2}{a_1} = \frac{\frac{-1}{6}}{\frac{1}{3}} = \frac{-1}{6} \times \frac{3}{1} = \frac{-1}{2}$

$$\frac{a_3}{a_2} = \frac{\frac{1}{12}}{\frac{-1}{6}} = \frac{1}{12} \times \frac{6}{-1} = \frac{-1}{2}$$

$$\frac{a_2}{a_1} = \frac{a_3}{a_2} = \cdots.$$

$\therefore$ *Given list of terms form a G.P where* $a = \frac{1}{3}, r = \frac{-1}{2}$

$$a_4 = ar^3 = \frac{1}{3} \times \left(\frac{-1}{2}\right)^3 = \frac{1}{3} \times \frac{-1}{8} = \frac{-1}{24}$$

$$a_5 = ar^4 = \frac{1}{3} \times \left(\frac{-1}{2}\right)^4 = \frac{1}{3} \times \frac{1}{16} = \frac{1}{48}$$

$$a_6 = ar^5 = \frac{1}{3} \times \left(\frac{-1}{2}\right)^5 = \frac{1}{3} \times \frac{-1}{32} = \frac{-1}{96}$$

Next three terms are $\frac{-1}{24}, \frac{1}{48}, \frac{-1}{96}$

(iii) 5, 55, 555, ...

Sol: $\frac{a_2}{a_1} = \frac{55}{5} = 11$

$$\frac{a_3}{a_2} = \frac{555}{55} = \frac{111}{11}$$

$$\frac{a_2}{a_1} \neq \frac{a_3}{a_2}$$

$\therefore$ Given list of terms not form a G.P

(iv) –2, –6, –18 .....

Sol: $\frac{a_2}{a_1} = \frac{-6}{-2} = 3$

$$\frac{a_3}{a_2} = \frac{-18}{-6} = 3$$

$$\frac{a_2}{a_1} = \frac{a_3}{a_2} = \cdots.$$

$\therefore$ Given list of terms form a G.P

Next three terms are $(-18) \times 3, (-18) \times 3^2, (-18) \times 3^3$

$= -54, -162, -486$

(v) $\mathbf{\frac{1}{2}, \frac{1}{4}, \frac{1}{6}, \ldots\ldots}$

$$Sol: \frac{a_2}{a_1} = \frac{\frac{1}{4}}{\frac{1}{2}} = \frac{1}{4} \times \frac{2}{1} = \frac{1}{2}$$

$$\frac{a_3}{a_2} = \frac{\frac{1}{6}}{\frac{1}{4}} = \frac{1}{6} \times \frac{4}{1} = \frac{2}{3}$$

$$\frac{a_2}{a_1} \neq \frac{a_3}{a_2}$$

$\therefore$ Given list of terms not form a G.P

(vi) $\mathbf{3, -3^2, 3^3, \ldots\ldots = 3, -9, 27, \ldots}$

$$Sol: \frac{a_2}{a_1} = \frac{-9}{3} = -3$$

$$\frac{a_3}{a_2} = \frac{27}{-9} = -3$$

$$\frac{a_2}{a_1} = \frac{a_3}{a_2} = \cdots.$$

$\therefore$ Given list of terms form a G.P

Next three terms are $-3^4, 3^5, -3^6$

(vii) $x, 1, \frac{1}{x}, \ldots\ldots$

$$Sol: \frac{a_2}{a_1} = \frac{1}{x}$$

$$\frac{a_3}{a_2} = \frac{\frac{1}{x}}{1} = \frac{1}{x}$$

$$\frac{a_2}{a_1} = \frac{a_3}{a_2} = \cdots.$$

$\therefore$ Given list of terms form a G.P

Next three terms are $\frac{1}{x^2}, \frac{1}{x^3}, \frac{1}{x^4}$

$(viii)$ $\mathbf{\frac{1}{\sqrt{2}}, -2, \frac{8}{\sqrt{2}}, \ldots}$

$$Sol: \frac{a_2}{a_1} = \frac{-2}{\frac{1}{\sqrt{2}}} = -2\sqrt{2}$$

$$\frac{a_3}{a_2} = \frac{\frac{8}{\sqrt{2}}}{-2} = \frac{8}{-2\sqrt{2}} = -2\sqrt{2}$$

$$\frac{a_2}{a_1} = \frac{a_3}{a_2} = \cdots.$$

∴ Given list of terms form a G.P

Next three terms are $-16, 32\sqrt{2}, -128$

(ix) 0.4, 0.04, 0.004, ...

$$Sol: \frac{a_2}{a_1} = \frac{0.04}{0.4} = \frac{1}{10}$$

$$\frac{a_3}{a_2} = \frac{0.004}{0.04} = \frac{1}{10}$$

$$\frac{a_2}{a_1} = \frac{a_3}{a_2} = \cdots.$$

∴ Given list of terms form a G.P

Next three terms are 0.0004,0.00004,0.000004

4. Find x so that $x, x + 2, x + 6$ are consecutive terms of a geometric progression.

Sol: $x, x + 2, x + 6$ are in GP

$$\Rightarrow \frac{a_2}{a_1} = \frac{a_3}{a_2}$$

$$\Rightarrow \frac{x + 2}{x} = \frac{x + 6}{x + 2}$$

$$\Rightarrow (x + 2)^2 = x(x + 6)$$

$$\Rightarrow x^2 + 4x + 4 - x^2 - 6x = 0$$

$$\Rightarrow -2x + 4 = 0$$

$$\Rightarrow 2x = 4 \Rightarrow x = 2$$

n^{th} *term an of a GP with first term 'a' and common ratio 'r' is given by* $a_n = ar^{n-1}$.

Example-20. Find the 20th and n th term of the GP. $\frac{5}{2}, \frac{5}{4}, \frac{5}{8}, \ldots$

$$Sol: \quad a = \frac{5}{2}, r = \frac{a_2}{a_1} = \frac{\frac{5}{4}}{\frac{5}{2}} = \frac{5}{4} \times \frac{2}{5} = \frac{1}{2}$$

$$20^{th}\ term = a_{20} = ar^{19}$$

$$= \frac{5}{2} \times \left(\frac{1}{2}\right)^{19} = \frac{5}{2^{20}}$$

$$n^{th}\ term = a_n = ar^{n-1}$$

$$= \frac{5}{2} \times \left(\frac{1}{2}\right)^{n-1} = \frac{5}{2^n}$$

Example-21. Which term of the GP : $\mathbf{2, 2\sqrt{2}, 4 \ \ldots\ldots is\ 128}$?

Sol: $a = 2, \quad r = \frac{a_2}{a_1} = \frac{2\sqrt{2}}{2} = \sqrt{2}$

$let\ a_n = 128$

$ar^{n-1} = 128$

$2 \times (\sqrt{2})^{n-1} = 128$

$(2)^{\frac{n-1}{2}} = \frac{128}{2} = 64 = 2^6$

$\frac{n-1}{2} = 6 \Rightarrow n - 1 = 12 \Rightarrow n = 13$

Hence 128 is the 13^{th} term of the GP

Example-22. In a GP the 3^{rd} term is 24 and 6^{th} term is 192. Find the 10^{th} term.

Sol: $a_3 = 24 \Rightarrow ar^2 = 24 \rightarrow (1)$

$a_6 = 192 \Rightarrow ar^5 = 192 \rightarrow (2)$

$(2) \div (1) \Rightarrow \frac{ar^5}{ar^2} = \frac{192}{24} = 8$

$\Rightarrow r^3 = 2^3$

$\Rightarrow r = 2$

Substitute r=2 in (1) we get

$a \times 2^2 = 24$

$a \times 4 = 24$

$a = \frac{24}{4} = 6$

10^{th} term$=ar^9 = 6 \times 2^9 = 6 \times 512 = 3072$

EXERCISE-6.5

1. **For each geometric progression find the common ratio 'r', and then find $\boldsymbol{a_n}$**

(i) $\mathbf{3, \frac{3}{2}, \frac{3}{4}, \frac{3}{8}, \ldots\ldots}$

Sol: $a = 3, \quad r = \frac{a_2}{a_1} = \frac{\frac{3}{2}}{3} = \frac{3}{2 \times 3} = \frac{1}{2}$

$a_n = ar^{n-1} = 3 \times \left(\frac{1}{2}\right)^{n-1}$

(ii) 2, –6, 18, –54

$$Sol: \ a = 3, \qquad r = \frac{a_2}{a_1} = \frac{-6}{2} = -3$$

$$a_n \ = \ ar^{n-1} = 3 \times (-3)^{n-1}$$

(iii) $-1, -3, -9, -27$ ....

$$Sol: \ a = 3, \qquad r = \frac{a_2}{a_1} = \frac{-3}{-1} = 3$$

$$a_n \ = \ ar^{n-1} = (-1) \times (3)^{n-1}$$

(iv) $5, 2, \frac{4}{5}, \frac{8}{25}, \ldots.$

$$Sol: \ a = 5, \qquad r = \frac{a_2}{a_1} = \frac{2}{5}$$

$$a_n \ = \ ar^{n-1} = 5 \times \left(\frac{2}{5}\right)^{n-1}$$

2. **Find the 10th and nth term of GP. : 5, 25, 125, ...**

$$\text{Sol: } a = 5, \qquad r = \frac{a_2}{a_1} = \frac{25}{5} = 5$$

$$a_{10} \ = \ ar^9 = 5 \times 5^9 = 5^{10}$$

$$a_n \ = \ ar^{n-1} = 5 \times 5^{n-1} = 5^n$$

3. **Find the indicated term of each geometric Progression**

(i) $a_1 = 9; r = \frac{1}{3}; \quad find\ a_7$

$$Sol: \quad a_7 = ar^6 = 9 \times \left(\frac{1}{3}\right)^6 = 3^2 \times \frac{1}{3^6} = \frac{1}{3^4}$$

(ii) $a_1 = -12; r = \frac{1}{3}; \quad find\ a_6$

$$Sol: \quad a_6 = ar^5 = -12 \times \left(\frac{1}{3}\right)^5 = -4 \times 3 \times \frac{1}{3^5} = \frac{-4}{3^4}$$

4. **Which term of the GP.**

(i) 2, 8, 32, ... is 512 ?

$$Sol: \ a = 2, \qquad r = \frac{a_2}{a_1} = \frac{8}{2} = 4$$

$$let \ a_n \ = \ 512$$

$$ar^{n-1} = 512$$

$$2 \times 4^{n-1} = 512$$

$$4^{n-1} = \frac{512}{2} = 256 = 4^4$$

$$n - 1 = 4 \Rightarrow n = 5$$

512 is the 5th term of given GP.

(ii) $\sqrt{3}, 3, 3\sqrt{3}, \dots\dots$ **is 729**

Sol: $a = \sqrt{3}, \quad r = \frac{a_2}{a_1} = \frac{3}{\sqrt{3}} = \frac{\sqrt{3}\times\sqrt{3}}{\sqrt{3}} = \sqrt{3}$

$let\ a_n = 729$

$ar^{n-1} = 3^7$

$\sqrt{3}\times(\sqrt{3})^{n-1} = 3^6$

$(\sqrt{3})^n = 3^6$

$3^{\frac{n}{2}} = 3^6 \Rightarrow \frac{n}{2} = 6 \Rightarrow n = 12$

729 is the 12th term of the given GP.

(iii) $\frac{1}{3}, \frac{1}{9}, \frac{1}{27} \dots\dots$ **is** $\frac{1}{2187}$?

Sol: $a = \frac{1}{3}, \quad r = \frac{a_2}{a_1} = \frac{\frac{1}{9}}{\frac{1}{3}} = \frac{1}{9}\times\frac{3}{1} = \frac{1}{3}$

$let\ a_n = \frac{1}{2187}$

$ar^{n-1} = \frac{1}{2187}$

$\frac{1}{3}\times\left(\frac{1}{3}\right)^{n-1} = \left(\frac{1}{3}\right)^7$

$\left(\frac{1}{3}\right)^n = \left(\frac{1}{3}\right)^7 \Rightarrow n = 7$

$\frac{1}{2187}$ is the 7th term of the GP

5. Find the 12th term of a GP. whose 8th term is 192 and the common ratio is 2.

Sol: common ratio $(r)= 2$

8th term = 192 $\Rightarrow ar^7 = 192$

$a\times 2^7 = 192$

$a = \frac{192}{128} = \frac{3}{2}$

12th term $= ar^{11} = \frac{3}{2}\times 2^{11} = 3\times 2^{10} = 3\times 1024 = 3072$

6. The 4th term of a geometric progression is $\frac{2}{3}$ and the 7th term is $\frac{16}{81}$. Find the geometric series.

Sol: 4th term=23$\Rightarrow ar^3 = \frac{2}{3} \quad \rightarrow (1)$

7th term = 16 81$\Rightarrow ar^6 = \frac{16}{81} \rightarrow (2)$

$$(2) \div (1) \Rightarrow \frac{ar^6}{ar^3} = \frac{\frac{16}{81}}{\frac{2}{3}} = \frac{16}{81} \times \frac{3}{2} = \frac{8}{27}$$

$$r^3 = \left(\frac{2}{3}\right)^3 \Rightarrow r = \frac{2}{3}$$

substitute $r = \frac{2}{3}\ in\ (1)$

$$ar^3 = \frac{2}{3} \Rightarrow a \times \left(\frac{2}{3}\right)^3 = \frac{2}{3}$$

$$\Rightarrow a = \frac{2}{3} \times \frac{27}{8} = \frac{9}{4}$$

GP is $a, ar, ar^2, ar^3, \ldots ..$

$$\Rightarrow \frac{9}{4}, \quad \frac{9}{4} \times \frac{2}{3}, \quad \frac{9}{4} \times \left(\frac{2}{3}\right)^2, \quad \frac{9}{4} \times \left(\frac{2}{3}\right)^3, \ldots$$

$$\Rightarrow \frac{9}{4}, \frac{3}{2}, 1, \frac{2}{3}, ..$$

7. If the geometric progressions 162, 54, 18 …. . and $\frac{2}{81}, \frac{2}{27}, \frac{2}{9}, \ldots$ have their n th term equal. Find the value of n.

Sol: 1st GP: 162, 54, 18 … ..

$$a = 162, \qquad r = \frac{a_2}{a_1} = \frac{54}{162} = \frac{1}{3}$$

$$a_n = ar^{n-1} = 162 \times \left(\frac{1}{3}\right)^{n-1}$$

2^{nd} GP: $\frac{2}{81}, \frac{2}{27}, \frac{2}{9}, \ldots$

$$a = \frac{2}{81}, \qquad r = \frac{a_2}{a_1} = \frac{\frac{2}{27}}{\frac{2}{81}} = \frac{2}{27} \times \frac{81}{2} = 3$$

$$a_n = ar^{n-1} = \frac{2}{81} \times (3)^{n-1}$$

Given their nth terms are equal

$$\Rightarrow \frac{2}{81} \times (3)^{n-1} = 162 \times \left(\frac{1}{3}\right)^{n-1}$$

$$\Rightarrow (3)^{n-1} \times (3)^{n-1} = 162 \times \frac{81}{2} = 81 \times 81 = 3^8$$

$$\Rightarrow (3)^{2n-2} = 3^8$$

$$\Rightarrow 2n - 2 = 8$$

$$\Rightarrow 2n = 8 + 2 = 10$$

$\Rightarrow n = 5$ $\quad\therefore$ The 5th terms of given two GPs are equal.