CHAPTER 5

X CLASS-2023-24
QUADRATIC EQUATIONS (NOTES)

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1. Standard form of quadratic equation in variable x is $\boldsymbol{ax^2 + bx + c = 0}$, where a, b, c are real numbers and $a \neq 0$.
2. $y = ax^2 + bx + c$ is called a quadratic function.
3. Uses of Quadratic functions.
   (i) When the rocket is fired upward, then the path of the rocket is defined by a 'quadratic function.'
   (ii) Shapes of the satellite dish, reflecting mirror in a telescope, lens of the eye glasses and orbits of the celestial objects are defined by the quadratic equations.
   (iii) The path of a projectile is defined by quadratic function.
   (iv) When the breaks are applied to a vehicle, the stopping distance is calculated by using quadratic equation
4. Roots of quadratic equation: A real number α is called a root of the quadratic equation $ax^2 + bx + c = 0$, if $a\alpha^2 + b\alpha + c = 0$
5. Quadratic formula: The roots of a quadratic equation $ax^2 + bx + c = 0$ are given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

6. Nature of roots:

   The nature of roots of a quadratic equation $ax^2 + bx + c = 0$ depends on $b^2 - 4ac$ is called the **discriminant**
   (i) $If\ \boldsymbol{b^2 - 4ac > 0}\ then\ the\ roots\ are\ real\ and\ distinct$
   (ii) $If\ \boldsymbol{b^2 - 4ac = 0}\ then\ the\ roots\ are\ real\ and\ equal$
   (iii) $If\ \boldsymbol{b^2 - 4ac < 0}\ then\ the\ roots\ are\ not\ real$

TRY THIS

Check whether the following equations are quadratic or not ?

(i) $x^2 - 6x - 4 = 0$

Sol: Quadratic equation

(ii) $x^3 - 6x^2 + 2x - 1 = 0$

Sol: Not a quadratic equation(Cubic equation)

(iii) $7x = 2x^2$

Sol: $2x^2 - 7x = 0$

Quadratic equation

(iv) $x^2 + \frac{1}{x^2} = 2$

Sol: $x^2 + \frac{1}{x^2} = 2$

$\Rightarrow \dfrac{x^4 + 1}{x^2} = 2$

$\Rightarrow x^4 + 1 = 2x^2$

$\Rightarrow x^4 - 2x^2 + 1 = 0$ Not a quadratic equation

$v)$ $(2x + 1)(3x + 1) = b(x - 1)(x - 2)$

Sol: $(2x + 1)(3x + 1) = b(x - 1)(x - 2)$

$2x(3x + 1) + 1(3x + 1) = b[x(x - 2) - 1(x - 2)]$

$6x^2 + 2x + 3x + 1 = b[x^2 - 2x - x + 2]$

$6x^2 + 5x + 1 = b[x^2 - 3x + 2]$

$6x^2 + 5x + 1 = bx^2 - 3bx + 2b$

$6x^2 - bx^2 + 5x + 3bx + 1 - 2b = 0$

$(6 - b)x^2 + (5 + 3b)x + 1 - 2b = 0$

This is a quadratic equation.

(vi) $3y^2 = 192$

Sol: $3y^2 - 192 = 0$

This is a quadratic equation

Example-1.

i. Raju and Rajendar together have 45 marbles. Both of them lost 5 marbles each, and the product of the number of marbles now they have is 124. We would like to find out how many marbles they had previously. Represent the situation mathematically.

Sol: Total marbles=45

Let the number of marbles at Raju $= x$

Then the number of marbles at Rajendar $= 45 - x$

If both of them lost 5 marbles each then

The number of marbles at Raju $= x - 5$

The number of marbles at Rajendar $= 45 - x - 5 = 40 - x$

The product of remaining marbles=124

$\Rightarrow (x - 5)(40 - x) = 124$

$\Rightarrow 40x - x^2 - 200 + 5x = 124$

$\Rightarrow -x^2 + 45x - 200 - 124 = 0$

$\Rightarrow -x^2 + 45x - 324 = 0$

$\Rightarrow x^2 - 45x + 324 = 0$ (multiply with -1)

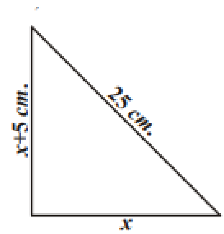
ii. The hypotenuse of a right triangle is 25 cm. We know that the difference in lengths of the other two sides is 5 cm. We would like to find out the length of the two sides?

Sol: Let the length of smaller side $= x\ cm$

The length of larger side $= (x+5)cm$

Length of hypotenuse $= 25\ cm$

In a right angle triangle we know that



$$(side)^2 + (side)^2 = (hypotenuse)^2$$

$$(x)^2 + (x+5)^2 = (25)^2$$

$$x^2 + x^2 + 2 \times x \times 5 + 5^2 = 625$$

$$2x^2 + 10x + 25 - 625 = 0$$

$$2x^2 + 10x + -600 = 0$$

$$x^2 + 5x - 300 = 0$$

Required quadratic equation: $x^2 + 5x - 300$

Example-2. Check whether the following are quadratic equations:

i. $\boldsymbol{(x-2)^2 + 1 = 2x - 3}$

Sol: $(x-2)^2 + 1 = 2x - 3$

$\Rightarrow x^2 - 4x + 4 + 1 - 2x + 3 = 0$

$\Rightarrow x^2 - 6x + 8 = 0$

The given equation is a quadratic equation.

ii. $\boldsymbol{x(x+1) + 8 = (x+2)(x-2)}$

Sol: $x(x+1) + 8 = (x+2)(x-2)$

$\Rightarrow x^2 + x + 8 = x^2 - 2^2$

$\Rightarrow x^2 + x + 8 - x^2 + 4 = 0$

$\Rightarrow x + 12 = 0$

The given equation is not a quadratic equation.

iii. $\boldsymbol{x(2x+3) = x^2 + 1}$

Sol: $x(2x+3) = x^2 + 1$

$\Rightarrow 2x^2 + 3x - x^2 - 1 = 0$

$\Rightarrow x^2 + 3x - 1 = 0$

The given equation is a quadratic equation.

iv. $\boldsymbol{(x+2)^3 = x^3 - 4}$

Sol: $(x+2)^3 = x^3 - 4$

$\Rightarrow x^3 + 2^3 + 3 \times x \times 2(x+2) = x^3 - 4$

$(a+b)^3 = a^3 + b^3 + 3ab(a+b)$

$\Rightarrow x^3 + 8 + 6x(x+2) = x^3 - 4$

$\Rightarrow x^3 + 8 + 6x^2 + 12x - x^3 + 4 = 0$

$\Rightarrow 6x^2 + 12\text{x} + 12 = 0$

The given equation is a quadratic equation.

EXERCISE - 5.1

1. **Check whether the following are quadratic equations**:

i. $\boldsymbol{(x+1)^2 = 2(x-3)}$

Sol: $(x+1)^2 = 2(x-3)$

$\Rightarrow x^2 + 2x + 1 = 2x - 6$

$\Rightarrow x^2 + 2x + 1 - 2x + 6 = 0$

$\Rightarrow x^2 + 7 = 0$

The given equation is a quadratic equation.

ii. $\boldsymbol{x^2 - 2x = (-2)(3-x)}$

Sol: $x^2 - 2x = (-2)(3-x)$

$\Rightarrow x^2 - 2x = -6 + 2x$

$\Rightarrow x^2 - 2x + 6 - 2x = 0$

$\Rightarrow x^2 - 4x + 6 = 0$

The given equation is a quadratic equation.

iii. $\boldsymbol{(x-2)(x+1) = (x-1)(x+3)}$

Sol: $(x-2)(x+1) = (x-1)(x+3)$

$\Rightarrow x^2 + x - 2x - 2 = x^2 + 3x - x - 3$

$\Rightarrow x^2 - x - 2 = x^2 + 2x - 3$

$\Rightarrow x^2 - x - 2 - x^2 - 2x + 3 = 0$

$\Rightarrow -3x + 1 = 0$

The given equation is not a quadratic equation.

iv. $\boldsymbol{(x-3)(2x+1) = x(x+5)}$

Sol: $(x-3)(2x+1) = x(x+5)$

$\Rightarrow 2x^2 + x - 6x - 3 = x^2 + 5x$

$\Rightarrow 2x^2 - 5x - 3 - x^2 - 5x = 0$

$\Rightarrow x^2 - 10x - 3 = 0$

The given equation is a quadratic equation.

v. $\boldsymbol{(2x-1)(x-3) = (x+5)(x-1)}$

Sol: $(2x-1)(x-3) = (x+5)(x-1)$

$\Rightarrow 2x^2 - 6x - x + 3 = x^2 - x + 5x - 5$

$\Rightarrow 2x^2 - 7x + 3 = x^2 + 4x - 5$

$\Rightarrow 2x^2 - 7x + 3 - x^2 - 4x + 5 = 0$

$\Rightarrow x^2 - 11x + 8 = 0$

The given equation is a quadratic equation.

vi. $\boldsymbol{x^2 + 3x + 1 = (x - 2)^2}$

Sol: $x^2 + 3x + 1 = (x - 2)^2$

$\Rightarrow x^2 + 3x + 1 = x^2 - 4x + 4$

$\Rightarrow x^2 + 3x + 1 - x^2 + 4x - 4 = 0$

$\Rightarrow 7x - 3 = 0$

The given equation is not a quadratic equation.

vii. $\boldsymbol{(x + 2)^3 = 2x(x^2 - 1)}$

Sol: $(x + 2)^3 = 2x(x^2 - 1)$

$\Rightarrow x^3 + 2^3 + 3 \times x \times 2(x + 2) = 2x^3 - 2x$

$\Rightarrow x^3 + 8 + 6x(x + 2) = 2x^3 - 2x$

$\Rightarrow x^3 + 8 + 6x^2 + 12x - 2x^3 + 2x = 0$

$\Rightarrow -x^3 + 6x^2 + 14x + 8 = 0$

The given equation is not a quadratic equation.

viii. $\boldsymbol{x^3 - 4x^2 - x + 1 = (x - 2)^3}$

Sol: $x^3 - 4x^2 - x + 1 = (x - 2)^3$

$\Rightarrow x^3 - 4x^2 - x + 1 = x^3 - 2^3 - 3 \times x \times 2(x - 2)$

$\Rightarrow x^3 - 4x^2 - x + 1 = x^3 - 8 - 6x^2 + 12x$

$\Rightarrow x^3 - 4x^2 - x + 1 - x^3 + 8 + 6x^2 - 12x = 0$

$\Rightarrow 2x^2 - 13x + 9 = 0$

The given equation is a quadratic equation.

2. **i. The area of a rectangular plot is 528 m² . The length of the plot (in metres) is one more than twice its breadth. We need to find the length and breadth of the plot.**

Sol: Let breadth of rectangular plot $(b) = x\ m$

Length of rectangular plot $(l) = (2x + 1)m$

Given area of the rectangular plot $= 528\ m^2$

$$l \times b = 528$$

$$(2x + 1) \times x = 528$$

$$2x^2 + x - 528 = 0$$

This is the required quadratic equation.

ii. The product of two consecutive positive integers is 306. We need to find the integers.

Sol: Let the two consecutive positive integers be $x, x + 1$

Given the product of two consecutive positive integers $= 306$

$x \times (x+1) = 306 \Longrightarrow x^2 + x - 306 = 0$

This is the required quadratic equation

iii. Rohan's mother is 26 years older than him. The product of their ages after 3 years will be 360 years. We need to find Rohan's present age.

Sol: Let Rohan's age=x years

Rohan's mother age=$(x+26)years$

After 3 years

Rohan's age=$x+3$ years

Rohan's mother age=$(x+26+3) = (x+29)years$

Given the product of their ages after 3 years=360 years

$\Rightarrow (x+3)(x+29) = 360$

$\Rightarrow x^2 + 29x + 3x + 87 - 360 = 0$

$\Rightarrow x^2 + 32x - 273 = 0$

This is the required quadratic equation.

iv. A train travels a distance of 480 km at a uniform speed. If the speed had been 8 km/h less, then it would have taken 3 hours more to cover the same distance. We need to find the speed of the train.

Sol: Let the speed of the train= $x\ km/h$

Distance= 480 km

$$Time(T_1) = \frac{Distance}{Speed} = \frac{480}{x} h$$

If the speed had been 8 km/h less then speed=$(x-8)\ km/h$

$$Time(T_2) = \frac{Distance}{Speed} = \frac{480}{x-8} h$$

Difference of times$(T_2 - T_1)$ =3h

$$\frac{480}{x-8} - \frac{480}{x} = 3$$

$$480\left(\frac{1}{x-8} - \frac{1}{x}\right) = 3$$

$$\frac{x-(x-8)}{x(x-8)} = \frac{3}{480}$$

$$\frac{x-x+8}{x^2-8x} = \frac{1}{160}$$

$$\frac{8}{x^2-8x} = \frac{1}{160}$$

$x^2 - 8x = 160 \times 8$

$x^2 - 8x = 1280$

$x^2 - 8x - 1280 = 0$

This is the required quadratic equation to find the speed of the train.

Solution of a Quadratic Equation by Factorisation

(i) A real number α is called a root of the quadratic equation $ax^2 + bx + c = 0$, if $a\alpha^2 + b\,\alpha + c = 0$ .We also say that x = α is a solution of the quadratic equation.

Example-3. Find the roots of the equation $2x^2 - 5x + 3 = 0$, by factorisation.

Sol: $2x^2 - 5x + 3 = 0$

$2x^2 - 2x - 3x + 3 = 0$

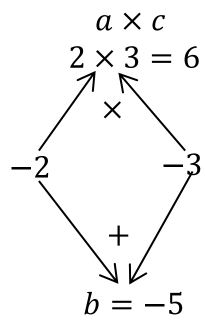
$2x(x-1) - 3(x-1) = 0$

$(x-1)(2x-3) = 0$

$x - 1 = 0 \;\; or \;\; 2x - 3 = 0$

$x = 1 \;\; or \;\; x = \frac{3}{2}$

$\therefore 1 \; and \; \frac{3}{2} \; are \; the \; roots \; of \; the \; equation \;\; 2x^2 - 5x + 3 = 0$



Try This

Verify that 1 and $\frac{3}{2}$ are the roots of the equation $2x^2 - 5x + 3 = 0$.

Sol: Let $P(x) = 2x^2 - 5x + 3$

$P(1) = 2(1)^2 - 5(1) + 3$

$= 2 - 5 + 3$

$= 5 - 5 = 0$

$P\left(\frac{3}{2}\right) = 2\left(\frac{3}{2}\right)^2 - 5\left(\frac{3}{2}\right) + 3$

$= 2 \times \frac{9}{4} - 5 \times \frac{3}{2} + 3$

$= \frac{9}{2} - \frac{15}{2} + 3 = \frac{-6}{2} + 3 = -3 + 3 = 0$

$P(1) = 0 \;\; and \; P\left(\frac{3}{2}\right) = 0$.

$So \; 1 \; and \; \frac{3}{2} \; are \; the \; roots \; of \; the \; Q.E \; 2x^2 - 5x + 3 = 0$

Example 4 : Find the roots of the quadratic equation $x - \frac{1}{3x} = \frac{1}{6}$.

$Sol: x - \frac{1}{3x} = \frac{1}{6}$

$\Rightarrow \frac{3x^2 - 1}{3x} = \frac{1}{6}$

$\Rightarrow 6(3x^2 - 1) = 3x \times 1$

$\Rightarrow 18x^2 - 3x - 6 = 0$

$\Rightarrow 6x^2 - x - 2 = 0$

$\Rightarrow 6x^2 - 4x + 3x - 2 = 0$

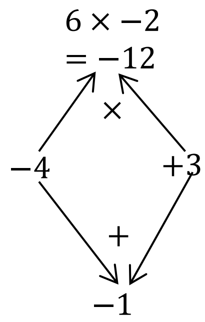
$\Rightarrow 2x(3x - 2) + 1(3x - 2) = 0$

$\Rightarrow (3x - 2)(2x + 1) = 0$

$\Rightarrow 3x - 2 = 0 \quad or \quad 2x + 1 = 0$

$3x = 2 \quad or \quad 2x = -1$

$x = \frac{2}{3} \quad or \quad x = \frac{-1}{2}$



$\therefore$ The roots of $x - \frac{1}{3x} = \frac{1}{6}$ are $\frac{2}{3}$ and $\frac{-1}{2}$

EXERCISE - 5.2

1. Find the roots of the following quadratic equations by factorisation.

(i) $x^2 - 3x - 10 = 0$

Sol: $x^2 - 3x - 10 = 0$

$x^2 - 2x + 5x - 10 = 0$

$x(x - 2) + 5(x - 2) = 0$

$(x - 2)(x + 5) = 0$

$x - 2 = 0 \quad or \quad x + 5 = 0$

$x = 2 \quad or \quad x = -5$

The roots of $x^2 - 3x - 10 = 0$ are 2 and -5

(ii) $2x^2 + x - 6 = 0$

Sol: $2x^2 + x - 6 = 0$

$2x^2 - 3x + 4x - 6 = 0$

$x(2x - 3) + 2(2x - 3) = 0$

$(2x - 3)(x + 2) = 0$

$2x - 3 = 0 \quad or \quad x + 2 = 0$

$x = \frac{3}{2} \quad or \quad x = -2$

The roots of $2x^2 + x - 6 = 0$ are $\frac{3}{2}$ and -2

(iii) $\sqrt{2}x^2 + 7x + 5\sqrt{2} = 0$

Sol: $\sqrt{2}x^2 + 7x + 5\sqrt{2} = 0$

$\sqrt{2} \times 5\sqrt{2} = 5 \times 2 = 10$
$2 \times 5 = 10$
$2 + 5 = 7$

$\sqrt{2}x^2 + 2x + 5x + 5\sqrt{2} = 0$

$\sqrt{2}x(x + \sqrt{2}) + 5(x + \sqrt{2}) = 0$

$(x + \sqrt{2})(\sqrt{2}x + 5) = 0$

$x + \sqrt{2} = 0 \quad or \quad \sqrt{2}x + 5 = 0$

$x = -\sqrt{2} \quad or \quad x = \frac{-5}{\sqrt{2}}$

The roots of $\sqrt{2}x^2 + 7x + 5\sqrt{2} = 0$ *are* $-\sqrt{2}$ *and* $\frac{-5}{\sqrt{2}}$

(iv) $\mathbf{2x^2 - x + \frac{1}{8} = 0}$

Sol: $2x^2 - x + \frac{1}{8} = 0$

Multiply with '8'

$8 \times 2x^2 - 8 \times x + 8 \times \frac{1}{8} = 8 \times 0$

$16x^2 - 8x + 1 = 0$

$16x^2 - 4x - 4x + 1 = 0$

$4x(4x - 1) - 1(4x - 1) = 0$

$(4x - 1)(4x - 1) = 0$

$4x - 1 = 0 \quad or \quad 4x - 1 = 0$

$x = \frac{1}{4} \quad or \quad x = \frac{1}{4}$

The roots of $2x^2 - x + \frac{1}{8} = 0$ *are* $\frac{1}{4}$ *and* $\frac{1}{4}$.

(v) $\mathbf{100x^2 - 20x + 1 = 0}$

Sol: $100x^2 - 20x + 1 = 0$

$100x^2 - 10x - 10x + 1 = 0$

$10x(10x - 1) - 1(10x - 1) = 0$

$(10x - 1)(10x - 1) = 0$

$10x - 1 = 0 \quad or \quad 10x - 1 = 0$

$x = \frac{1}{10} \quad or \quad x = \frac{1}{10}$

The roots of $100x^2 - 20x + 1 = 0$ *are* $\frac{1}{10}$ *and* $\frac{1}{10}$

(vi) $\mathbf{x(x + 4) = 12}$

Sol: $x(x + 4) = 12$

$x^2 + 4x - 12 = 0$

$x^2 - 2x + 6x - 12 = 0$

$x(x - 2) + 6(x - 2) = 0$

$(x - 2)(x + 6) = 0$

$x - 2 = 0 \quad or \quad x + 6 = 0$

$x = 2 \quad or \quad x = -6$

The roots of $x(x + 4) = 12$ *are* 2 *and* -6.

(vii) $\mathbf{3x^2 - 5x + 2 = 0}$

Sol: $3x^2 - 5x + 2 = 0$

$3x^2 - 3x - 2x + 2 = 0$

$3x(x-1) - 2(x-1) = 0$

$(x-1)(3x-2) = 0$

$x - 1 = 0 \quad or \quad 3x - 2 = 0$

$x = 1 \quad or \quad x = \frac{2}{3}$

$The\ roots\ of\ 3x^2 - 5x + 2 = 0\ are\ 1\ and\ \frac{2}{3}.$

(viii) $\mathbf{x - \frac{3}{x} = 2}$

Sol: $x - \frac{3}{x} = 2$

$\frac{x^2 - 3}{x} = 2$

$x^2 - 3 = 2x$

$x^2 - 2x - 3 = 0$

$x^2 - 3x + 1x - 3 = 0$

$x(x-3) + 1(x-3) = 0$

$(x-3)(x+1) = 0$

$x - 3 = 0 \quad or \quad x + 1 = 0$

$x = 3 \quad or \quad x = -1$

The roots of $x - \frac{3}{x} = 2\ are\ \ 3\ and - 1$

(ix) $\mathbf{3(x-4)^2 - 5(x-4) = 12}$

Sol: $3(x-4)^2 - 5(x-4) = 12$

$3(x^2 - 8x + 16) - 5x + 20 - 12 = 0$

$3x^2 - 24x + 48 - 5x + 8 = 0$

$3x^2 - 29x + 56 = 0$

$3x^2 - 21x - 8x + 56 = 0$

$3x(x-7) - 8(x-7) = 0$

$(x-7)(3x-8) = 0$

$x - 7 = 0 \quad or \quad 3x - 8 = 0$

$x = 7 \quad or \quad x = \frac{8}{3}$

The roots of $3(x-4)^2 - 5(x-4) = 12\ \ are\ 7\ and\ \frac{8}{3}.$

2. Find two numbers whose sum is 27 and product is 182.

Sol: Let one number $= x$, The second number$= 27 - x$

Product of numbers=182

$x(27 - x) = 182$

$27x - x^2 = 182$

$-x^2 + 27x - 182 = 0$

$x^2 - 27x + 182 = 0$

$x^2 - 13x - 14x + 182 = 0$

$x(x - 13) - 14(x - 13) = 0$

$(x - 13)(x - 14) = 0$

$x - 13 = 0 \quad or \quad x - 14 = 0$

$x = 13 \quad or \quad x = 14$

If $x = 13$ the required numbers are 13 and 14.

If $x = 14$ the required numbers are 14 and 13.

3. Find two consecutive positive integers, sum of whose squares is 613.

Sol: Let the two consecutive positive integers be $x, x + 1$.

Sum of whose squares $= 613$

$x^2 + (x + 1)^2 = 613$

$x^2 + x^2 + 2x + 1 - 613 = 0$

$2x^2 + 2x - 612 = 0$

$x^2 + x - 306 = 0$

$x^2 - 17x + 18x - 306 = 0$

$x(x - 17) + 18(x - 17) = 0$

$(x - 17)(x + 18) = 0$

$x = 17 \quad or \quad x = -18$

$\therefore x = 17$ ($since\ x\ is\ a\ positive\ integer\ so\ x \neq -18$)

The required two consecutive positive integers are 17 , 18.

4. **The altitude of a right triangle is 7 cm less than its base. If the hypotenuse is 13 cm, find the other two sides.**

Sol: Base of a right triangle (AB)$=x$

The altitude $(BC) = x - 7$ cm

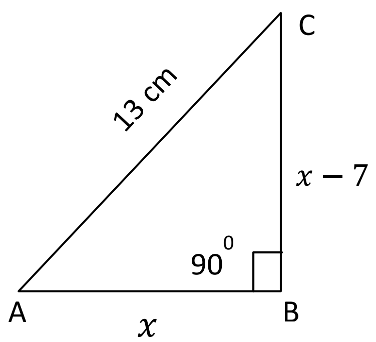
The hypotenuse (AC)= 13 cm

From Pythagoras theorem

$AB^2 + BC^2 = AC^2$

$x^2 + (x - 7)^2 = 13^2$

$x^2 + x^2 - 14x + 49 - 169 = 0$



$2x^2 - 14x - 120 = 0$

$x^2 - 7x - 60 = 0$

$x^2 - 12x + 5x - 60 = 0$

$x(x - 12) + 5(x - 12) = 0$

$(x - 12)(x + 5) = 0$

$x - 12 = 0 \quad or \quad x + 5 = 0$

$x = 12 \quad or \quad x = -5$

$\therefore x = 12 \quad (since\ side\ of\ a\ triangle\ is\ positive\ iteger\ so\ x \neq -5)$

The other two sides are 12 cm, (12-7) cm i.e 12 cm , 5 cm.

5. **A cottage industry produces a certain number of pottery articles in a day. It was observed on a particular day that the cost of production of each article (in rupees) was 3 more than twice the number of articles produced on that day. If the total cost of production on that day was Rs 90, find the number of articles produced and the cost of each article.**

Sol: Let the number of articles produced=x

The cost of each article=$Rs\ (2x + 3)$

Given the total cost of production on that day=Rs 90

$x(2x + 3) = 90$

$2x^2 + 3x - 90 = 0$

$2x^2 - 12x + 15x - 90 = 0$

$2x(x - 6) + 15(x - 6) = 0$

$(x - 6)(2x + 15) = 0$

$x - 6 = 0 \quad or \quad 2x + 15 = 0$

$x = 6 \quad or \quad x = \frac{-15}{2}$

$\therefore \quad x = 6 \quad (\ Number\ of\ articles\ is\ always\ can'tbe\ negative)$

The number of articles produced= x =6

The cost of each article=$Rs\ (2x + 3)$=$Rs\ (2 \times 6 + 3) = Rs$ 15.

6. **Find the dimensions of a rectangle whose perimeter is 28 meters and whose area is 40 square meters.**

Sol: Let the length of the rectangle=l , breadth =b

Perimeter of the rectangle=28 m

$2(l + b) = 28$

$\Rightarrow l + b = \frac{28}{2} \Rightarrow l + b = 14 \Rightarrow b = 14 - l$

Area of the square=40 square meters.

$\Rightarrow l \times b = 40$

$\Rightarrow l(14-l)=40$

$\Rightarrow 14l-l^2-40=0$

$\Rightarrow -l^2+14l-40=0$

$\Rightarrow l^2-14l+40=0$

$\Rightarrow l^2-10l-4l+40=0$

$\Rightarrow l(l-10)-4(l-10)=0$

$\Rightarrow (l-10)(l-4)=0$

$\Rightarrow l-10=0 \quad or \quad l-4=0$

$\Rightarrow l=10 \quad or \quad l=4$

$If\ l=10\ m\ then\ b=14-10=4\ m$

$If\ l=4\ m\ then\ b=14-4=10\ m$

The dimensions of the rectangle are 10 m and 4 m.

7. **The base of a triangle is 4cm longer than its altitude. If the area of the triangle is 48 sq.cm then find its base and altitude.**

Sol: Let altitude (h)=x

The base of a triangle(b)=$(x+4)$

The area of the triangle = 48 sq.cm

$\frac{1}{2}\times b\times h=48$

$(x+4)\times x=48\times 2$

$x^2+4x-96=0$

$x^2-8x+12x-96=0$

$x(x-8)+12(x-8)=0$

$(x-8)(x+12)=0$

$x-8=0 \quad or \quad x+12=0$

$x=8 \quad or \quad x=-12$

$\therefore x=8$ (Altitude can'tbe negative)

Altitude of the triangle=$x=8\ cm$

Base of the triangle=$x+4=8+4=12\ cm$.

8. **Two trains leave a railway station at the same time. The first train travels towards west and the second train towards north. The first train travels 5 km/hr faster than the second train. If after two hours they are 50 km. apart find the average speed of each train.**

Sol: Let the speed of second train=$x\ km/hr$

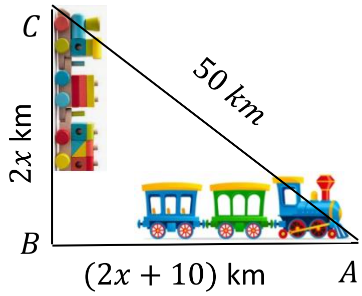
The speed of first train=$(x+5)km/hr$

Time (t)=2 hr

Distance travelled by second train=$s\times t=2\times x=2x\ km$

Distance travelled by first train$= s \times t = 2 \times (x+5) = (2x+10)km$

By Pythagoras theorem



$AB^2 + BC^2 = AC^2$

$(2x+10)^2 + (2x)^2 = 50^2$

$4x^2 + 40x + 100 + 4x^2 = 2500$

$8x^2 + 40x + 100 - 2500 = 0$

$8x^2 + 40x - 2400 = 0$

$x^2 + 5x - 300 = 0$ (Dividing with 8)

$x^2 - 15x + 20x - 300 = 0$

$x(x-15) + 20(x-15) = 0$

$(x-15)(x+20) = 0$

$x - 15 = 0 \quad or \quad x + 20 = 0$

$x = 15 \quad or \quad x = -20$

$\therefore x = 15$ ($speed\ of\ the\ train\ is\ can't negative$)

The average speed of second train =15 km/hr.

The average speed of first train =(15+5) =20 km/hr

9. **In a class of 60 students, each boy contributed rupees equal to the number of girls and each girl contributed rupees equal to the number of boys. If the total money then collected was ₹1600. How many boys are there in the class?**

Sol: Total number of students=60

Let the number of boys$=x$

The number of girls$=60-x$

Money contributed by the boys$=x \times (60-x) = 60x - x^2$

Money contributed by the boys$=(60-x) \times x = 60x - x^2$

Total money collected=₹1600

$60x - x^2 + 60x - x^2 = 1600$

$-2x^2 + 120x - 1600 = 0$

$x^2 - 60x + 800 = 0$($dividing\ by\ '-2'$)

$x^2 - 40x - 20x + 800 = 0$

$x(x-40) - 20(x-40) = 0$

$(x-40)(x-20) = 0$

$x - 40 = 0 \quad or \quad x - 20 = 0$

$x = 40 \quad or \quad x = 20$

$\therefore$ The number of boys in the class=40 or 20.

10. A motor boat heads upstream a distance of 24km on a river whose current is running at 3 km per hour. The trip up and back takes 6 hours. Assuming that the motor boat maintained a constant speed, what was its speed?

Sol: Let the speed of the boat in water=$x\ km/hr$

The speed of the river=3 km/hr

The distance of river=24 km

The speed of the boat in upstream= $(x-3)\ km/hr$

$$Time\ taken\ to\ upstream\ (T_1) = \frac{Distance}{Speed} = \frac{24}{x-3}\ hr$$

The speed of the boat in downstream= $(x+3)\ km/hr$

$$Time\ taken\ to\ downstream\ (T_2) = \frac{Distance}{Speed} = \frac{24}{x+3}\ hr$$

Total time taken=6 hr

$$\frac{24}{x-3} + \frac{24}{x+3} = 6$$

$$24\left(\frac{1}{x-3} + \frac{1}{x+3}\right) = 6$$

$$\frac{1}{x-3} + \frac{1}{x+3} = \frac{6}{24}$$

$$\frac{x+3+x-3}{(x-3)(x+3)} = \frac{1}{4}$$

$$\frac{2x}{x^2-9} = \frac{1}{4}$$

$$x^2 - 9 = 4 \times 2x$$

$$x^2 - 8x - 9 = 0$$

$$x^2 - 9x + 1x - 9 = 0$$

$$x(x-9) + 1(x-9) = 0$$

$$(x-9)(x+1) = 0$$

$$x - 9 = 0 \quad or \quad x + 1 = 0$$

$$x = 9 \quad or \quad x = -1$$

$$\therefore x = 9\ (\ speed\ of\ the\ boat\ is\ can't\ negative)$$

The speed of the boat in water= 9 km/hr.

Solution of a Quadratic Equation by Completing the Square

Example: Find the roots of the equation $3x^2 - 5x + 2 = 0$ by the method of completing the square.

Sol: Given $3x^2 - 5x + 2 = 0$

Diving both sides by 3

$$\frac{3x^2}{3} - \frac{5x}{3} + \frac{2}{3} = 0$$

$$x^2 - \frac{5x}{3} = -\frac{2}{3}$$

$$x^2 - 2 \times x \times \frac{5}{6} = -\frac{2}{3}$$

Adding $\left(\frac{5}{6}\right)^2$ to both sides

$$x^2 - 2 \times x \times \frac{5}{6} + \left(\frac{5}{6}\right)^2 = -\frac{2}{3} + \left(\frac{5}{6}\right)^2$$

$$\left(x - \frac{5}{6}\right)^2 = -\frac{2}{3} + \frac{25}{36}$$

$$\left(x - \frac{5}{6}\right)^2 = \frac{(12 \times -2) + 25}{36}$$

$$\left(x - \frac{5}{6}\right)^2 = \frac{-24 + 25}{36}$$

$$\left(x - \frac{5}{6}\right)^2 = \frac{1}{36}$$

$$x - \frac{5}{6} = \pm\sqrt{\frac{1}{36}}$$

$$x - \frac{5}{6} = \pm\frac{1}{6}$$

$$x = \frac{5}{6} + \frac{1}{6} \quad or \quad \frac{5}{6} - \frac{1}{6}$$

$$x = \frac{6}{6} \ \ or \ \ \frac{4}{6}$$

$$x = \ 1 \ \ or \ \frac{2}{3}$$

The roots of given equation are $1 \ \ and \ \ \frac{2}{3}$

Example-6. Find the roots of the equation $5x^2 - 6x - 2 = 0$ by the method of completing the square.

Sol: Given $5x^2 - 6x - 2 = 0$

Diving both sides by 5

$$\frac{5x^2}{5} - \frac{6x}{5} - \frac{2}{5} = 0$$

$$x^2 - \frac{6x}{5} = \frac{2}{5}$$

$$x^2 - 2 \times x \times \frac{3}{5} = \frac{2}{5}$$

Adding $\left(\frac{3}{5}\right)^2$ to both sides

$$x^2 - 2 \times x \times \frac{3}{5} + \left(\frac{3}{5}\right)^2 = \frac{2}{5} + \left(\frac{3}{5}\right)^2$$

$$\left(x - \frac{3}{5}\right)^2 = \frac{2}{5} + \frac{9}{25}$$

$$\left(x - \frac{3}{5}\right)^2 = \frac{(2 \times 5) + 9}{25}$$

$$\left(x - \frac{3}{5}\right)^2 = \frac{10 + 9}{25}$$

$$\left(x - \frac{3}{5}\right)^2 = \frac{19}{25}$$

$$\left(x - \frac{3}{5}\right)^2 = \pm\sqrt{\frac{19}{25}}$$

$$x - \frac{3}{5} = \pm\frac{\sqrt{19}}{5}$$

$$x = \frac{3}{5} + \frac{\sqrt{19}}{5} \quad or \quad \frac{3}{5} - \frac{\sqrt{19}}{5}$$

$$x = \frac{3 + \sqrt{19}}{5} \, or \, \frac{3 - \sqrt{19}}{5}$$

The roots of given equation are $\frac{3+\sqrt{19}}{5}$ or $\frac{3-\sqrt{19}}{5}$

Example-7. Find the roots of $4x^2 + 3x + 5 = 0$ by the method of completing the square.

Sol: Given $4x^2 + 3x + 5 = 0$

Diving both sides by 4

$$\frac{4x^2}{4} + \frac{3x}{4} + \frac{5}{4} = 0$$

$$x^2 + \frac{3x}{4} = -\frac{5}{4}$$

$$x^2 + 2 \times x \times \frac{3}{8} = -\frac{5}{4}$$

Adding $\left(\frac{3}{8}\right)^2$ to both sides

$$x^2 + 2 \times x \times \frac{3}{8} + \left(\frac{3}{8}\right)^2 = -\frac{5}{4} + \left(\frac{3}{8}\right)^2$$

$$\left(x + \frac{3}{8}\right)^2 = -\frac{5}{4} + \frac{9}{64}$$

$$\left(x - \frac{3}{8}\right)^2 = \frac{(-5 \times 16) + 9}{64}$$

$$\left(x-\frac{3}{8}\right)^2 = \frac{-80+9}{64}$$

$$\left(x-\frac{3}{8}\right)^2 = \frac{-71}{64}$$

$x-\frac{3}{8} = \sqrt{\frac{-71}{64}}$ *it is not a real number.*

Therefore, the given equation has no real roots.

Do This

Solve the equations by completing the square.

(i) $\boldsymbol{x^2 - 10x + 9 = 0}$

Sol: $x^2 - 10x + 9 = 0$

$x^2 - 10x = -9$

$x^2 - 2 \times x \times 5 = -9$

Adding 5^2 to both sides

$x^2 - 2 \times x \times 5 + 5^2 = -9 + 5^2$

$(x-5)^2 = -9 + 25$

$(x-5)^2 = 16$

$x - 5 = \sqrt{16}$

$x - 5 = \pm 4$

$x = 5 + 4 \quad or \quad x = 5 - 4$

$x = 9 \;\; or \;\; x = 1$

The roots of given equation are 9 and 1.

(ii) $\boldsymbol{x^2 - 5x + 5 = 0}$

Sol: $x^2 - 5x + 5 = 0$

$x^2 - 5x = -5$

$$x^2 - 2 \times x \times \frac{5}{2} = -5$$

Adding $\left(\frac{5}{2}\right)^2$ to both sides

$$x^2 - 2 \times x \times \frac{5}{2} + \left(\frac{5}{2}\right)^2 = -5 + \left(\frac{5}{2}\right)^2$$

$$\left(x-\frac{5}{2}\right)^2 = -5 + \frac{25}{4}$$

$$\left(x-\frac{5}{2}\right)^2 = \frac{-20+25}{4} = \frac{5}{4}$$

$$x - \frac{5}{2} = \sqrt{\frac{5}{4}}$$

$$x-\frac{5}{2}=\pm\frac{\sqrt{5}}{2}$$

$$x=\frac{5}{2}+\frac{\sqrt{5}}{2} \quad or \quad x=\frac{5}{2}-\frac{\sqrt{5}}{2}$$

$$x=\frac{5+\sqrt{5}}{2} \quad or \quad x=\frac{5-\sqrt{5}}{2}$$

The roots of given equation are $\frac{5+\sqrt{5}}{2}$ and $\frac{5-\sqrt{5}}{2}$.

(iii) $x^2+7x-6=0$

Sol: $x^2+7x-6=0$

$$x^2+7x=6$$

$$x^2+2\times x\times\frac{7}{2}=6$$

Adding $\left(\frac{7}{2}\right)^2$ to both sides

$$x^2+2\times x\times\frac{7}{2}+\left(\frac{7}{2}\right)^2=6+\left(\frac{7}{2}\right)^2$$

$$\left(x+\frac{7}{2}\right)^2=6+\frac{49}{4}$$

$$\left(x+\frac{7}{2}\right)^2=\frac{24+49}{4}=\frac{73}{4}$$

$$x+\frac{7}{2}=\sqrt{\frac{73}{4}}$$

$$x+\frac{7}{2}=\pm\frac{\sqrt{73}}{2}$$

$$x=-\frac{7}{2}+\frac{\sqrt{73}}{2} \quad or \quad x=-\frac{7}{2}-\frac{\sqrt{73}}{2}$$

$$x=\frac{-7+\sqrt{73}}{2} \quad or \quad x=\frac{-7-\sqrt{73}}{2}$$

The roots of given equation are $\frac{-7+\sqrt{73}}{2}$ and $\frac{-7-\sqrt{73}}{2}$.

Quadratic formula (formula for finding the roots of a quadratic equation)

If $b^2-4ac\geq$ then the roots of a quadratic equation $ax^2+bx+c=0$ are given by

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

Example-8: The area of a rectangular plot is 528 m² . The length of the plot (in metres) is one more than twice its breadth. Find the length and breadth of the plot.

Sol: Let breadth of rectangular plot $(b)=x\ m$

Length of rectangular plot $(l)=(2x+1)m$

Given area of the rectangular plot $= 528\, m^2$

$$l \times b = 528$$

$$(2x + 1) \times x = 528$$

$2x^2 + x - 528 = 0 \quad a = 2, b = 1, c = -528$

$b^2 - 4ac = 1^2 - 4 \times 2 \times (-528) = 1 + 4224 = 4225 > 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-1 \pm \sqrt{4225}}{2 \times 2}$$

$$x = \frac{-1 \pm 65}{4}$$

$$x = \frac{-1 + 65}{4} \quad or \quad \frac{-1 - 65}{4}$$

$$x = \frac{64}{4} \quad or \quad \frac{-66}{4}$$

$$x = 16 \quad or \quad \frac{-33}{2}$$

$\therefore x = 16$ (Breath of plot can't be negative)

So, the breadth of the plot$= x = 16$ m.

The length of the plot $= (2x + 1) = 2 \times 16 + 1 = 32 + 1 =$33m.

Example-9. Find two consecutive odd positive integers, sum of whose squares is 290.

Sol: Let first odd positive integer $= x$

Then, the second odd positive integer$= x + 2$

Sum of whose squares $= 290$

$x^2 + (x + 2)^2 = 290$

$x^2 + x^2 + 4x + 4 - 290 = 0$

$2x^2 + 4x - 286 = 0$

$x^2 + 2x - 143 = 0 \quad a = 1, b = 2, c = -143$

$b^2 - 4ac = 2^2 - 4 \times 1 \times (-143) = 4 + 572 = 576 > 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-2 \pm \sqrt{576}}{2 \times 1}$$

$$x = \frac{-2 \pm 24}{2}$$

$$x = \frac{-2 + 24}{2} \quad or \quad \frac{-2 - 24}{2}$$

$$x = \frac{22}{2} \quad or \quad \frac{-26}{2}$$

$x = 11 \quad or \quad -13$

$\therefore x = 11$ ($given\ x\ is\ positive\ integer\ x \neq -13$)

Thus, the two consecutive odd integers are 11 and 11+2$\Rightarrow$ 11 and 13

Example-10. A rectangular park is to be designed whose breadth is 3 m less than its length. Its area is to be 4 square metres more than the area of a park that has already been made in the shape of an isosceles triangle with its base as the breadth of the rectangular park and of altitude 12 m. Find its length and breadth.

Sol: Let the length of the rectangular park=$x\ m$

So, its breadth =$(x-3)m$

Area of the rectangle=$x(x-3) = (x^2 - 3x)\ m^2$

Now, base of the isosceles triangle= $(x-3)m$

Altitude(height) =12 m

Area of isosceles triangle=$\frac{1}{2} \times base \times height$

$= \frac{1}{2} \times (x-3) \times 12 = (6x - 18)m^2$

Given area of rectangle=Area of triangle+4

$x^2 - 3x = 6x - 18 + 4$

$x^2 - 3x - 6x + 18 - 4 = 0$

$x^2 - 9x + 14 = 0$

$x^2 - 7x - 2x + 14 = 0$

$(x-7)(x-2) = 0$

$x = \quad 7\ or\ 2$

$\therefore x = 7 \quad (since\ x > 3)$

So, the $length\ of\ the\ park = 7\ m \quad and\ \ breadth = 7 - 3 = 4m.$

Example-11. Find the roots of the following quadratic equations, if they exist, using the quadratic formula:

(i) $\boldsymbol{x^2 + 4x + 5 = 0}$

Sol: $a = 1, b = 4, c = 5$

$b^2 - 4ac = 4^2 - 4 \times 1 \times 5 = 16 - 20 = -4 < 0$

So, there are no real roots for the given equation.

(ii) $\boldsymbol{2x^2 - 2\sqrt{2}x + 1 = 0}$

Sol: $a = 2, b = -2\sqrt{2}, c = 1$

$b^2 - 4ac = \left(-2\sqrt{2}\right)^2 - 4 \times 2 \times 1 = 8 - 8 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{2\sqrt{2} \pm \sqrt{0}}{2 \times 2} = \frac{2\sqrt{2} \pm 0}{4}$$

$$= \frac{2\sqrt{2} + 0}{4} \quad or \quad \frac{2\sqrt{2} - 0}{4} = \frac{2\sqrt{2}}{4} \quad or \quad \frac{2\sqrt{2}}{4}$$

$$x = \frac{\sqrt{2}}{2} \ or \ \frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}} \quad or \quad \frac{1}{\sqrt{2}}$$

So, the roots are $\frac{1}{\sqrt{2}}$, $\frac{1}{\sqrt{2}}$

Example-12. Find the roots of the following equations:

(i) $\boldsymbol{x + \frac{1}{x} = 3 \ (x \neq 0)}$

Sol: $x + \dfrac{1}{x} = 3$

$$\frac{x^2 + 1}{x} = 3$$

$$x^2 + 1 = 3x$$

$$x^2 - 3x + 1 = 0 \quad a = 1, b = -3, c = 1$$

$$b^2 - 4ac = (-3)^2 - 4 \times 1 \times 1 = 9 - 4 = 5 > 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{3 \pm \sqrt{5}}{2 \times 1} = \frac{3 \pm \sqrt{5}}{2}$$

$$x = \frac{3 + \sqrt{5}}{2} \quad or \quad \frac{3 - \sqrt{5}}{2}$$

The roots of the given equation are $\frac{3+\sqrt{5}}{2}$ and $\frac{3-\sqrt{5}}{2}$.

$\boldsymbol{(ii)}$ $\boldsymbol{\dfrac{1}{x} - \dfrac{1}{x-2} = 3 \ (x \neq 0, 2)}$

Sol: $\dfrac{1}{x} - \dfrac{1}{x-2} = 3$

$$\frac{x - 2 - x}{x(x - 2)} = 3$$

$$-2 = 3x(x - 2)$$

$$3x^2 - 6x + 2 = 0 : \ a = 3, b = -6, c = 2$$

$$b^2 - 4ac = (-6)^2 - 4 \times 3 \times 2 = 36 - 24 = 12 > 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{6 \pm \sqrt{12}}{2 \times 3} = \frac{6 \pm 2\sqrt{3}}{6} = \frac{2\left(3 \pm \sqrt{3}\right)}{6} = \frac{3 \pm \sqrt{3}}{3}$$

$$x = \frac{3 + \sqrt{3}}{3} \quad or \quad \frac{3 - \sqrt{3}}{3}$$

The roots of the given equation are $\frac{3+\sqrt{3}}{3}$ *and* $\frac{3-\sqrt{3}}{3}$.

Example-13. A motor boat whose speed is 18 km/h in still water. It takes 1 hour more to go 24 km upstream than to return downstream to the same spot. Find the speed of the stream.

Sol: : Let the speed of the stream=$x\ km/h$

Given speed of the boat=18 km/h

Relative speed of the boat in upstream= $(18-x)\ km/h$

$$Time\ taken\ to\ go\ upstream = \frac{distance}{speed} = \frac{24}{18-x}\ h$$

Relative speed of the boat in downstream= $(18+x)\ km/h$

$$Time\ taken\ to\ go\ downstream = \frac{distance}{speed} = \frac{24}{18+x}\ h$$

According to the problem

$$\frac{24}{18-x} - \frac{24}{18+x} = 1$$

$$24\left(\frac{18+x-18+x}{(18-x)18+x}\right) = 1$$

$$\frac{2x}{324-x^2} = \frac{1}{24}$$

$$2x \times 24 = 324 - x^2$$

$$x^2 + 48x - 324 = 0\ ;\ \ a = 1, b = 48, c = -324:$$

$$b^2 - 4ac = (48)^2 - 4 \times 1 \times (-324) = 2304 + 1296 = 3600 > 0$$

$$x = \frac{-48 \pm \sqrt{3600}}{2 \times 1} = \frac{-48 \pm 60}{2}$$

$$x = \frac{-48+60}{2}\ \ or\ \ \frac{-48-60}{2}$$

$$x = \frac{12}{2}\ \ or\ \ \frac{-108}{2}$$

$$x = 6\ \ or\ \ -54$$

$$\therefore\ \ x = 6\ (\ speed\ of\ the\ streem\ can'tbe\ negative)$$

The speed of the stream = 6 km/h.

Exercise - 5.3

1. **Find the roots of the following quadratic equations, if they exist, by the method of completing the square:**

(i) $\mathbf{2x^2 + x - 4 = 0}$

Sol: $2x^2 + x - 4 = 0$

Dividing both sides by 2

$$\frac{2x^2}{2} + \frac{x}{2} - \frac{4}{2} = 0$$

$$x^2 + \frac{x}{2} = 2$$

$$x^2 + 2 \times x \times \frac{1}{4} = 2$$

Adding $\left(\frac{1}{4}\right)^2$ to both sides

$$x^2 + 2 \times x\frac{1}{4} + \left(\frac{1}{4}\right)^2 = 2 + \left(\frac{1}{4}\right)^2$$

$$\left(x + \frac{1}{4}\right)^2 = 2 + \frac{1}{16}$$

$$\left(x + \frac{1}{4}\right)^2 = \frac{33}{4}$$

$$x + \frac{1}{4} = \pm\sqrt{\frac{33}{4}}$$

$$x + \frac{1}{4} = \pm\frac{\sqrt{33}}{4}$$

$$x = -\frac{1}{4} + \frac{\sqrt{33}}{4} \quad or \quad x = -\frac{1}{4} - \frac{\sqrt{33}}{4}$$

$$x = \frac{-1 + \sqrt{33}}{4} \quad or \quad x = \frac{-1 - \sqrt{33}}{4}$$

The roots of $2x^2 + x - 4 = 0$ are $\frac{-1+\sqrt{33}}{4}$ and $\frac{-1-\sqrt{33}}{4}$

(ii) $\mathbf{4x^2 + 4\sqrt{3}x + 3 = 0}$

Sol: $4x^2 + 4\sqrt{3}x + 3 = 0$

Dividing both sides by 4

$$\frac{4x^2}{4} + \frac{4\sqrt{3}x}{4} + \frac{3}{4} = 0$$

$$x^2 + \sqrt{3}x = -\frac{3}{4}$$

$$x^2 + 2 \times x \times \frac{\sqrt{3}}{2} = -\frac{3}{4}$$

Adding $\left(\frac{\sqrt{3}}{2}\right)^2$ to both sides

$$x^2 + 2 \times x \times \frac{\sqrt{3}}{2} + \left(\frac{\sqrt{3}}{2}\right)^2 = -\frac{3}{4} + \left(\frac{\sqrt{3}}{2}\right)^2$$

$$\left(x + \frac{\sqrt{3}}{2}\right)^2 = -\frac{3}{4} + \frac{3}{4}$$

$$\left(x + \frac{\sqrt{3}}{2}\right)^2 = 0$$

$$x+\frac{\sqrt{3}}{2}=0 \Rightarrow x=-\frac{\sqrt{3}}{2}$$

The roots of $4x^2+4\sqrt{3}x+3=0$ *are* $-\frac{\sqrt{3}}{2},-\frac{\sqrt{3}}{2}$

(iii) $\mathbf{5x^2-7x-6=0}$

Sol: $5x^2-7x-6=0$

Dividing both sides by 5

$$\frac{5x^2}{5}-\frac{7x}{5}-\frac{6}{5}=0$$

$$x^2-\frac{7x}{5}=\frac{6}{5}$$

$$x^2-2\times x\times\frac{7}{10}=\frac{6}{5}$$

Adding $\left(\frac{7}{10}\right)^2$ to both sides

$$x^2-2\times x\times\frac{7}{10}+\left(\frac{7}{10}\right)^2=\frac{6}{5}+\left(\frac{7}{10}\right)^2$$

$$\left(x-\frac{7}{10}\right)^2=\frac{6}{5}+\frac{49}{100}$$

$$\left(x-\frac{7}{10}\right)^2=\frac{120+49}{4}=\frac{169}{100}$$

$$x-\frac{7}{10}=\pm\sqrt{\frac{169}{100}}$$

$$x-\frac{7}{10}=\pm\frac{13}{10}$$

$$x=\frac{7}{10}+\frac{13}{10} \quad or \quad x=\frac{7}{10}-\frac{13}{10}$$

$$x=\frac{7+13}{10} \quad or \quad x=\frac{7-13}{10}$$

$$x=\frac{20}{10}=2 \ \ or \ \ x=\frac{-6}{10}=-\frac{3}{5}$$

The roots of $5x^2-7x-6=0$ *are* 2 *and* $-\frac{3}{5}$.

(iv) $x^2+5=-6x$

Sol: $x^2-6x+5=0$

$$x^2-6x=-5$$

$$x^2-2\times x\times 3=-5$$

Adding $(3)^2$ to both sides

$$x^2-2\times x\times 3+(3)^2=-5+(3)^2$$

$$(x-3)^2=-5+9=4$$

$x - 3 = \sqrt{4}$

$x - 3 = \pm 2$

$x = 3 \pm 2$

$x = 3 + 2 \;\; or \;\; x = 3 - 2$

$x = 5 \;\; or \;\; x = 1$

$The\ roots\ of\ x^2 + 5 = -6x\ \ are\ \ 5\ and\ 1.$

2. Find the roots of the quadratic equations by applying the quadratic formula.

(i) $2x^2 + x - 4 = 0$

Sol: $a = 2, b = 1, c = -4$

$b^2 - 4ac = (1)^2 - 4 \times 2 \times (-4) = 1 + 32 = 33 > 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-1 \pm \sqrt{33}}{2 \times 2} = \frac{-1 \pm \sqrt{33}}{4}$$

$$x = \frac{-1 + \sqrt{33}}{4} \;\; or \;\; \frac{-1 - \sqrt{33}}{4}$$

The roots of the given equation are $\frac{-1+\sqrt{33}}{4}$ and $\frac{-1-\sqrt{33}}{4}$.

(ii) $\mathbf{4x^2 + 4\sqrt{3}x + 3 = 0}$

Sol: $a = 4, b = 4\sqrt{3}, c = 3$

$b^2 - 4ac = \left(4\sqrt{3}\right)^2 - 4 \times 4 \times 3 = 48 - 48 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-4\sqrt{3} \pm \sqrt{0}}{2 \times 4}$$

$$x = \frac{-4\sqrt{3}}{2 \times 4}, \frac{-4\sqrt{3}}{2 \times 4}$$

$$x = -\frac{\sqrt{3}}{2} \;,\;\; -\frac{\sqrt{3}}{2}$$

(iii) $\mathbf{5x^2 - 7x - 6 = 0}$

Sol: $a = 5, b = -7, c = -6$

$b^2 - 4ac = (-7)^2 - 4 \times 5 \times (-6) = 49 + 120 = 169 > 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{7 \pm \sqrt{169}}{2 \times 5} = \frac{7 \pm 13}{10}$$

$$x = \frac{7 + 13}{10} \;\; or \;\; \frac{7 - 13}{10}$$

$$x = \frac{20}{10} \ or \ \frac{-6}{10}$$

$$x = 2 \ or \ -\frac{3}{5}$$

(iv) $\boldsymbol{x^2 + 5 = -6x}$

Sol: $x^2 - 6x + 5 = 0$

$a = 1, b = -6, c = 5$

$b^2 - 4ac = (-6)^2 - 4 \times 1 \times 5 = 36 - 20 = 16 > 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-6 \pm \sqrt{16}}{2 \times 1} = \frac{-6 \pm 4}{2}$$

$$x = \frac{-6 + 4}{2} \ or \ \frac{-6 - 4}{2}$$

$$x = \frac{-2}{2} \ or \ \frac{-10}{2}$$

$x = -1 \ or \ -5$

3. Find the roots of the following equations:

(i) $x - \frac{1}{x} = 3 \, , \ x \neq 0$

Sol: $x - \frac{1}{x} = 3$

$$\frac{x^2 - 1}{x} = 3$$

$x^2 - 1 = 3x$

$x^2 - 3x - 1 = 0 \quad a = 1, b = -3, c = -1$

$b^2 - 4ac = (-3)^2 - 4 \times 1 \times (-1) = 9 + 4 = 13 > 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{3 \pm \sqrt{13}}{2 \times 1} = \frac{3 \pm \sqrt{13}}{2}$$

$$x = \frac{3 + \sqrt{13}}{2} \ or \ \frac{3 - \sqrt{13}}{2}$$

The roots of the given equation are $\frac{3+\sqrt{13}}{2}$ and $\frac{3-\sqrt{13}}{2}$.

$(\boldsymbol{ii})$ $\boldsymbol{\frac{1}{x+4} - \frac{1}{x-7} = \frac{11}{30}}$, $\boldsymbol{x \neq -4, 7}$

Sol: $\frac{1}{x+4} - \frac{1}{x-7} = \frac{11}{30}$

$$\frac{x-7-x-4}{(x+4)(x-7)}=\frac{11}{30}$$

$$\frac{-11}{x^2-3x-28}=\frac{11}{30}$$

$$x^2-3x-28=-11\times\frac{30}{11}$$

$$x^2-3x-28=-30$$

$$x^2-3x-28+30=0$$

$$x^2-3x+2=0$$

$$(x-1)(x-2)=0$$

$$x-1=0 \;\; or \;\; x-2=0$$

$$x=1 \;\; or \;\; x=2$$

4. **The sum of the reciprocals of Rehman's ages, (in years) 3 years ago and 5 years from now is $\frac{1}{3}$. Find his present age.**

Sol: Let Rehman's present age=x *years*

3 years ago Rehman's age=$(x-3)$ years

$$Reciprocal\ of\ it=\frac{1}{x-3}$$

5 years from now Rehman's age=$(x+5)$ years

$$Reciprocal\ of\ it=\frac{1}{x+5}$$

$$The\ sum\ of\ the\ reciprocals=\frac{1}{3}$$

$$\frac{1}{x-3}+\frac{1}{x+5}=\frac{1}{3}$$

$$\frac{x+5+x-3}{(x-3)(x+5)}=\frac{1}{3}$$

$$\frac{2x+2}{x^2+2x-15}=\frac{1}{3}$$

$$1\times(x^2+2x-15)=3\times(2x+2)$$

$$x^2+2x-15=6x+6$$

$$x^2+2x-15-6x-6=0$$

$$x^2-4x-21=0$$

$$(x-7)(x+3)=0$$

$-7\times3=-21$
$-7+3=-4$

$$x=7 \;\; or \;\; x=-3$$

$$\therefore x=7 \;\; (\,age\ can't\ be\ negative)$$

Present age of Rehman=7 years

5. **In a class test, the sum of Moulika's marks in Mathematics and English is 30. If she got 2 marks more in Mathematics and 3 marks less in English, the product of her marks would have been 210. Find her marks in the two subjects.**

Sol: The sum of Moulika's marks in Mathematics and English $= 30$

Let Moulika's marks in Mathematics$=x$

Moulika's marks in English$=30 - x$

If she got 2 marks more in Mathematics then her marks$=x + 2$

If she got 3 marks less in English then her marks$=30 - x - 3 = 27 - x$

Product of these marks$=210$

$(x + 2)(27 - x) = 210$

$27x - x^2 + 54 - 2x - 210 = 0$

$-x^2 + 25x - 156 = 0$

$x^2 - 25x + 156 = 0$

$(x - 12)(x - 13) = 0$

$-12 \times -13 = 156$
$-12 - 13 = -25$

$x = 12 \;\; or \;\; x = 13$

If $x = 12$ then marks in Mathematics$=12$,

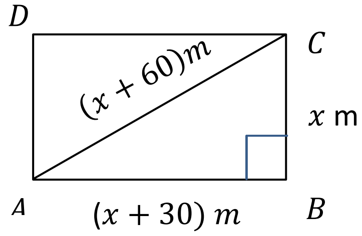
Marks in English$=30 - 12 = 18$

If $x = 13$ then marks in Mathematics$=13$,

Marks in English$=30 - 13 = 17$

6. **The diagonal of a rectangular field is 60 metres more than the shorter side. If the longer side is 30 metres more than the shorter side, find the sides of the field.**

Sol: Let the shorter side of a rectangular field $=x\ m$



Longer side$=(x + 30)\ m$

The diagonal$=(x + 60)\ m$

By Pythagoras theorem

$(x + 30)^2 + x^2 = (x + 60)^2$

$x^2 + 60x + 900 + x^2 = x^2 + 120x + 3600$

$x^2 + 60x + 900 + x^2 - x^2 - 120x - 3600 = 0$

$x^2 - 60x - 2700 = 0$

$-90 \times 30 = -2700$
$-90 + 30 = -60$

$(x - 90)(x + 30) = 0$

$x - 90 = 0 \;\; or \;\; x + 30 = 0$

$x = 90 \;\; or \;\; x = -30$

$\therefore x = 90 \;\; (side\ of\ a\ rectangle\ can't\ be\ negative)$

Length$=x + 30 = 90 + 30 = 120\ m$

Breadth$=x = 90\ m.$

7. **The difference of squares of two numbers is 180. The square of the smaller number is 8 times the larger number. Find the two numbers.**

Sol: Let the larger number=x

$(\text{smallara number})^2 = 8x$

Given $(\text{larger number})^2 - (\text{smallara number})^2 = 180$

$x^2 - 8x = 180$

$x^2 - 8x - 180 = 0$

$-18 \times 10 = -180$ $-18 + 10 = -8$

$(x - 18)(x + 10) = 0$

$x - 18 = 0 \quad or \quad x + 10 = 0$

$x = 18 \quad or \quad x = -10$

$\therefore x = 18$ (x can't be negative)

The larger number=$x = 18$

smallara number $= \sqrt{8x} = \sqrt{8 \times 18} = \sqrt{144} = 12.$

8. **A train travels 360 km at a uniform speed. If the speed had been 5 km/h more, it would have taken 1 hour less for the same journey. Find the speed of the train.**

Sol: Let the speed of the train=$x\ km/h$

Distance=360 km

$$Time = \frac{Distance}{speed} = \frac{360}{x}\ h$$

If the speed had been 5 km/h more then speed of the train=$(x + 5)km/h$

$$Time = \frac{Distance}{speed} = \frac{360}{x+5}\ h$$

Difference of times=1h

$$\frac{360}{x} - \frac{360}{x+5} = 1$$

$$360\left(\frac{1}{x} - \frac{1}{x+5}\right) = 1$$

$$\frac{x + 5 - x}{x(x+5)} = \frac{1}{360}$$

$x(x + 5) = 5 \times 360$

$x^2 + 5x - 1800 = 0$

$-40 \times 45 = -1800$ $-40 + 45 = 5$

$(x - 40)(x + 45) = 0$

$x - 40 = 0 \quad or \quad x + 45 = 0$

$x = 40 \quad or \quad x = -45$

$\therefore x = 40$ $(speed\ can't\ be\ negative)$

The speed of the train=40 km/h.

9. **Two water taps together can fill a tank in $9\frac{3}{8}$ hours. The tap of larger diameter takes 10 hours less than the smaller one to fill the tank separately. Find the time in which each tap can separately fill the tank.**

Sol: Time taken by the tap of smaller diameter to fill the tank=$x\ h$

Time taken by the tap of larger diameter to fill the tank=$(x-10)\ h$

Tank filled by the smaller diameter tap in 1 hour=$\frac{1}{x}$

Tank filled by the larger diameter tap in 1 hour=$\frac{1}{x-10}$

Time taken by both taps to fill the tank=$9\frac{3}{8}\ h=\frac{75}{8}h$

Total work done=1

$$\frac{75}{8}\left(\frac{1}{x}+\frac{1}{x-10}\right)=1$$

$$\frac{x-10+x}{x(x-10)}=\frac{8}{75}$$

$$\frac{2x-10}{x^2-10x}=\frac{8}{75}$$

$$8\times(x^2-10x)=75(2x-10)$$

$$8x^2-80x-150x+750=0$$

$$8x^2-230x+750=0$$

$$4x^2-115x+375=0$$

$$4x^2-100x-15x+375=0$$

$$4x(x-25)-15(x-25)=0$$

$$(x-25)(4x-15)=0$$

$$x-25=0\ \ or\ \ 4x-15=0$$

$$x=25\ \ or\ \ x=\frac{15}{4}=4\frac{1}{4}$$

$$\therefore x=25\ (\ since\ x>10)$$

$$4\times375=1500$$
$$(-100)(-15)=1500$$
$$-100-15=-115$$

Time taken by the tap of smaller diameter to fill the tank=$x=25h$

Time taken by the tap of larger diameter to fill the tank=$(x-10)=(25-10)=15\ h$

10. **An express train takes 1 hour less than a passenger train to travel 132 km between Mysore and Bangalore (without taking into consideration the time they stop at intermediate stations). If the average speed of the express train is 11km/h more than that of the passenger train, find the average speed of the two trains.**

Sol: Let the average speed of the passenger train=$x\ km/h$

Then the average speed of the express train= $(x+11)\ km/h$

Distance travelled=132 km

$$Time\ taken\ to\ passenger\ train = \frac{Distance}{speed} = \frac{132}{x}\ h$$

$$Time\ taken\ to\ exprees\ train = \frac{132}{x+11}\ h$$

Difference of times = 1h

$$\frac{132}{x} - \frac{132}{x+11} = 1$$

$$132\left(\frac{1}{x} - \frac{1}{x+11}\right) = 1$$

$$\frac{x+11-x}{x(x+11)} = \frac{1}{132}$$

$$\frac{11}{x^2+11x} = \frac{1}{132}$$

$$x^2 + 11x = 11 \times 132$$

$$x^2 + 11x - 1452 = 0$$

$-33 \times 44 = -1452$
$-33 + 44 = 11$

$$(x-33)(x+44) = 0$$

$$x - 33 = 0\ \ or\ \ x + 44 = 0$$

$$x = 33\ \ or\ \ x = -44$$

$$\therefore x = 33\ (\ speed\ can't\ be\ negative)$$

The average speed of passenger train=33 km/h

The average speed of express train=33+11=44 km/h.

11. **Sum of the areas of two squares is 468 m² . If the difference of their perimeters is 24 m, find the sides of the two squares.**

Sol: Let the side of the smaller square= $x\ m$

Perimeter of the smaller square=$4 \times side = 4x\ m$

Perimeter of the larger square=$(4x + 24) = 4(x+6)m$

$$Side\ of\ the\ larger\ square = \frac{Perimter}{4} = \frac{4(x+6)}{4} = (x+6)m$$

Given Sum of the areas of two squares is= 468 m²

$$x^2 + (x+6)^2 = 468$$

$$x^2 + x^2 + 12x + 36 - 468 = 0$$

$$2x^2 + 12x - 432 = 0$$

$$x^2 + 6x - 216 = 0$$

$-12 \times 18 = -432$
$-12 + 18 = 6$

$$(x-12)(x+18) = 0$$

$$x = 12\ \ or\ \ x = -18$$

$$\therefore x = 12\ \ (\ side\ can't\ be\ negative)$$

Sides of two squares are 12m ,12+6=18m.

12. If a polygon of 'n' sides has $\frac{1}{2}n(n-3)$diagonals. How many sides will a polygon having 65 diagonals? Is there a polygon with 50 diagonals?

$Sol: \frac{1}{2}n(n-3) = 65$

$n^2 - 3n = 2 \times 65$

$n^2 - 3n - 130 = 0$

$-13 \times 10 = -130$ $-13 + 10 = -3$

$(n-13)(n+10) = 0$

$n - 13 = 0 \quad or \quad n + 10 = 0$

$n = 13 \quad or \quad n = -10$

$\therefore n = 13$ ($sides\ of\ a\ polygon\ can't\ be\ negative$)

The number of sides of the polygon with 65 diagonals=13.

$Let \ \frac{1}{2}n(n-3) = 50$

$n^2 - 3n - 100 = 0$

$b^2 - 4ac = (-3)^2 - 4 \times 1 \times (-100) = 9 + 400 = 409$ it is not a perfect square.

There is no real value for n.

$\therefore$ There can't be a polygon with 50 diagonals.

Nature of Roots

The nature of roots of a quadratic equation $ax^2 + bx + c = 0$ depends on $\boldsymbol{b^2 - 4ac}$ is called the **discriminant(D or** Δ)of the Q.E.

(i) If $b^2 - 4ac > 0$ then the roots are distinct and real.

(ii) If $b^2 - 4ac = 0$ then the roots are equal and real.

(iii) If $b^2 - 4ac < 0$ then no real roots.

Example-14. Find the discriminant of the quadratic equation $2x^2 - 4x + 3 = 0$, and hence find the nature of its roots.

Sol: Given Q.E is $2x^2 - 4x + 3 = 0 \quad a = 2, b = -4, c = 3$

$b^2 - 4ac = (-4)^2 - 4 \times 2 \times 3 = 16 - 24 = -8 < 0$

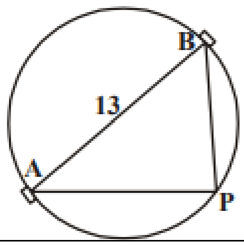
So, the given equation has no real roots.

Example-15. A pole has to be erected at a point on the boundary of a circular park of diameter 13 metres in such a way that the differences of its distances from two diametrically opposite fixed gates A and B on the boundary is 7 metres. Is it possible to do so? If yes, at what distances from the two gates should the pole be erected?

Sol: Let P be the required location of the pole.

Let BP= $x\ m$ then AP= $(x+7)m$ and AB= 13 m

We know that angle in semicircle=90^0. So $\angle APB = 90^0$



$AP^2 + BP^2 = AB^2$ (By Pythagoras theorem)

$(x+7)^2 + x^2 = 13^2$

$x^2 + x^2 + 14x + 49 - 169 = 0$

$2x^2 + 14x - 120 = 0$

$x^2 + 7x - 60 = 0$

$-5 \times 12 = -60$ $-5 + 12 = 7$

$(x-5)(x+12) = 0$

$x - 5 = 0$ or $x + 12 = 0$

$x = 5$ or $x = -12$

$\therefore x = 5$ (*distance can't be negative*)

Thus, the pole has to be erected on the boundary of the park at a distance of 5m from the gate B and 12m from the gate A.

Example-16. Find the discriminant of the equation $3x^2 - 2x + \frac{1}{3} = 0$ and hence find the nature of its roots. Find them, if they are real.

Sol: Given Q.E is $3x^2 - 2x + \frac{1}{3} = 0$: $a = 3, b = -2, c = \frac{1}{3}$

$b^2 - 4ac = (-2)^2 - 4 \times 3 \times \frac{1}{3} = 4 - 4 = 0$

So, the roots are equal and real.

The roots are $\frac{-b}{2a}, \frac{-b}{2a} \Rightarrow \frac{2}{2\times 3}, \frac{2}{2\times 3} \Rightarrow \frac{1}{3}, \frac{1}{3}$.

EXERCISE - 5.4

1. **Find the nature of the roots of the following quadratic equations. If real roots exist, find them:**

(i) $2x^2 - 3x + 5 = 0$

Sol: $a = 2, b = -3, c = 5$

$b^2 - 4ac = (-3)^2 - 4 \times 2 \times 5 = 9 - 40 = -31 < 0$

So, the Q.E has no real roots.

(ii) $3x^2 - 4\sqrt{3}x + 4 = 0$

Sol: $a = 3, b = -4\sqrt{3}, c = 4$

$b^2 - 4ac = \left(-4\sqrt{3}\right)^2 - 4 \times 3 \times 4 = 48 - 48 = 0$

So, the roots are real and equal

The roots are $\frac{-b}{2a}, \frac{-b}{2a} \Rightarrow \frac{4\sqrt{3}}{2\times 3}, \frac{4\sqrt{3}}{2\times 3} \Rightarrow \frac{2\sqrt{3}}{3}, \frac{2\sqrt{3}}{3}$

(iii) $2x^2 - 6x + 3 = 0$

Sol: $a = 2, b = -6, c = 3$

$b^2 - 4ac = (-6)^2 - 4 \times 2 \times 3 = 36 - 24 = 12 > 0$

So, the roots are real and distinct.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{6 \pm \sqrt{12}}{2 \times 2} = \frac{6 \pm 2\sqrt{3}}{4} = \frac{2(3 \pm \sqrt{3})}{4} = \frac{3 \pm \sqrt{3}}{2}$$

$$\textit{The roots are } \frac{3 + \sqrt{3}}{2}, \frac{3 - \sqrt{3}}{2}$$

2. **Find the values of k for each of the following quadratic equations, so that they have two equal roots.**

(i) $\mathbf{2x^2 + kx + 3 = 0}$

Sol: $a = 2, b = k, c = 3$

If the Q.E has equal roots then $b^2 - 4ac = 0$

$k^2 - 4 \times 2 \times 3 = 0$

$k^2 = 24 \Rightarrow k = \pm\sqrt{24} = \pm\sqrt{4 \times 6} = \pm 2\sqrt{6}$

(ii) $\mathbf{kx(x - 2) + 6 = 0}$

Sol: $kx^2 - 2kx + 6 = 0$

$a = k, b = -2k, c = 6$

If the Q.E has equal roots then $b^2 - 4ac = 0$

$(-2k)^2 - 4 \times k \times 6 = 0$

$4k^2 - 24k = 0$

$4k(k - 6) = 0$

$4k = 0 \;\; or \;\; k - 6 = 0$

$k = 0 \;\; or \;\; k = 6$

$\therefore k = 6 \; (\; if \; k = 0 \; then \; a = 0 \; , b = 0 \; it \; is \; not \; a \; Q.E)$

3. **Is it possible to design a rectangular mango grove whose length is twice its breadth, and the area is 800 m ^{2} ? If so, find its length and breadth.**

Sol: Let the breadth (b) $= x \; m$

Length (l) $= 2x \; m$

Given area of the rectangular grove=800m^2

$x \times 2x = 800$

$$x^2 = \frac{800}{2} = 400 \Rightarrow x = \sqrt{400} = 20$$

Yes , it is possible

Length of the mango grove=$2 \times 20 = 40m$

Breadth of the mango grove=$20 \; m$.

4. **The sum of the ages of two friends is 20 years. Four years ago, the product of their ages in years was 48. Is the situation possible? If so, determine their present ages.**

Sol:

	First friend	Second friend
Present age(in years)	x	$20-x$
Four years ago age	$x-4$	$20-x-4=16-x$

Four years ago, the product of their ages=48

$(x-4)(16-x)=48$

$16x-x^2-64+4x-48=0$

$-x^2+20x-112=0$

$x^2-20x+112=0$

$a=1, b=-20, c=112$

$b^2-4ac=(-20)^2-4\times1\times112=400-448=-48<0$

The roots are not real. So, the situation is not possible.

5. **Is it possible to design a rectangular park of perimeter 80 m. and area 400 m ^{2} ? If so, find its length and breadth.**

Sol: Let length of rectangular park(l)=x m

Perimeter of the park=80 m

$2(l+b)=80$

$x+b=\frac{80}{2}=40\Rightarrow b=40-x$

Area of park=400 m^2

$x(40-x)=400$

$40x-x^2-400=0$

$x^2-40x+400=0$

$(x-20)(x-20)=0$

$x-20=0\ \Rightarrow x=20$

Length of the rectangular park=20 m

Breadth of the rectangular park=$40-20$ =20m

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