CHAPTER 4

X CLASS

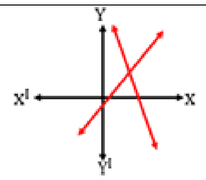
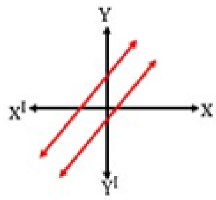
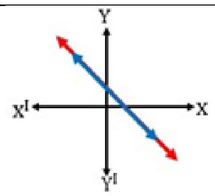
PAIR OF LINEAR EQUATIONS IN TWO VARIABLES-GRAPHS

PREPARED BY: BALABHADRA SURESH

1. An equation of the form $ax + by + c = 0$ where a, b, c are real numbers and where at least one of a or b is not zero (i.e. $a^2 + b^2 \neq 0$), is called a linear equation in two variables x and y
2. Two linear equations in the same two variables are called a pair of linear equations in two variables

   $a_1x + b_1y + c_1 = 0({a_1}^2 + {b_1}^2 \neq 0)$

   $a_2x + b_2y + c_2 = 0({a_2}^2 + {b_2}^2 \neq 0)$
3.

Comparison of ratios	Graphical representation	Algebraic interpretation	Solution	Graph
$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$	Intersecting lines	Consistent	Unique solution	
$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$	Parallel lines	In consistent	No solution	 -
$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$	Coincident lines	Consistent	Infinite number of solutions	

Example-1. Check whether the given pair of equations represent intersecting, parallel or coincident lines. Find the solution if the equations are consistent.

2x + y - 5 = 0, 3x - 2y - 4 = 0

Sol: $2x + y - 5 = 0 \quad (a_1 = 2, \quad b_1 = 1, \quad c_1 = -5)$

$3x - 2y - 4 = 0 \quad (a_2 = 3, \quad b_2 = -2, \quad c_2 = -4)$

$\frac{a_1}{a_2} = \frac{2}{3}; \quad \frac{b_1}{b_2} = \frac{1}{-2}; \quad \frac{c_1}{c} = \frac{-5}{-4} = \frac{5}{4}$

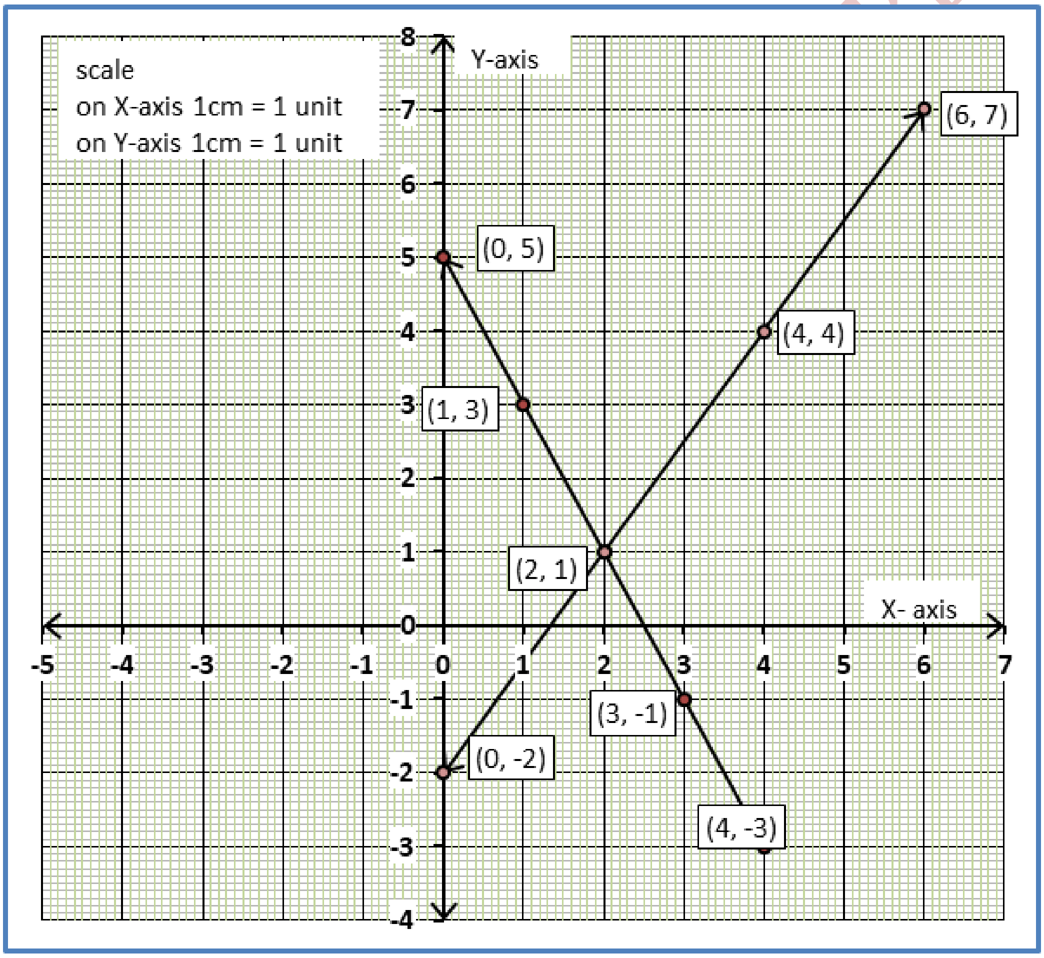
$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

∴ Given pair of equations represent intersecting lines and hence, consistent pair of linear equations.

$2x + y - 5 = 0 \Rightarrow y = 5 - 2x$		
x	$y = 5 - 2x$	(x, y)
0	$y = 5 - 2(0) = 5 - 0 = 5$	$(0,5)$
1	$y = 5 - 2(1) = 5 - 2 = 3$	$(1,3)$
3	$y = 5 - 2(3) = 5 - 6 = -1$	$(3,-1)$
4	$y = 5 - 2(4) = 5 - 8 = -3$	$(4,-3)$

$3x - 2y - 4 = 0 \Rightarrow 2y = 3x - 4 \Rightarrow y = \frac{3x-4}{2}$		
x	$y = \frac{3x-4}{2}$	(x, y)
0	$y = \frac{3(0)-4}{2} = \frac{0-4}{2} = \frac{-4}{2} = -2$	$(0,-2)$
2	$y = \frac{3(2)-4}{2} = \frac{6-4}{2} = \frac{2}{2} = 1$	$(2,1)$
4	$y = \frac{3(4)-4}{2} = \frac{12-4}{2} = \frac{8}{2} = 4$	$(4,4)$
6	$y = \frac{3(6)-4}{2} = \frac{18-4}{2} = \frac{14}{2} = 7$	$(6,7)$

The unique solution of this pair of equations is (2,1).



Example-2. Check whether the following pair of equations is consistent.

3x + 4y = 2 and 6x + 8y = 4. Verify by a graphical representation.

Sol: $3x + 4y - 2 = 0$ ($a_1 = 3,\quad b_1 = 4,\quad c_1 = -2$)

$6x + 8y - 4 = 0$ $(a_2 = 6, \quad b_2 = 8, \quad c_2 = -4)$

$$\frac{a_1}{a_2} = \frac{3}{6} = \frac{1}{2}; \quad \frac{b_1}{b_2} = \frac{4}{8} = \frac{1}{2}; \quad \frac{c_1}{c_2} = \frac{-2}{-4} = \frac{1}{2}$$

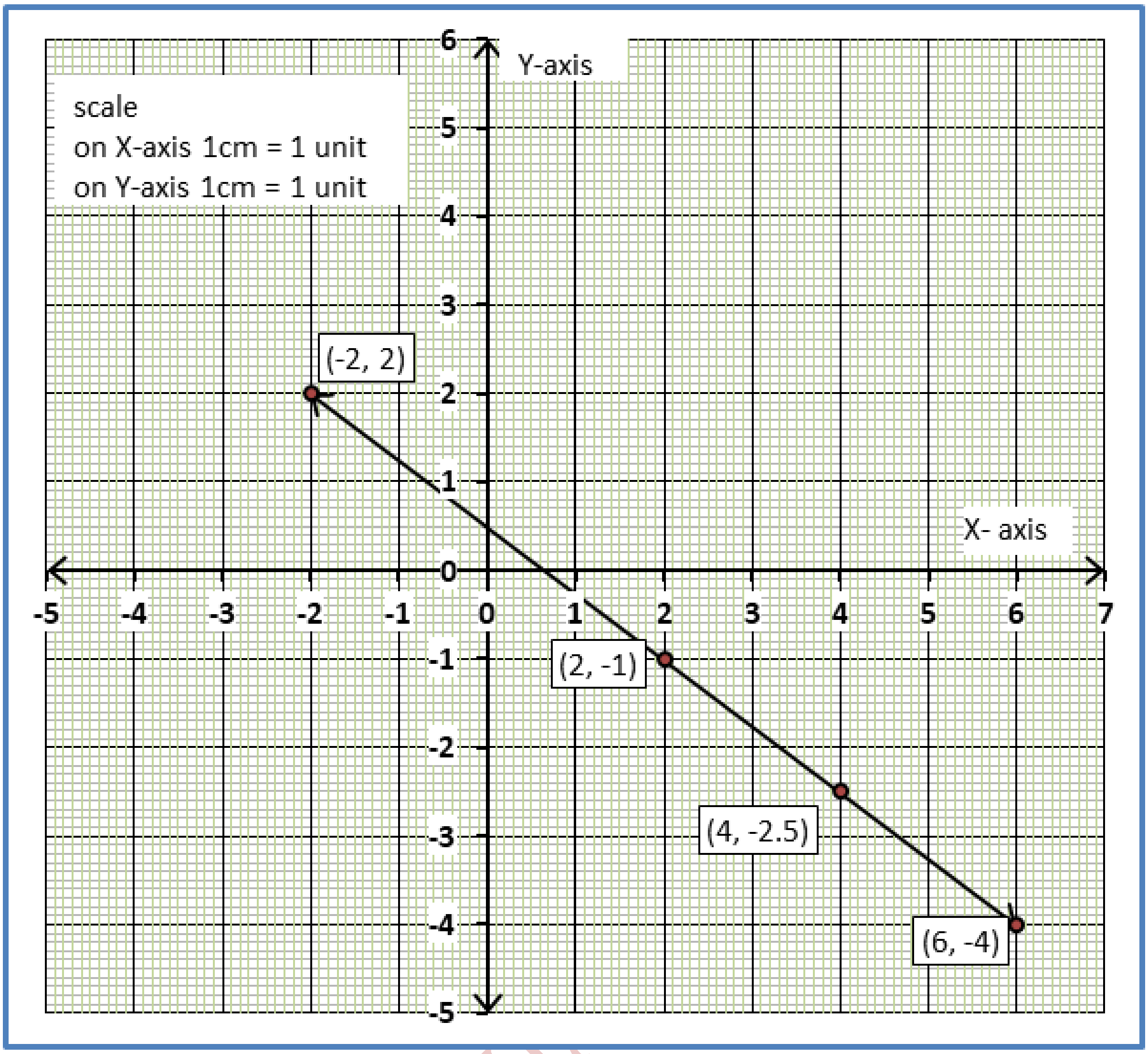
$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$\therefore$ They are coincident lines. So, the pair of linear equations is dependent and have infinitely many solutions.

$3x + 4y = 2 \Rightarrow 4y = 2 - 3x \Rightarrow y = \frac{2-3x}{4}$		
x	$y = \frac{2-3x}{4}$	(x, y)
-2	$y = \frac{2-3(-2)}{4} = \frac{2+6}{4} = \frac{8}{4} = 2$	$(-2,2)$
2	$y = \frac{2-3(2)}{4} = \frac{2-6}{4} = \frac{-4}{4} = -1$	$(2,-1)$
4	$y = \frac{2-3(4)}{4} = \frac{2-12}{4} = \frac{-10}{4} = -2.5$	$(4,-2.5)$
6	$y = \frac{2-3(6)}{4} = \frac{2-18}{4} = \frac{-16}{4} = -4$	$(6,-4)$

$6x + 8y = 4 \Rightarrow 8y = 4 - 6x \Rightarrow y = \frac{4-6x}{8}$		
x	$y = \frac{4-6x}{8}$	(x, y)
-2	$y = \frac{4-6(-2)}{8} = \frac{4+12}{8} = \frac{16}{8} = 2$	$(-2,2)$
2	$y = \frac{4-6(2)}{8} = \frac{4-12}{8} = \frac{-8}{8} = -1$	$(2,-1)$
4	$y = \frac{4-6(4)}{8} = \frac{4-24}{8} = \frac{-20}{8} = -2.5$	$(4,-2.5)$
6	$y = \frac{4-6(6)}{8} = \frac{4-36}{8} = \frac{-32}{8} = -4$	$(6,-4)$

.



Example-3. Check whether the equations 2x-3y = 5 and 4x-6y = 15 are consistent. Also verify by graphical representation.

Sol: $2x - 3y - 5 = 0$; $(a_1 = 2, \quad b_1 = -3, \quad c_1 = -5)$

$4x - 6y - 15 = 0$ $(a_2 = 4, \quad b_2 = -6, \quad c_2 = -15)$

$$\frac{a_1}{a_2} = \frac{2}{4} = \frac{1}{2}; \ \frac{b_1}{b_2} = \frac{-3}{-6} = \frac{1}{2}; \ \frac{c_1}{c_2} = \frac{-5}{-15} = \frac{1}{3}$$

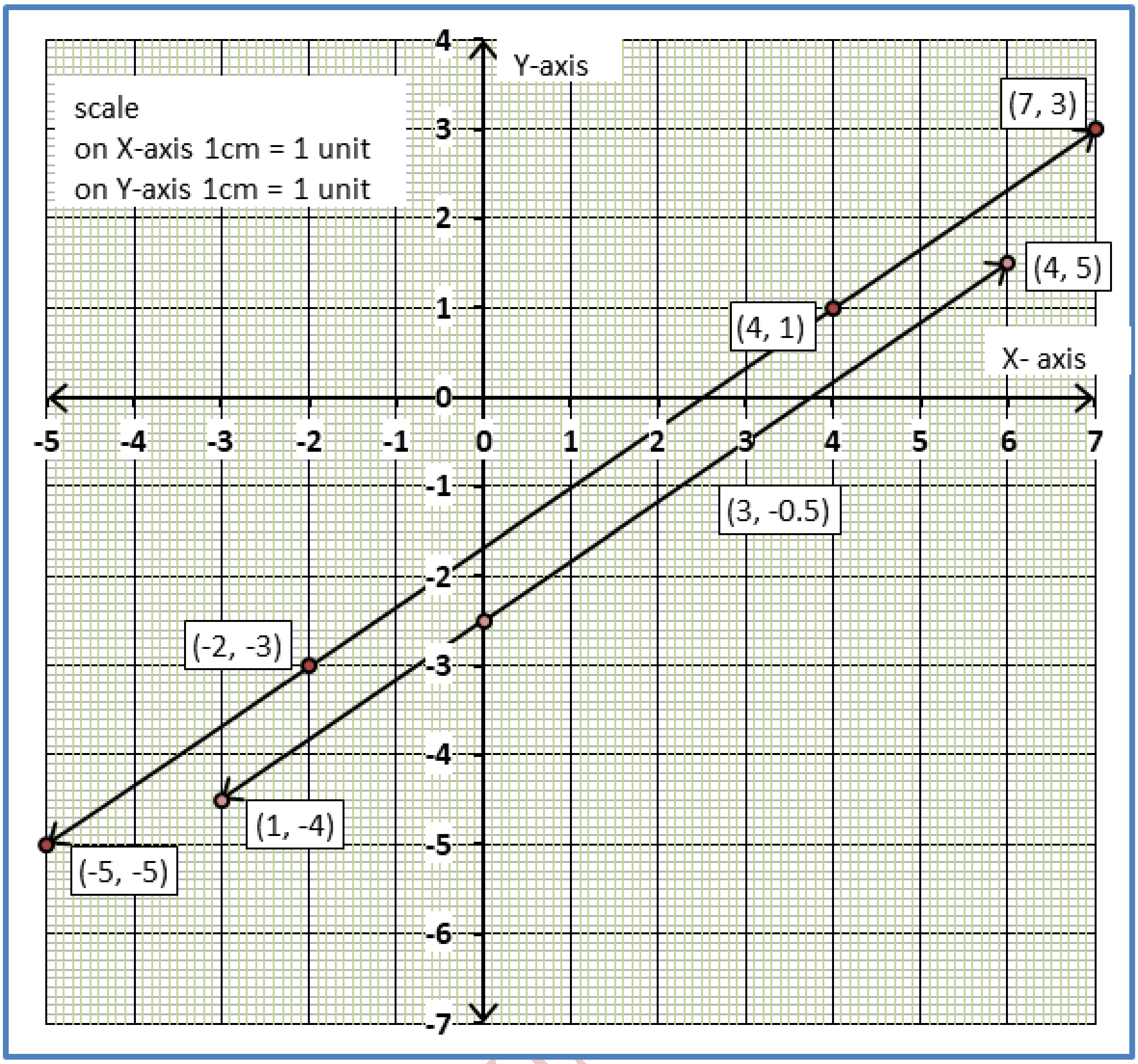
$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

So the equations are inconsistent. They have no solutions and its graph is of parallel lines.

$2x - 3y = 5 \Rightarrow 3y = 2x - 5 \Rightarrow y = \frac{2x-5}{3}$		
x	$y = \frac{2x-5}{3}$	(x, y)
-5	$y = \frac{2(-5)-5}{3} = \frac{-10-5}{3} = \frac{-15}{3} = -5$	$(-5, -5)$

-2	$y=\frac{2(-2)-5}{3}=\frac{-4-5}{3}=\frac{-9}{3}=-3$	$(-2,-3)$
4	$y=\frac{2(4)-5}{3}=\frac{8-5}{3}=\frac{3}{3}=1$	$(4,1)$
7	$y=\frac{2(7)-5}{3}=\frac{14-5}{3}=\frac{9}{3}=3$	$(7,3)$

$4x-6y=15 \Rightarrow 6y=4x-15 \Rightarrow y=\frac{4x-15}{6}$		
x	$y=\frac{4x-15}{6}$	(x,y)
-3	$y=\frac{4(-3)-15}{6}=\frac{-12-15}{6}=\frac{-27}{6}=\frac{-9}{2}=-4.5$	$(-3,-4.5)$
0	$y=\frac{4(0)-15}{6}=\frac{0-15}{6}=\frac{-15}{6}=-2.5$	$(0,-2.5)$
3	$y=\frac{4(3)-15}{6}=\frac{12-15}{6}=\frac{-3}{6}=-0.5$	$(3,-0.5)$
6	$y=\frac{4(6)-15}{6}=\frac{24-15}{6}=\frac{9}{6}=1.5$	$(6,1.5)$



(i) Solve graphically $2x + 3y = 1$; $3x - y = 7$

Sol: Given equations : $2x + 3y - 1 = 0$ $\quad a_1 = 2,\ b_1 = 3, c_1 = -1$

$3x - y - 7 = 0$ $\quad a_2 = 3, b_2 = -1, c_2 = -7$

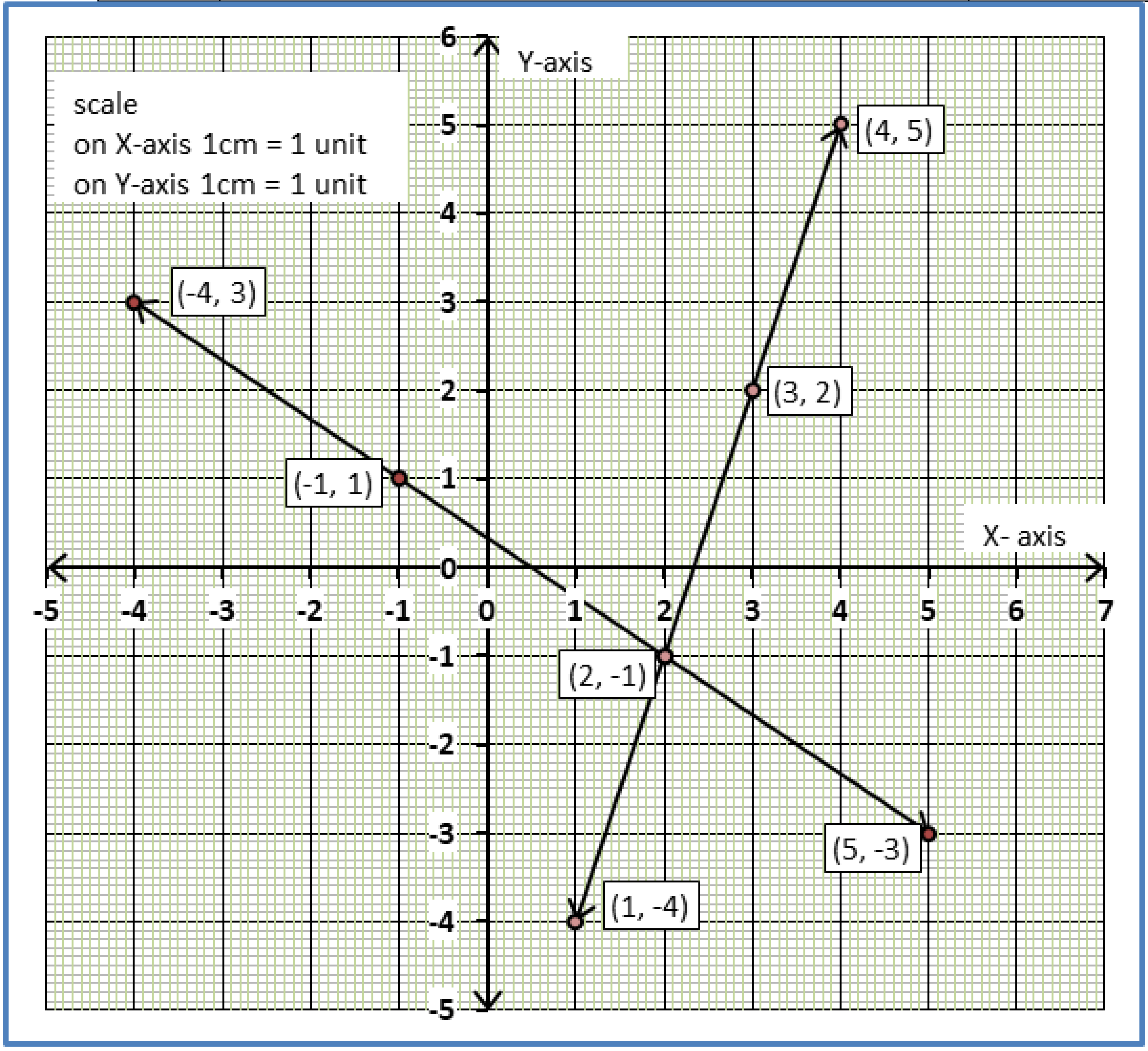
$\frac{a_1}{a_2} = \frac{2}{3}$; $\frac{b_1}{b_2} = \frac{3}{-1} = -3 \Rightarrow \frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

$\Rightarrow$ The given lines are intersecting lines, consistent pair of linear equation .

$2x + 3y = 1 \Rightarrow 3y = 1 - 2x \Rightarrow y = \frac{1-2x}{3}$		
x	$y = \frac{1-2x}{3}$	(x, y)
-1	$y = \frac{1-2(-1)}{3} = \frac{1+2}{3} = \frac{3}{3} = 1$	$(-1,1)$
-4	$y = \frac{1-2(-4)}{3} = \frac{1+8}{3} = \frac{9}{3} = 3$	$(-4,3)$
2	$y = \frac{1-2(2)}{3} = \frac{1-4}{3} = \frac{-3}{3} = -1$	$(2,-1)$

5	$y = \frac{1-2(5)}{3} = \frac{1-10}{3} = \frac{-9}{3} = -3$	$(5,-3)$

$3x - y = 7 \Rightarrow y = 3x - 7$		
x	$y = 3x - 7$	(x, y)
0	$y = 3(0) - 7 = 0 - 7 = -7$	$(0, -7)$
2	$y = 3(2) - 7 = 6 - 7 = -1$	$(2, -1)$
4	$y = 3(4) - 7 = 8 - 7 = 1$	$(4,1)$
6	$y = 3(6) - 7 = 12 - 7 = 5$	$(6,5)$



The unique solution of this pair of equations is (2,-1).

(ii) solve graphically $x + 2y = 6$ and $2x + 4y = 12$

Sol: Given equations : $x + 2y - 6 = 0$ $\quad (a_1 = 1,\ b_1 = 2, c_1 = -6)$

$2x + 4y - 12 = 0$ $\quad (a_2 = 2, b_2 = 4, c_2 = -12)$

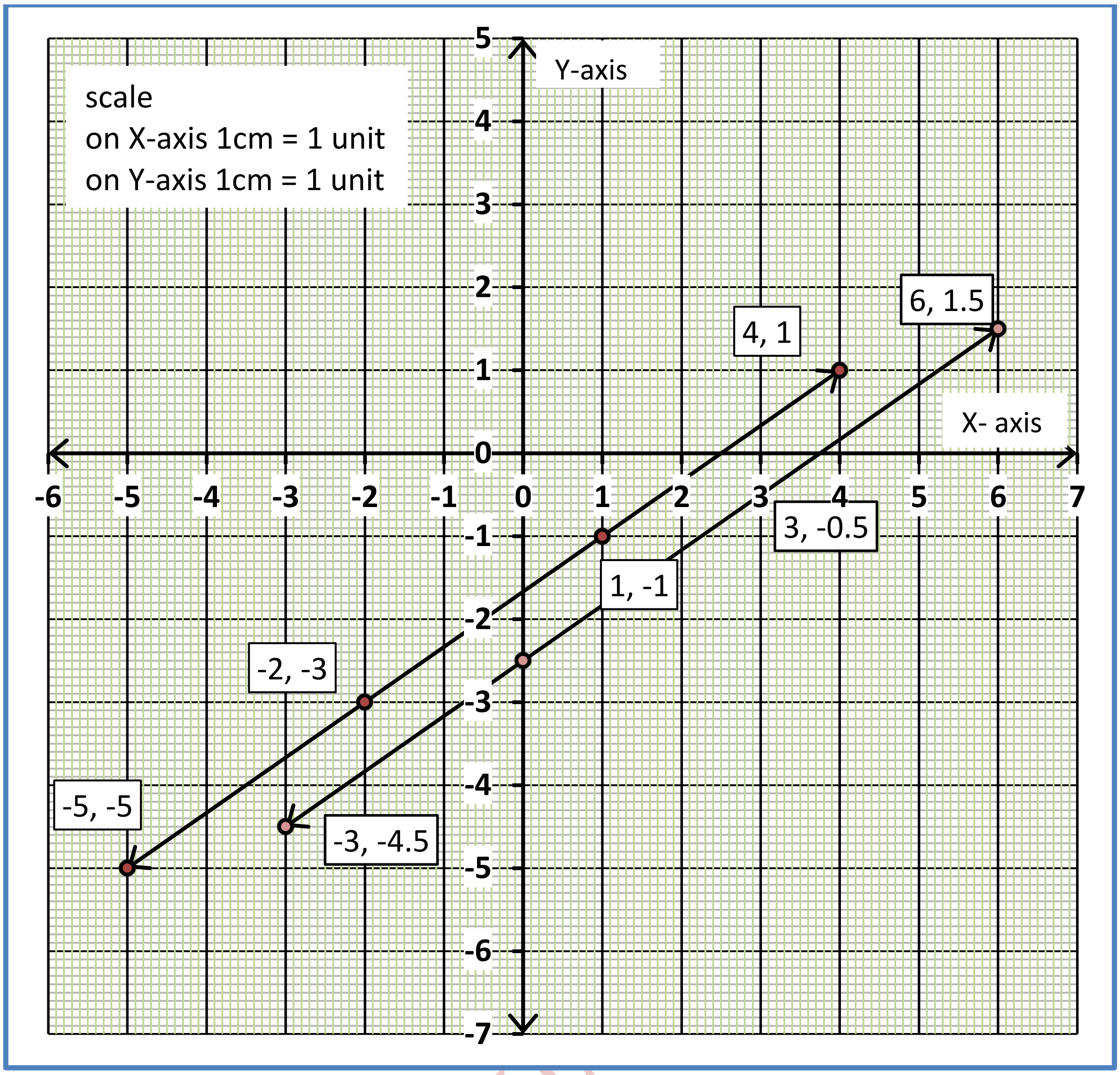
$\frac{a_1}{a_2} = \frac{1}{2}; \quad \frac{b_1}{b_2} = \frac{2}{4} = \frac{1}{2}; \quad \frac{c_1}{c_2} = \frac{-6}{-12} = \frac{1}{2}$

$\Rightarrow \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

$\Rightarrow$ The given lines are parallele lines.

$x + 2y = 6 \Rightarrow 2y = 6 - x \Rightarrow y = \frac{6-x}{2}$		
x	$y = \frac{6-x}{2}$	(x, y)
0	$y = \frac{6-0}{2} = \frac{6}{2} = 3$	(0,3)
2	$y = \frac{6-2}{2} = \frac{4}{2} = 2$	(2,2)
4	$y = \frac{6-4}{2} = \frac{2}{2} = 1$	(4,1)
−2	$y = \frac{6-(-2)}{2} = \frac{6+2}{2} = \frac{8}{2} = 4$	(−2,4)

$2x + 4y = 12 \Rightarrow 4y = 12 - 2x \Rightarrow y = \frac{12-2x}{4}$		
x	$y = \frac{12-2x}{4}$	(x, y)
2	$y = \frac{12-2(2)}{4} = \frac{12-4}{4} = \frac{8}{4} = 2$	(2,2)
4	$y = \frac{12-2(4)}{4} = \frac{12-8}{4} = \frac{4}{4} = 1$	(4,1)
6	$y = \frac{12-2(6)}{4} = \frac{12-12}{4} = \frac{0}{4} = 0$	(6,0)



(iii) solve graphically 3x + 2y = 6 and 6x + 4y = 18

Sol: Given equations : $3x + 2y - 6 = 0$ $\quad a_1 = 3,\ b_1 = 2, c_1 = -6$

$6x + 4y - 18 = 0 \quad a_2 = 6, b_2 = 4, c_2 = -18$

$\frac{a_1}{a_2} = \frac{3}{6} = \frac{1}{2};\ \ \frac{b_1}{b_2} = \frac{2}{4} = \frac{1}{2};\ \ \frac{c_1}{c_2} = \frac{-6}{-18} = \frac{1}{3}$

$\Rightarrow \frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

$\Rightarrow$ The given lines are parallel lines . The equations have no solution.

$3x + 2y = 6 \Rightarrow 2y = 6 - 3x \Rightarrow y = \frac{6-3x}{2}$		
x	$y = \frac{6-3x}{2}$	(x, y)
0	$y = \frac{6-3(0)}{2} = \frac{6-0}{2} = \frac{6}{2} = 3$	(0,3)

2	$y = \frac{6-3(2)}{2} = \frac{6-6}{2} = \frac{0}{2} = 0$	(2,0)
4	$y = \frac{6-3(4)}{2} = \frac{6-12}{2} = \frac{-6}{2} = -3$	(4, −3)
−2	$y = \frac{6-3(-2)}{2} = \frac{6+6}{2} = \frac{12}{2} = 6$	(−2,4)

$6x + 4y = 18 \Rightarrow 4y = 18 - 6x \Rightarrow y = \frac{18-6x}{4}$		
x	$y = \frac{18-6x}{4}$	(x, y)
1	$y = \frac{18-6(1)}{4} = \frac{18-6}{4} = \frac{12}{4} = 3$	(1,3)
3	$y = \frac{18-6(3)}{4} = \frac{18-18}{4} = \frac{0}{4} = 0$	(3,0)
−1	$y = \frac{18-6(-1)}{4} = \frac{18+6}{4} = \frac{24}{4} = 6$	(−1,6)

EXERCISE - 4.1

2. Check whether the following equations are consistent or inconsistent. Solve them graphically.

a) 3x+2y=8; 2x-3y=1

Sol: Given equations :

$3x + 2y - 5 = 0 \qquad a_1 = 3,\ b_1 = 2, c_1 = -5$

$2x - 3y - 7 = 0 \qquad a_2 = 2, b_2 = -3, c_2 = -7$

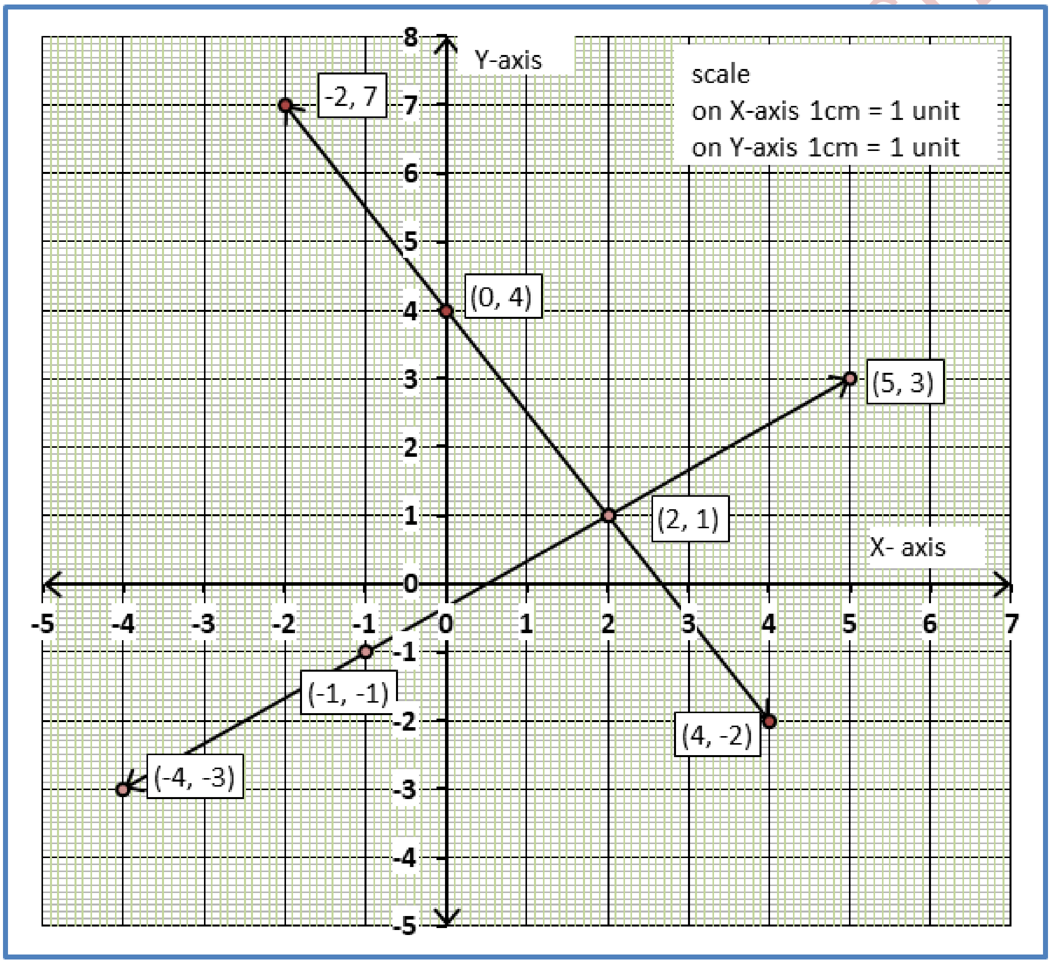
$\frac{a_1}{a_2} = \frac{3}{2};\ \ \frac{b_1}{b_2} = \frac{2}{-3} \Rightarrow \frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

$\Rightarrow$ The given equations are consistent . The pair of equations have one solution.

$3x + 2y = 8 \Rightarrow 2y = 8 - 3x \Rightarrow y = \frac{8-3x}{2}$		
x	$y = \frac{8-3x}{2}$	(x, y)
−2	$y = \frac{8-3(-2)}{2} = \frac{8+6}{2} = \frac{14}{2} = 7$	(−2,7)
0	$y = \frac{8-3(0)}{2} = \frac{8-0}{2} = \frac{8}{2} = 4$	(0,4)
2	$y = \frac{8-3(2)}{2} = \frac{8-6}{2} = \frac{2}{2} = 1$	(2,1)
4	$y = \frac{8-3(4)}{2} = \frac{8-12}{2} = \frac{-4}{2} = -2$	(4, −2)

$2x-3y=1 \Rightarrow 3y=2x-1 \Rightarrow y=\dfrac{2x-1}{3}$		
x	$y=\dfrac{2x-1}{3}$	(x,y)
2	$y=\dfrac{2(2)-1}{3}=\dfrac{4-1}{3}=\dfrac{3}{3}=1$	(2,1)
5	$y=\dfrac{2(5)-1}{3}=\dfrac{10-1}{3}=\dfrac{9}{3}=3$	(5,3)
-1	$y=\dfrac{2(-1)-1}{3}=\dfrac{-2-1}{3}=\dfrac{-3}{3}=-1$	$(-1,-1)$
-4	$y=\dfrac{2(-4)-1}{3}=\dfrac{-8-1}{3}=\dfrac{-9}{3}=-3$	$(-4,-3)$

The unique solution of this pair of equations is (2,1)



b) $2x-3y=8;\ 4x-6y=9$

Sol: Given equations : $2x-3y-8=0$ $\quad a_1=2,\ b_1=-3, c_1=-8$

$4x-6y-9=0 \quad a_2=4, b_2=-6, c_2=-9$

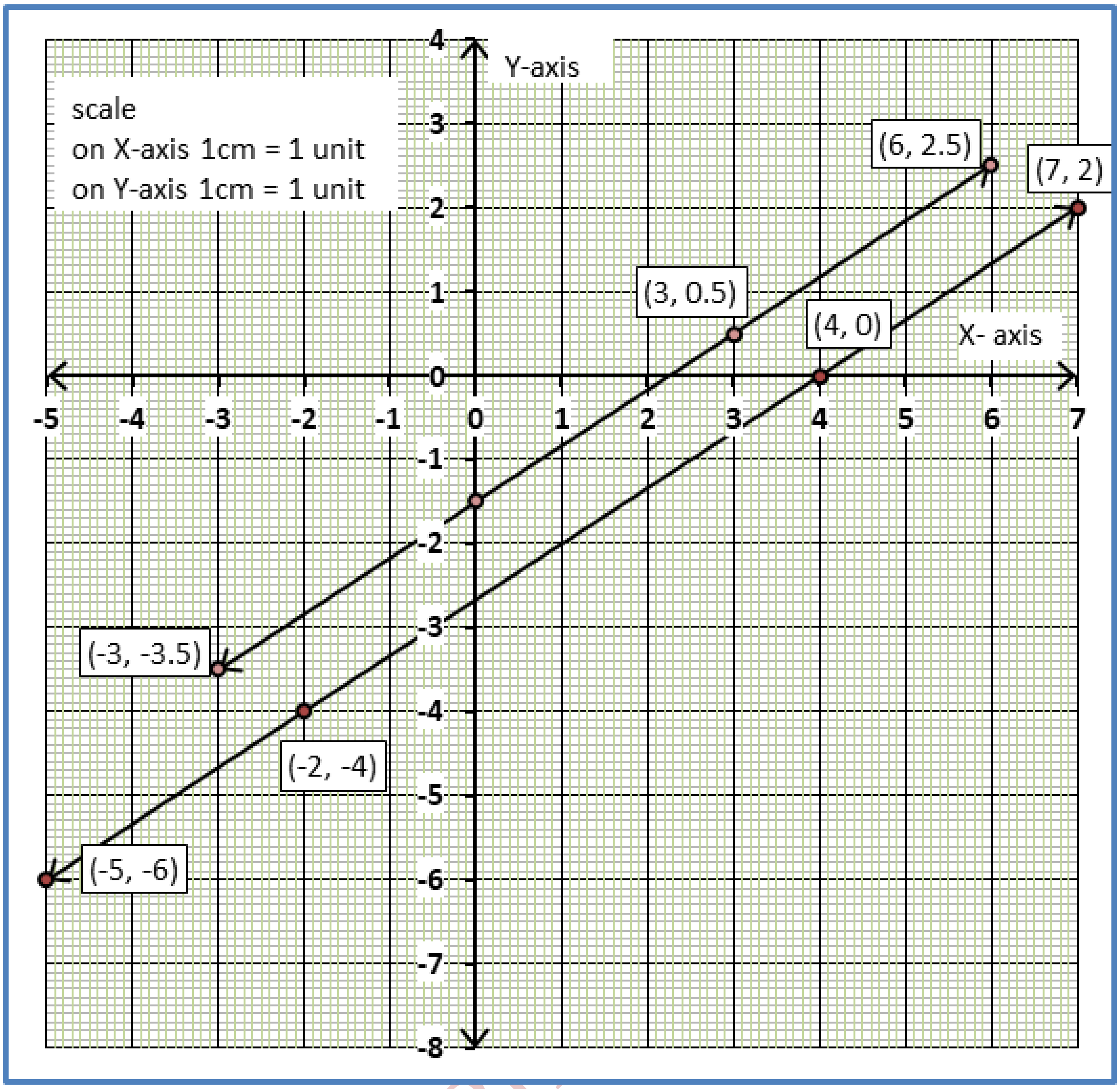
$\frac{a_1}{a_2} = \frac{2}{4} = \frac{1}{2}$; $\frac{b_1}{b_2} = \frac{-3}{-6} = \frac{1}{2}$; $\frac{c_1}{c_2} = \frac{-8}{-9} = \frac{8}{9}$

$\Rightarrow \frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

⇒ The given equations are *in*consistent(parallel lines) . The pair of equations have no solution.

$2x - 3y = 8 \Rightarrow 3y = 2x - 8 \Rightarrow y = \frac{2x-8}{3}$		
x	$y = \frac{2x-8}{3}$	(x, y)
7	$y = \frac{2(7)-8}{3} = \frac{14-8}{3} = \frac{6}{3} = 2$	(7,2)
4	$y = \frac{2(4)-8}{3} = \frac{8-8}{3} = \frac{0}{3} = 0$	(4,0)
−2	$y = \frac{2(-2)-8}{3} = \frac{-4-8}{3} = \frac{-12}{3} = -4$	(−2, −4)
−5	$y = \frac{2(-5)-8}{3} = \frac{-10-8}{3} = \frac{-18}{3} = -6$	(−5, −6)

$4x - 6y = 9 \Rightarrow 6y = 4x - 9 \Rightarrow y = \frac{4x-9}{6}$		
x	$y = \frac{4x-9}{6}$	(x, y)
0	$y = \frac{4(0)-9}{6} = \frac{0-9}{6} = \frac{-9}{6} = -1.5$	(0, −1.5)
3	$y = \frac{4(3)-9}{6} = \frac{12-9}{6} = \frac{3}{6} = 0.5$	(3,0.5)
6	$y = \frac{4(6)-9}{6} = \frac{24-9}{6} = \frac{15}{6} = 2.5$	(6,2.5)
−3	$y = \frac{4(-3)-9}{6} = \frac{-12-9}{6} = \frac{-21}{6} = -3.5$	(−3, −3.5)



c) $\frac{3}{2}x + \frac{5}{3}y = 7$; $9x - 10y = 12$

Sol: Given equations: $\frac{3}{2}x + \frac{5}{3}y - 7 = 0$

$9x + 10y - 42 = 0$ $a_1 = 9$, $b_1 = 10, c_1 = -42$

$9x - 10y - 12 = 0$ $a_2 = 9, b_2 = -10, c_2 = -12$

$\frac{a_1}{a_2} = \frac{9}{9} = 1$; $\frac{b_1}{b_2} = \frac{10}{-10} = -1$

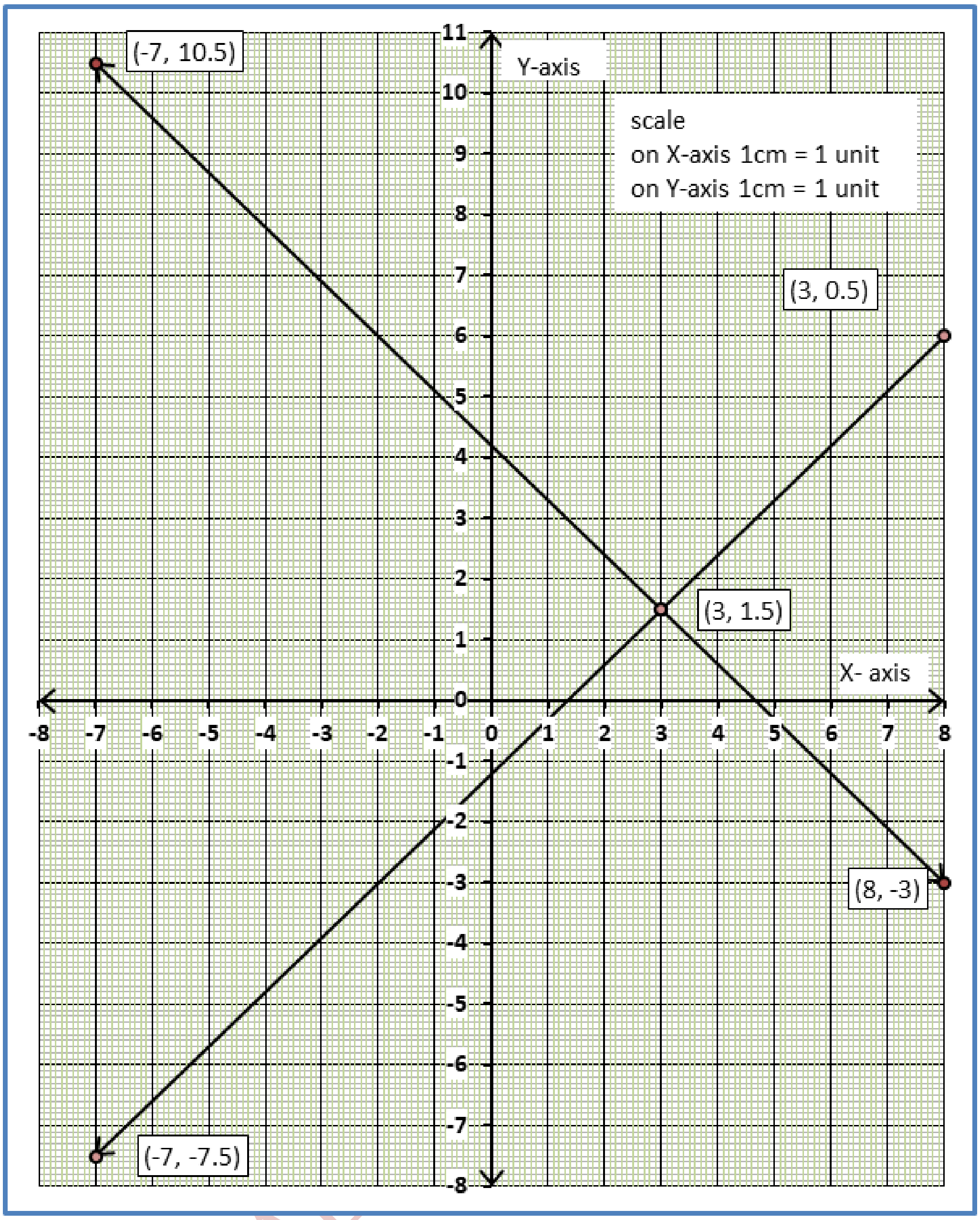
$\Rightarrow \frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

$\Rightarrow$ The given equations are consistent . The pair of equations has one solution.

$9x + 10y = 42 \Rightarrow 10y = 42 - 9x \Rightarrow y = \frac{42 - 9x}{10}$		
x	$y = \frac{42 - 9x}{10}$	(x, y)

3	$y=\frac{42-9(3)}{10}=\frac{42-27}{10}=\frac{15}{10}=1.5$	$(3,1.5)$
8	$y=\frac{42-9(8)}{10}=\frac{42-72}{10}=\frac{-30}{10}=-3$	$(8,-3)$
-7	$y=\frac{42-9(-7)}{10}=\frac{42+63}{10}=\frac{105}{10}=10.5$	$(-7,10.5)$

$9x-10y=12 \Rightarrow 10y=9x-12 \Rightarrow y=\frac{9x-12}{10}$		
x	$y=\frac{9x-12}{10}$	(x,y)
3	$y=\frac{9(3)-12}{10}=\frac{27-12}{10}=\frac{15}{10}=1.5$	$(3,1.5)$
8	$y=\frac{9(8)-12}{10}=\frac{72-12}{10}=\frac{60}{10}=6$	$(8,6)$
-7	$y=\frac{9(-7)-12}{10}=\frac{-63-12}{10}=\frac{-75}{10}=-7.5$	$(-7,-7.5)$



(d) $5x - 3y = 11$; $-10x + 6y = -22$

Sol: $5x - 3y - 11 = 0$ $(a_1 = 5, \quad b_1 = -3, \quad c_1 = -11)$

$-10x + 6y + 22 = 0$ $(a_2 = -10, \quad b_2 = 6, \quad c_2 = 22)$

$$\frac{a_1}{a_2} = \frac{5}{-10} = \frac{-1}{2}; \quad \frac{b_1}{b_2} = \frac{-3}{6} = \frac{-1}{2}; \quad \frac{c_1}{c_2} = \frac{-11}{22} = \frac{-1}{2}$$

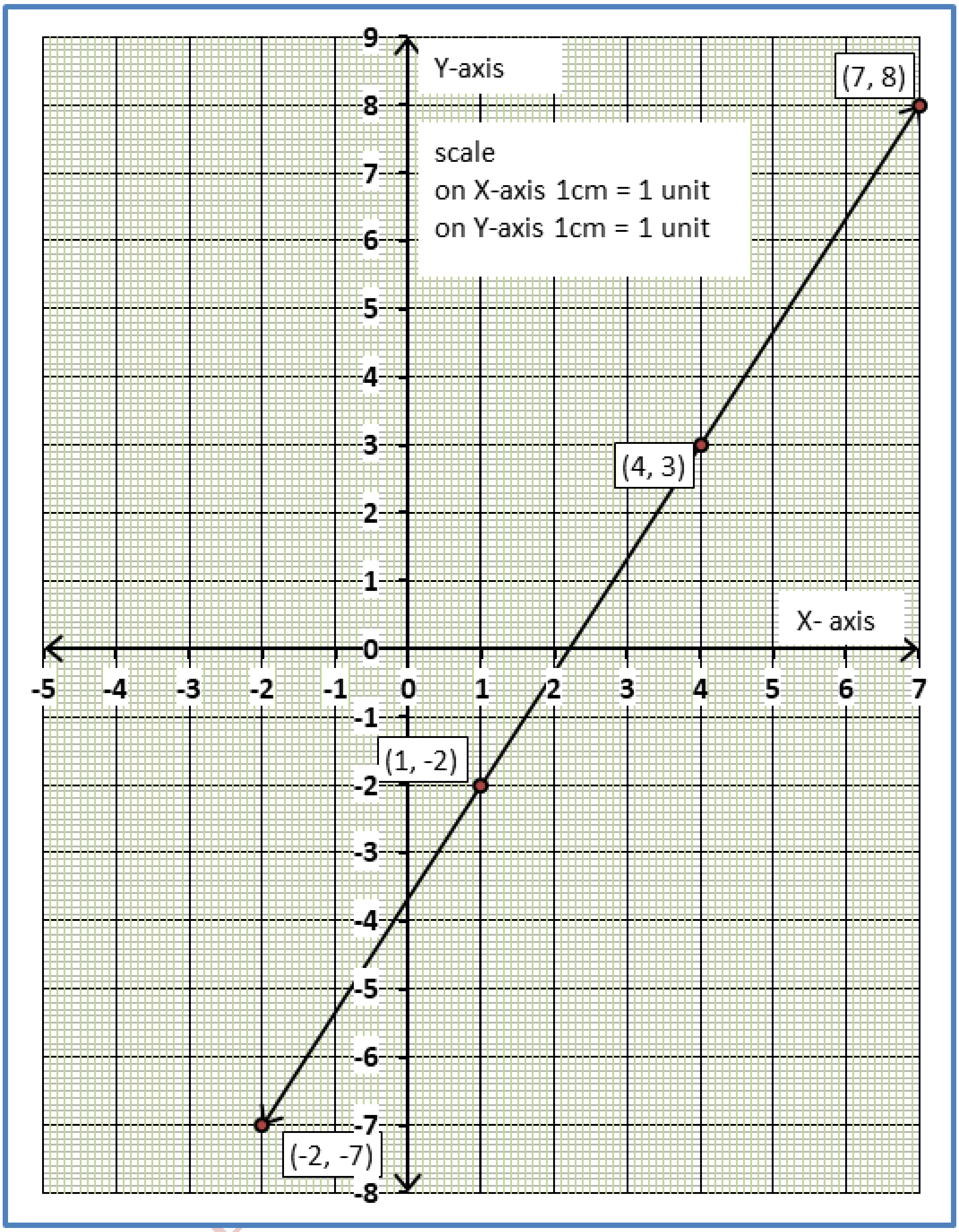
$$\Rightarrow \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$\Rightarrow$ The given lines are coinsident lines . The equations have infinitely many solutions.

$$5x - 3y = 11 \Rightarrow 3y = 5x - 11 \Rightarrow y = \frac{5x - 11}{3}$$

x	$y=\frac{5x-11}{3}$	(x,y)
-2	$y=\frac{5(-2)-11}{3}=\frac{-10-11}{3}=\frac{-21}{3}=-7$	$(-2,-7)$
1	$y=\frac{5(1)-11}{3}=\frac{5-11}{3}=\frac{-6}{3}=-2$	$(1,-2)$
4	$y=\frac{5(4)-11}{3}=\frac{20-11}{3}=\frac{9}{3}=3$	$(4,3)$
7	$y=\frac{5(7)-11}{3}=\frac{35-11}{3}=\frac{24}{3}=8$	$(7,8)$

$-10x+6y=-22 \Rightarrow 6y=10x-22 \Rightarrow y=\frac{10x-22}{6}$		
x	$y=\frac{10x-22}{6}$	(x,y)
-2	$y=\frac{10(-2)-22}{6}=\frac{-20-22}{6}=\frac{-42}{6}=-7$	$(-2,-7)$
1	$y=\frac{10(1)-22}{6}=\frac{10-22}{6}=\frac{-12}{6}=-2$	$(1,-2)$
4	$y=\frac{10(4)-22}{6}=\frac{40-22}{6}=\frac{18}{6}=3$	$(4,3)$
7	$y=\frac{10(7)-22}{6}=\frac{70-22}{6}=\frac{48}{6}=8$	$(7,8)$



(e) $2x + y - 6 = 0;\;\; 4x - 2y - 4 = 0$

Sol: $2x + y - 6 = 0 \;\; (a_1 = 2, \quad b_1 = 1, \quad c_1 = -6)$

$4x - 2y - 4 = 0 \;\; (a_2 = 4, \quad b_2 = -2, \quad c_2 = -4)$

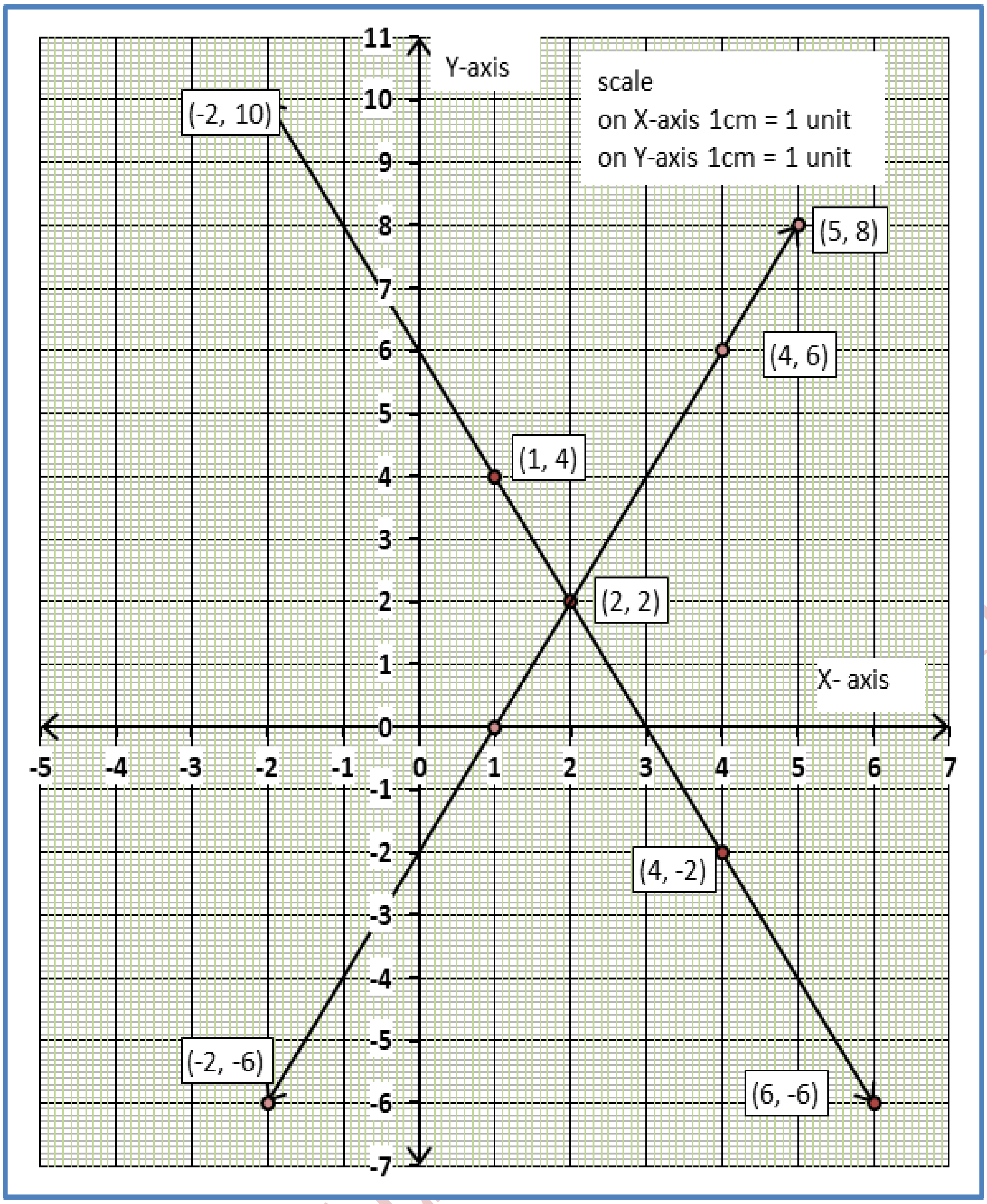
$\frac{a_1}{a_2} = \frac{2}{4} = \frac{1}{2};\;\; \frac{b_1}{b_2} = \frac{1}{-2} = \frac{-1}{2};\;\; \frac{c_1}{c_2} = \frac{-6}{-4} = \frac{3}{2}$

$\Rightarrow \frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

$\Rightarrow$ The given lines are intersecting lines . The equations have unique solution.

$2x+y-6=0 \Rightarrow y=-2x+6$		
x	$y=-2x+6$	(x,y)
-2	$y=-2(-2)+6=4+6=10$	$(-2,10)$
1	$y=-2(1)+6=-2+6=4$	$(1,4)$
4	$y=-2(4)+6=-8+6=-2$	$(4,-2)$
6	$y==-2(6)+6=-12+6=-6$	$(6,-6)$

$4x-2y-4=0 \Rightarrow 2y=4x-4 \Rightarrow y=\frac{4x-4}{2}=2x-2$		
x	$y=2x-2$	(x,y)
-2	$y=2(-2)-2=-4-2=-6$	$(-2,-6)$
1	$y=2(1)-2=2-2=0$	$(1,0)$
4	$y=2(4)-2=8-2=6$	$(4,6)$
5	$y=2(5)-2=10-2=8$	$(5,8)$



The unique solution of the equation is (2,2)

***f*) $x + y = 5;\ 2x + 2y = 10$**

Sol: $x + y - 5 = 0 \quad (a_1 = 1, \quad b_1 = 1, \quad c_1 = -5)$

$2x + 2y - 10 = 0 \ (a_2 = 2, \quad b_2 = 2, \quad c_2 = -10)$

$$\frac{a_1}{a_2} = \frac{1}{2};\ \frac{b_1}{b_2} = \frac{1}{2};\ \frac{c_1}{c_2} = \frac{-5}{-10} = \frac{1}{2}$$

$$\Rightarrow \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

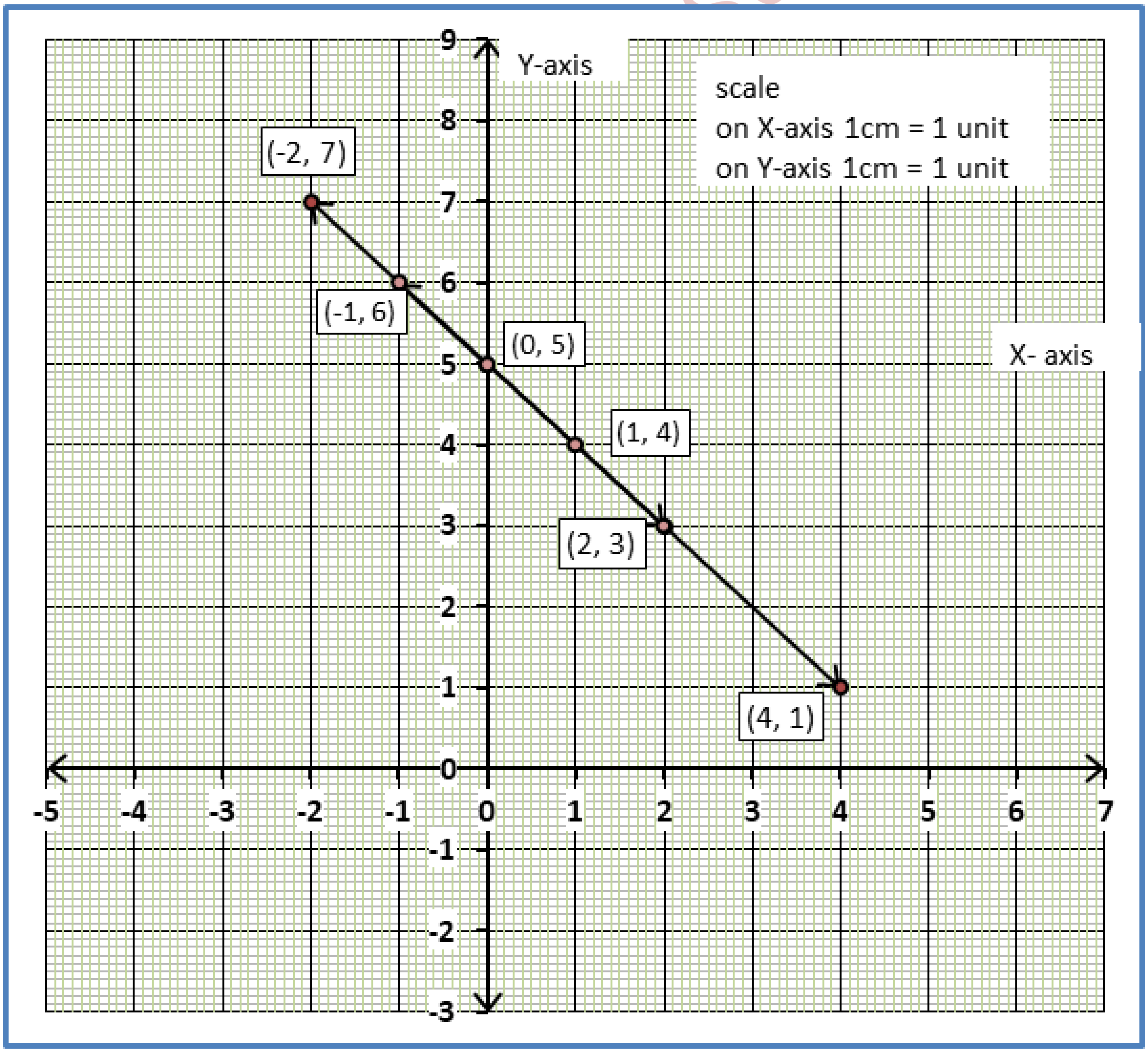
$\Rightarrow$ The given equations are consistent(coinsident lines) . The pair of equations has infinitely many solutions.

$x + y = 5 \Rightarrow y = 5 - x$		
x	$y = 5 - x$	(x, y)
0	$y = 5 - 0 = 5$	(0,5)

2	$y = 5 - 2 = 3$	$(2,3)$
4	$y = 5 - 4 = 1$	$(4,1)$
-2	$y = 5 - (-2) = 5 + 2 = 7$	$(-2,7)$

$$2x + 2y = 10 \Rightarrow 2y = 10 - 2x \Rightarrow y = \frac{10 - 2x}{2}$$

x	$y = \frac{10 - 2x}{2}$	(x, y)
0	$y = \frac{10 - 2(0)}{2} = \frac{10 - 0}{2} = \frac{10}{2} = 5$	$(0,5)$
2	$y = \frac{10 - 2(2)}{2} = \frac{10 - 4}{2} = \frac{6}{2} = 3$	$(2,3)$
-1	$y = \frac{10 - 2(-1)}{2} = \frac{10 + 2}{2} = \frac{12}{2} = 6$	$(-1,6)$
1	$y = \frac{10 - 2(1)}{2} = \frac{10 - 2}{2} = \frac{8}{2} = 4$	$(1,4)$



$g)\ x - y = 8;\ 3x - 3y = 16$

Sol: $x - y - 8 = 0 \quad (a_1 = 1, \quad b_1 = -1, \quad c_1 = -8)$

$3x - 3y - 16 = 0\ (a_2 = 3, \quad b_2 = -3, \quad c_2 = -16)$

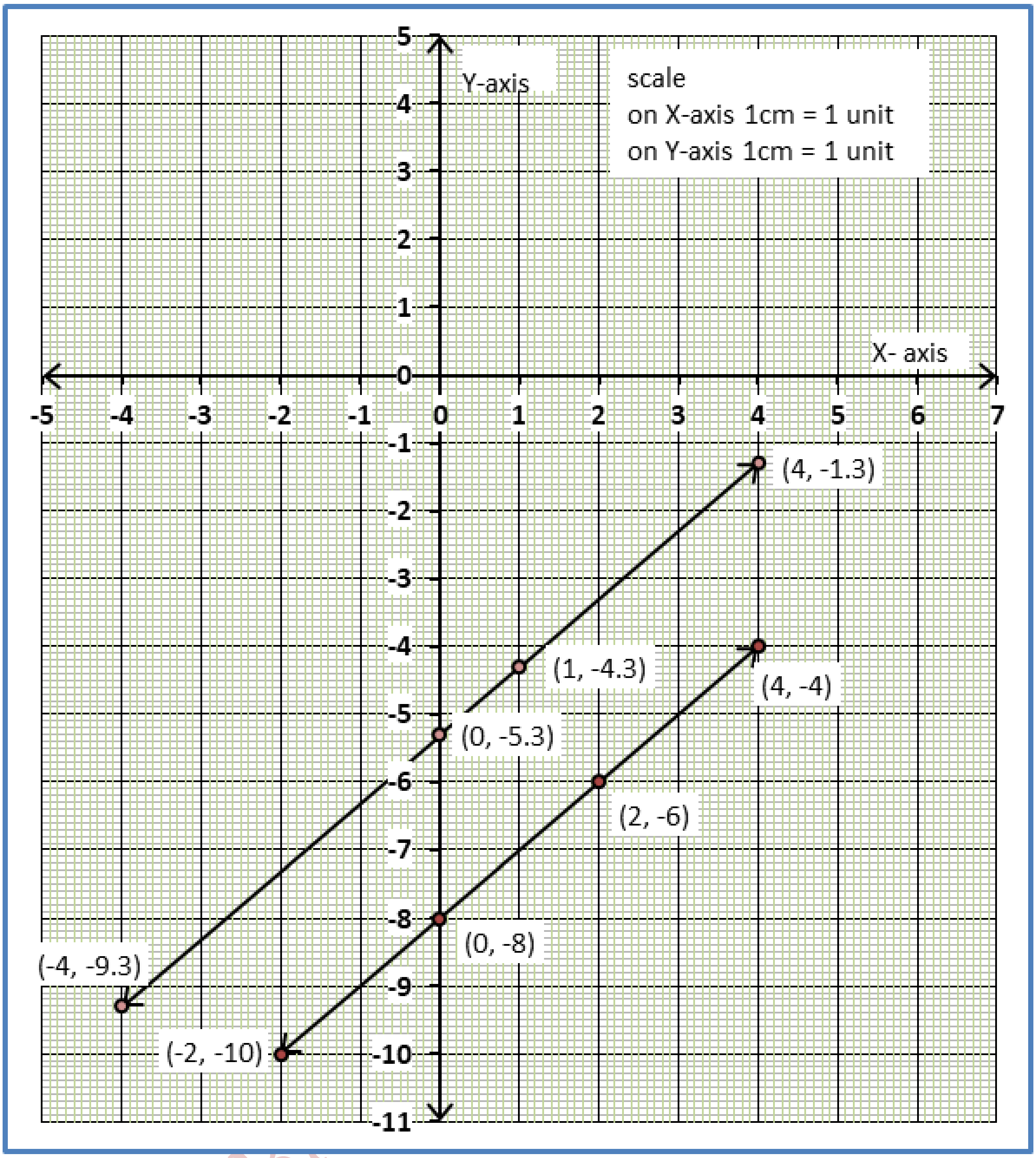
$$\frac{a_1}{a_2} = \frac{1}{3};\ \ \frac{b_1}{b_2} = \frac{-1}{-3} = \frac{1}{3};\ \ \frac{c_1}{c_2} = \frac{-8}{-16} = \frac{1}{2}$$

$$\Rightarrow \frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

$\Rightarrow$ The given equations are *in*consistent(parallel lines) . The pair of equations have no solution.

$x - y = 8 \Rightarrow y = x - 8$		
x	$y = x - 8$	(x, y)
0	$y = 0 - 8 = -8$	$(0, -8)$
2	$y = 2 - 8 = -6$	$(2, -6)$
4	$y = 4 - 8 = -4$	$(4, -4)$
-2	$y = -2 - 8 = -10$	$(-2, -10)$

$3x - 3y = 16 \Rightarrow 3y = 3x - 16 \Rightarrow y = \frac{3x - 16}{3}$		
x	$y = \frac{3x - 16}{3}$	(x, y)
0	$y = \frac{3(0) - 16}{3} = \frac{0 - 16}{3} = \frac{-16}{3} = -5.3$	$(0, -5.3)$
2	$y = \frac{3(2) - 16}{3} = \frac{6 - 16}{3} = \frac{-10}{3} = -3.3$	$(2, -3.3)$
-4	$y = \frac{3(-4) - 16}{3} = \frac{-12 - 16}{3} = \frac{-28}{3} = -9.3$	$(-4, -9.3)$
4	$y = \frac{3(4) - 16}{3} = \frac{12 - 16}{3} = \frac{-4}{3} = -1.3$	$(4, -1.3)$

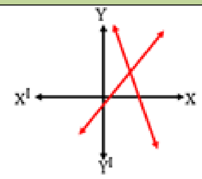
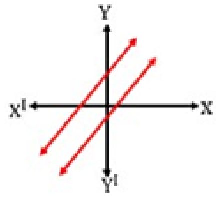
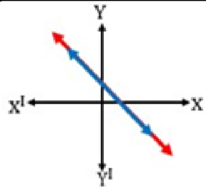


Y-axis
scale
on X-axis 1cm = 1 unit
on Y-axis 1cm = 1 unit
X- axis
(4, -1.3)
(1, -4.3)
(0, -5.3)
(4, -4)
(2, -6)
(0, -8)
(-4, -9.3)
(-2, -10)

CHAPTER 4

X CLASS-2023-24
PAIR OF LINEAR EQUATIONS IN TWO VARIABLES (NOTES)
PREPARED BY: BALABHADRA SURESH

1. An equation of the form $ax + by + c = 0$ where a, b, c are real numbers and where at least one of a or b is not zero (i.e. $a^2 + b^2 \neq 0$), is called a linear equation in two variables x and y
2. Two linear equations in the same two variables are called a pair of linear equations in two variables
   $a_1x + b_1y + c_1 = 0({a_1}^2 + {b_1}^2 \neq 0)$
   $a_2x + b_2y + c_2 = 0({a_2}^2 + {b_2}^2 \neq 0)$
3.

Comparison of ratios	Graphical representation	Algebraic interpretation	Solution	Graph
$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$	Intersecting lines	Consistent	Unique solution	
$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$	Parallel lines	In consistent	No solution	
$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$	Coincident lines	Consistent	Infinite number of solutions	

TRY THIS

1. **For what value of 'p' the following pair of equations has a unique solution.** $\boldsymbol{2x + py = -5}$ **and** $\boldsymbol{3x + 3y = -6}$

Sol: $2x + py = -5$

$2x + py + 5 = 0$; $a_1 = 2,\ b_1 = p, c_1 = 5$

$3x + 3y = -6$

$3x + 3y + 6 = 0$; $a_2 = 3, b_2 = 3, c_2 = 6$

If given pair of equations has unique solution then

$\frac{a_1}{a_2} \neq \frac{b_1}{b_2} \Rightarrow \frac{2}{3} \neq \frac{p}{3} \Rightarrow p \neq 2$

Hence, given lines have unique solution for all real values of p except 2.

2. **Find the value of 'k' for which the pair of equations $2x - ky + 3 = 0, 4x + 6y - 5 = 0$ represents parallel lines**

Sol: $2x - ky + 3 = 0$; $a_1 = 2,\ b_1 = -k,\ c_1 = 3$

$4x + 6y - 5 = 0$; $a_2 = 4,\ b_2 = 6,\ c_2 = -5$

If given pair of equations represent parallel lines then

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \Rightarrow \frac{2}{4} = \frac{-k}{6}$$

$$\Rightarrow -k \times 4 = 2 \times 6$$

$$\Rightarrow -k = \frac{12}{4} = 3$$

$$\Rightarrow k = -3$$

3. **For what value of 'k', the pair of equation $3x + 4y + 2 = 0$ and $9x + 12y + k = 0$ represents coincident lines.**

Sol: $3x + 4y + 2 = 0$; $a_1 = 3,\ b_1 = 4,\ c_1 = 2$

$9x + 12y + k = 0$; $a_2 = 9,\ b_2 = 12,\ c_2 = k$

If given pair of equations represents coincident lines then

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} \Rightarrow \frac{3}{9} = \frac{4}{12} = \frac{2}{k}$$

$$\Rightarrow k \times 3 = 2 \times 9$$

$$\Rightarrow k = \frac{18}{3} = 6$$

4. **For what positive values of 'p' the following pair of liner equations have infinitely many solutions? $px + 3y - (p - 3) = 0$; $12x + py - p = 0$**

Sol: $px + 3y - (p - 3) = 0$; $a_1 = p,\ b_1 = 3,\ c_1 = -(p-3)$

$12x + py - p = 0$; $a_2 = 12,\ b_2 = p,\ c_2 = -p$

If given pair of liner equations have infinitely many solutions then

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} \Rightarrow \frac{p}{12} = \frac{3}{p} = \frac{-(p-3)}{-p}$$

$$\Rightarrow \frac{p}{12} = \frac{3}{p}$$

$$\Rightarrow p^2 = 36$$

$$\Rightarrow p = 6$$

Example-4. In a garden there are some bees and flowers. If one bee sits on each flower then one bee will be left. If two bees sit on each flower, one flower will be left. Find the number of bees and number of flowers.

Solu: Let the number of bees = x and the number of flowers = y

If one bee sits on each flower then one bee will be left.

So, x = y + 1

$x - y - 1 = 0 \rightarrow (1)$

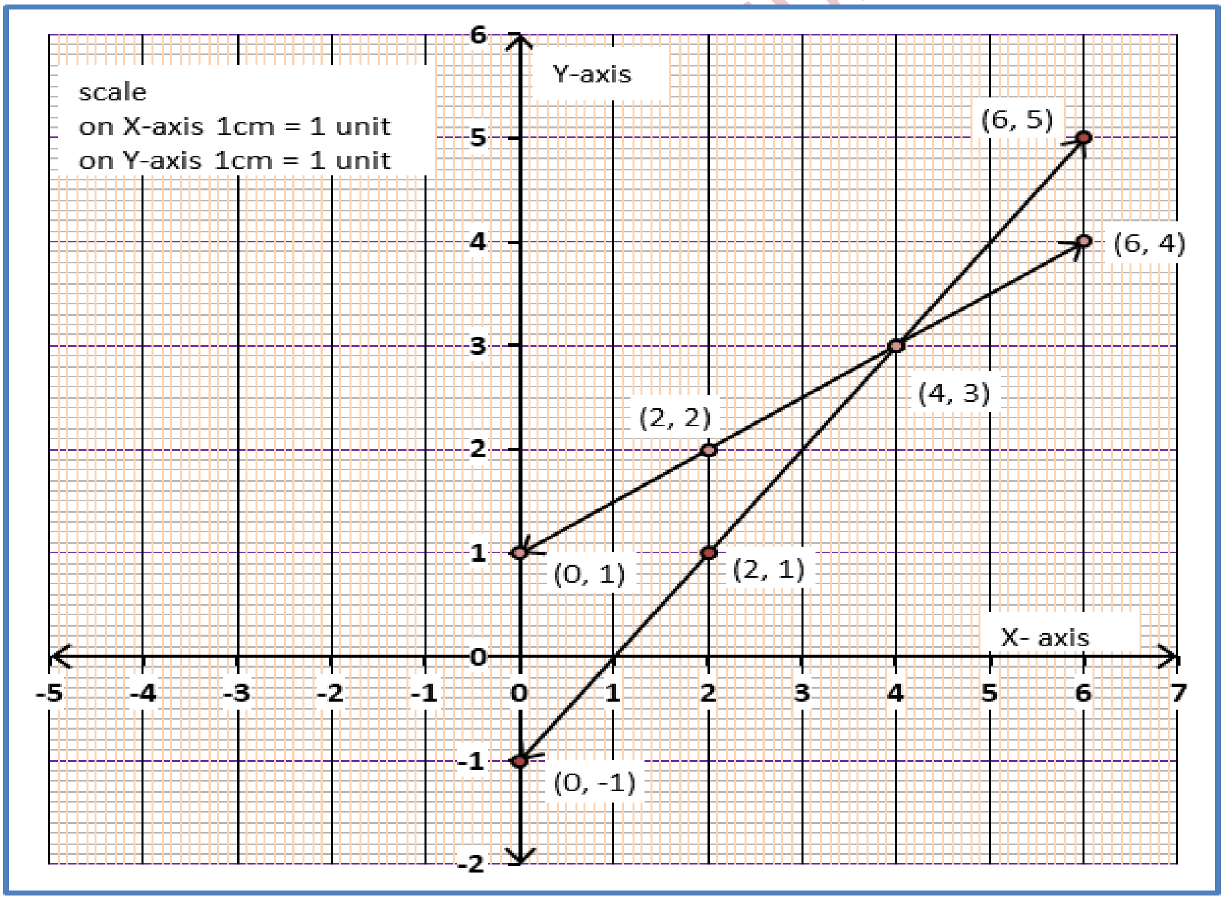
If two bees sit on each flower, one flower will be left.

So, $x = 2(y - 1)$

$x - 2y + 2 = 0 \rightarrow (2)$

$x - y - 1 = 0 \Rightarrow y = x - 1$		
x	$y = x - 1$	(x, y)
0	$y = 0 - 1 = -1$	$(0, -1)$
2	$y = 2 - 1 = 1$	(2,1)
4	$y = 4 - 1 = 3$	(4,3)
6	$y = 6 - 1 = 5$	(6,5)

$x - 2y + 2 = 0 \Rightarrow 2y = x + 2 \Rightarrow y = \frac{x+2}{2}$		
x	$y = \frac{x+2}{2}$	(x, y)
0	$y = \frac{0+2}{2} = \frac{2}{2} = 1$	(0,1)
2	$y = \frac{2+2}{2} = \frac{4}{2} = 2$	(2,2)
4	$y = \frac{4+2}{2} = \frac{6}{2} = 3$	(4,3)
6	$y = \frac{6+2}{2} = \frac{8}{2} = 4$	(6,4)



Solution (4,3)

Therefore, there are 4 bees and 3 flowers.

Example-5. The perimeter of a rectangular plot is 32m. If the length is increased by 2m and the breadth is decreased by 1m, the area of the plot remains the same. Find the length and breadth of

the plot.

Sol: Let length $= l$ and breadth $= b$

Perimeter $= 2(\mathrm{l}+\mathrm{b}) = 32$ m

$l + b = 16 \ \rightarrow (1)$

If the length is increased by 2m and the breadth is decreased by 1m

New length $= L = (l+2)m$ and breadth $= B = (b-1)m$

The area of the plot remains the same

$L \times B = l \times b$

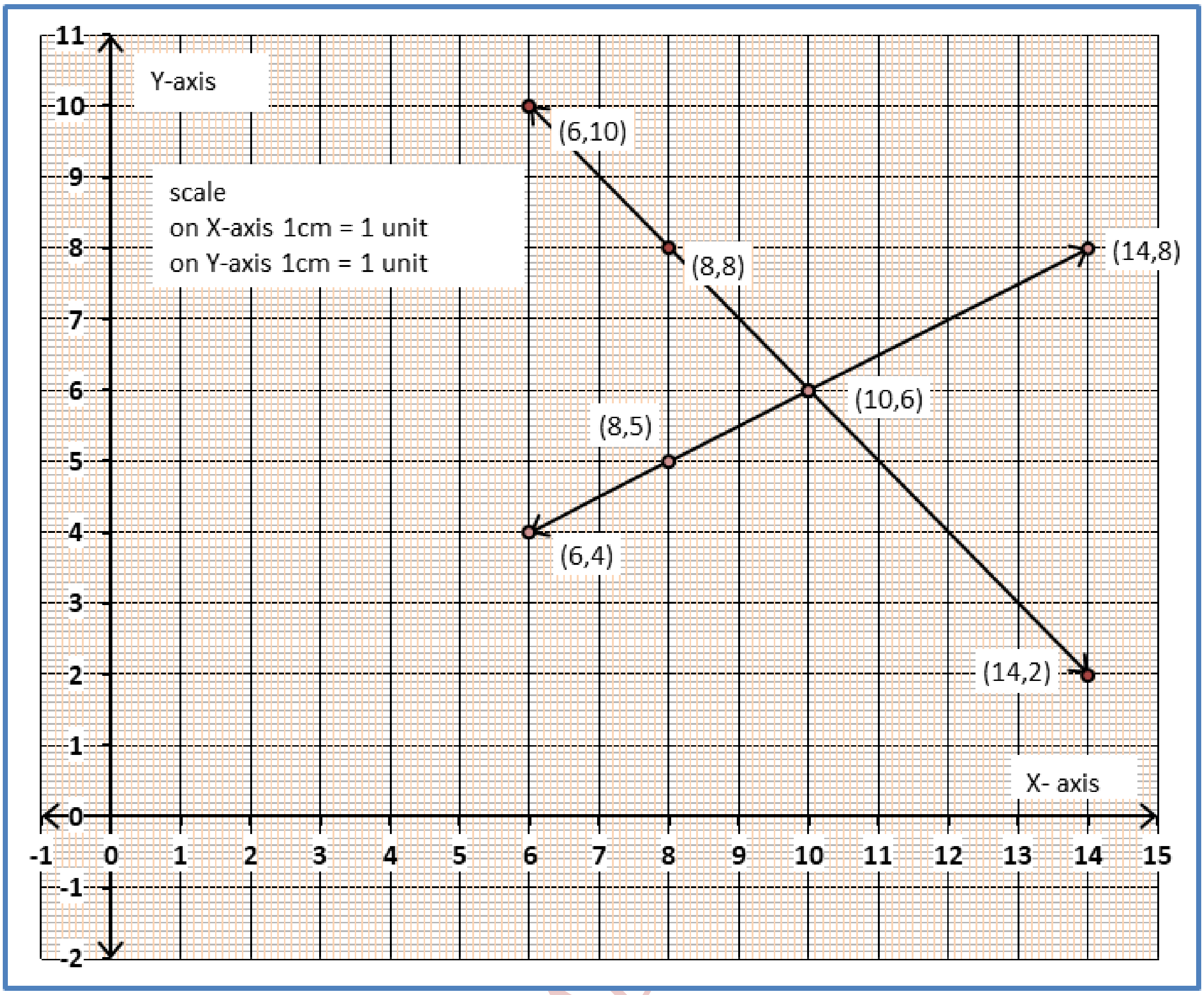
$(l+2)(b-1) = lb$

$lb - l + 2b - 2 = lb$

$l - 2b + 2 = 0 \rightarrow (2)$

$l + b = 16 \Rightarrow l = 16 - b$		
x	$b = 16 - l$	(l, b)
6	$b = 16 - 6 = 10$	(6,10)
8	$b = 16 - 8 = 8$	(8,8)
10	$b = 16 - 10 = 6$	(10,6)
14	$b = 16 - 14 = 2$	(14,2)

$l - 2b + 2 = 0 \Rightarrow 2b = l + 2 \Rightarrow b = \frac{l+2}{2}$		
l	$b = \frac{l+2}{2}$	(l, b)
6	$y = \frac{6+2}{2} = \frac{8}{2} = 4$	(6,4)
8	$y = \frac{8+2}{2} = \frac{10}{2} = 5$	(8,5)
10	$y = \frac{10+2}{2} = \frac{12}{2} = 6$	(10,6)
14	$y = \frac{14+2}{2} = \frac{16}{2} = 8$	(14,8)



So, original length of the plot is 10m and its breadth is 6m

Exercise - 4.1

1. **By comparing the ratios $\frac{a_1}{a_2}, \frac{b_1}{b_2}, \frac{c_1}{c_2}$ find out whether the lines represented by the following pairs of linear equations intersect at a point, are parallel or are coincident.**

a) $5x - 4y + 8 = 0;\ 7x + 6y - 9 = 0$

Sol: $5x - 4y + 8 = 0$; $(a_1 = 5,\ b_1 = -4, c_1 = 8)$

$7x + 6y - 9 = 0$; $(a_2 = 7,\ b_2 = 6,\ c_2 = -9)$

$\frac{a_1}{a_2} = \frac{5}{7};\quad \frac{b_1}{b_2} = \frac{-4}{6} = \frac{-2}{3};\quad \frac{c_1}{c_2} = \frac{8}{-9} = \frac{-8}{9}$

$\frac{a_1}{a_2} \neq \frac{b_1}{b_2} \Rightarrow$ *Given pairs of linear equations intersect at a point.*

b) $\boldsymbol{9x + 3y + 12 = 0;\ 18x + 6y + 24 = 0}$

Sol: $9x + 3y + 12 = 0$; $a_1 = 9,\ b_1 = 3, c_1 = 12$

$18x + 6y + 24 = 0$; $a_2 = 18,\ b_2 = 6,\ c_2 = 24$

$\frac{a_1}{a_2} = \frac{9}{18} = \frac{1}{2};\quad \frac{b_1}{b_2} = \frac{3}{6} = \frac{1}{2};\quad \frac{c_1}{c_2} = \frac{12}{24} = \frac{1}{2}$

$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} \Rightarrow Given$ pairs of linear equations are coincident

c) $\mathbf{6x - 3y + 10 = 0;\ 2x - y + 9 = 0}$

Sol: $6x - 3y + 10 = 0$; $a_1 = 6,\ b_1 = -3, c_1 = 10$

$2x - y + 9 = 0$; $a_2 = 2,\ b_2 = -1,\ c_2 = 9$

$\frac{a_1}{a_2} = \frac{6}{2} = 3;$ $\frac{b_1}{b_2} = \frac{-3}{-1} = 3;$ $\frac{c_1}{c_2} = \frac{10}{9}$

$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2} \Rightarrow Given$ pairs of linear equations are parallel

ALGEBRAIC METHODS OF FINDING THE SOLUTIONS FOR A PAIR OF LINEAR EQUATIONS

SUBSTITUTION METHOD

Do This

Solve each pair of equation by using the substitution method.

1. $\mathbf{3x - 5y = -1;\ x - y = -1}$

Sol: $3x - 5y = -1 \rightarrow (1)$

$x - y = -1 \rightarrow (2)$

From (2) : $x = -1 + y$

Substituting in equation (1) we get

$3(-1 + y) - 5y = -1$

$-3 + 3y - 5y = -1$

$-2y = -1 + 3$

$-2y = 2$

$y = \frac{2}{-2} = -1$

Substitute $y = -1$ in equation (2)

$x = -1 - 1 = -2$

Therefore, required solution is $x = -2$ *and* $y = -1$.

2. $\mathbf{x + 2y = -1;\ 2x - 3y = 12}$

Sol: $x + 2y = -1 \rightarrow (1)$

$2x - 3y = 12 \rightarrow (2)$

From (1): $x = -1 - 2y$

Substituting in equation (2) we get

$2(-1 - 2y) - 3y = 12$

$-2 - 4y - 3y = 12$

$-7y = 12 + 2 = 14$

$y = \frac{14}{-7} = -2$

Substitute $y = -2$ in equation (1)

$x = -1 - 2y = -1 - 2 \times (-2) = -1 + 4 = 3$

Therefore, required solution is $x = 3$ *and* $y = -2$

3. $\mathbf{2x + 3y = 9;\ 3x + 4y = 5}$

Sol: $2x + 3y = 9 \rightarrow (1)$

$3x + 4y = 5 \rightarrow (2)$

From (1): $3y = 9 - 2x$

$$y = \frac{9 - 2x}{3}$$

Substituting in equation (2) we get

$$3x + 4\left(\frac{9 - 2x}{3}\right) = 5$$

$$\frac{9x + 36 - 8x}{3} = 5$$

$x + 36 = 15$

$x = 15 - 36$

$x = -21$

Substitute $x = -21$ in equation (1)

$$y = \frac{9 - 2x}{3} = \frac{9 - 2(-21)}{3} = \frac{9 + 42}{3} = \frac{51}{3} = 17$$

Therefore, required solution is $x = -21$ *and* $y = 17$

4. $\boldsymbol{x + \frac{6}{y} = 6\ ;\ 3x - \frac{8}{y} = 5}$

Sol: $x + \frac{6}{y} = 6 \rightarrow (1)$

$3x - \frac{8}{y} = 5 \rightarrow (2)$

Let $\frac{1}{y} = a$

$(1) \Rightarrow x + 6a = 6 \rightarrow (3)$

$(2) \Rightarrow 3x - 8a = 5 \rightarrow (4)$

From(3): $x = 6 - 6a$

Substituting in equation (4) we get

$3(6 - 6a) - 8a = 5$

$18 - 18a - 8a = 5$

$-26a = 5 - 18 = -13$

$$a = \frac{-13}{-26} = \frac{1}{2}$$

$$\therefore \frac{1}{y} = \frac{1}{2} \Rightarrow y = 2$$

Substitute $y = 2$ in equation (1)

$$x + \frac{6}{2} = 6 \Rightarrow x + 3 = 6$$

$$\Rightarrow x = 6 - 3 = 3$$

Therefore, required solution is $x = 3$ *and* $y = 2$

5. $\mathbf{0.2x + 0.3y = 1.3;\ 0.4x + 0.5y = 2.3}$

Sol: $0.2x + 0.3y = 1.3$

Multiply with 10

$$2x + 3y = 13 \rightarrow (1)$$

$$0.4x + 0.5y = 2.3$$

Multiply with 10

$$4x + 5y = 23 \rightarrow (2)$$

From(1): $2x = 13 - 3y$

$$\Rightarrow x = \frac{13 - 3y}{2}$$

Substituting in equation (2) we get

$$4\left(\frac{13 - 3y}{2}\right) + 5y = 23$$

$$26 - 6y + 5y = 23$$

$$-y = 23 - 26 = -3$$

$$y = 3$$

Substitute $y = 3$ in (1)

$$x = \frac{13 - 3y}{2} = \frac{13 - 3 \times 3}{2} = \frac{13 - 9}{2} = \frac{4}{2} = 2$$

Therefore, required solution is $x = 2$ *and* $y = 3$

6. $\mathbf{\sqrt{2}x + \sqrt{3}y = 0;\ \sqrt{3}x - \sqrt{8}y = 0}$

Sol: $\sqrt{2}x + \sqrt{3}y = 0 \rightarrow (1)$

$$\sqrt{3}x - \sqrt{8}y = 0 \rightarrow (2)$$

From(1): $\sqrt{3}y = -\sqrt{2}x$

$$y = \frac{-\sqrt{2}x}{\sqrt{3}}$$

Substituting in equation (2) we get

$$\sqrt{3}x - \sqrt{8}\left(\frac{-\sqrt{2}x}{\sqrt{3}}\right) = 0$$

$$\Longrightarrow \sqrt{3}x + \frac{4x}{\sqrt{3}} = 0 \Longrightarrow \frac{3x + 4x}{\sqrt{3}} = 0$$

$$3x + 4x = 0$$

$$7x = 0$$

$$x = 0$$

$$y = \frac{-\sqrt{2}x}{\sqrt{3}} = 0$$

Therefore, required solution is $x = 0$ *and* $y = 0$.

ELIMINATION METHOD

Do This

Solve each of the following pairs of equations by the elimination method.

1. $\boldsymbol{8x + 5y = 9;\ 3x + 2y = 4}$

Sol: $8x + 5y = 9 \rightarrow (1)$

$3x + 2y = 4 \rightarrow (2)$

$2 \times (1) \Rightarrow 16x + 10y = 18$

$5 \times (2) \Rightarrow 15x + 10y = 20$

$\quad (-) \quad (-) \quad (-)$

$\quad x \qquad = -2$

Substitute $x = -2$ in equation (1)

$8(-2) + 5y = 9$

$-16 + 5y = 9$

$5y = 9 + 16$

$5y = 25$

$y = \frac{25}{5} = 5$

Therefore, the required solution is $x = -2, y = 5$.

2. $\boldsymbol{2x + 3y = 8;\ 4x + 6y = 7}$

Sol: $2x + 3y = 8 \rightarrow (1)$

$4x + 6y = 7 \rightarrow (2)$

$2 \times (1) \Rightarrow 4x + 6y = 16$

$1 \times (2) \Rightarrow 4x + 6y = 7$

$0 = 9$ it is not possible.

So, the given pair of equations has no solutions.

3. $\boldsymbol{3x + 4y = 25;\ 5x - 6y = -9}$

Sol: $3x + 4y = 25 \rightarrow (1)$

$5x - 6y = -9 \rightarrow (2)$

$3 \times (1) \Rightarrow 9x + 12y = 75$

$2 \times (2) \Rightarrow 10x - 12y = -18$

$19x \qquad = 57$

$$x = \frac{57}{19} = 3$$

Substitute $x = 3$ in equation (1)

$3(3) + 4y = 25$

$9 + 4y = 25$

$4y = 25 - 9 = 16$

$$y = \frac{16}{4} = 4$$

Therefore, the required solution is $x = 3, y = 4$.

TRY THIS

Solve the given pair of linear equations

$(a - b)x + (a + b)y = a^2 - 2ab - b^2 \rightarrow (1)$

$(a + b)(x + y) = a^2 + b^2 \rightarrow (2)$

$(1) \Rightarrow (a - b)x + (a + b)y = a^2 - 2ab - b^2$

$(2) \Rightarrow (a + b)x + (a + b)y = a^2 + b^2$

$(a - b)x - (a + b)x = a^2 - 2ab - b^2 - a^2 - b^2$

$ax - bx - ax - bx = -2ab - 2b^2$

$-2bx = -2b(a + b)$

$$x = \frac{-2b(a + b)}{-2b} = (a + b)$$

Substitute $x = (a + b)$ in equation (1)

$(a - b)(a + b) + (a + b)y = a^2 - 2ab - b^2$

$a^2 - b^2 + (a + b)y = a^2 - 2ab - b^2$

$(a + b)y = a^2 - 2ab - b^2 - a^2 + b^2$

$(a + b)y = -2ab$

$$y = \frac{-2ab}{a + b}$$

$$Therefore, the\ required\ solution\ is\ x = a + b, y = \frac{-2ab}{a + b}$$

Example-8. Rubina went to a bank to withdraw ₹2000. She asked the cashier to give the cash in ₹50 and ₹100 notes only. Rubina got 25 notes in all. Can you tell how many notes each of D50 and ₹100 she received?

Sol: Let the number of ₹50 notes $= x$

The number of ₹100 notes $= y$

Total notes=25

$x + y = 25 \rightarrow (1)$

Value of notes=₹ 2000

$50x + 100y = 2000$

$x + 2y = 40 \rightarrow (2)$

$(2) \Rightarrow x + 2y = 40$

$(1) \Rightarrow x + \ \ y = 25$

$\ \ (-)\ (-)\ \ (-)$

$y = 15$

Substitute y=15 in equ (1)

$x + 15 = 25$

$x = 25 - 15 = 10$

∴Rubina received ten ₹50 notes and fifteen ₹100 rupee notes.

Example-9. In a competitive exam, 3 marks are to be awarded for every correct answer and for every wrong answer, 1 mark will be deducted. Madhu scored 40 marks in this exam. Had 4 marks been awarded for each correct answer and 2 marks deducted for each incorrect answer, Madhu would have scored 50 marks. How many questions were there in the test? (Madhu attempted all the questions)

Sol: Let the number of correct answers $= x$ and wrong answers $= y$

When 3 marks are given for each correct answer and 1 mark deducted for each wrong answer, his score = 40 marks.

$3x - y = 40 \rightarrow (1)$

If 4 marks were given for each correct answer and 2 marks deducted for each wrong answer his score =50 marks

$4x - 2y = 50 \rightarrow (2)$

$2 \times equ(1) \Rightarrow 6x - 2y = 80$

$equ\ (2) \Rightarrow 4x - 2y = 50$

$2x = 30$

$x = 15$

Substitute $x = 15$ in equ (1)

$3(15) - y = 40$

$45 - y = 40$

$y = 45 - 40 = 5$

∴ Total number of questions = 15 + 5 = 20.

Example-10. Mary told her daughter, "Seven years ago, I was seven times as old as you were then. Also, three years from now, I shall be three times as old as you will be." Find the present age of Mary and her daughter.

Sol:

	Mary's age	daughter's age

Present	x	y
Seven years ago	$x-7$	$y-7$
Three years from now	$x+3$	$y+3$

Seven years ago:

Mary's age=7× daughter's age

$x - 7 = 7(y - 7)$

$x - 7 = 7y - 49$

$x - 7y = -42 \longrightarrow (1)$

Three years from now:

Mary's age=3× daughter's age

$x + 3 = 3(y + 3)$

$x + 3 = 3y + 9$

$x - 3y = 6 \rightarrow (2)$

$equ(1) \Rightarrow x - 7y = -42$

$equ(2) \Rightarrow x - 3y = \quad 6$

$(-) \ (-) \quad (-)$

$-4y = -48$

$$y = \frac{-48}{-12} = 4$$

Substitute the value of y in equation (2)

$x - 3(12) = 6$

$x = 6 + 36 = 42$

∴ Mary's present age is 42 years and her daughter's age is 12 years.

Example-11. A publisher is planning to produce a new textbook. The fixed costs (reviewing, editing, typesetting and so on) are D 31.25 per book. Besides that, he also spends another D 320000 in producing the book. The wholesale price (the amount received by the publisher) is D 43.75 per book. How many books must the publisher sell to break even, i.e., so that the costs will equal revenues?

Sol: Let the number of books printed $= x$

Break even point $= y$

Cost equation is given by : $y = 320000 + 31.25x \rightarrow (1)$

Revenue equation is given by : $y = 43.75x \rightarrow (2)$

From (1) and (2)

$43.75x = 320000 + 31.25x$

$43.75x - 31.25x = 320000$

$12.5x = 320000$

$$x = \frac{320000}{12.5} = 25{,}600$$

Thus, the publisher will break even when 25,600 books are printed and sold

EXERCISE - 4.2

Form a pair of linear equations for each of the following problems and find their solution

1. **The ratio of incomes of two persons is 9 : 7 and the ratio of their expenditures is 4 : 3. If each of them manages to save ₹2000 per month, find their monthly income.**

Sol: The ratio of incomes of two persons = 9 : 7

Let their incomes be $9x$ and $7x$

The ratio of their expenditures = 4 : 3

Let their expenditures be $4y$ and $3y$

Given each of them manages to save ₹2000 per month

$9x - 4y = 2000 \rightarrow (1)$

$7x - 3y = 2000 \rightarrow (2)$

$3 \times (1) \Rightarrow 27x - 12y = 6000$

$4 \times (2) \Rightarrow 28x - 12y = 8000$

$-x = -2000$

$x = 2000$

Substitute $x = 2000$ in (1)

$9(2000) - 4y = 2000$

$18000 - 4y = 2000$

$-4y = 2000 - 18000$

$-4y = -16000$

$4y = 16000$

$y = \frac{16000}{4} = 4000$

Their incomes are 9×2000 *and* 7×2000

$\Rightarrow$ ₹18000 *and* ₹ 14000

2. **The sum of a two digit number and the number obtained by reversing the digits is 66. If the digits of the number differ by 2, find the number. How many such numbers are there?**

Sol: Let the unit place digit be x and tens place digit be y

The number=$10y + x$

The number obtained by reversing the digits=$10x + y$

From the problem

$(10y + x) + (10x + y) = 66$

$11x + 11y = 66$

$x + y = 6 \rightarrow (1)$

Given the digits of the number differ by 2

$x - y = 2 \rightarrow (2)$

$(1) \Rightarrow x + y = 6$

$(2) \Rightarrow x - y = 2$

$$\begin{aligned} 2x &= 8 \end{aligned}$$

$$x = \frac{8}{2} = 4$$

Substitute $x = 4$ in (1)

$4 + y = 6$

$y = 6 - 4 = 2$

The number is $10y + x$ and $10x + y$

$\Rightarrow 10 \times 2 + 4 \quad and \quad 10 \times 4 + 2$

$\Rightarrow 24 \ and \ 42$

3. **The larger of two supplementary angles exceeds the smaller by 18°. Find the angles**

Sol: Let the two supplementary angles be x and y $(x > y)$

$x + y = 180^0 \rightarrow (1)$

From problem

$x = y + 18^0$

$x - y = 18^0 \rightarrow (2)$

$(1) + (2) \Rightarrow x + y = 180^0$

$$\begin{aligned} x - y &= 18^0 \\ \hline 2x &= 198^0 \end{aligned}$$

$$x = \frac{198}{2} = 99^0$$

Substitute x value in (1)

$99^0 + y = 180^0$

$y = 180^0 - 99^0 = 81^0$

The angles are $99^0 \ and \ 81^0$

4. **The taxi charges in Hyderabad are fixed, along with the charge for the distance covered. For a distance of 10 km., the charge paid is D220. For a journey of 15 km. the charge paid is ₹310. i. What are the fixed charges and charge per km?**

ii. How much does a person have to pay for travelling a distance of 25 km?

Sol: Let the fixed charge=x

Let the charge for 1km=y

The charge paid for 10 km=220

$x + 10y = 220 \rightarrow (1)$

The charge paid for 15 km=220

$x + 15y = 310 \rightarrow (2)$

$(2) - (1) \Rightarrow x + 15y = 310$

$$\begin{aligned} x + 10y &= 220 \\ (-)\ (-) \quad &\quad (-) \end{aligned}$$

$$5y \quad = 90$$

$$y = \frac{90}{5} = 18$$

Substitute y=18 in (1)

$x + 10(18) = 220$

$x + 180 = 220$

$x = 220 - 180 = 40$

∴ Fixed charge= x = ₹40

Charge for 1 km= y = ₹18

ii. Charge for travelling a distance of 25 km=$x + 25y = 40 + 25 \times 18 = 40 + 450 =$ ₹490

5. **A fraction becomes $\frac{4}{5}$ if 1 is added to both numerator and denominator. If, however, 5 is subtracted from both numerator and denominator, the fraction becomes $\frac{1}{2}$. What is the fraction?**

Sol: Let the fraction=$\frac{x}{y}$

From the problem

$\frac{x+1}{y+1} = \frac{4}{5}$ *and* $\frac{x-5}{y-5} = \frac{1}{2}$

$\Rightarrow 5(x+1) = 4(y+1)$ *and* $2(x-5) = 1(y-5)$

$\Rightarrow 5x + 5 = 4y + 4$ *and* $2x - 10 = y - 5$

$\Rightarrow 5x - 4y = 4 - 5$ *and* $2x - y = -5 + 10$

$\Rightarrow 5x - 4y = -1 \rightarrow (1)$ *and* $2x - y = 5 \rightarrow (2)$

$(1) \Rightarrow 5x - 4y = -1$

$4 \times (2) \Rightarrow 8x - 4y = 20$

$(-)\ (+)\ \ (-)$

$-3x = -21$

$3x = 21$

$$x = \frac{21}{3} = 7$$

Substitute $x = 7$ in (2)

$2(7) - y = 5$

$14 - y = 5$

$y = 14 - 5 = 9$

∴ The fraction $= \frac{7}{9}$

6. **Places A and B are 100 km apart on a highway. One car starts from A and another from B at the same time at different speeds. If the cars travel in the same direction, they meet in 5 hours. If they travel towards each other, they meet in 1 hour. What are the speeds of the two cars?**

Sol: Let the speed of car starts form A=x km/h

Let the speed of car starts form B=y km/h

$Time \times speed = Distance$

When the cars travelled in same direction relative speed= $(x - y)$,they meet in 5 hours.

$5(x - y) = 100 \Rightarrow x - y = 20 \rightarrow (1)$

When the cars travelled towards each other relative speed= $(x + y)$, they meet in 1hour.

$1(x + y) = 100 \Rightarrow x + y = 100 \rightarrow (2)$

$(1) + (2) \Rightarrow 2x = 120$

$$x = \frac{120}{2} = 60$$

Substitute $x = 60$ in (2)

$60 + y = 100$

$y = 100 - 60 = 40$

∴ The speed of first car=60 km/h and second car=40 km/h.

7. **Two angles are complementary. The larger angle is 3° less than twice the measure of the smaller angle. Find the measure of each angle.**

Sol: Let the larger angle = x and smaller angle= y

Sum of complementary angles=90^0

$x + y = 90^0 \rightarrow (1)$

From the problem

$x = 2y - 3^0$

$x - 2y = -3^0 \rightarrow (2)$

$(1) - (2) \Rightarrow x + y = 90^0$

$x - 2y = -3^0$

$3y = 93^0$

$$y = \frac{93^0}{3} = 31^0$$

Substitute $y = 31^0$ in (1)

$x + 31^0 = 90^0$

$x = 90^0 - 31^0 = 59^0$

∴ $\textit{The angles are } 59^0 \textit{ and } 31^0$

8. **An algebra textbook has a total of 1382 pages. It is broken up into two parts. The second part of the book has 64 pages more than the first part. How many pages are in each part of the book?**

Sol: Let the pages in first part= x

Let the pages in second part= y

Total pages= 1382

$x + y = 1382 \rightarrow (1)$

$y = x + 64$

$x - y = -64 \rightarrow (2)$

$(1) + (2) \Rightarrow x + y + x - y = 1382 - 64$

$2x = 1318$

$x = \frac{1318}{2} = 659$

Substitute $x = 659$ in (1)

$659 + y = 1382$

$y = 1382 - 659 = 723$

∴ The number of pages in the first part=659.

The number of pages in the second part=723.

9. **A chemist has two solutions of hydrochloric acid in stock. One is 50% solution and the other is 80% solution. How much of each should be used to obtain 100ml of a 68% solution**

Sol: Let the first solution= $x\ ml$ and second solution $= y\ ml$

$x + y = 100 \rightarrow (1)$

From problem

$50\% of\ x + 80\%\ of\ y = 68\%\ of\ 100$

$\frac{50x}{100} + \frac{80y}{100} = \frac{68}{100} \times 100$

$50x + 80y = 6800$

$5x + 8y = 680 \rightarrow (2)$

$8 \times (1) \Rightarrow 8x + 8y = 800$

$(2) \Rightarrow 5x + 8y = 680$

$(-)\ (+)\ (-)$

$3x = 120$

$x = \frac{120}{3} = 40$

Substitute x=40 in (1)

$40 + y = 100$

$y = 100 - 40 = 60$

∴ The first solution=40 ml

The second solution=60 ml

10. **Suppose you have ₹12000 to invest. You have to invest some amount at 10% and the rest at 15%. How much should be invested at each rate to yield 12% on the total amount invested?**

Sol: Let the amount to be saved at 10%=x

Let the amount to be saved at 15%=y

From problem

$x + y = 12000 \rightarrow (1)$

$10\%\ of\ x + 15\%\ of\ y = 12\%\ of\ 12000$

$\frac{10x}{100} + \frac{15y}{100} = \frac{12}{100} \times 12000$

$10x + 15y = 144000 \rightarrow (2)$

$(2) \Rightarrow 10x + 15y = 144000$

$10 \times (1) \Rightarrow 10x + 10y = 120000$

$\quad (-) \quad (-) \qquad (-)$

$\qquad 5y \quad = 24000$

$$y = \frac{24000}{5} = 4800$$

Substitute y=4800 in (1)

$x + 4800 = 12000$

$x = 12000 - 4800 = 7200$

The amount saved at 10%= ₹7200

The amount saved in 15%= ₹4800

Example-12. Solve the following pair of equations.

$\frac{2}{x} + \frac{3}{y} = 13\ and\ \frac{5}{x} - \frac{4}{y} = -2$

Sol: $\frac{2}{x} + \frac{3}{y} = 13; \qquad \frac{5}{x} - \frac{4}{y} = -2$

$2\left(\frac{1}{x}\right) + 3\left(\frac{1}{y}\right) = 13; \quad 5\left(\frac{1}{x}\right) - 4\left(\frac{1}{y}\right) = -2$

$Let\ \frac{1}{x} = a\ \ and\ \ \frac{1}{y} = b$

$2a + 3b = 13 \rightarrow (1)$

$5a - 4b = -2 \rightarrow (2)$

$4 \times (1) \Rightarrow 8a + 12b = 52$

$3 \times (2) \Rightarrow 15a - 12b = -6$

$23a = 46$

$a = \frac{46}{23} = 2$

Substitute $a = 2$ in (1)

$2 \times 2 + 3b = 13$

$4 + 3b = 13$

$3b = 13 - 4$

$3b = 9$

$b = 3$

$But\ \frac{1}{x} = a \Rightarrow \frac{1}{x} = 2 \Rightarrow x = \frac{1}{2}$

$\frac{1}{y} = b \Rightarrow \frac{1}{y} = 3 \Rightarrow y = \frac{1}{3}$

Solution $x = \frac{1}{2}$ and $y = \frac{1}{3}$

Example-13. Kavitha thought of constructing 2 more rooms in her house. She enquired about the labour. She came to know that 6 men and 8 women could finish this work in 14 days. But she wish to complete the work in 10 days. When she enquired, she was told that 8 men and 12 women could finish the work in 10 days. Find out that how much time would be taken to finish the work if one man or one woman worked alone?

Solu : Let the time taken by one man to finish the work = x days.

Work done by one man in one day $= \frac{1}{x}$

Let the time taken by one woman to finish the work = y days.

Work done by one woman in one day $= \frac{1}{y}$

Now, 8 men and 12 women can finish the work in 10 days.

So work done by 8 men and 12 women in one day $= \frac{1}{10}$

$8 \times \frac{1}{x} + 12 \times \frac{1}{y} = \frac{1}{10}$

$\frac{8}{x} + \frac{12}{y} = \frac{1}{10} \Rightarrow 10\left(\frac{8}{x} + \frac{12}{y}\right) = 1 \Rightarrow \frac{80}{x} + \frac{120}{y} = 1 \rightarrow (1)$

Also, 6 men and 8 women can finish the work in 14 days.

$\frac{6}{x} + \frac{8}{y} = \frac{1}{14} \Rightarrow 14\left(\frac{6}{x} + \frac{8}{y}\right) = 1 \Rightarrow \frac{84}{x} + \frac{112}{y} = 1 \rightarrow (2)$

$Let\ \frac{1}{x} = a\ and\ \frac{1}{y} = b$

$Equ\ (1) \Rightarrow 80a + 120b = 1 \rightarrow (3)$

$Equ\ (2) \Rightarrow 84a + 112b = 1 \rightarrow (4)$

LCM of 120,112=1680

$Equ\ (4) \times 15 \Rightarrow 1260a + 1680b = 15$

$Equ\ (3) \times 14 \Rightarrow 1120a + 1680b = 14$

$(-) \quad (-) \quad (-)$

$140a = 1$

$a = \frac{1}{140}$

$$Substitute\ a = \frac{1}{140}\ in\ equ(3)$$

$$80 \times \frac{1}{140} + 120b = 1$$

$$\frac{4}{7} + 120b = 1$$

$$120b = 1 - \frac{4}{7} = \frac{7-4}{7} = \frac{3}{7}$$

$$b = \frac{3}{7 \times 120} = \frac{1}{7 \times 40} = \frac{1}{280}$$

$$\frac{1}{x} = \frac{1}{140}\ \ and\ \frac{1}{y} = \frac{1}{280}$$

$$x = 140\ and\ y = 280$$

So one man alone can finish the work in 140 days and one woman alone can finish the work in 280 days.

Example-14. A man travels 370 km partly by train and partly by car. If he covers 250 km by train and the rest by car, it takes him 4 hours. But if he travels 130 km by train and the rest by car, it takes 18 minutes more. Find the speed of the train and that of the car.

Sol: Let the speed of the train $= x\ km/hr$.

Let the speed of the car $= y\ km/hr$

Distance covered by train=250km

$$Time = \frac{Distance}{Speed}$$

Time spent travelling by train $= \frac{250}{x}\ hrs$

Distance covered by car=370-250=120km

Time spent travelling by car $= \frac{120}{y}\ hrs$

Total time taken=4 hours

$$\frac{250}{x} + \frac{120}{y} = 4 \to (1)$$

Similarly

$$\frac{130}{x} + \frac{240}{y} = 4\frac{3}{10} \to (2)$$

$$18\ minutes = \frac{18}{60}hrs = \frac{3}{10}hrs$$

$$Let\ \frac{1}{x} = a\ \ and\ \frac{1}{y} = b$$

$$Equ\ (1) \Rightarrow 250a + 120b = 4 \to (3)$$

$$Equ\ (2) \Rightarrow 130a + 240b = \frac{43}{10} \to (4)$$

$$Equ\ (3) \times 2 \Rightarrow 500a + 240b = 8$$

$$Equ\ (4) \times 1 \Rightarrow 130a + 240b = \frac{43}{10}$$

$$(-) \quad (-) \quad\quad (-)$$

$$370a = 8 - \frac{43}{10}$$

$$370a = \frac{37}{10}$$

$$a = \frac{37}{10 \times 370} = \frac{1}{100}$$

$$Substitute\ a = \frac{1}{100}\ in\ equ(3)$$

$$250 \times \frac{1}{100} + 120b = 4$$

$$\frac{5}{2} + 120b = 4$$

$$120b = 4 - \frac{5}{2} = \frac{8-5}{2} = \frac{3}{2}$$

$$b = \frac{3}{2 \times 120} = \frac{1}{80}$$

$$\frac{1}{x} = \frac{1}{100}\ \ and\ \frac{1}{y} = \frac{1}{80}$$

$$x = 100\ and\ y = 80$$

So, speed of train was 100 km/hr and speed of car was 80 km/hr.

EXERCISE - 4.3

Solve each of the following pairs of equations by reducing them to a pair of linear equations.

1. $\mathbf{\frac{5}{x-1} + \frac{1}{y-2} = 2\ ;\ \frac{6}{x-1} - \frac{3}{y-2} = 1}$

Sol:

$$5\left(\frac{1}{x-1}\right) + 1\left(\frac{1}{y-2}\right) = 2 \rightarrow (1) \qquad 6\left(\frac{1}{x-1}\right) - 3\left(\frac{1}{y-2}\right) = 1 \rightarrow (2)$$

$$Let\ \frac{1}{x-1} = a\ \ and\ \ \frac{1}{y-2} = b$$

$$5a + b = 2 \rightarrow (3) \qquad 6a - 3b = 1 \rightarrow (4)$$

$$3 \times (3) \Rightarrow 15a + 3b = 6$$

$$(4) \Rightarrow 6a - 3b = 1$$

$$21a = 7$$

$$a = \frac{7}{21} = \frac{1}{3}$$

$$Substitute\ a = \frac{1}{3}\ in\ equation\ (4)$$

$$6\left(\frac{1}{3}\right) - 3b = 1$$

$2-3b=1$

$-3b=1-2$

$-3b=-1$

$b=\frac{-1}{-3}=\frac{1}{3}$

$Now\ a=\frac{1}{3}\Rightarrow\frac{1}{x-1}=\frac{1}{3}$

$\Rightarrow x-1=3$

$\Rightarrow x=3+1=4$

$b=\frac{1}{3}\Rightarrow\frac{1}{y-2}=\frac{1}{3}$

$\Rightarrow y-2=3$

$\Rightarrow y=3+2=5$

Solution $x=4\ and\ y=5$

ii) $\boldsymbol{\frac{x+y}{xy}=2,\ \ \frac{x-y}{xy}=6}$

$Sol:\ \frac{x+y}{xy}=2\ \Rightarrow\frac{x}{xy}+\frac{y}{xy}=2\Rightarrow\frac{1}{y}+\frac{1}{x}=2\rightarrow(1)$

$\frac{x-y}{xy}=6\ \Rightarrow\frac{x}{xy}-\frac{y}{xy}=6\Rightarrow\frac{1}{y}-\frac{1}{x}=6\rightarrow(2)$

$Let\ \frac{1}{y}=a\ and\ \frac{1}{x}=b$

$(1)\Rightarrow a+b=2\rightarrow(3)$

$(2)\Rightarrow a-b=6\rightarrow(4)$

$(3)+(4)\Rightarrow a+b=2$

$a-b=6$

$2a=8$

$\Rightarrow a=\frac{8}{2}=4$

Substitute $a=4$ in equation (3)

$4+b=2$

$b=2-4=-2$

$But\ \frac{1}{x}=b\Rightarrow\frac{1}{x}=-2\Rightarrow x=\frac{-1}{2}$

$\frac{1}{y}=a\Rightarrow\frac{1}{y}=4\Rightarrow y=\frac{1}{4}$

Solution $x=\frac{-1}{2}$ and $y=\frac{1}{4}$

iii) $\boldsymbol{\frac{2}{\sqrt{x}}+\frac{3}{\sqrt{y}}=2;\frac{4}{\sqrt{x}}-\frac{9}{\sqrt{y}}=-1}$

$Sol: \frac{2}{\sqrt{x}} + \frac{3}{\sqrt{y}} = 2 \Rightarrow 2\left(\frac{1}{\sqrt{x}}\right) + 3\left(\frac{1}{\sqrt{y}}\right) = 2 \rightarrow (1)$

$\frac{4}{\sqrt{x}} - \frac{9}{\sqrt{y}} = -1 \Rightarrow 4\left(\frac{1}{\sqrt{x}}\right) - 9\left(\frac{1}{\sqrt{y}}\right) = -1 \rightarrow (2)$

$Let \frac{1}{\sqrt{x}} = a \ \ and \ \frac{1}{\sqrt{y}} = b$

$(1) \Rightarrow 2a + 3b = 2 \rightarrow (3)$

$(2) \Rightarrow 4a - 9b = -1 \rightarrow (4)$

$3 \times (3) \Rightarrow 6a + 9b = 6$

$(4) \Rightarrow 4a - 9b = -1$

$10a \quad = 5$

$a = \frac{5}{10} = \frac{1}{2}$

Substitute $a = \frac{1}{2}$ in (3)

$2 \times \frac{1}{2} + 3b = 2$

$1 + 3b = 2$

$3b = 1 \Rightarrow b = \frac{1}{3}$

$Now \ \frac{1}{\sqrt{x}} = a \ \Rightarrow \frac{1}{\sqrt{x}} = \frac{1}{2} \Rightarrow \sqrt{x} = 2 \Rightarrow x = 4$

$\frac{1}{\sqrt{y}} = b \Rightarrow \frac{1}{\sqrt{y}} = \frac{1}{3} \Rightarrow \sqrt{y} = 3 \Rightarrow y = 9$

Solution is $x = 4$ and $y = 9$

iv) $\boldsymbol{6x + 3y = 6xy; 2x + 4y = 5xy}$

Sol:

$6x + 3y = 6xy$	$2x + 4y = 5xy$
$\frac{6x}{xy} + \frac{3y}{xy} = \frac{6xy}{xy},$	$\frac{2x}{xy} + \frac{4y}{xy} = \frac{5xy}{xy},$
$\frac{6}{y} + \frac{3}{x} = 6$	$\frac{2}{y} + \frac{4}{x} = 5$
$6\left(\frac{1}{y}\right) + 3\left(\frac{1}{x}\right) = 6$	$2\left(\frac{1}{y}\right) + 4\left(\frac{1}{x}\right) = 5$

$Let \ \frac{1}{x} = a \ \ and \ \frac{1}{y} = b \ then$

$6b + 3a = 6 \rightarrow (1)$

$2b + 4a = 5 \rightarrow (2)$

$3 \times (2) \Rightarrow 6b + 12a = 15$

$$(1) \Rightarrow 6b + 3a \quad = 6$$
$$\underline{\text{v}(-)\ (-) \qquad (-)}$$
$$9a \qquad = 9$$

$$a = 1$$

Substitute $a = 1$ in (1)

$6b + 3 \times 1 = 6$

$6b = 6 - 3 = 3$

$b = \frac{3}{6} = \frac{1}{2}$

$But\ \frac{1}{x} = a \ \Rightarrow \frac{1}{x} = 1 \Rightarrow x = 1$

$\frac{1}{y} = b \Rightarrow \frac{1}{y} = \frac{1}{2} \Rightarrow y = 2$

Solution $x = 1 \ \ and \ \ y = 2$

$\boldsymbol{v)}\ \boldsymbol{\frac{5}{x+y} - \frac{2}{x-y} = -1; \frac{15}{x+y} + \frac{7}{x-y} = 10}$

Sol: $\frac{5}{x+y} - \frac{2}{x-y} = -1;$ $\qquad \frac{15}{x+y} + \frac{7}{x-y} = 10$

$5\left(\frac{1}{x+y}\right) - 2\left(\frac{1}{x-y}\right) = -1; \quad 15\left(\frac{1}{x+y}\right) + 7\left(\frac{1}{x-y}\right) = 10$

$Let \frac{1}{x+y} = a \ \ and \ \ \frac{1}{x-y} = b \ \ then$

$5a - 2b = -1 \rightarrow (1)$

$15a + 7b = 10 \rightarrow (2)$

$7 \times (1) \Rightarrow 35a - 14b = -7$

$2 \times (2) \Rightarrow 30a + 14b = 20$

$$65a \qquad = 13$$

$$a = \frac{13}{65} = \frac{1}{5}$$

Substitute $a = \frac{1}{5}$ in (2)

$15 \times \left(\frac{1}{5}\right) + 7b = 10$

$3 + 7b = 10$

$7b = 10 - 3 = 7$

$b = 1$

$But\ \frac{1}{x+y} = a \ \Rightarrow \frac{1}{x+y} = \frac{1}{5} \Rightarrow x + y = 5 \rightarrow (4)$

$\frac{1}{x-y} = b \Rightarrow \frac{1}{x-y} = 1 \Rightarrow x - y = 1 \rightarrow (5)$

$(4)+(5) \Rightarrow 2x = 6 \Rightarrow x = 3$

Substitute x=3 in (4)

$3 + y = 5 \Rightarrow y = 5 - 3 = 2$

Solution $x = 3$ and $y = 2$

vi) $\boldsymbol{\frac{2}{x} + \frac{3}{y} = 13; \frac{5}{x} - \frac{4}{y} = -2}$

Sol: $\frac{2}{x} + \frac{3}{y} = 13; \quad \frac{5}{x} - \frac{4}{y} = -2$

$2\left(\frac{1}{x}\right) + 3\left(\frac{1}{y}\right) = 13; \quad 5\left(\frac{1}{x}\right) - 4\left(\frac{1}{y}\right) = -2$

$Let\ \frac{1}{x} = a\ \ and\ \ \frac{1}{y} = b$

$2a + 3b = 13 \rightarrow (1)$

$5a - 4b = -2 \rightarrow (2)$

$4 \times (1) \Rightarrow 8a + 12b = 52$

$3 \times (2) \Rightarrow 15a - 12b = -6$

$23a = 46$

$a = \frac{46}{23} = 2$

Substitute $a = 2$ in (1)

$2 \times 2 + 3b = 13$

$4 + 3b = 13$

$3b = 13 - 4$

$3b = 9$

$b = 3$

$But\ \frac{1}{x} = a \ \Rightarrow \frac{1}{x} = 2 \ \Rightarrow x = \frac{1}{2}$

$\frac{1}{y} = b \Rightarrow \frac{1}{y} = 3 \Rightarrow y = \frac{1}{3}$

Solution $x = \frac{1}{2}$ and $y = \frac{1}{3}$

vii) $\boldsymbol{\frac{10}{x+y} + \frac{2}{x-y} = 4; \frac{15}{x+y} - \frac{5}{x-y} = -2}$

Sol: $\frac{10}{x+y} + \frac{2}{x-y} = 4; \quad \frac{15}{x+y} - \frac{5}{x-y} = 10$

$10\left(\frac{1}{x+y}\right) + 2\left(\frac{1}{x-y}\right) = 4; \quad 15\left(\frac{1}{x+y}\right) - 5\left(\frac{1}{x-y}\right) = -2$

$Let \frac{1}{x+y} = a\ \ and\ \ \frac{1}{x-y} = b\ \ then$

$10a + 2b = 4 \rightarrow (1)$

$15a - 5b = -2 \rightarrow (2)$

$5 \times (1) 50a + 10b = 20$

$2 \times (2) 30a - 10b = -4$

$80a \qquad = 16$

$a = \frac{16}{80} = \frac{1}{5}$

Substitute $a = \frac{1}{5}$ $in\ (1)$

$10 \times \frac{1}{5} + 2b = 4$

$2 + 2b = 4$

$2b = 4 - 2 = 2$

$b = 1$

$But\ \frac{1}{x+y} = a\ \ and\ \ \frac{1}{x-y} = b$

$\frac{1}{x+y} = \frac{1}{5}\ \ and\ \ \frac{1}{x-y} = 1$

$x + y = 5 \rightarrow (3)\ and\ \ x - y = 1 \rightarrow (4)$

$(3) + (4) \Rightarrow 2x = 6 \Rightarrow x = 3$

$Substitute\ x = 3\ in\ (3)$

$3 + y = 5$

$y = 5 - 3 = 2$

$Solution\ x = 3\ \ and\ \ y = 2$

viii) $\mathbf{\frac{1}{3x+y} + \frac{1}{3x-y} = \frac{3}{4}; \frac{1}{2(3x+y)} - \frac{1}{2(3x-y)} = \frac{-1}{8}}$

Sol: $\frac{1}{3x+y} + \frac{1}{3x-y} = \frac{3}{4}; \frac{1}{2(3x+y)} - \frac{1}{2(3x-y)} = \frac{-1}{8}$

$Let\ \frac{1}{3x+y} = a\quad and\quad \frac{1}{3x-y} = b\ \ then$

$a + b = \frac{3}{4} \Rightarrow 4a + 4b = 3 \rightarrow (1)$

$\frac{a}{2} - \frac{b}{2} = \frac{-1}{8} \Rightarrow 4a - 4b = -1 \rightarrow (2)$

$(1) + (2) \Rightarrow 8a = 2 \Rightarrow a = \frac{2}{8} = \frac{1}{4}$

$Substitute\ a = \frac{1}{4}\ \ in\ (1)$

$4 \times \frac{1}{4} + 4b = 3$

$1 + 4b = 3$

$4b = 3 - 1 = 2$

$$b = \frac{2}{4} = \frac{1}{2}$$

$$But \;\; \frac{1}{3x+y} = a \quad and \quad \frac{1}{3x-y} = b$$

$$\frac{1}{3x+y} = \frac{1}{4} \quad and \quad \frac{1}{3x-y} = \frac{1}{2}$$

$3x + y = 4 \rightarrow (3) \quad 3x - y = 2 \rightarrow (4)$

$(3) + (4) \Rightarrow 6x = 6 \Rightarrow x = 1$

$Substitute\ x = 1\ in\ (3)$

$3 \times 1 + y = 4$

$y = 4 - 3 = 1$

Solution $x = 1$ and $y = 1$

2. Formulate the following problems as a pair of equations and then find their solutions.

i. **A boat goes 30 km upstream and 44 km downstream in 10 hours. In 13 hours it can go 40 km upstream and 55 km downstream. Determine the speed of the stream and that of the boat in still water**

Sol: Let the speed of boat in still water=$x\ km/h$

The speed of stream=$y\ \ km/h$

Relative speed of boat in upstream=$(x - y)km/h$

Relative speed of boat in downstream=$(x + y)km/h$

$$Time = \frac{distance}{speed}$$

Given boat goes 30 km upstream and 44 km downstream in 10 hours

$$\frac{30}{x-y} + \frac{44}{x+y} = 10 \rightarrow (1)$$

Boat goes 40 km upstream and 55 km downstream in 13 hours

$$\frac{40}{x-y} + \frac{55}{x+y} = 13 \rightarrow (2)$$

$$Let\ \frac{1}{x-y} = a\ \ and\ \ \frac{1}{x+y} = b\ then$$

$30a + 44b = 10 \rightarrow (3)$

$40a + 55b = 13 \rightarrow (4)$

$4 \times (4) \Rightarrow 160a + 220b = 52$

$5 \times (5) \Rightarrow 150a + 220b = 50$

$\quad\quad\quad\;\; (-) \quad (-) \quad\quad (-)$

$\quad\quad\quad\;\; 10a \quad\quad\quad = 2$

$$a = \frac{2}{10} = \frac{1}{5}$$

Substitute $a=\frac{1}{5}$ in (3)

$30\times\frac{1}{5}+44b=10$

$6+44b=10$

$44b=10-6=4$

$b=\frac{4}{44}=\frac{1}{11}$

$But\ \frac{1}{x-y}=a\Rightarrow\frac{1}{x-y}=\frac{1}{5}\Rightarrow x-y=5\rightarrow(5)$

$\frac{1}{x+y}=b\Rightarrow\frac{1}{x+y}=\frac{1}{11}\Rightarrow x+y=11\rightarrow(6)$

$(5)+(6)\Rightarrow 2x=16\Rightarrow x=\frac{16}{2}=8$

Substitute $x=8$ in (6)

$8+y=11\Rightarrow y=11-8=3$

Speed of boat in still water=$x=8\ km/h$

Speed of stream=$y=3\ km/h$

ii. **Rahim travels 600 km to his home partly by train and partly by car. He takes 8 hours if he travels 120 km by train and rest by car. He takes 20 minutes more if he travels 200 km by train and rest by car. Find the speed of the train and the car.**

Sol: Let speed of the train=$x\ km/h$

Speed of car=$y\ km/h$

Total travel distance=600 km

$Time=\frac{distance}{speed}$

Time for travels 120km by train + 480 km by car=8 h

$\frac{120}{x}+\frac{480}{y}=8\rightarrow(1)$

Time for travels 200km by train + 400 km by car=8 h+20 minutes

$\frac{200}{x}+\frac{400}{y}=8+\frac{20}{60}=8+\frac{1}{3}=\frac{25}{3}\rightarrow(2)$

$Let\ \frac{1}{x}=a\ \ and\ \ \frac{1}{y}=b$

$120a+480b=8\Rightarrow 15a+60b=1\rightarrow(3)$(Divided by 8)

$200a+400b=\frac{25}{3}\Rightarrow 24a+48b=1\rightarrow(4)$ (Dived by 25 multiply 3)

$4\times(3)\Rightarrow 60a+240b=4$

$5\times(4)\Rightarrow 120a+240b=5$

$(-)\quad(-)\quad(-)$

$-60a\quad=-1$

$$a = \frac{1}{60}$$

Substitute $a = \frac{1}{60}$ in (3)

$$15 \times \frac{1}{60} + 60b = 1$$

$$\frac{1}{4} + 60b = 1$$

$$60b = 1 - \frac{1}{4} = \frac{3}{4}$$

$$b = \frac{3}{4 \times 60} = \frac{1}{80}$$

$$But\ \frac{1}{x} = a \Rightarrow \frac{1}{x} = \frac{1}{60} \Rightarrow x = 60$$

$$\frac{1}{y} = b \Rightarrow \frac{1}{y} = \frac{1}{80} \Rightarrow y = 80$$

Speed of train=60 km/h

Speed of car =80 km/h

iii. **2 women and 5 men can together finish an embroidery work in 4 days while 3 women and 6 men can finish it in 3 days. Find the time taken by 1 woman alone and 1 man alone to finish the work.**

Sol: Let time taken by 1 woman to complete the work=$x\ days$

Time taken by 1 man to complete the work=$y\ days$

$$Work\ done\ by\ 1\ woman\ in\ 1\ day = \frac{1}{x}$$

$$Work\ done\ by\ 1\ man\ in\ 1\ day = \frac{1}{y}$$

2 women and 5 men can together finish work in 4 days

$$\frac{2}{x} + \frac{5}{y} = \frac{1}{4} \Rightarrow \frac{8}{x} + \frac{20}{y} = 1 \rightarrow (1)$$

3 women and 6 men can finish work in 3 days

$$\frac{3}{x} + \frac{6}{y} = \frac{1}{3} \Rightarrow \frac{9}{x} + \frac{18}{y} = 1 \rightarrow (2)$$

$$Let\ \frac{1}{x} = a\ \ and\ \ \frac{1}{y} = b$$

$(1) \Rightarrow 8a + 20b = 1 \rightarrow (3)$

$(2) \Rightarrow 9a + 18b = 1 \rightarrow (4)$

$9 \times (3) \Rightarrow 72a + 180b = 9$

$8 \times (4) \Rightarrow 72a + 144b = 8$

$(-)\quad (-)\qquad (-)$

$36b = 1$

$$b = \frac{1}{36}$$

$Substitute \quad b = \frac{1}{36} \quad in\ (3)$

$$8a + 20 \times \left(\frac{1}{36}\right) = 1$$

$$8a + \frac{5}{9} = 1$$

$$8a = 1 - \frac{5}{9} = \frac{4}{9}$$

$$a = \frac{4}{9 \times 8} = \frac{1}{18}$$

$But \ \frac{1}{x} = a \ \ and \ \ \frac{1}{y} = b$

$$\frac{1}{x} = \frac{1}{18} \quad and \quad \frac{1}{y} = \frac{1}{36}$$

$x = 18 \ and \ y = 36$

Time taken by 1 woman alone=18 days

Time taken by 1 man alone=36 days

Exercise - 4.1

3. **Neha went to a 'sale' to purchase some pants and skirts. When her friend asked her how many of each she had bought, she answered "The number of skirts are two less than twice the number of pants purchased. Also the number of skirts is four less than three times the number of pants purchased." Help her friend to find how many pants and skirts Neha bought.**

Sol: Let the number of plants=x and number of skirts=y

The number of skirts is two less than twice the number of pants.

$y = 2x - 2$

$2x - y = 2 \rightarrow (1)$

The number of skirts is four less than three times the number of pants.

$y = 3x - 4$

$3x - y = 4 \rightarrow (2)$

$(1) + (2) \Rightarrow$

$$\begin{array}{rcl} 2x - y &=& 2 \\ 3x - y &=& 4 \\ (-) \quad (+) & & (-) \\ \hline -x \quad\quad &=& -2 \\ \hline \end{array}$$

$\therefore x = 2$

Substitute $x = 2$ in equ(1)

$y = 2x - 2 = 2 \times 2 - 2 = 4 - 2 = 2$

∴ Number of pants=2;   Number of skirts=2.

4. **10 students of Class-X took part in a mathematics quiz. If the number of girls is 4 more than the number of boys then, find the number of boys and the number of girls who took part in the quiz.**

Sol: Let the number of boys=x and the number of girls=y

Total number of students=10

$\Rightarrow x + y = 10 \rightarrow (1)$

The number of girls is 4 more than the number of boys.

$\Rightarrow y = x + 4$

$\Rightarrow x - y = -4 \rightarrow (2)$

$(1) + (2)$

$$\begin{array}{rl} x + y &= 10 \\ x - y &= -4 \\ \hline 2x &= 6 \\ \hline \end{array}$$

$x = \frac{6}{2} = 3$

Substitute $x = 3$ in equ (1)

$3 + y = 10$

$y = 10 - 3 = 7$

∴ The number of boys=3 and the number of girls=7.

5. **5 pencils and 7 pens together cost ₹50 whereas 7 pencils and 5 pens together cost ₹46. Find the cost of one pencil and that of one pen.**

Sol: Let the cost of pencil=₹ x

And the cost of pen=₹ y

5 pencils + 7 pens =50

$5x + 7y = 50 \rightarrow (1)$

7 pencils + 5 pens =46

$7x + 5y = 46 \rightarrow (2)$

$$\begin{array}{l} Equ(1) \times 5 \Longrightarrow 25x + 35y = 250 \\ Equ(2) \times 7 \Longrightarrow 49x + 35y = 322 \\ \qquad\qquad\quad (-) \quad (-) \qquad\quad (-) \\ \hline \qquad\qquad\quad -24x \qquad\;\; = -72 \\ \hline \end{array}$$

$x = \frac{72}{24} = 3$

Substitute $x = 3$ in equ (1)

$5 \times 3 + 7y = 50$

$15 + 7y = 50$

$7y = 50 - 15 = 35$

$y = \frac{35}{7} = 5$

∴ Cost of pencil=₹3 and cost of pen=₹5.

6. **Half the perimeter of a rectangular garden, whose length is 4m more than its width, is 36m. Find the dimensions of the garden.**

Sol: Let the length of the garden=$x\ cm$ and width of the garden $=y\ cm$

Length is 4m more than its width

$x = y + 4 \Rightarrow x - y = 4 \rightarrow (1)$

Half the perimeter of a rectangular garden=36 cm

$\frac{1}{2} \times 2(x + y) = 36 \Rightarrow x + y = 36 \rightarrow (2)$

$$\begin{array}{rl} x - y = & 4 \\ x + y = & 36 \\ \hline 2x \quad = & 40 \\ \hline \end{array}$$

$x = \frac{40}{2} = 20$

Substitute $x = 20$ in equ (2)

$20 + y = 36$

$y = 36 - 20 = 16$

∴ Length=20cm and width=16 cm.

7. **We have a linear equation 2x + 3y - 8 = 0. Write another linear equation in two variables such that the geometrical representation of the pair so formed is intersecting lines. Now, write two more linear equations so that one forms a pair of parallel lines and the second forms coincident line with the given equation.**

Sol: Given linear equation: $2x + 3y - 8 = 0$

(i) Intersecting line:$3x + 5y + 6 = 0$

(ii) Parallel line:$4x + 6y + 15 = 0$

(iii) Coincident line:$4x + 6y - 16 = 0$

8. **The area of a rectangle gets reduced by 80 sq units if its length is reduced by 5 units and**

breadth is increased by 2 units. If we increase the length by 10 units and decrease the breadth by 5 units, the area will increase by 50 sq units. Find the length and breadth of the rectangle.

Sol: Let the length of rectangle=x units and breadth=y units.

Area of rectangle=$x \times y = xy\ sq\ units$

From problem

$(x-5) \times (y+2) = xy - 80$

$\Rightarrow xy + 2x - 5y - 10 = xy - 80$

$\Rightarrow 2x - 5y = -80 + 10$

$\Rightarrow 2x - 5y = -70 \rightarrow (1)$

$(x+10) \times (y-5) = xy + 50$

$\Rightarrow xy - 5x + 10y - 50 = xy + 50$

$\Rightarrow -5x + 10y = 50 + 50$

$\Rightarrow -5x + 10y = 100 \rightarrow (2)$

$$\begin{aligned} Equ(1) \times 2 \Longrightarrow \quad 4x - 10y &= -140 \\ Equ(2) \times 1 \Longrightarrow -5x + 10y &= \quad 100 \\ \hline -x \qquad\quad &= -40 \end{aligned}$$

$x = 40$

Substitute $x = 40$ in equ (1)

$2 \times 40 - 5y = -70$

$80 - 5y = -70$

$5y = 80 + 70 = 150$

$y = \frac{150}{5} = 30$

$\therefore$ Length=40 cm and breadth=30 cm.

9. **In X class, if three students sit on each bench, one student will be left. If four students sit on each bench, one bench will be left. Find the number of students and the number of benches in that class.**

Sol: Let the number of benches=x and the number of students=y

From problem

$y = 3x + 1 \Rightarrow 3x - y = -1 \rightarrow (1)$

$y = 4(x-1) \Rightarrow y = 4x - 4 \Rightarrow 4x - y = 4 \rightarrow (2)$

$$\begin{aligned} 3x - y &= -1 \\ 4x - y &= \quad 4 \\ (-) \ (+) & \quad (-) \\ \hline -x \qquad &= -5 \end{aligned}$$

$\therefore x = 5$

Substitute $x = 5$ in equ (1)

$3 \times 5 - y = -1$

$15 - y = -1$

$y = 15 + 1 = 16$

The number of students=16 and the number of benches=5.