CHAPTER

2

X CLASS-2023-24

2.SETS(Notes)

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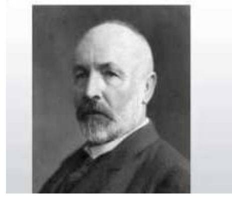
Georg Cantor



1. Set theory was introduced by Georg Cantor.
2. A set is a collection of things that have something in common or follow a rule.
3. The things in the set are called “elements”.
4. Set notation uses braces { } with elements separated by commas.
5. Sets are generally denoted by capital letters of English alphabet A, B, C......
6. Set of natural numbers N= {1, 2, 3, 4, 5, 6,………}
7. Set of whole numbers W={0,1,2,3,4,5,6,……}
8. Set of Integers Z={…….-3,-2,-1,0,1,2,3,4…..}
9. Set of even numbers E= {2, 4, 6, 8, 10, ...}
10. Set of odd numbers O = {1, 3, 5, 7, 9, …….}
11. Set of prime numbers P= {2, 3, 5, 7, 11, 13, 17, 19, …..}
12. Set of composite numbers C= {4, 6, 8, 9, 10, 12, 14, 16, ……}
13. a is an element of a set A then we write $a \in$ A (a is belongs to A)
14. a is not an element of a set A then we write $a \notin$ A (a is does not belong to A)

    Example: $A = \{2, 5, 6, 8\}$ then $2 \in A, 5 \in A, 6 \in A, 8 \in A, 1 \notin A, 3 \notin A$

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Write the following sets.

1) Set of the first five positive integers.

   $\{1, 2, 3, 4, 5\}$

2) Set of multiples of 5 which are more than 100 and less than 125.

   $\{105, 110, 115, 120\}$

3) Set of first five cubic numbers.

   $\{1, 8, 27, 64, 125\}$

4) Set of digits in the Ramanujan number.

   $\{ 1, 7, 2, 9\}$

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Set of natural numbers N ,set of integers Z, set of rational numbers Q, and set of real numbers R

Some numbers are given below. Decide the numbers to which number sets they belong to and does not belong to and express with correct symbols.

i) 1

   1 ∈ N, 1 ∈ Z, 1 ∈ Q, 1 ∈ R

ii) 0

$0 \notin N, \quad 0 \in Z, \quad 0 \in Q, \quad 0 \in R$

iii) -4

$-4 \notin N, \quad -4 \in Z, \quad -4 \in Q, \quad -4 \in R$

iv) $\frac{5}{6}$

$\frac{5}{6} \notin N, \quad \frac{5}{6} \notin Z, \quad \frac{5}{6} \in Q, \quad \frac{5}{6} \in R$

v) $1.\overline{3}$

$1.\overline{3} \notin N, \quad 1.\overline{3} \notin Z, \quad 1.\overline{3} \in Q, \quad 1.\overline{3} \in R$

vi) $\sqrt{2}$

$\sqrt{2} \notin N, \quad \sqrt{2} \notin Z, \quad \sqrt{2} \notin Q, \quad \sqrt{2} \in R$

vii) $\log 2$

$\log 2 \notin N, \quad \log 2 \notin Z, \quad \log 2 \notin Q, \quad \log 2 \in R$

viii) 0.03

$0.03 \notin N, \quad 0.03 \notin Z, \quad 0.03 \in Q, \quad 0.03 \in R$

ix) π

$\pi \notin N, \quad \pi \notin Z, \quad \pi \notin Q, \quad \pi \in R$

x) $\sqrt{-4}$

$\sqrt{-4} \notin N, \quad \sqrt{-4} \notin Z, \quad \sqrt{-4} \notin Q, \quad \sqrt{-4} \notin R$

ROSTER FORM:

The elements of the set, separated by commas, inside a set of curly brackets. This way of describing a set is called **roster form.**

SET BUILDER FORM:

When we write a set by defining its elements with a “common property”, we can say that the set is in the “set builder form”

Example: roaster form A = {3,6,9,12,15,18}

Set builder form $A = \{x: x \text{ is a multiple of } 3 \text{ and } x < 20\}(or)\{x: x = 3n, n \in N, n \leq 6\}$

Roster form	Set builder form
V = {a, e, i, o, u}	V = {x : x is a vowel in the english alphabet}
A = {-2,-1,0,1,2}	A = {x : $-2 \leq x \leq 2, x \in Z$ }
$B = \{1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}\}$	$B = \{x : x = \frac{1}{n}, n \in N, n \leq 5\}$
C = {2,5,10,17}	$C = \{x : x = n^2 + 1, n \in N, n \leq 4\}$

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1. **List the elements of the following sets.**

(i) G = {all the factors of 20}

G = {1, 2, 4, 5, 10, 20 }

(ii) F = {the multiples of 4 between 17 and 61 which are divisible by 7}

$F = \{28,56\}$

(iii) S = {x : x is a letter in the word 'MADAM'}

S = {M, A, D}

(iv) P = {x : x is a whole number between 3.5 and 6.7}

P = {4, 5, 6}

2. Write the following sets in the roster form.

(i) B is the set of all months in a year having 30 days

B = $\{April, June, september, November\}$

(ii) P is the set of all prime numbers smaller than 10.

P= {2, 3, 5, 7}

(iii) X is the set of the colours of the rainbow.

X = {Violet, Indigo, Blue, Green, Yellow, Orange, Red }

3. A is the set of factors of 12. Which one of the following is not a member of A.

$A = \{1,2,3,4,6,12\}$ 5 is not a member of A.

TRY THIS

2. Match roster forms with the set builder form.

(i)	{P, R, I, N, C, A, L}	(a)	$\{x : x$ is a positive integer and is a divisor of 18$\}$
(ii)	{0}	(b)	$\{x : x$ is an integer and $x^2 - 9 = 0\}$
(iii)	{1, 2, 3, 6, 9, 18}	(c)	$\{x : x$ is an integer and $x + 1 = 1\}$
(iv)	{3, -3}	(d)	$\{x : x$ is a letter of the word PRINCIPAL$\}$

Solution: (i)-(d), (ii)-(c), (iii)-(a), (iv)-(b)

EXERCISE - 2.1

1. Which of the following are sets? Justify your answer.

(i) The collection of all the months of a year beginning with the letter "J".

Sol: It is a set

{ January, June, July}

(ii) The collection of ten most talented writers of India.

Sol: Not a set.

Not a well-defined collection because the criteria for determining a writer's talent may vary from person to person. Hence, this collection is not a set.

(iii) A team of eleven best cricket batsmen of the world.

Sol: Not a set

Not a well-defined collection because the criteria for determining a batmen's talent may vary from person to person. Hence, this collection is not a set.

(iv) The collection of all boys in your class.

Sol: It is a set

The collection of all boys in your class is a well-defined collection because any boy who belongs to this collection can be easily identified.

2. **If A={0, 2, 4, 6}, B = {3, 5, 7} and C = {p, q, r}then fill the appropriate symbol, ∈ or ∉ in the blanks.**

   (i) 0 ∈ A (ii) 3 ∉ C (iii) 4 ∉ B (iv) 8 ∉ A (v) p ∈ C (vi) 7 ∈ B

3. **Express the following statements using symbols.**

   (i) The elements 'x' does not belong to 'A'.

   Sol: $x \notin A$

   (ii) 'd' is an element of the set 'B'.

   Sol: $d \notin B$

   (iii) '1' belongs to the set of Natural numbers N.

   Sol: 1 ∈ N

   (iv) '8' does not belong to the set of prime numbers P.

   Sol: 8 ∉ P

4. **State whether the following statements are true or false. Justify your answer.**

   (i) 5 ∉ set of prime numbers.

   Sol: False. 5 is a prime number.

   (ii) S = {5, 6, 7} implies 8 ∈ S.

   Sol: False. 8 is does not belongs to S.

   (iii) -5 ∉ W where 'W' is the set of whole numbers.

   Sol: True. -5 is not a whole number.

   (iv) $\frac{8}{11} \in Z$ where 'Z' is the set of integers.

   Sol: False. $\frac{8}{11}$ is not an integer.

5. **Write the following sets in roster form.**

   (i) B = {x : x is a natural number smaller than 6}

   Sol: B={1,2,3,4,5}

   (ii) C = {x : x is a two-digit natural number such that the sum of its digits is 8}

   Sol: $C = \{17, 26, 35, 44, 53, 62, 71, 80\}$

   (iii) D = {x : x is a prime number which is a divisor of 60}

   Sol: $D = \{2, 3, 5\}$

   (iv) E = {x : x is an alphabet in BETTER}.

Sol: $E = \{B, E, T, R\}$

6. **Write the following sets in the set-builder form.**

(i) {3, 6, 9, 12}

Sol: $\{x: x\ is\ a\ multiple\ of\ 3\ , x < 13\}$ (or) $\{x: x = 3n, n \in N\ and\ n \leq 4\}$

(ii) {2, 4, 8, 16, 32}

Sol: $\{x: x = 2^n, n \in N, n \leq 5\}$

(iii) {5, 25, 125, 625}

Sol: $\{x: x = 5^n, n \in N, n \leq 4\}$

(iv) {1, 4, 9, 16, 25, ….. 100}

Sol: $\{x: x = n^2, n \in N, n \leq 10\}$

7. **Write the following sets in roster form.**

(i) A = {x : x is a natural number greater than 50 but smaller than 100}

Sol: $A = \{51, 52, 53, \ldots \ldots \ldots \ldots, 98, 99\}$

(ii) B = {x : x is an integer, $x^2 = 4$}.

Sol: $B = \{-2, 2\}$

(iii) D = {x : x is a letter in the word "LOYAL"}

Sol: D = $\{L, O, Y, A\}$

8. Match the roster form with set builder form.

(i) {1, 2, 3, 6}	(c) {x : x is a natural number and divisor of 6}
(ii) {2, 3}	(a) {x : x is prime number and a divisor of 6}
(iii) {M, A, T, H, E, I, C, S}	(d) {x : x is a letter of the word MATHEMATICS}
(iv) {1, 3, 5, 7, 9}	(b) {x : x is an odd natural number smaller than 10}

EMPTY SET (or) NULL SET:

A set which does not contain any element is called an empty set, or a Null set, or a void set.

Empty set is denoted by the symbol ϕ or { }.

ϕ and {0} are two different sets. {0} is a set containing an element 0 while ϕ has no elements (null set).

Example: (i) A = {x : 1 < x < 2, x is a natural number}

(ii) B = {x : $x^2 - 2 = 0$ and x is a rational number}

(iii) C = {x : x is natural number smaller than 1}

(iv) D = {x : x is a odd number divisible by 2}

SUBSET:

i. If every element of set A is an element of set B then we say that A is sub set of B.

ii. Denoted by $A \subseteq B$

iii. A,B are two sets then $A \subset B \Leftrightarrow (x \in A \Rightarrow x \in B)$

iv. Empty set ϕ is a sub set of all sets.

v. Every set is a subset to itself.

vi. $N \subset W \subset Q \subset R$ and $Q' \subset R$

vii. Number of elements in a set are 'n' then number of subsets to the set $=2^n$

Do This — Page No-33

1. A = {1, 2, 3, 4}, B = {2, 4}, C = {1, 2, 3, 4, 7}, ϕ = { }. Fill in the blanks with $\subset$ or $\not\subset$.

   (i) A $\not\subset$ B (ii) C $\not\subset$ A (iii) B $\subset$ A (iv) A $\subset$ C (v) B $\subset$ C (vi) ϕ $\subset$ B

2. State which of the following statement are true.

   (i) { } = ϕ - True (ii) ϕ = 0 – False (iii) 0 = { 0 }- False

Try This — Page No-33

1. A = {set of quadrilaterals}, B = {square, rectangle, trapezium, rhombus}. State whether A $\subset$B or B $\subset$ A. Justify your answer.

Sol: B $\subset$ A and A $\not\subset$B

Justification: All elements of B are quadrilaterals which are belongs to A. So B $\subset$ A

Kite is a quadrilateral which does not belongs to A. so A $\not\subset$B

2. If A = {a, b, c, d}. How many subsets does the set A have?

Sol: Number of elements in set A=4

Number of subsets to the set A=2^4=16

3. P is the set of factors of 5, Q is the set of factors of 25 and R is the set of factors of 125. Which one of the following is false?

Sol: $P = \{1,5\}$, $Q = \{1,5,25\}$ $R = \{1,5,25,125\}$

(A) $P \subset Q \rightarrow True$ (B) $Q \subset R \rightarrow True$ (C) $R \subset P \rightarrow False$ (D) $P \subset R \rightarrow True$

4. A is the set of prime numbers less than 10, B is the set of odd numbers less than 10 and C is the set of even numbers less than 10. Which of the following statements are true?

Sol: $A = \{2,3,5,7\}$ $B = \{1,3,5,7,9\}$ $C = \{2,4,6,8\}$

(i) $A \subset B \rightarrow False$ (ii) $B \subset A \rightarrow False$ (iii) $A \subset C \rightarrow False$

(iv) $C \subset A \rightarrow False$ (v) $B \subset C \rightarrow False$ (vi) $\phi \subset A \rightarrow True$

2.5 Venn diagrams

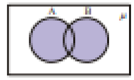
i. Venn-diagram is one of the ways of representing the relationships between sets.

ii. These diagrams are introduced by **John Venn** and **Leonhard Euler .** So it is also called Venn-Euler diagram.

UNION OF SETS :

The union of A and B is the set which consists of all the elements of A and B. The symbol '∪' is used to denote the union. We write A ∪ B.

$A \cup B = \{x : x \in A \ \ or \ \ x \in B\}$



Example-1. Let A = {2, 5, 6, 8} and B = {5, 7, 9, 1}. Find A ∪ B.

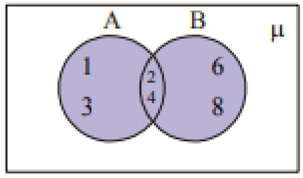
Sol: A ∪ B = {2, 5, 6, 8} ∪ {5, 7, 9, 1}

= {1, 2, 5, 6, 7, 8, 9}.

Example-2 . Let A = {a, e, i, o, u} and B = {a, i, u}. Show that A ∪ B = A.

Sol: $A \cup B = \{a, e, i, o, u\} \cup \{a, i, u\}$

$= \{a, e, i, o, u\} =$ A.



A ∪ B = {1, 2, 3, 4, 6, 8}

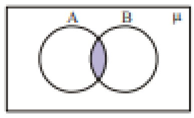
Example-3: If A = {1, 2, 3, 4} and B = {2, 4, 6, 8}. Find $A \cup B$.

Sol: $A \cup B = \{1, 2, 3, 4\} \cup \{2, 4, 6, 8\}$

$= \{1, 2, 3, 4, 6, 8\}$

INTERSECTION OF SETS :

The intersection of sets A and B is the set of all elements which are common in both A and B. We denote intersection symbolically by as A ∩B (read as "A intersection B").



A∩B

$A \cap B = \{x : x \in A \ \ and \ \ x \in B\}$

Example-4. Find A ∩ B when A = {5, 6, 7, 8} and B = {7, 8, 9, 10}

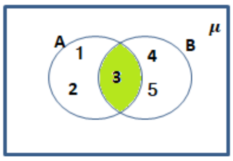
Sol: $A \cap B = \{5, 6, 7, 8\} \cap \{7, 8, 9, 10\}$

$= \{7, 8\}$

Example-5. If A = {1, 2, 3} and B = {3, 4, 5}, then illustrate A ∩ B in Venn-diagrams.

Sol: $A \cap B = \{1, 2, 3\} \cap \{3, 4, 5\}$

$= \{3\}$



DISJOINT SETS

If two sets have no common elements then they are called disjoint sets.

If $A \cap B = \emptyset$ then A,B are disjoint sets.

Examples: (i) A = {1, 2, 3}   B = {7, 8,10}

Do Th Page No-37

1. **Let A = {1, 3, 7, 8} and B = {2, 4, 7, 9}. Find A ∩ B.**

Sol: $A \cap B = \{1, 3, 7, 8\} \cap \{2, 4, 7, 9\} = \{7\}$

2. **If A = {6, 9, 11}; B = { }, find A ∪ ∅ .**

Sol: $A \cup \emptyset = \{6, 9, 11\} \cup \{ \ \ \} = \{6, 9, 11\} = A$

3. **A = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}; B = {2, 3, 5, 7}. Find A ∩ B.**

Sol: $A \cap B = \{1,2,3,4,5,6,7,8,9,10\} \cap \{2,3,5,7\}$

$= \{2,3,5,7\} = B$

$\therefore \ A \cap B = B$

4. **If A = {4, 5, 6}; B = {7, 8} then show that A ∪ B = B ∪ A.**

Sol: $A \cup$ B = {4,5,6} ∪ {7,8} = {4,5,6,7,8}

$B \cup A = \{7,8\} \cup \{4,5,6\} = \{4,5,6,7,8\}$

$\therefore\ A \cup B = B \cup A$

TRY THIS Page No-37

1. **List out some sets A and B and choose their elements such that A and B are disjoint.**

Sol: (i) $A = \{1, 2, 3, 4\}$ *and* $B = \{5, 6, 7, 8\}$

(ii) $A = \{even\ numbers\}$ *and* $B = \{odd\ numbers\}$

(iii) $P = \{a, e, i\}$ *and* $Q = \{b, c, d, f, g\}$

2. **If A = {2, 3, 5}, find A ∪ ∅ and ∅ ∪ A and compare.**

Sol: $A \cup \emptyset = \{2,3,4\} \cup \{\ \} = \{2,3,4\} = A$

$\emptyset \cup A = \{\ \} \cup \{2,3,4\} = \{2,3,4\} = A$

$\therefore\ A \cup \emptyset = \emptyset \cup A = A$

3. **If A = {1, 2, 3, 4} and B = {1, 2, 3, 4, 5, 6, 7, 8}, then find $A \cup B$ and $A \cap B$. What do you notice about the result?**

Sol: $A \cup B = \{1,2,3,4\} \cup \{1,2,3,4,5,6,7,8\}$

$= \{1,2,3,4,5,6,7,8\} = B$

$A \cap B = \{1,2,3,4\} \cap \{1,2,3,4,5,6,7,8\}$

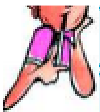
$= \{1,2,3,4\} = A$

If $A \subset B$ then $A \cup B = B$ and $A \cap B = A$.

4. **Let A = {1, 2, 3, 4, 5, 6} and B = {2, 4, 6, 8, 10}. Find the intersection of A and B.**

Sol: $A \cap B = \{1,2,3,4,5,6\} \cap \{2,4,6,8,10\}$

$= \{2,4,6\}$



THINK AND DISCUSS

The intersection of any two disjoint sets is a null set. Justify your answer.

Sol: Yes. The intersection of any two disjoint sets is a null set.

$A \cap B$ means set of common elements of A and B . If A and B are disjoint sets then they have no common elements. So $A \cap B = \emptyset$.

DIFFERENCE OF SETS

The difference set of sets A and B as the set of elements which belong to A but do not belong to B.

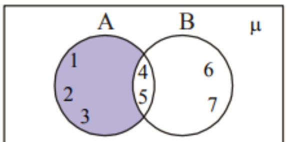
We denote the difference of A and B by A – B or simply "A minus B".

$A - B = \{x : x \in A\ and\ x \notin B\}$.

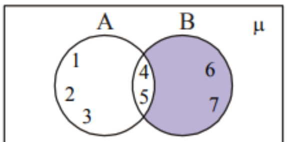
Example-6. Let A = {1, 2, 3, 4, 5}; B = {4, 5, 6, 7}. Find A – B and B – A. Are they equal?

Sol: A – B = {1, 2, 3, 4, 5} – {4, 5, 6, 7}

= {1, 2, 3}



A – B = {1, 2, 3}



B – A = {6, 7}

$B - A = \{4, 5, 6, 7\} - \{1, 2, 3, 4, 5\}$

$= \{6, 7\}$

$A - B \neq B - A$

Do This Page No-38

2. If V = {a, e, i, o, u} and B = {a, i, k, u}, find V – B and B – V.

Sol: $V - B = \{a, e, i, o, u\} - \{a, i, k, u\}$

$= \{e, o\}$

$B - V = \{a, i, k, u\} - \{a, e, i, o, u\}$

$= \{k\}$

Think - Discuss

The sets A – B, B – A and A ∩ B are mutually disjoint sets. Use examples to observe if this is true.

Sol: Let $A = \{1, 2, 3, 4, 5\}$ $B = \{4, 5, 6, 7\}$

$A - B = \{1, 2, 3, 4, 5\} - \{4, 5, 6, 7\}$

$= \{1, 2, 3\}$

$B - A = \{4, 5, 6, 7\} - \{1, 2, 3, 4, 5\}$

$= \{6, 7\}$

$A \cap B = \{1, 2, 3, 4, 5\} \cap \{4, 5, 6, 7\}$

$= \{4, 5\}$

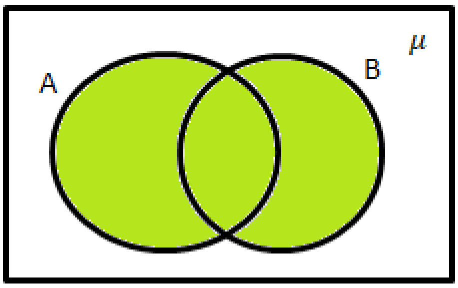
$(A - B) \cap (B - A) = \{1, 2, 3\} \cap \{6, 7\} = \emptyset$

$(B - A) \cap (A \cap B) = \{6, 7\} \cap \{4, 5\} = \emptyset$

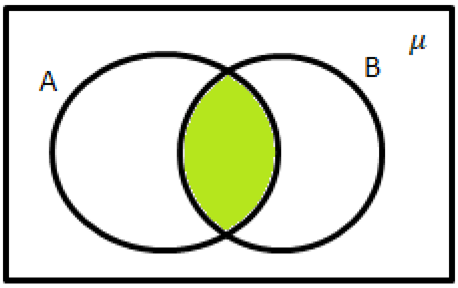
$(A - B) \cap (A \cap B) = \{1, 2, 3\} \cap \{4, 5\} = \emptyset$

∴ The sets A – B, B – A and A ∩ B are mutually disjoint sets.

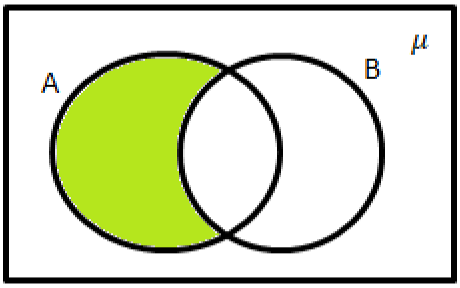
VEN DIAGRAMS:



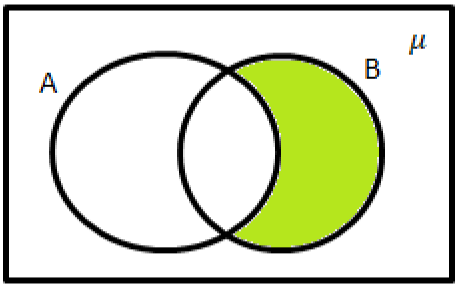
$A \cup B$



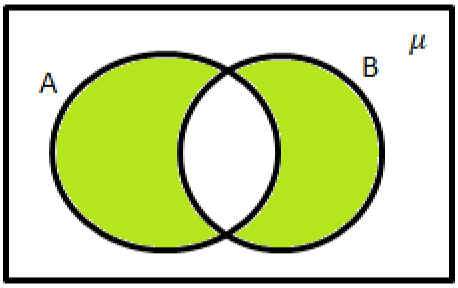
$A \cap B$



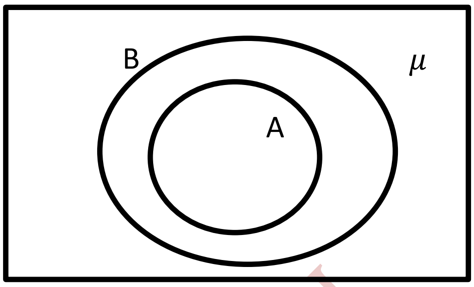
$A - B$



$B - A$



$A \Delta B = (A \cup B) - (A \cap B) =$
$(A - B) \cup (B - A)$



$A \subset B$

EXERCISE - 2.2

1. **If A = {1, 2, 3, 4}; B = {1, 2, 3, 5, 6} then find A ∩ B and B ∩ A. Are they equal?**

   Sol: $A \cap B = \{1,2,3,4\} \cap \{1,2,3,5,6\}$
   $= \{1,2,3\}$
   $B \cap A = \{1,2,3,5,6\} \cap \{1,2,3,4\}$
   $= \{1,2,3\}$
   $\therefore A \cap B = B \cap A$

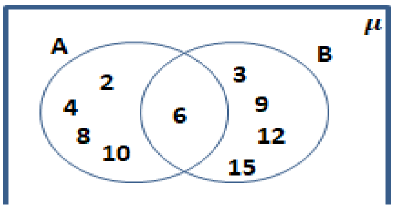
2. **A = {0, 2, 4}, find A ∩ ϕ and A ∩ A. Comment.**

   Sol: $A \cap \emptyset = \{0,2,4\} \cap \{\ \} = \{\ \} = \emptyset$

   $A \cap A = \{0,2,4\} \cap \{0,2,4\} = \{0,2,4\} = A$
   $\therefore A \cap \emptyset = \emptyset$ and $A \cap A = A$

3. **If A = {2, 4, 6, 8, 10} and B = {3, 6, 9, 12, 15}, find A – B and B – A.**

   Sol: $A - B = \{2,4,6,8,10\} - \{3,6,9,12,15\}$
   $= \{2,4,8,10\}$
   $B - A = \{3,6,9,12,15\} - \{2,4,6,8,10\}$
   $= \{3,9,12,15\}$



4. **If A and B are two sets such that A ⊂ B then what is A ∪ B?**

Sol: If A ⊂ B then what is A ∪ B=B.

5. If A = {x : x is a natural number}, B = {x : x is an even natural number}
   C = {x : x is an odd natural number} and D = {x : x is a prime number}

Find $A \cap B$, $A \cap C$, $A \cap D$, $B \cap C$, $B \cap D$, $C \cap D$.

Sol: A={1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,...}

B={2,4,6,8,10,12,14,.......}

C={1,3,5,7,9,11,13,15,....}

D={2,3,5,7,11,13,..........}

$A \cap B$ ={1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,...} ∩ {2,4,6,8,10,12,14, … … . }

={2,4,6,8,10,12,14,.......} =B

$A \cap C$ = {1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,...} ∩ {1,3,5,7,9,11,13,15,....}

= {1,3,5,7,9,11,13,15,....} = C

$A \cap D$ ={1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,...} ∩ {2,3,5,7,11,13,..........}

={2,3,5,7,11,13,..........} = D

$B \cap C$ ={2,4,6,8,10,12,14,.......} ∩ {1,3,5,7,9,11,13,15, … . }

={ } =∅

$B \cap D$ = {2,4,6,8,10,12,14,.......} ∩ {2,3,5,7,11,13, … … … . }

= { 2}

$C \cap D$ = {1,3,5,7,9,11,13,15, … . } ∩ {2,3,5,7,11,13, … … … . }

= {3,5,7,11,13, … … … . }

6. If A = {3, 6, 9, 12, 15, 18, 21}; B = {4, 8, 12, 16, 20} C = {2, 4, 6, 8, 10, 12, 14, 16}; D = {5, 10, 15, 20} find

(i) $A - B$ (ii) $A - C$ (iii) $A - D$ (iv) $B - A$ (v) $C - A$

(vi) $D - A$ (vii) $B - C$ (viii) $B - D$ (ix) $C - B$ (x) $D - B$

Sol: (i) $A - B$ = {3,6,9,12,15,18,21} − {4,8,12,16,20}

= {3,6,9,15,18,21}

(ii) $A - C$ = {3,6,9,12,15,18,21} − {2,4,6,8,10,12,14,16}

= {3,9,15,18,21}

(iii) $A - D$ = {3,6,9,12,15,18,21} − {5,10,15,20}

= {3,6,9,12,18,21}

(iv) $B - A$ = {4,8,12,16,20} − {3,6,9,12,15,18,21}

= {4,8,16,20}

(v) $C - A$ = {2,4,6,8,10,12,14,16} − {3,6,9,12,15,18,21}

= {2,4,8,10,14,16}

(vi) D − A = {5,10,15,20} − {3,6,9,12,15,18,21}

= {5,10,20}

(vii) B − C = {4,8,12,16,20} − {2,4,6,8,10,12,14,16}

$= \{20\}$

(viii) $B - D = \{4,8,12,16,20\} - \{5,10,15,20\}$

$= \{4,8,12,16\}$

(ix) $C - B = \{2,4,6,8,10,12,14,16\} - \{4,8,12,16,20\}$

$= \{2,6,10,14\}$

(x) $D - B = \{5,10,15,20\} - \{4,8,12,16,20\}$

$= \{5,10,15\}$

7. **State whether each of the following statement is true or false. Justify you answers.**

(i) {2, 3, 4, 5} and {3, 6} are disjoint sets.

Sol: False. Both sets have a common element 3 so they are not disjoint sets.

(ii) {a, e, i, o, u} and {a, b, c, d} are disjoint sets.

Sol: False. Both sets have a common element 'a' so they are not disjoint sets.

(iii) {2, 6, 10, 14} and {3, 7, 11, 15} are disjoint sets.

Sol: True . Both sets have no common elements so they are disjoint sets.

(iv) {2, 6, 10} and {3, 7, 11} are disjoint sets.

Sol: True . Both sets have no common elements so they are disjoint sets.

EQUAL SETS

Two sets A and B are said to be equal if every element in A belongs to B (i.e. $A \subseteq B$) and every element in B belongs to A (i.e. $B \subseteq A$).

If $B \subseteq A$ and $A \subseteq B \Leftrightarrow A = B$. Here $\Leftrightarrow$ is the symbol read as ' if and only if' ("iff")

Example-7. If A = {p, q, r} and B = {q, p, r}, then check whether A=B or not.

Sol: The elements are same in both the sets. Therefore A=B.

Examples-8. If A = {1, 2, 3, ….} and N is a set of natural numbers, then check whether A and N are equal?

Sol: The elements are same in both the sets. Therefore A=N.

Example-9. Consider the sets A = {p, q, r, s} and B = {1, 2, 3, 4}. Are they equal?

Solution : A and B do not contain the same elements. So, $A \neq B$

Example-10. Let A be the set of prime numbers smaller than 6 and P the set of prime factors of 30. Check if A and P are equal.

Solution : The set of prime numbers less than 6, A = { 2,3,5}

The prime factors of 30 are 2, 3 and 5. So, P = { 2,3,5}

The elements are same in both the sets. Therefore A=P.

Example-11. Show that the sets A and B are equal, where

A = {x : x is a letter in the word 'ASSASSINATION'}

B = {x : x is a letter in the word STATION}

Sol: $A = \{A, S, I, N, T, O\}$

$B = \{S, T, A, I, O, N\}$

The elements of A and B are same so A = B.

Example-12. Consider the sets ϕ , A = {1, 3}, B = {1, 5, 9}, C = {1, 3, 5, 7, 9}. Insert the symbol $\subset$ or $\not\subset$ between each of the following pair of sets. (i) ϕ ..... B (ii) A ..... B (iii) A ..... C (iv) B C

Solution: (i) $\phi \subset$ B, as ϕ is a subset of every set.

(ii) A $\not\subset$ B, for 3 $\in$ A but 3 $\notin$ B.

(iii) A $\subset$ C as each element of A is also an element of C.

(iv) B $\subset$ C as each element of B is also an element of C.

EXERCISE - 2.3

1. **Which of the following sets are equal?**

   (i) A = {x : x is a letter in the word FOLLOW} ={F, O, L, W}

   (ii) B = {x : x is a letter in the word FLOW} ={F, L, O, W}

   (iii) C = {x : x is a letter in the word WOLF}={W, O, L, F}

Sol: The elements are same in A,B and C . So A=B=C(A,B,C are equal sets)

2. **Consider the following sets and fill up the blank in the statement given below with = or ≠ so as to make the statement true.**

Sol: A = {1, 2, 3}; B = {The first three natural numbers}={1, 2, 3}

C = {a, b, c, d}; D = {d, c, a, b} E = {a, e, i, o, u};

F = {set of vowels in English Alphabet} = {a, e, i, o, u}

(i) A = B (ii) A ≠ E (iii) C = D (iv) D ≠ F

(v) F ≠ A (vi) D ≠ E (vii) F ≠ B

3. In each of the following, state whether A = B or not.

(i) A = {a, b, c, d} B = {d, c, a, b}

Sol: The elements of A and B are same . So A = B

(ii) A = {4, 8, 12, 16} B = {8, 4, 16, 18}

Sol: $12 \in A$ *but* $12 \notin B$. So A ≠ B

(iii) A = {2, 4, 6, 8, 10} B = {x : x is a positive even integer and x ≤10}={2,4,6,8,10}

Sol: The elements of A and B are same . So A = B

(iv) A = {x : x is a multiple of 10} B = {10, 15, 20, 25, 30, ...}

Sol:$15 \in B$ *and* 15 *is not a multiple of* $10 \Rightarrow 15 \notin A$. So A ≠ B.

4. State the reasons for the following :

(i) {1, 2, 3, ..., 10} ≠ {x : x ∈ N and 1 < x < 10}={2,3,4,5,6,7,8,9}

Sol:$10 \in$ *LHS set and* $10 \notin$ *RHS* set

(ii) {2, 4, 6, 8, 10} ≠ {x : x = 2n+1 and n ∈ N}={3,5,7,9,...}

Sol: $2 \in LHS\ set\ and\ 2 \notin RHS$ set.

(iii) {5, 15, 30, 45} ≠ {x : x is a multiple of 15}={15,30,45,...}

Sol: $5 \in LHS\ set\ and\ 5 \notin RHS$ set.

(iv) {2, 3, 5, 7, 9} ≠ {x : x is a prime number}

Sol: 9 is not a prime number.

5. **List all the subsets of the following sets.**

(i) B = {p, q}

Sol: Number of elements in B=2. Number of subsets to B=2^2=4

Subsets of B are

∅, {p}, {q}, B

(ii) C = {x, y, z}

Sol: Number of elements in C=3. Number of subsets to C=2^3=8

Subsets of C are

$\emptyset, \{x\}, \{y\}, \{z\}, \{x, y\}, \{x, z\}, \{y, z\}$, C

(iii) D = {a, b, c, d}

Sol: Number of elements in D=4. Number of subsets to D=2^4=16

Subsets of D are

$\emptyset, \{a\}, \{b\}, \{c\}, \{d\}, \{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{c, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}$, D

(iv) E = {1, 4, 9, 16}

Sol: Number of elements in E=4. Number of subsets to E=2^4=16

Subsets of E are

$\emptyset, \{1\}, \{4\}, \{9\}, \{16\}, \{1,4\}, \{1,9\}, \{1,16\}, \{4,9\}, \{4,16\}, \{9,16\}, \{1,4,9\}, \{1,4,16\}, \{1,9,16\}, \{4,9,16\}$, E

(v) F = {10, 100, 1000}

Sol: Number of elements in F=3. Number of subsets to F=2^3=8

Subsets of F are

$\emptyset, \{10\}, \{100\}, \{1000\}, \{10,100\}, \{10,1000\}, \{100,1000\}$, F

Finite & Infinite sets:

The finite set is countable and contains a finite number of elements.

The set which is not finite is known as the infinite set.

Example-13. State which of the following sets are finite or infinite.

(i) {x : x ∈ N and (x – 1) (x – 2) = 0}

Sol: The set is {1,2}. Hence, it is finite.

(ii) {x : x ∈ N and x^2 = 4}

Sol: $x^2 = 4 \Rightarrow x = \sqrt{4} = 2$(x is a natural number)

the set is{2}. Hence, it is finite.

(iii) {x : x ∈ N and 2x – 2 = 0}

Sol: $2x - 2 = 0 \Rightarrow 2x = 2 \Rightarrow x = \frac{2}{2} = 1$.

the set is {1}. Hence, it is finite.

(iv) {x : x ∈ N and x is prime}

Sol:{2,3,5,7,..........} There are infinitely many prime numbers. Hence, set is infinite.

(v) {x : x ∈ N and x is odd}

Sol: {1,3,5,7,9.....} There are infinitely many odd numbers. Hence, set is infinite.

Cardinal number of a set:

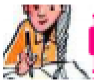
The number of elements in a set is called the cardinal number of the set.

The cardinal number of the set A is denoted as n(A).

Example: i)A={1,5,7,9} then n(A)=4

ii) B={a,e,i,o,u} then n(B)=5

iii)$\phi = \{\ \}$ then $n(\phi)=0$



Do These

1. Which of the following are empty sets? Justify your answer.

(i) Set of integers which lie between 2 and 3.

Sol: Empty set. There are no integers lie between 2 and 3.

(ii) Set of natural numbers that are smaller than 1.

Sol: Empty set. There are no natural numbers that are smaller than 1.

(iii) Set of odd numbers that leave remainder zero, when divided by 2.

Sol: Empty set. There are no odd numbers that leave remainder zero, when divided by 2.

2 . State which of the following sets are finite and which are infinite. Give reasons for your answer.

(i) A = {x : x ∈ N and x < 100}

Sol: A={1,2,3,4,5,.....,98,99} . The number of elements in A is 99.

So A is finite set.

(ii) B = {x : x ∈ N and x ≤ 5}.

Sol: B={1,2,3,4,5}. The number of elements in A is 5.

So B is finite set.

(iii) $C = \{1^2, 2^2, 3^2, \ldots\}$

Sol: The numbers of elements in C are not countable.

So C is infinite set.

(iv) D = {1, 2, 3, 4}

Sol: The number of elements in D is 4.

So D is finite set.

v) {x : x is a day of the week}.

Sol: The number of days of a week is 7 is countable. So it is finite set.

3. Tick the set which is infinite.

(A) The set of whole numbers < 10= {0, 1, 2, 3, 4, 5, 6, 7, 8, 9}-Finite set

(B) The set of prime number < 10 = {2, 3, 5, 7}-Finite set

(C) The set of integers < 10={9,8,7,6,5,4,3,2,1,0,-1,-2,………}-Infinite set.

(D) The set of factors of 10= {1, 2, 5, 10}-Finite set.

TRY THIS

1. **Which of the following sets are empty sets? Justify your answer.**

(i) A = {x : x^2 = 4 and 3x = 9}.

Sol: A is empty set.

$x^2 = 4 \Rightarrow x = \pm 2$ and $3x = 9 \Rightarrow x = \frac{9}{3} = 3$

There is no common x value for given two equations.

(ii) The set of all triangles in a plane having the sum of their three angles less than 180.

Sol: Empty set.

The sum of angles in a triangle is 180^0.

2. B = {x : x + 5 = 5} is not an empty set. Why?

Sol: $x + 5 = 5 \Rightarrow x = 5 - 5 \Rightarrow x = 0$.

$B = \{0\}$ is not an empty set

THINK & DISCUSS

An empty set is a finite set. Is this statement true or false? Why?

Sol: The number of elements in an empty set is '0'. So empty set is a finite set.

EXERCISE - 2.4

1. State which of the following sets are empty and which are not?

(i) The set of lines passing through a point.

Sol: Not empty.

There are infinite number of lines passing through a point.

(ii) Set of odd natural numbers divisible by 2.

Sol: Empty set.

There are no odd numbers divisible by 2.

(iii) {x : x is a natural number, x < 5 and x > 7}

Sol: Empty set

There are no natural numbers lees than 5 and greater than 7.

(iv) {x : x is a common point to any two parallel lines}

Sol: Empty set

Parallel lines have no common points.

(v) Set of even prime numbers.

Sol: Not empty

Set of even prime numbers={2}.

2. Which of the following sets are finite or infinite.

(i) The set of months in a year.

Sol: Finite set.

There are 12 months in a year.

(ii) {1, 2, 3, ..., 99, 100}

Sol: Finite set

The number of elements in the set are 100.

(iii) The set of prime numbers smaller than 99.

Sol: Finite set

Given set={2,3,5,7,11,13,17,19,23,29,31,37,41,43,47,53,59,61,67,71,73,79,83,89,97}.

There are 25 elements in the set.

3. State whether each of the following sets is finite or infinite.

(i) The set of letters in the English alphabet.

Sol: Finite set.

There are 26 letters in the English alphabet.

(ii) The set of lines which are parallel to the X-axis.

Sol: Infinite set.

We can draw infinite number of lines which are parallel to the X-axis.

(iii) The set of numbers which are multiplies of 5.

Sol: Infinite set.

Set of multiples of 5={5,10,15,20,25,…….} which are infinite.

(iv) The set of circles passing through the origin (0, 0).

Sol: Infinite set

We can draw infinite number of circles passing through the origin (0, 0).

Example-14. If A = {1, 2, 3, 4, 5}; B = {2, 4, 6, 8} then verify n(A∪B)=n(A)+n(B)-n($A \cap B$).

Sol: A = {1, 2, 3, 4, 5}   ∴ n(A)=5

$B = \{2, 4, 6, 8\}$ $\quad \therefore n(B)=4$

$A \cup B = \{1,2,3,4,5\} \cup \{2,4,6,8\}$

$= \{1,2,3,4,5,6,8\}$ $\quad \therefore n(A \cup B)=7$

$A \cap B = \{1,2,3,4,5\} \cap \{2,4,6,8\}$

$= \{2,4\}$ $\quad \therefore n(A \cap B)=2$

$$n(A) + n(B) - n(A \cap B) = 5 + 4 - 2 = 7 = n(A \cup B)$$

THINK & DISCUSS

1. **What is the relation between n(A), n(B), n(A ∩ B) and n(A∪B).**

   Sol: $n(A \cup B)=n(A)+n(B)-n(A \cap B)$.

2. **If A and B are disjoint sets, then how can you find n(A∪B).**

   Sol: If A and B are disjoint sets, then $A \cap B = \phi \Rightarrow n(A \cap B) = 0$

   If A and B are disjoint sets, then $n(A \cup B)=n(A)+n(B)$

Some important points:

1. If $A \subset B$ then (i) $A \cup B = B$ (ii) $A \cap B = A$ (iii) $A - B = \phi$.
2. $n(A \cup B) = n(A - B) + n(A \cap B) + n(B - A)$.
3. If A,B are disjoint sets then (i) $A - B = A$ (ii) $B - A = B$ (iii) $A \cap B = \phi$
4. $A \Delta B = (A - B) \cup (B - A) = (A \cup B) - (A \cap B)$.