

1. Father of statistics Sir Ronald Aylmer Fisher.
2. Father of Indian statistics Prasanta Chandra Mahalanobis .
3. Types of Measure of central tendency
 - i) Mean ii) Mode iii) Median
4. Mean of Ungrouped Data:

(i) $x_1, x_2, x_3, \dots, x_n$ are observations of the data then

$$\text{Mean } (\bar{x}) = \frac{\text{sum of observations}}{\text{Number of observations}} = \frac{x_1 + x_2 + x_3 + \dots + x_n}{n} = \frac{\sum x_i}{n}$$

(ii) Observations are $x_1, x_2, x_3, \dots, x_n$ and corresponding frequencies are $f_1, f_2, f_3, \dots, f_n$ then

$$\text{Mean}(\bar{x}) = \frac{f_1x_1 + f_2x_2 + f_3x_3 + \dots + f_nx_n}{f_1 + f_2 + f_3 + \dots + f_n} = \frac{\sum f_i x_i}{\sum f_i}$$

Example-1: Find the mean of the marks obtained by the students

Marks obtained (x_i)	10	20	36	40	50	56	60	70	72	80	88	92	95
Number of student (f_i)	1	1	3	4	3	2	4	4	1	1	2	3	1

Solution:

Marks obtained (x_i)	Number of students (f_i)	$f_i x_i$
10	1	10
20	1	20
36	3	108
40	4	160
50	3	150
56	2	112
60	4	240
70	4	280
72	1	72
80	1	80
88	2	176
92	3	276
95	1	95
Total	$\sum f_i = 30$	$\sum f_i x_i = 1779$

$$\begin{aligned} \text{Mean } (\bar{x}) &= \frac{\sum f_i x_i}{\sum f_i} \\ &= \frac{1779}{30} \\ &= 59.3 \end{aligned}$$

The mean marks are 59.3

Mean for grouped data- Direct Method:

$$\text{Mean } (\bar{x}) = \frac{\sum f_i x_i}{\sum f_i}$$

f_i = frequency of i^{th} class

x_i = class mark (mid value) of i^{th} class

Example: Find the mean of grouped data

Class interval	10-25	25-40	40-55	55-70	70-85	85-100
Number of students	2	3	7	6	6	6

Sol:

Class interval	Number of students (f_i)	Class Marks (x_i)	$f_i x_i$
10-25	2	17.5	35.0
25-40	3	32.5	97.5
40-55	7	47.5	332.5
55-70	6	62.5	375.0
70-85	6	77.5	465.0
85-100	6	92.5	555.0
Total	$\sum f_i = 30$		$\sum f_i x_i = 1860.0$

$$\begin{aligned}\text{Mean } (\bar{x}) &= \frac{\sum f_i x_i}{\sum f_i} \\ &= \frac{1860}{30} \\ &= 62\end{aligned}$$

Mean for grouped data- Assumed Mean Method:

$$\text{Mean}(\bar{x}) = a + \frac{\sum f_i d_i}{\sum f_i}$$

a = assumed mean (choose one among the x_i 's).

f_i = frequency of i^{th} class.

$d_i = x_i - a$.

x_i = class mark (mid value) of i^{th} class.

Example: Find the mean of grouped data

Class interval	10-25	25-40	40-55	55-70	70-85	85-100
Number of students	2	3	7	6	6	6

Sol:

Class interval	Number of students (f_i)	Class Marks (x_i)	$d_i = x_i - 47.5$ $d_i = x_i - a$	$f_i d_i$
10-25	2	17.5	-30	-60
25-40	3	32.5	-15	-45
40-55	7	47.5 (a)	0	0
55-70	6	62.5	15	90
70-85	6	77.5	30	180
85-100	6	92.5	45	270
Total	$\sum f_i = 30$			$\sum f_i d_i = 435$

$$\begin{array}{r} 14.5 \\ 30 \overline{) 435.0} \\ \underline{30} \\ 135 \\ \underline{120} \\ 150 \\ \underline{150} \\ 0 \end{array}$$

$$\text{Mean}(\bar{x}) = a + \frac{\sum f_i d_i}{\sum f_i} = 47.5 + \frac{435}{30} = 47.5 + 14.5 = 62$$

The mean of the marks obtained by the students is 62

Mean for grouped data- Step-deviation method:

$$\text{Mean}(\bar{x}) = a + \left(\frac{\sum f_i u_i}{\sum f_i} \right) \times h$$

a = assumed mean (choose one among the x_i 's).

f_i = frequency of i^{th} class.

$$u_i = \frac{x_i - a}{h}$$

x_i = class mark (mid value) of i^{th} class

h = class size .

Example: Find the mean of grouped data using Step-deviation method.(for above problem)

Sol:

Class interval	Number of students (f_i)	Class Marks (x_i)	$d_i = x_i - a$	$u_i = \frac{x_i - a}{h}$ $h = 15$	$f_i u_i$
10-25	2	17.5	-30	-2	-4
25-40	3	32.5	-15	-1	-3
40-55	7	47.5 $\rightarrow a$	0	0	0
55-70	6	62.5	15	1	6
70-85	6	77.5	30	2	12
85-100	6	92.5	45	3	18
Total	$\sum f_i = 30$				$\sum f_i u_i = 29$

$$\text{Mean}(\bar{x}) = a + \left(\frac{\sum f_i u_i}{\sum f_i} \right) \times h$$

$$= 47.5 + \frac{29}{30} \times 15$$

$$= 47.5 + \frac{29}{2}$$

$$= 47.5 + 14.5 = 62$$

$$\begin{array}{r} 14.5 \\ 2 \overline{) 29.0} \\ \underline{9} \\ 8 \\ \underline{10} \\ 10 \\ \underline{0} \end{array}$$

The mean of the marks obtained by the students is 62

Example-2: Find the mean percentage of female teachers using all the three methods.

Percentage of female teachers	15 - 25	25 - 35	35 - 45	45 - 55	55 - 65	65 - 75	75 - 85
Number of States/U.T.	6	11	7	4	4	2	1

Sol: Take $a = 50$

Percentage of female teachers C.I	Number of States/U.T. f_i	x_i	$d_i = x_i - 50$	$u_i = \frac{x_i - 50}{10}$	$f_i x_i$	$f_i d_i$	$f_i u_i$
15 - 25	6	20	-30	-3	120	-180	-18
25 - 35	11	30	-20	-2	330	-220	-22
35 - 45	7	40	-10	-1	280	-70	-7
45 - 55	4	50	0	0	200	0	0
55 - 65	4	60	10	1	240	40	4
65 - 75	2	70	20	2	140	40	4
75 - 85	1	80	30	3	80	30	3
Total	35				1390	-360	-36

From the above table, we obtain $\sum f_i = 35$, $\sum f_i x_i = 1390$, $\sum f_i d_i = -360$, $\sum f_i u_i = -36$.

Using the direct method, $\bar{x} = \frac{\sum f_i x_i}{\sum f_i} = \frac{1390}{35} = 39.71$.

Using the assumed mean method $\bar{x} = a + \frac{\sum f_i d_i}{\sum f_i} = 50 + \frac{-360}{35} = 50 - 10.29 = 39.71$.

Using the step-deviation method $\bar{x} = a + \left(\frac{\sum f_i u_i}{\sum f_i} \right) \times h = 50 + \frac{-36}{35} \times 10 = 39.71$.

Therefore, the mean percentage of female teachers in the primary schools of rural areas is 39.71.

Example -3: Find the mean number of wickets by choosing a suitable method.

Number of wickets	20 - 60	60 - 100	100 - 150	150 - 250	250 - 350	350 - 450
Number of bowlers	7	5	16	12	2	3

Sol:

Number of wickets	Number of bowlers (f_i)	x_i	$d_i = x_i - a$	$u_i = \frac{x_i - a}{h}$ ($h = 20$)	$f_i u_i$
20 - 60	7	40	-160	-8	-56
60 - 100	5	80	-120	-6	-30
100 - 150	16	125	-75	-3.75	-60
150 - 250	12	200 (a)	0	0	0
250 - 350	2	300	100	5	10
350 - 450	3	400	200	10	30
Total	$\sum f_i = 45$		$\sum f_i u_i = -146 + 40 =$		-106

$$\text{So } \bar{x} = a + \left(\frac{\sum f_i u_i}{\sum f_i} \right) \times h = 200 + \frac{-106}{45} \times 20 = 200 - 47.11 = 152.89$$

Exercise-14.1:

- Find the mean number of plants per house.

Number of plants	0 - 2	2 - 4	4 - 6	6 - 8	8 - 10	10 - 12	12 - 14
Number of houses	1	2	1	5	6	2	3

Sol:

Class interval	Frequency(f_i)	Class mark(x_i)	$f_i x_i$
0-2	1	1	1
2-4	2	3	6
4-6	1	5	5
6-8	5	7	35
8-10	6	9	54
10-12	2	11	22
12-14	3	13	39
	$\sum f_i = 20$		$\sum f_i x_i = 162$

$$\sum f_i = 20, \sum f_i x_i = 162$$

$$\text{Mean } (\bar{x}) = \frac{\sum f_i x_i}{\sum f_i} = \frac{162}{20} = 8.1$$

$$\begin{array}{r} 8.1 \\ 20 \overline{)162.0} \\ \underline{160} \\ 20 \\ \underline{20} \\ 0 \end{array}$$

2. Find the mean daily wages of the workers of the factory by using an appropriate method

Daily wages in Rupees	200 - 250	250 - 300	300 - 350	350 - 400	400- 450
Number of workers	12	14	8	6	10

Sol: Class interval are large . so we use step-deviation method

Class interval	Number of workers (f_i)	Class mark(x_i)	$u_i = \frac{x_i - a}{h}$	$f_i u_i$
200-250	12	225	-1	-12
250-300	14	275 $\rightarrow a$	0	0
300-350	8	325	1	8
350-400	6	375	2	12
400-450	10	425	3	30
	$\Sigma f_i = 50$			$\Sigma f_i u_i = -12 + 50 = 38$

$$a = 275, h = 50, \Sigma f_i = 50, \Sigma f_i u_i = 38$$

$$\begin{aligned} \text{Mean}(\bar{x}) &= a + \left(\frac{\Sigma f_i u_i}{\Sigma f_i} \right) \times h \\ &= 275 + \frac{38}{50} \times 50 \\ &= 275 + 38 \\ &= 313 \end{aligned}$$

The mean daily wages of the workers is ₹ 313

3. The mean pocket allowance is ₹ 18. Find the missing frequency f

Daily pocket allowance(in Rupees)	11 - 13	13 - 15	15 - 17	17 - 19	19 - 21	21 - 23	23 - 25
Number of children	7	6	9	13	f	5	4

Sol:

Class interval	Number of children (f_i)	Class mark(x_i)	$f_i x_i$
11-13	7	12	84
13-15	6	14	84
15-17	9	16	144
17-19	13	18	234
19-21	f	20	$20f$
21-23	5	22	110
23-25	4	24	96
	$\Sigma f_i = 44 + f$		$\Sigma f_i x_i = 752 + 20f$

Given mean pocket allowance is ₹ 18

$$\frac{\sum f_i x_i}{\sum f_i} = 18$$

$$\frac{752 + 20f}{44 + f} = 18$$

$$752 + 20f = 18(44 + f)$$

$$752 + 20f = 792 + 18f$$

$$20f - 18f = 792 - 752$$

$$2f = 40 \Rightarrow f = \frac{40}{2} \quad \therefore f = 20$$

4. Find the mean heart beats per minute for these women, choosing a suitable method.

Number of heart beats/minute	65-68	68-71	71-74	74-77	77-80	80-83	83-86
Number of women	2	4	3	8	7	4	2

Sol: We choose Assume mean method

Number of heart beats/minute	Number of women (f_i)	Class mark(x_i)	$d_i = x_i - a$	$f_i d_i$
65-68	2	66.5	-9	-18
68-71	4	67.5	-6	-24
71-74	3	72.5	-3	-9
74-77	8	75.5 $\rightarrow a$	0	0
77-80	7	78.5	3	21
80-83	4	81.5	6	24
83-86	2	84.5	9	18
	$\sum f_i = 30$			$\sum f_i d_i = -51 + 63 = 12$

$$\begin{aligned} \text{Mean}(\bar{x}) &= a + \frac{\sum f_i d_i}{\sum f_i} \\ &= 75.5 + \frac{12}{30} \\ &= 75.5 + \frac{2}{5} \\ &= 75.5 + 0.4 \\ &= 75.9 \end{aligned}$$

5. Find the mean number of oranges kept in each basket. Which method of finding the mean did you choose?

Number of oranges	10-14	15-19	20-24	25-29	30-34
Number of baskets	15	110	135	115	25

Sol: We choose step-deviation method

Number of oranges	Number of baskets (f_i)	Class mark(x_i)	$d_i = x_i - a$	$u_i = \frac{x_i - a}{h}$	$f_i u_i$
10-14	15	12	-10	-2	-30
15-19	110	17	-5	-1	-110
20-24	135	22 $\rightarrow a$	0	0	0
25-29	115	27	5	1	115
30-34	25	32	10	2	50
	$\Sigma f_i = 400$				$\Sigma f_i u_i = -140 + 165 = 25$

$$a = 22, h = 5, \Sigma f_i = 400, \Sigma f_i u_i = 25$$

$$\text{Mean}(\bar{x}) = a + \left(\frac{\Sigma f_i u_i}{\Sigma f_i} \right) \times h$$

$$= 22 + \frac{25}{400} \times 5$$

$$= 22 + \frac{5}{16}$$

$$= 22 + 0.31$$

$$= 22.31$$

$$\begin{array}{r} 0.31 \\ 16 \overline{)5.00} \\ \underline{48} \\ 20 \\ \underline{16} \\ 4 \end{array}$$

6. Find the mean daily expenditure on food by a suitable method.

Daily expenditure (in Rupees)	100-150	150-200	200-250	250-300	300-350
Number of house holds	4	5	12	2	2

Sol:

Class intervals	Frequency (f_i)	Class mark (x_i)	$d_i = (x_i - a)$ $a = 225$	$u_i = \frac{x_i - a}{h}$	$f_i u_i$
100-150	4	125	-100	-2	-8
150-200	5	175	-50	-1	-5
200-250	12	225	0	0	0
250-300	2	275	50	1	2
300-350	2	325	100	2	4

$$\Sigma f_i = 25$$

$$\Sigma f_i u_i = -7$$

$$a = 225, h = 50, \Sigma f_i = 25, \Sigma f_i u_i = -7$$

$$\text{Mean}(\bar{x}) = a + \left(\frac{\Sigma f_i u_i}{\Sigma f_i} \right) \times h$$

$$= 225 + \left(\frac{-7}{25} \right) \times 50$$

$$= 225 + (-7) \times 2$$

$$= 225 - 14$$

$$= 211$$

The mean daily expenditure on food is Rs. 211.

7. Find the mean concentration of SO₂ in the air.

Concentration of SO ₂ in ppm	0.00-0.04	0.04-0.08	0.08-0.12	0.12-0.16	0.16-0.20	0.20-0.24
Frequency	4	9	9	2	4	2

Sol:

Concentration of SO ₂ in ppm(class intervals)	Frequency (f _i)	Class mark (x _i)	f _i x _i
0.00-0.04	4	0.02	0.08
0.04-0.08	9	0.06	0.54
0.08-0.12	9	0.10	0.90
0.12-0.16	2	0.14	0.28
0.16-0.20	4	0.18	0.72
0.20-0.24	2	0.22	0.44
	∑ f _i = 30		∑ f _i x _i = 2.96

$$\begin{array}{r} 0.0986 \\ 30 \overline{) 2.96} \\ \underline{270} \\ 260 \\ \underline{240} \\ 200 \\ \underline{180} \\ 20 \end{array}$$

$$\sum f_i = 30, \quad \sum f_i x_i = 2.96$$

$$\text{Mean } (\bar{x}) = \frac{\sum f_i x_i}{\sum f_i} = \frac{2.96}{30} = 0.099$$

8. Find the mean number of days a student was present out of 56 days in the term .

Number of days	35-38	38-41	41-44	44-47	47-50	50-53	53-56
Number of students	1	3	4	4	7	10	11

Sol:

Class intervals	Frequency (f _i)	Class mark (x _i)	d _i = (x _i - a) a = 48.5	u _i = $\frac{x_i - a}{h}$	f _i u _i
35-38	1	36.5	-12	-4	-4
38-41	3	39.5	-9	-3	-9
41-44	4	42.5	-6	-2	-8
44-47	4	45.5	-3	-1	-4
47-50	7	48.5 → a	0	0	0
50-53	10	51.5	3	1	10
53-56	11	54.5	6	2	22
	∑ f _i = 40				∑ f _i u _i = -25 + 32 = 7

$$a = 48.5, h = 3, \quad \sum f_i = 40, \quad \sum f_i u_i = 7$$

$$\text{Mean}(\bar{x}) = a + \left(\frac{\sum f_i u_i}{\sum f_i} \right) \times h$$

$$= 48.5 + \frac{7}{40} \times 3$$

$$= 48.5 + \frac{21}{40}$$

$$= 48.5 + 0.5$$

$$\begin{array}{r} 0.5 \\ 40 \overline{) 21.0} \\ \underline{200} \\ 10 \end{array}$$

$$= 49$$

The mean number of days a student = 49

9. The following table gives the literacy rate (in percentage) of 35 cities. Find the mean literacy rate.

Literacy rate in %	45-55	55-65	65-75	75-85	85-95
Number of cities	3	10	11	8	3

Sol:

Literacy rate in % (class intervals)	Number of cities (f_i)	Class mark (x_i)	$u_i = \frac{x_i - a}{h}$	$f_i u_i$
45-55	3	50	-2	-6
55-65	10	60	-1	-10
65-75	11	70 $\rightarrow a$	0	0
75-85	8	80	1	8
85-95	3	90	2	6
	$\sum f_i = 35$		$\sum f_i u_i = -16 + 14 = -2$	

$$a = 70, \quad h = 10, \quad \sum f_i = 35, \quad \sum f_i u_i = -2$$

$$\text{Mean}(\bar{x}) = a + \left(\frac{\sum f_i u_i}{\sum f_i} \right) \times h$$

$$= 70 + \frac{(-2)}{35} \times 10$$

$$= 70 - \frac{4}{7}$$

$$= 70 - 0.57 = 69.43$$

\therefore The mean literacy rate = 69.43%

MODE

A mode is that value among the observations which occurs most frequently.

Example-4. The wickets taken by a bowler in 10 cricket matches are as follows: 2, 6, 4, 5, 0, 2, 1, 3, 2, 3. Find the mode of the data.

Sol: Arrange the observations in order

0, 1, 2, 2, 2, 3, 3, 4, 5, 6

2 is occurs most frequently. So, the mode of this data is 2.



Do This

1. Find the mode of the following data.

a) 5, 6, 9, 10, 6, 12, 3, 6, 11, 10, 4, 6, 7

Sol: Arrange the observations in order

3, 4, 5, 6, 6, 6, 6, 7, 9, 10, 10, 11, 12

6 is occurred most frequently. So, the mode of this data is 6.

b) 20, 3, 7, 13, 3, 4, 6, 7, 19, 15, 7, 18, 3.

Sol: Arrange the observations in order

3, 3, 3, 4, 6, 7, 7, 7, 13, 15, 18, 19, 20.

3 and 7 are occurred most frequently. So, the mode of this data is 3 and 7.

c) 2, 2, 2, 3, 3, 3, 4, 4, 4, 5, 5, 5, 6, 6, 6

Sol: No mode

2. Is the mode always at the centre of the data?

Sol: No.

3. Does the mode change, if another observation is added to the data in Example-4. Comment.

Sol: Yes. If we add 3 the mode is changed . Mode is 2 and 3

4. If the maximum value of an observation in the data in Example 4 is changed to 8, would the mode of the data be affected? Comment.

Sol: The maximum value 6 is changed to 8 then the mode is does not changed .

MODE FOR GROUPED DATA:

Fist we locate a class with the maximum frequency, called the modal class.

$$\text{Mode} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2} \right) \times h$$

Where l = lower boundary of the modal class

f_1 = frequency of the modal class

f_0 = frequency of the class preceding the modal class

f_2 = frequency of the class succeeding the modal class

h = size of the modal class

Example-5: Find the mode of this data.

Family size	1-3	3-5	5-7	7-9	9-11
Number of families	7	8	2	2	1

Solution: The maximum class frequency is 8. So, the modal class is 3-5.

Family size	Number of families
1-3	7 → f_0
$l = 3-5$	8 → f_1
5-7	2 → f_2
7-9	2
9-11	1

$$l = 3, f_1 = 8, f_0 = 7, f_2 = 2, h = 2$$

$$\begin{aligned} \text{Mode} &= l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2} \right) \times h \\ &= 3 + \left(\frac{8 - 7}{2 \times 8 - 7 - 2} \right) \times 2 \end{aligned}$$

$$= 3 + \left(\frac{1}{7}\right) \times 2$$

$$= 3 + \frac{2}{7} = 3 + 0.286 = 3.286$$

Therefore, the mode of the data is 3.286

Example-6. The marks distribution of 30 students in a mathematics examination are given in the adjacent table. Find the mode of this data. Also compare and interpret the mode and the mean.

Class interval	10-25	25-40	40-55	55-70	70-85	85-100
Number of students	2	3	7	6	6	6

Sol:

Class interval	Number of students (f_i)	Class Marks (x_i)	$d_i = x_i - a$	$u_i = \frac{x_i - a}{h}$ h = 15	$f_i u_i$
10-25	2	17.5	-30	-2	-4
25-40	3 $\rightarrow f_0$	32.5	-15	-1	-3
40-55	7 $\rightarrow f_1$	47.5 $\rightarrow a$	0	0	0
55-70	6 $\rightarrow f_2$	62.5	15	1	6
70-85	6	77.5	30	2	12
85-100	6	92.5	45	3	18
Total	$\Sigma f_i = 30$				$\Sigma f_i u_i = 29$

Mode: The highest frequency is 7. So the modal class is 40-45

$$l = 40, f_1 = 7, f_0 = 3, f_2 = 6, h = 15$$

$$\text{Mode} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2} \right) \times h$$

$$= 40 + \left(\frac{7 - 3}{2 \times 7 - 3 - 6} \right) \times 15$$

$$= 40 + \frac{4}{5} \times 15$$

$$= 40 + 4 \times 3$$

$$= 40 + 12 = 52$$

$$\text{Mean}(\bar{x}) = a + \left(\frac{\Sigma f_i u_i}{\Sigma f_i} \right) \times h$$

$$= 47.5 + \frac{29}{30} \times 15$$

$$= 47.5 + \frac{29}{2}$$

$$= 47.5 + 14.5 = 62$$

Interpretation : The mode marks is 52. The mean marks is 62. So, the maximum number of students obtained 52 marks, while on an average a student obtained 62 marks.

EXERCISE - 14.2

1. Find the mode and the mean of the data given below. Compare and interpret the two measures of central tendency

Age (in years)	5-15	15-25	25-35	35-45	45-55	55-65
Number of patients	6	11	21	23	14	5

Sol:

Age (in years)	Number of patients (f_i)	Class mark (x_i)	$f_i x_i$
5-15	6	10	60
15-25	11	20	220
25-35	21 $\rightarrow f_0$	30	630
35-45	23 $\rightarrow f_1$	40	920
45-55	14 $\rightarrow f_2$	50	700
55-65	5	60	300
	$\sum f_i = 80$		$\sum f_i x_i = 2830$

Mode: The highest frequency is 23. So the modal class is 35-45

$$l = 35, f_1 = 23, f_0 = 21, f_2 = 14, h = 10$$

$$\begin{aligned} \text{Mode} &= l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2} \right) \times h \\ &= 35 + \left(\frac{23 - 21}{2 \times 23 - 21 - 14} \right) \times 10 \\ &= 35 + \left(\frac{2}{46 - 35} \right) \times 10 \\ &= 35 + \frac{20}{11} = 35 + 1.8 = 36.8 \end{aligned}$$

$$\text{Mean } (\bar{x}) = \frac{\sum f_i x_i}{\sum f_i} = \frac{2830}{80} = 35.37$$

Interpretation : Mode = 36.8 years, Mean = 35.37 years, Maximum number of patients admitted in the hospital are of the age 36.8 years (approx.), while on an average the age of a patient admitted to the hospital is 35.37 years.

2. The following data gives the information on the observed life times (in hours) of 225 electrical components : Determine the modal lifetimes of the components.

Lifetimes (in hours)	0 - 20	20 - 40	40 - 60	60 - 80	80 - 100	100 - 120
Frequency	10	35	52	61	38	29

Sol:

Lifetimes (in hours)	Frequency
0-20	10
20-40	35
40-60	52 $\rightarrow f_0$
60-80	61 $\rightarrow f_1$
80-100	38 $\rightarrow f_2$
100-120	29

The highest frequency is 61. So the modal class is 60-80

$$l = 60, f_1 = 61, f_0 = 52, f_2 = 38, h = 20$$

$$\begin{aligned} \text{Mode} &= l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2} \right) \times h \\ &= 60 + \left(\frac{61 - 52}{2 \times 61 - 52 - 38} \right) \times 20 \\ &= 60 + \left(\frac{9}{122 - 90} \right) \times 20 \\ &= 60 + \frac{9 \times 20}{32} \\ &= 60 + \frac{45}{8} = 60 + 5.625 = 65.625 \text{ hours} \end{aligned}$$

3. Find the modal monthly expenditure of the families. Also, find the mean monthly expenditure :

Expenditure (in rupees)	1000-1500	1500-2000	2000-2500	2500-3000	3000-3500	3500-4000	4000-4500	4500-5000
Number of families	24	40	33	28	30	22	16	7

Sol:

Expenditure (in rupees)	Number of families (f_i)	Class mark (x_i)	$u_i = \frac{x_i - a}{h}$	$f_i u_i$
1000-1500	24 $\rightarrow f_0$	1250	-4	-96
1500-2000	40 $\rightarrow f_1$	1750	-3	-120
2000-2500	33 $\rightarrow f_2$	2250	-2	-66
2500-3000	28	2750	-1	-28
3000-3500	30	3250 $\rightarrow a$	0	0
3500-4000	22	3750	1	22
4000-4500	16	4250	2	32
4500-5000	7	4750	3	21
	$\Sigma f_i = 200$			$\Sigma f_i u_i = -310 + 75 = -235$

Mode: The highest frequency is 40. So the modal class is 1500-2000

$$l = 1500, f_1 = 40, f_0 = 24, f_2 = 33, h = 500$$

$$\text{Mode} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2} \right) \times h$$

$$\begin{aligned}
&= 1500 + \left(\frac{40 - 24}{2 \times 40 - 24 - 33} \right) \times 500 \\
&= 1500 + \left(\frac{16}{80 - 57} \right) \times 500 \\
&= 1500 + \frac{16 \times 500}{23} \\
&= 1500 + \frac{8000}{23} = 1500 + 347.83 = 1847.83
\end{aligned}$$

Mean:

$$a = 3250, \quad h = 500, \quad \sum f_i = 200, \quad \sum f_i u_i = -235$$

$$\begin{aligned}
\text{Mean}(\bar{x}) &= a + \left(\frac{\sum f_i u_i}{\sum f_i} \right) \times h \\
&= 3250 + \left(\frac{-235}{200} \right) \times 500 \\
&= 3250 - \frac{235 \times 5}{2} \\
&= 3250 - \frac{1175}{2} = 3250 - 587.5 = 2662.5
\end{aligned}$$

4. Find the mode and mean of this data. Interpret the two measures

Number of students	15-20	20-25	25-30	30-35	35-40	40-45	45-50	50-55
Number of States	3	8	9	10	3	0	0	2

Sol:

Number of students	Number of States	Class mark (x_i)	$u_i = \frac{x_i - a}{h}$	$f_i u_i$
15-20	3	17.5	-3	-9
20-25	8	22.5	-2	-16
25-30	9 $\rightarrow f_0$	27.5	-1	-9
30-35	10 $\rightarrow f_1$	32.5 $\rightarrow a$	0	0
35-40	3 $\rightarrow f_2$	37.5	1	3
40-45	0	42.5	2	0
45-50	0	47.5	3	0
50-55	2	52.5	4	8
	$\sum f_i = 35$		$\sum f_i u_i = -34 + 11 = -23$	

Mode: The highest frequency is 10. So the modal class is 30-35

$$l = 30, f_1 = 10, f_0 = 9, f_2 = 3, h = 5$$

$$\begin{aligned}
\text{Mode} &= l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2} \right) \times h \\
&= 30 + \left(\frac{10 - 9}{2 \times 10 - 9 - 3} \right) \times 5 \\
&= 30 + \left(\frac{1}{20 - 12} \right) \times 5
\end{aligned}$$

$$= 30 + \frac{5}{8} = 30 + 0.625 = 30.625$$

Mean:

$$a = 32.5, \quad h = 5, \quad \sum f_i = 35, \quad \sum f_i u_i = -23$$

$$\text{Mean}(\bar{x}) = a + \left(\frac{\sum f_i u_i}{\sum f_i} \right) \times h$$

$$= 32.5 + \left(\frac{-23}{35} \right) \times 5$$

$$= 32.5 - \frac{23}{7} = 32.5 - 3.3 = 29.2$$

Interpretation : Mode : 30.625 , Mean = 29.2. Most states/U.T. have a student teacher ratio of 30.6 and on an average, this ratio is 29.2.

5. The given distribution shows the number of runs scored by some top batsmen of the world in one-day international cricket matches. Find the mode of the data. ↗

Runs	3000-4000	4000-5000	5000-6000	6000-7000	7000-8000	8000-9000	9000-10000	10000-11000
Number of batsmen	4	18	9	7	6	3	1	1

Sol:

Runs	Number of batsmen
3000-4000	4 → f_0
4000-5000	18 → f_1
5000-6000	9 → f_2
6000-7000	7
7000-8000	6
8000-9000	3
9000-10000	1
10000-11000	1

The highest frequency is 18. So the modal class is 4000-5000

$$l = 4000, f_1 = 18, f_0 = 4, f_2 = 9, h = 1000$$

$$\text{Mode} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2} \right) \times h$$

$$= 4000 + \left(\frac{18 - 4}{2 \times 18 - 4 - 9} \right) \times 1000$$

$$= 4000 + \left(\frac{14}{36 - 13} \right) \times 1000$$

$$= 4000 + \frac{14000}{23} = 4000 + 608.7 = 4608.7 \text{ runs}$$

6. A student noted the number of cars passing through a spot on a road for 100 periods, each of 3 minutes, and summarised this in the table given below. Find the mode of the data.

Number of cars	0 - 10	10 - 20	20 - 30	30 - 40	40 - 50	50 - 60	60 - 70	70 - 80
Frequency	7	14	13	12	20	11	15	8

Sol:

Number of cars	Frequency
0-10	7
10-20	14
20-30	13
30-40	$12 \rightarrow f_0$
40-50	$20 \rightarrow f_1$
50-60	$11 \rightarrow f_2$
60-70	15
70-80	8

The highest frequency is 20. So the modal class is 40-50

$$l = 40, f_1 = 20, f_0 = 12, f_2 = 11, h = 10$$

$$\begin{aligned} \text{Mode} &= l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2} \right) \times h \\ &= 40 + \left(\frac{20 - 12}{2 \times 20 - 12 - 11} \right) \times 10 \\ &= 40 + \left(\frac{8}{40 - 23} \right) \times 10 = 40 + \frac{80}{17} = 40 + 4.7 = 44.7 \text{ cars} \end{aligned}$$

MEDIAN OF UNGROUPED DATA:

- Median is the value of the middle – most observation in the data.
- we first arrange the data values or the observations in ascending order. Then

(i) If n is odd, the median = $\left(\frac{n+1}{2}\right)^{\text{th}}$ observation.

(ii) If n is even, the median will be the average of the $\left(\frac{n}{2}\right)^{\text{th}}$ and $\left(\frac{n}{2} + 1\right)^{\text{th}}$ observations

$$\text{(i.e.) If } n \text{ is even, the median} = \frac{\left(\frac{n}{2}\right)^{\text{th}} \text{ observation} + \left(\frac{n}{2} + 1\right)^{\text{th}} \text{ observation}}{2}$$

Example: Find median of the given data.

Marks obtained	20	29	28	33	42	38	43	25
Number of students	6	28	24	15	2	4	1	20

Sol: First, we arrange the marks in ascending order and prepare a frequency table. To find the position of these middle values, we construct cumulative frequency.

Marks obtained	Number of students	Cumulative frequency
20	6	6
upto 25	6 + 20 = 26	26
upto 28	26 + 24 = 50	50
upto 29	50 + 28 = 78	78
upto 33	78 + 15 = 93	93
upto 38	93 + 4 = 97	97
upto 42	97 + 2 = 99	99
upto 43	99 + 1 = 100	100

Here' n'=100. Which is even

$$\frac{n}{2} = \frac{100}{2} = 50 \quad \text{and} \quad \frac{n}{2} + 1 = 50 + 1 = 51$$

50th observation is 28 and 51st observation is 29

$$\text{Median} = \frac{28+29}{2} = \frac{57}{2} = 28.5$$

MEDIAN OF GROUPED DATA:

The class whose cumulative frequency exceeds $\frac{n}{2}$ for the first time. This is called the median class.

$$\text{Median} = l + \left(\frac{\frac{n}{2} - cf}{f} \right) \times h$$

where l = lower boundary of median clas,

n = number of observations,

cf = cumulative frequency of class preceding the median class,

f = frequency of median class,

h = class size (size of the median class).

Example: Find median of the given data

Marks	0-10	10-20	20-30	30-40	40-50	50-60	60-70	70-80	80-90	90-100
Number of students	5	3	4	3	3	4	7	9	7	8

Sol:

Marks	Number of students (f)	Cumulative frequency (cf)
0-10	5	5
10-20	3	8
20-30	4	12
30-40	3	15
40-50	3	18
50-60	4	22 $\rightarrow cf$
60-70	7 $\rightarrow f$	29
70-80	9	38
80-90	7	45
90-100	8	53
	$n = 53$	

$$\frac{n}{2} = \frac{53}{2} = 26.5.$$

From cumulative frequency 60-70 is the median class.

$$l = 60, \quad cf = 22, \quad f = 7, \quad h = 10$$

$$\begin{aligned} \text{Median} &= l + \left(\frac{\frac{n}{2} - cf}{f} \right) \times h \\ &= 60 + \left(\frac{26.5 - 22}{7} \right) \times 10 \\ &= 60 + \left(\frac{4.5}{7} \right) \times 10 \\ &= 60 + \frac{45}{7} = 60 + 6.43 = 66.43 \end{aligned}$$

Example – 7: Find the median height of the girls.

Height (in cm)	Number of girls
Less than 140	4
Less than 145	11
Less than 150	29
Less than 155	40
Less than 160	46
Less than 165	51

Sol:

Class intervals	Frequency	Cumulative frequency
Below 140	4	4
140-145	7	11 → cf
$l \leftarrow$ 145-150	18 → f	29
150-155	11	40
155-160	6	46
160-165	5	51

$n = 51, \frac{n}{2} = \frac{51}{2} = 25.5$. So median class is 145 - 150

$l = 145, cf = 11, f = 18, h = 5$

$$\begin{aligned} \text{Median} &= l + \left(\frac{\frac{n}{2} - cf}{f} \right) \times h \\ &= 145 + \left(\frac{25.5 - 11}{18} \right) \times 5 \\ &= 145 + \frac{14.5 \times 5}{18} \\ &= 145 + \frac{72.5}{18} = 145 + 4.03 = 149.03 \end{aligned}$$

So, the median height of the girls is 149.03 cm.

Example-8. The median of the following data is 525. Find the values of x and y , if the total frequency is 100. Here, CI stands for class interval and Fr for frequency.

CI	0-100	100-200	200-300	300-400	400-500	500-600	600-700	700-800	800-900	900-1000
Fr	2	5	x	12	17	20	y	9	7	4

Sol:

Class intervals	Frequency	Cumulative frequency
0-100	2	2
100-200	5	7
200-300	x	$7+x$
300-400	12	$19+x$
400-500	17	$36+x$
500-600	20	$56+x$
600-700	y	$56+x+y$
700-800	9	$65+x+y$
800-900	7	$72+x+y$
900-1000	4	$76+x+y$

Given $n=100$

So, $76+x+y=100$

$$x+y=100-76$$

$$x+y=24\text{-----(1)}$$

The median is 525, which lies in the class 500 – 600

$$\text{So, } l = 500, f = 20, cf = 36 + x, h = 100$$

$$\text{Median} = l + \left(\frac{\frac{n}{2} - cf}{f} \right) \times h$$

$$525 = 500 + \left(\frac{50 - [36 + x]}{20} \right) \times 100$$

$$525 = 500 + (50 - 36 - x) \times 5$$

$$525 - 500 = (14 - x) \times 5$$

$$\frac{25}{5} = 14 - x$$

$$\Rightarrow 5 = 14 - x$$

$$\Rightarrow x = 14 - 5$$

$$\Rightarrow x = 9$$

$$\text{From (1) : } 9 + y = 24 \Rightarrow y = 24 - 9 \Rightarrow y = 15$$

Which measure would be best suited for a particular requirement.

1. The mean is the most frequently used measure of central tendency because it takes into account all the observations, and lies between the extremes.
2. extreme values in the data affect the mean.
3. where individual observations are not important, especially extreme values, and we wish to find out a 'typical' observation, the median is more appropriate.
4. finding the typical productivity rate of workers, average wage in a country, etc. These are situations where extreme values may exist. So, rather than the mean, we take the median as a better measure of central tendency.
5. In situations which require establishing the most frequent value or most popular item, the mode is the best choice,

Example: To find the most popular T.V. programme being watched, the consumer item in greatest demand, the colour of the vehicle used by most of the people, etc

EXERCISE - 14.3

1. Find the median, mean and mode of the data and compare them

Monthly consumption (in units)	65-85	85-105	105-125	125-145	145-165	165-185	185-205
Number of consumers	4	5	13	20	14	8	4

Sol:

Monthly consumption	Number of consumers	Class mark (x_i)	$u_i = \frac{x_i - a}{h}$	$f_i u_i$	Cumulative frequency
---------------------	---------------------	-------------------------	---------------------------	-----------	----------------------

65-85	4	75	-3	-12	4
85-105	5	95	-2	-10	9
105-125	13 → f_0	115	-1	-13	22 → cf
125-145	20 → $f_1(f)$	135 → a	0	0	42
145-165	14 → f_2	155	1	14	56
165-185	8	175	2	16	64
185-205	4	195	3	12	68
	$n = \sum f_i = 68$		$\sum f_i u_i = -35 + 42 = 7$		

Mean:

$$a = 135, \sum f_i u_i = 7, \sum f_i = 68, h = 20$$

$$\text{Mean}(\bar{x}) = a + \left(\frac{\sum f_i u_i}{\sum f_i} \right) \times h$$

$$= 135 + \left(\frac{7}{68} \right) \times 20$$

$$= 135 + \frac{7 \times 5}{17}$$

$$= 135 + \frac{35}{17} = 135 + 2.06 = 137.06$$

Mode: Highest frequency is 20. So, modal class = 125-145

$$l = 125, f_1 = 20, f_0 = 13, f_2 = 14, h = 20$$

$$\text{Mode} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2} \right) \times h$$

$$= 125 + \left(\frac{20 - 13}{2 \times 20 - 13 - 14} \right) \times 20$$

$$= 125 + \left(\frac{7 \times 20}{40 - 27} \right)$$

$$= 125 + \frac{140}{13}$$

$$= 125 + 10.77 = 135.77$$

Median:

$$n = 68, \frac{n}{2} = \frac{68}{2} = 34. \text{ So median class is } 125 - 145$$

$$l = 125, cf = 22, f = 20, h = 20$$

$$\text{Median} = l + \left(\frac{\frac{n}{2} - cf}{f} \right) \times h$$

$$= 125 + \left(\frac{34 - 22}{20} \right) \times 20$$

$$= 125 + 12 = 137$$

Median = 137 units, Mean = 137.06 units, Mode = 135.77 units. The three measures are approximately the same in this case.

2. If the median of 60 observations, given below is 28.5, find the values of x and y.

Class interval	0-10	10-20	20-30	30-40	40-50	50-60
Frequency	5	x	20	15	y	5

Sol:

Class Interval	Frequency	Cumulative Frequency
0-10	5	5
10-20	X	5+x
20-30	20	25+x
30-40	15	40+x
40-50	Y	40+x+y
50-60	5	45+x+y
	n= 45+x+y	

Given that $n=60$. So,

$$45 + x + y = 60$$

$$x + y = 60 - 45 \Rightarrow x + y = 15 \text{-----}$$

(1)

Since Median=28.5.

Therefore Median class=20-30

$$l = 20, \frac{n}{2} = 30, f = 20, cf = 5 + x, h = 10$$

$$\text{Median} = l + \left(\frac{\frac{n}{2} - cf}{f} \right) \times h$$

$$28.5 = 20 + \left(\frac{30 - [5+x]}{20} \right) \times 10$$

$$28.5 = 20 + \left(\frac{30 - 5 - x}{2} \right)$$

$$28.5 - 20 = \frac{25 - x}{2}$$

$$8.5 = \frac{25 - x}{2}$$

$$8.5 \times 2 = 25 - x$$

$$17 = 25 - x$$

$$x = 25 - 17$$

$$\therefore x = 8$$

$$\text{From (1) } x + y = 15$$

$$8 + y = 15$$

$$y = 15 - 8$$

$$y = 7$$

3. A life insurance agent found the following data about distribution of ages of 100 policy holders. Calculate the median age.

Age (in years)	Below 20	Below 25	Below 30	Below 35	Below 40	Below 45	Below 50	Below 55	Below 60
Number of policy holders	2	6	24	45	78	89	92	98	100

Sol:

Age(in years)	Number of policy holders (Frequency)	Cumulative Frequency
Below 20	2	2

20-25	4	6
25-30	18	24
30-35	21	45 → <i>cf</i>
35-40	33 → <i>f</i>	78
40-45	11	89
45-50	3	92
50-55	6	98
55-60	2	100
	$n = \sum f_i = 100$	

$n = 100$, $\frac{n}{2} = \frac{100}{2} = 50$. So median class is 35-40

$l = 35$, $cf = 45$, $f = 33$, $h = 5$

$$\begin{aligned} \text{Median} &= l + \left(\frac{\frac{n}{2} - cf}{f} \right) \times h \\ &= 35 + \left(\frac{50 - 45}{33} \right) \times 5 \\ &= 35 + \frac{5 \times 5}{33} \\ &= 35 + \frac{25}{33} = 35 + 0.76 = 35.76 \end{aligned}$$

4. Find the median length of the leaves.

Length (in mm)	118-126	127-135	136-144	145-153	154-162	163-171	172-180
Number of leaves	3	5	9	12	5	4	2

Sol:

Length (in mm)	Number of leaves	Cumulative Frequency
118-126	3	3
127-135	5	8
136-144	9	17 → <i>cf</i>
145-153	12 → <i>f</i>	29
154-162	5	34
163-171	4	38
172-180	2	40
	$n = \sum f_i = 40$	

$n = 40$, $\frac{n}{2} = \frac{40}{2} = 20$. So median class is 35-40

$$l = \frac{144 + 145}{2} = \frac{289}{2} = 144.5, \quad cf = 17, \quad f = 12, \quad h = 9$$

$$\text{Median} = l + \left(\frac{\frac{n}{2} - cf}{f} \right) \times h$$

$$\begin{aligned}
 &= 144.5 + \left(\frac{20 - 17}{12}\right) \times 9 \\
 &= 144.5 + \frac{3 \times 9}{12} \\
 &= 144.5 + \frac{9}{4} = 144.5 + 2.25 = 146.75.
 \end{aligned}$$

The median length of the leaves=146.75 mm

5. The following table gives the distribution of the life-time of 400 neon lamps. Find the median life time of a lamp.

Life time (in hours)	1500- 2000	2000- 2500	2500- 3000	3000- 3500	3500- 4000	4000- 4500	4500- 5000
Number of lamps	14	56	60	86	74	62	48

Sol:

Life time(in hours)	Number of lamps	Cumulative frequency
1500-2000	14	14
2000-2500	56	70
2500-3000	60	130 → <i>cf</i>
3000-3500	86 → <i>f</i>	216
3500-4000	74	290
4000-4500	62	352
4500-5000	48	400
	$n = \sum f_i = 400$	

$n = 400$, $\frac{n}{2} = \frac{400}{2} = 200$. So median class is 3000-3500

$l = 3000$, $cf = 130$, $f = 86$, $h = 500$

$$\begin{aligned}
 \text{Median} &= l + \left(\frac{\frac{n}{2} - cf}{f}\right) \times h \\
 &= 3000 + \left(\frac{200 - 130}{86}\right) \times 500 \\
 &= 3000 + \frac{70 \times 250}{43} \\
 &= 3000 + \frac{17500}{43} = 3000 + 406.98 = 3406.98
 \end{aligned}$$

The median life time of a lamp=3406.98 hours.

6. Determine the median number of letters in the surnames. Find the mean number of letters in the surnames? Also, find the modal size of the surnames

Number of letters	1-4	4-7	7-10	10-13	13-16	16-19
Number of surnames	6	30	40	16	4	4

Sol:

Number of letters	Number of surnames	Class mark (x_i)	$u_i = \frac{x_i - a}{h}$	$f_i u_i$	Cumulative frequency
1-4	6	2.5	-2	-12	6
4-7	30 $\rightarrow f_0$	5.5	-1	-30	36 $\rightarrow cf$
7-10	40 $\rightarrow f_1(f)$	8.5 $\rightarrow a$	0	0	76
10-13	16 $\rightarrow f_2$	11.5	1	16	92
13-16	4	14.5	2	8	96
16-19	4	17.5	3	12	100
	$n = \sum f_i = 100$		$\sum f_i u_i = -42 + 36 = -6$		

Median:

$n = 100$, $\frac{n}{2} = \frac{100}{2} = 50$. So median class is 7-10.

$l = 7$, $cf = 36$, $f = 40$, $h = 3$

$$\begin{aligned} \text{Median} &= l + \left(\frac{\frac{n}{2} - cf}{f} \right) \times h \\ &= 7 + \left(\frac{50 - 36}{40} \right) \times 3 \\ &= 7 + \frac{14 \times 3}{40} \\ &= 7 + \frac{21}{20} = 7 + 1.05 = 8.05 \end{aligned}$$

Mean:

$a = 8.5$, $\sum f_i u_i = -6$, $\sum f_i = 100$, $h = 3$

$$\begin{aligned} \text{Mean}(\bar{x}) &= a + \left(\frac{\sum f_i u_i}{\sum f_i} \right) \times h \\ &= 8.5 + \left(\frac{-6}{100} \right) \times 3 \\ &= 8.5 - \frac{18}{100} = 8.5 - 0.18 = 8.32. \end{aligned}$$

Mode: Highest frequency is 40. So, modal class = 7-10

$l = 7$, $f_1 = 40$, $f_0 = 30$, $f_2 = 16$, $h = 3$

$$\begin{aligned} \text{Mode} &= l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2} \right) \times h \\ &= 7 + \left(\frac{40 - 30}{2 \times 40 - 30 - 16} \right) \times 3 \\ &= 7 + \frac{10 \times 3}{80 - 46} \\ &= 7 + \frac{30}{34} = 7 + \frac{15}{17} = 7 + 0.88 = 7.88 \end{aligned}$$

Median = 8.05, Mean = 8.32, Modal size = 7.88

7. The distribution below gives the weights of 30 students of a class. Find the median weight of the

students.

Weight (in kg)	40-45	45-50	50-55	55-60	60-65	65-70	70-75
Number of students	2	3	8	6	6	3	2

Sol:

Weight (in kg)	Number of students	Cumulative frequency
40-45	2	2
45-50	3	5
50-55	8	13 → cf
55-60	6 → f	19
60-65	6	25
65-70	3	28
70-75	2	30
	$n = \sum f_i = 30$	

$n = 30$, $\frac{n}{2} = \frac{30}{2} = 15$. So median class is 55-60.

$l = 55$, $cf = 13$, $f = 6$, $h = 5$

$$\begin{aligned}\text{Median} &= l + \left(\frac{\frac{n}{2} - cf}{f} \right) \times h \\ &= 55 + \left(\frac{15 - 13}{6} \right) \times 5 \\ &= 55 + \frac{2 \times 5}{6} = 55 + \frac{5}{3} = 55 + 1.67 = 56.67\end{aligned}$$

Median weight = 56.67 kg.

GRAPHICAL REPRESENTATION OF CUMULATIVE FREQUENCY DISTRIBUTION:

1. Representing a cumulative frequency distribution graphically as a cumulative frequency curve, or an ogive of the less than type and of the more than type.
2. While drawing ogives, class boundaries are taken on X-axis and corresponding cumulative frequencies are taken on Y-axis.
3. The median of grouped data can be obtained graphically as the x-coordinate of the point of intersection of the two ogives of the data.

Less than cumulative frequency curve:

Take the upper boundaries of the class intervals as x-coordinate and their corresponding less than cumulative frequencies as y-coordinate. Plot the points on a graph and join them by a free hand smooth curve. The curve we get is called a less than cumulative frequency curve, or an ogive.

More than cumulative frequency curve:

Take the lower boundaries of the class intervals as x-coordinate and their corresponding more than cumulative frequencies as y-coordinate. Plot the points on a graph and join them by a free hand smooth curve. The curve we get is called a more than cumulative

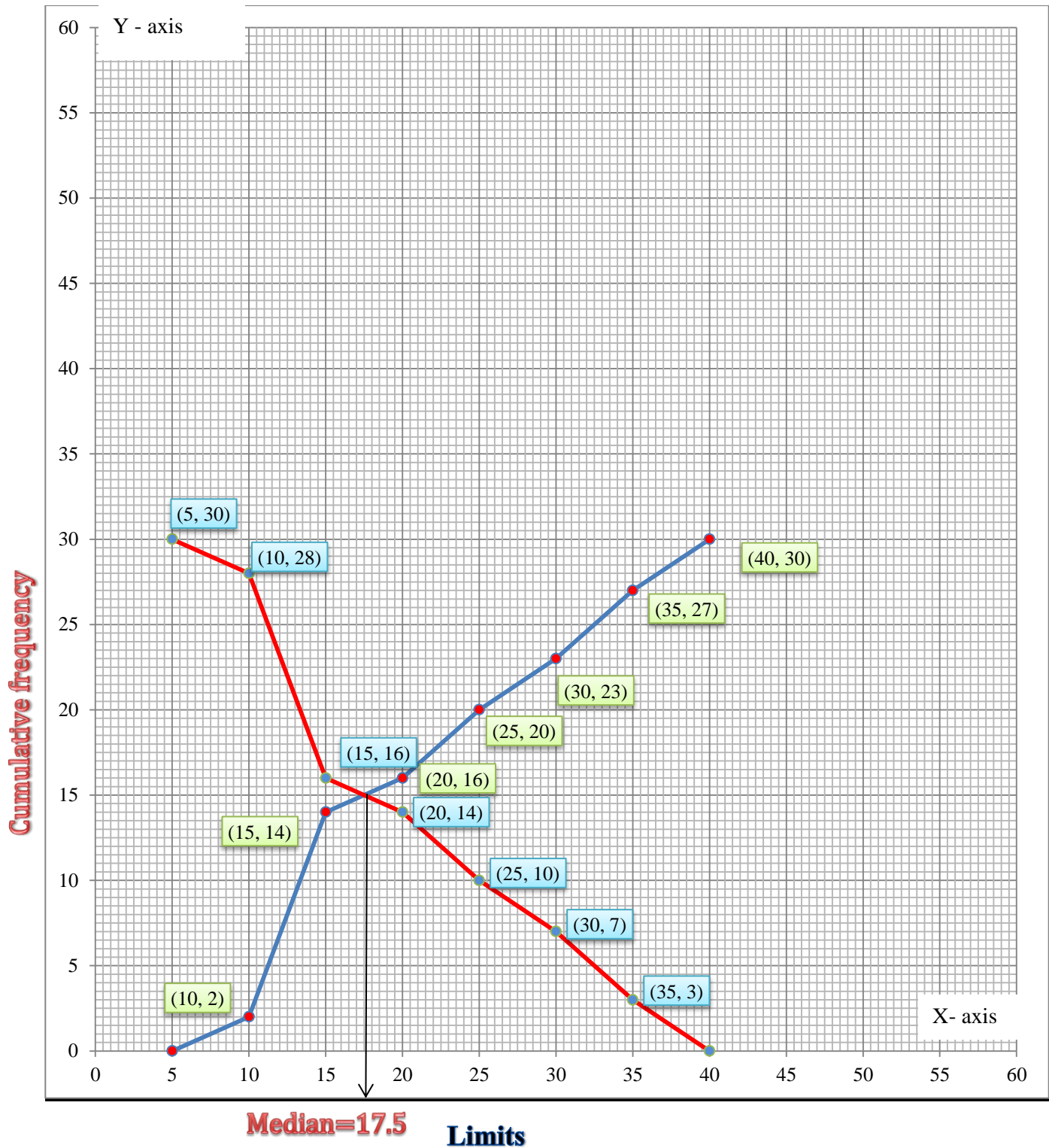
frequency curve, or an ogive.

Example-9. The annual profits earned by 30 shops in a locality give rise to the following distribution: Draw both ogives for the data above. Hence obtain the median profit.

Profit (in lakhs)	Number of shops (frequency)
More than or equal to 5	30
More than or equal to 10	28
More than or equal to 15	16
More than or equal to 20	14
More than or equal to 25	10
More than or equal to 30	7
More than or equal to 35	3

Sol:

Classes	frequency	Upper boundary	Less than cumulative frequency	points	Lower boundary	Greater than cumulative frequency	points
5-10	2	10	2 ↓	(10,2)	5	30	(5,30)
10-15	12	15	14	(15,14)	10	28	(10,28)
15-20	2	20	16	(20,16)	15	16	(15,16)
20-25	4	25	20	(25,20)	20	14	(20,14)
25-30	3	30	23	(30,23)	25	10	(25,10)
30-35	4	35	27	(35,27)	30	7	(30,7)
35-40	3	40	30	(40,30)	35	3 ↑	(35,3)



EXERCISE - 14.4

1. The following distribution gives the daily income of 50 workers of a factory

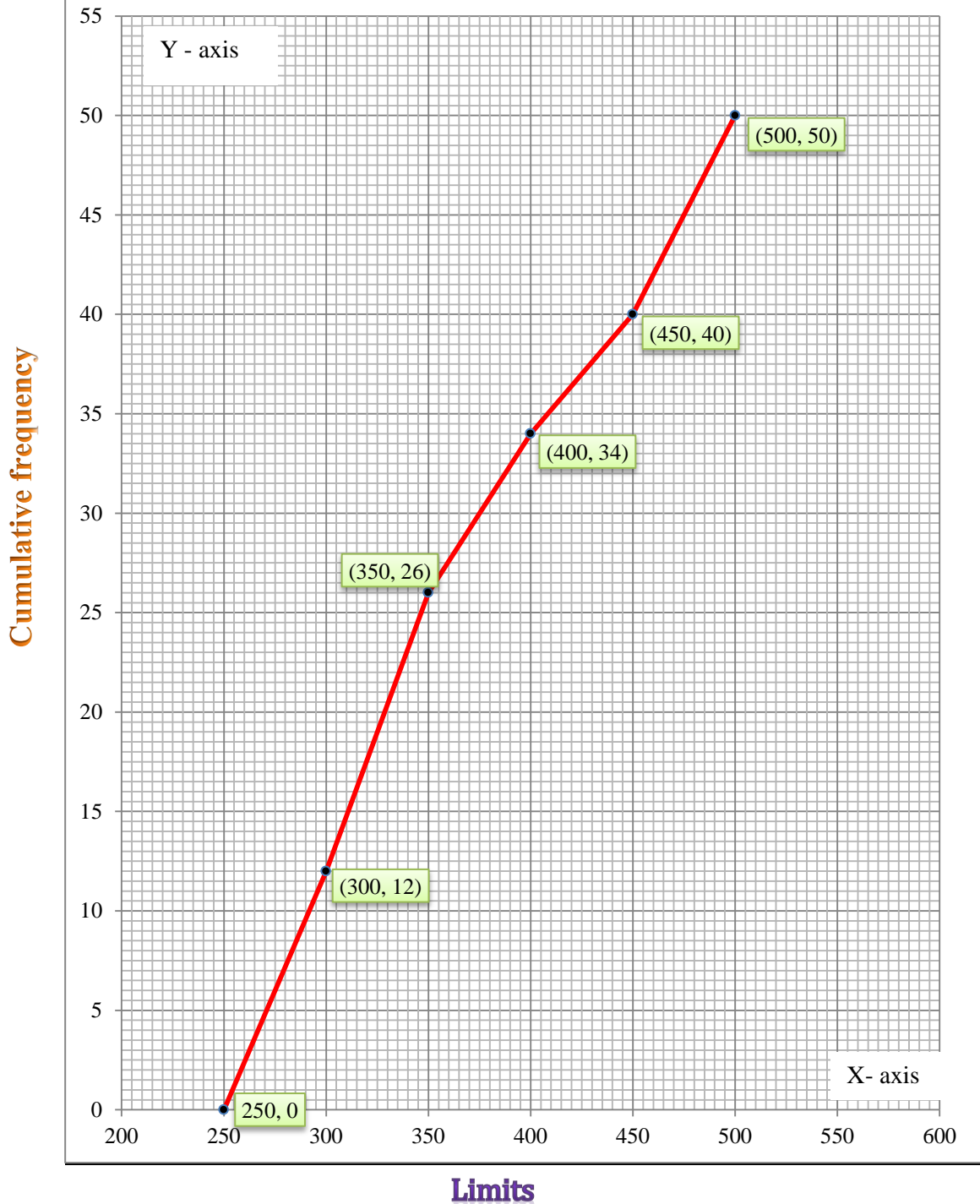
Daily income (in Rupees)	250-300	300-350	350-400	400-450	450-500
Number of workers	12	14	8	6	10

Convert the distribution above to a less than type cumulative frequency distribution, and draw

its ogive.

Sol:

Daily income (in Rupees)	Number of workers	Upper boundary	Less than cumulative frequency	points
250-300	12	300	12	(300,12)
300-350	14	350	26	(350,26)
350-400	8	400	34	(400,34)
400-450	6	450	40	(450,40)
450-500	10	500	50	(500,50)

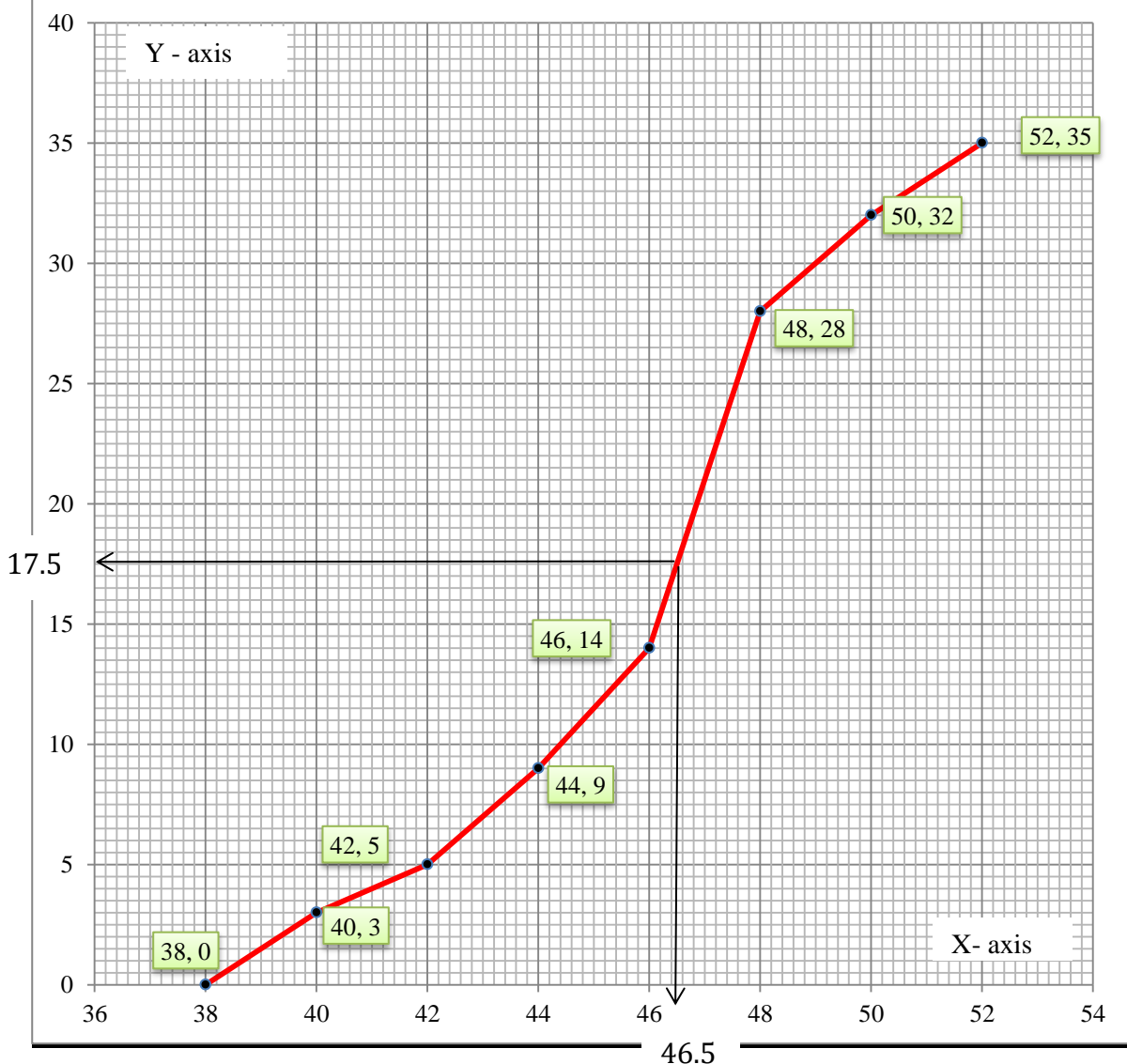


2. During the medical check-up of 35 students of a class, their weights were recorded as follows : Draw a less than type ogive for the given data. Hence obtain the median weight from the graph and verify the result by using the formula.

Weight (in kg)	Number of students
Less than 38	0
Less than 40	3
Less than 42	5
Less than 44	9
Less than 46	14
Less than 48	28
Less than 50	32
Less than 52	35

Sol:

Weight (in kg)	Number of students(f)	Less than cumulative frequency(y)	Upper bound(x)	Point(x,y)
0-38	0	0	38	(38,0)
38-40	3	3	40	(40,3)
40-42	2	5	42	(42,5)
42-44	4	9	44	(44,9)
44-46	5	14 \rightarrow cf	46	(46,14)
46-48	14 \rightarrow f	28	48	(48,28)
48-50	4	32	50	(50,32)
50-52	3	35	52	(52,35)



$$\frac{n}{2} = \frac{35}{2} = 17.5$$

Now Y-coordinate=17.5. From graph Corresponding X- coordinate= 46.5.

∴ Median=46.5

Median (Using formula):

$$\frac{n}{2} = \frac{35}{2} = 17.5. \text{ So median class } = 46-48.$$

$$l = 46, \quad cf = 14, \quad f = 14, \quad h = 2$$

$$\begin{aligned} \text{Median} &= l + \left(\frac{\frac{n}{2} - cf}{f} \right) \times h \\ &= 46 + \left(\frac{17.5 - 14}{14} \right) \times 2 \\ &= 46 + \frac{3.5}{7} = 46 + 0.5 = 46.5 \end{aligned}$$

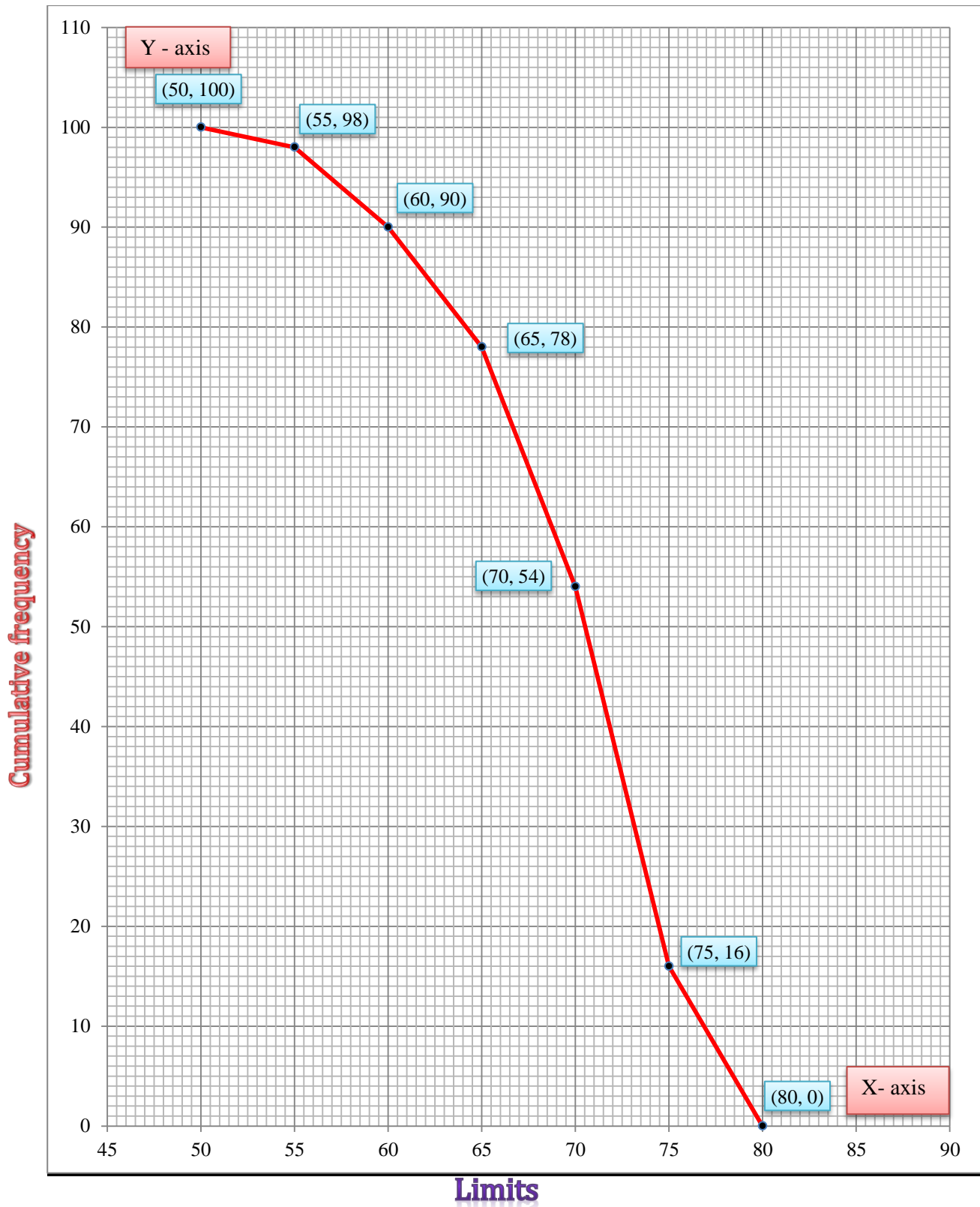
3. The following table gives production yield per hectare of wheat of 100 farms of a village

Production yield (Qui/Hec)	50-55	55-60	60-65	65-70	70-75	75-80
Number of farmers	2	8	12	24	38	16

Change the distribution to a more than type distribution, and draw its ogive.

Sol:

Production yield (Qui/Hec)	Number of farmers(f)	Lower boundary	Greater than cumulative frequency	points
50-55	2	50	100	(50,100)
55-60	8	55	98	(55,98)
60-65	12	60	90	(60,90)
65-70	24	65	78	(65,78)
70-75	38	70	54	(70,54)
75-80	16	75	16	(75,16)



THANK YOU
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