- 1. The line of sight is the line drawn from the eye of an observer to the point in the object viewed by the observer.
- 2. "The line of sight is above the horizontal line and angle between the line of sight and the horizontal line is called angle of elevation".
- 3. "The line of sight is below the horizontal line and angle between the line of sight and the horizontal line is called angle of depression".



- 4. Theodolites are used to measure angles of elevation or depression in the process of survey.
- Example-1. The top of a clock tower is observed at angle of elevation of α and the foot of the tower is at the distance of d meters from the observer. Draw the diagram for this data.
- Sol: Height of clock tower=AB

Observer point=C

Distance between foot of the tower and

observer=BC=d m

Angle of elevation= $\angle ACB = \alpha$

Example-2. Rinky observes a flower on the ground from the balcony of the first floor of a building at an angle of depression β° . The height of the first floor

of the building is x meters. Draw the diagram for this data.

Solution :

The height of the first floor of the building=AB = x m

Flower point=C, Angle of depression= $\angle DAC = \beta$

 $\angle DAC = \angle ACB = \beta$ (Alternate interior angles)





Example-3. A large balloon has been tied with a rope and it is floating in the air. A person has observed the balloon from the top of a building at angle of elevation of θ_1 and foot of the rope at an angle of depression of θ_2 . The height of the building is h feet. Draw the diagram for this data.

Solution:

Length of rope=AC

Building height=DE=h feet

Angle of elevation= $\angle BDC = \theta_1$

Angle of depression= $\angle ADB = \theta_2$

 $\angle ADB = \angle DAE = \theta_2$ (Alternate interior angles)



- 1. Draw diagram for the following situations:
- (i) A person is flying a kite at an angle of elevation α and the length of thread from his hand to kite is '*l*'.

Sol:

Length of thread=BC=l

Angle of elevation= $\angle ACB = \alpha$

(ii) A person observes two banks of a river at angles of depression θ_1 and θ_2 ($\theta_1 < \theta_2$) from the top of a tree of height h which is at a side of the river. The width of the river is 'd'.

Solution:

Height of tree=AB=h

Width of river=CD=*d*



α

А

- You are observing top of your school building at an angle of elevation α from a point which is at d meter distance from foot of the building. Which trigonometric ratio would you like to consider to find the height of the building?
- Sol: Height of the building=AB=h m

FHINK - DISCUSS

Distance from foot of the building to observing point=AC=d m

We have adjacent side .We want opposite side

So we consider tangent ratio

$$\tan \alpha = \frac{AB}{AC} = \frac{h}{d} \Rightarrow h = d \times \tan \alpha$$

2. A ladder of length x meter is leaning against a wall making angle θ with the ground. Which trigonometric ratio would you like to consider to find the height of the point on the wall at which the ladder is touching?

Sol: Length of ladder=BC=x m

The height of the point on the wall at which the ladder is touching=

We have hypotenuse . We want opposite side

So we consider sine ratio

$$\sin \theta = \frac{h}{x} \Rightarrow h = x \times \sin \theta$$

- Example-4. A boy observed the top of an electric pole at an angle of elevation of 60° when the observation point is 8 meters away from the foot of the pole. Find the height of the pole
- Sol: Height of electric pole=AB = h m

Observation point from the foot of the pole=AC = 8 m

Angle of elevation= $\angle ACB = 60^{\circ}$

$$\tan 60^{\circ} = \frac{AB}{AC}$$

$$\sqrt{3} = \frac{h}{8} \Rightarrow h = 8\sqrt{3} m$$

- Example-5. Rajender observes a person standing on the ground from a helicopter at an angle of depression 45°. If the helicopter flies at a height of 50 meters from the ground, what is the distance of the person from Rajender?
- Sol: Height of helicopter flies from the ground=AB=50 m

 $\angle PBC = \angle ACB = 45^{\circ}$ (Alternate interior angles)

distance of the person from Rajender =BC=x m

$$\sin 45^{\circ} = \frac{AB}{BC}$$
$$\frac{1}{\sqrt{2}} = \frac{50}{x}$$
$$x = 50\sqrt{2}$$

The distance from the person to Rajendar is $50\sqrt{2} m$



В

h

A

В

A

hm

А

h m

d

x m

h m

(60⁰

8m

С



1. A tower stands vertically on the ground. From a point which is 15 meter away from the foot of the tower, the angle of elevation of the top of the tower is 45°. What is the height of the tower?

Sol: Height of the tower=AB=h m

Observing point=C AC= 15 m Angle of elevation= $\angle ACB = 45^{\circ}$ $\tan 45^{\circ} = \frac{AB}{AC}$ $1 = \frac{h}{15} \Rightarrow h = 15$

$$C \xrightarrow{45^{0}}{15 m} A^{B}$$

x m

6m

30⁰

В

A

ym

The height of the tower=15 m

2. A tree breaks due to storm and the broken part bends so that the top of the tree touches the ground by making 30° angle with the ground. The distance between the foot of the tree and the top of the tree on the ground is 6m. Find the height of the tree before falling down.

Sol: Let length of remaining part
$$=AB=ym$$

Length of broken part=BC=x m

Angle of elevation = $\angle ACB = 30^{\circ}$

$$\cos 30^{\circ} = \frac{AC}{BC}$$

$$\frac{\sqrt{3}}{2} = \frac{6}{x} \Rightarrow x \times \sqrt{3} = 6 \times 2 \Rightarrow x = \frac{12}{\sqrt{3}} = \frac{12 \times \sqrt{3}}{\sqrt{3} \times \sqrt{3}} = \frac{12 \times \sqrt{3}}{3} = 4\sqrt{3}$$

$$\tan 30^{\circ} = \frac{AB}{AC}$$

$$\frac{1}{\sqrt{3}} = \frac{y}{6} \Rightarrow y \times \sqrt{3} = 6 \times 1 \Rightarrow y = \frac{6}{\sqrt{3}} = \frac{6 \times \sqrt{3}}{\sqrt{3} \times \sqrt{3}} = \frac{6 \times \sqrt{3}}{3} = 2\sqrt{3}$$

$$x + y = 4\sqrt{3} + 2\sqrt{3} = 6\sqrt{3}$$

The height of the tree before falling down= $6\sqrt{3} m$

3. A contractor wants to set up a slide for the children to play in the park. He wants to set it up at the height of 2 m and by making an angle of 30^o with the ground. What should be the length of the slide?

Length of slide=BC=x m

Angle of elevation= $\angle ACB = 30^{\circ}$

$$\sin 30^{\circ} = \frac{AB}{BC}$$
$$\frac{1}{2} = \frac{2}{x} \Rightarrow x = 2 \times 2 = 4$$

The length of slide=4 m

4. Length of the shadow of a 15 meter high pole is $5\sqrt{3}$ meters at 7 o'clock in the morning. Then, what is the angle of elevation of the Sun rays with the ground at the time?

x m

30⁰

θ

 $5\sqrt{3} m$

x m

С

30⁰

10 m

2m

15 m

A

Sol: Height of the pole=AB=15 m

Length of shadow=AC= $15\sqrt{3}m$

Let the angle of elevation $= \angle ACB = \theta$

$$\tan \theta = \frac{AB}{AC} = \frac{15}{5\sqrt{3}} = \frac{3}{\sqrt{3}} = \sqrt{3}$$
$$\Rightarrow \theta = 60^{\circ}$$

The angle of elevation of the Sun rays with the ground $= 60^{\circ}$

5. You want to erect a pole of height 10 m with the support of three ropes. Each rope has to make an angle 30° with the pole. What should be the total length of the rope required.

Sol: Height of pole=AB=10 m

Let length of each rope=BC=x mAngle made by the rope with the pole= 30°

$$\cos 30^{\circ} = \frac{AB}{BC}$$

$$\frac{\sqrt{3}}{2} = \frac{10}{x} \Rightarrow \sqrt{3} \times x = 2 \times 10 \Rightarrow x = \frac{20}{\sqrt{3}}$$

The length of each rope $=\frac{20}{\sqrt{3}}m$

Required length of rope =
$$3 \times \frac{20}{\sqrt{3}} = \sqrt{3} \times \sqrt{3} \times \frac{20}{\sqrt{3}} = 20\sqrt{3}m$$

6. Suppose you are shooting an arrow from the top of a building at a height of 6 m to a target on the ground at an angle of depression of 60°. What is the distance between you and the object?

Sol: Height of building=AB=6 m Target place=C Angle of depression=60⁰ $\angle PBC = \angle ACB = 60^{\circ}$ (Alternate interior angles) C
A
B
60⁰
B
6 m
C
C
A

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$$\sin 60^{\circ} = \frac{AB}{BC}$$
$$\frac{\sqrt{3}}{2} = \frac{6}{x}$$
$$\Rightarrow x = \frac{12}{\sqrt{3}} = \frac{12}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{12 \times \sqrt{3}}{3} = 4\sqrt{3}$$

Required distance= $4\sqrt{3} m$

7. An electrician wants to repair an electric connection on a pole of height 9 m. He needs to reach 1.8 m below the top of the pole to do repair work. What should be the length of the ladder which he should use, when he climbs it at an angle of 60° with the ground? What will be the distance between foot of the ladder and foot of the pole?

Sol: Height of pole=AD=9 m
Electrician reached point=B
BD = 1.8 m and AB = 9 - 1.8 = 7.2 m
Let length of ladder=BC=x m and
Required distance=AC= d m

$$\sin 60^{\circ} = \frac{AB}{BC}$$

 $\Rightarrow x = \frac{7.2 \times 2}{\sqrt{3}} = \frac{7.2 \times 2}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{7.2 \times 2 \times \sqrt{3}}{3} = 2.4 \times 2 \times \sqrt{3} = 4.8 \times 1.732 = 8.3136m$
 $\tan 60^{\circ} = \frac{AB}{AC} \Rightarrow \sqrt{3} = \frac{7.2 \times \sqrt{3}}{\sqrt{3}} = \frac{7.2 \times \sqrt{3}}{3} = 2.4 \times 1.732 = 4.1568 m$
8. A boat has to cross a river. It crosses the river by making an angle of 60° with the bank of the

river due to the stream of the river and travels a distance of 600m to reach the another side of the river. What is the width of the river?

Sol: Let width of the river=AC=
$$x m$$

Distance travelled by boat=BC=600 m
Angle made by boat= $\angle ACB = 60^{\circ}$
 $\cos 60^{\circ} = \frac{AC}{\pi a}$

BC



С

$$\frac{1}{2} = \frac{x}{600}$$
$$\Rightarrow x = \frac{600}{2} = 300$$

The width of the river = 300 m

9. An observer of height 1.8 m is13.2 m away from a palm tree. The angle of elevation of the top of the tree from his eyes is 45°. What is the height of the palm tree?

Sol: Height of observer=CD=AE=1.8 m



The height of the palm tree=AB=x + 1.8 = 13.2 + 1.8 = 15 m

10. In the adjacent figure, AC = 6 cm, AB = 5 cm and $\angle BAC = 30^{\circ}$. Find the area of the triangle.

Sol: Let
$$BD \perp AC$$

From $\triangle ADB$
 $\sin 30^{\circ} = \frac{BD}{AB}$
 $\frac{1}{2} = \frac{BD}{5}$
 $BD = \frac{5}{2}m$
Area of $\triangle ABC = \frac{1}{2} \times AC \times BD = \frac{1}{2} \times 6 \times \frac{5}{2} = \frac{30}{4} = 7.5 m^2$
Solution for Two Triangles

Ex: Suppose you are observing the top of the palm tree at an angle of elevation 45°. The angle of elevation changes to 30° when you move 11 m away from the tree. Find height of the tree.



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From
$$\triangle ACD$$

 $\tan 30^{\circ} = \frac{CD}{AC}$
 $\frac{1}{\sqrt{3}} = \frac{h}{11+x}$
 $\Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{11+h} (from (1) x = h)$
 $\Rightarrow \sqrt{3}h = 11 + h$
 $\Rightarrow \sqrt{3}h - h = 11$
 $\Rightarrow h(\sqrt{3} - 1) = 11$
 $\Rightarrow h = \frac{11}{\sqrt{3} - 1}$

The height of the palm tree $=\frac{11}{\sqrt{3}-1}m$

Example-6. Two men on either side of a temple of 30 meter height observe its top at the angles of elevation 30^o and 60^o respectively. Find the distance between the two men.

Sol: Height of the temple=BD=30 m
Observers places=A,C

$$\angle BAD = 30^{\circ}, \angle BCD = 60^{\circ}$$

Let $AD = x m$ and $CD = d m$
From ΔBAD
 $\tan 30^{\circ} = \frac{BD}{AD}$
 $\frac{1}{\sqrt{3}} = \frac{30}{x}$
 $x = 30\sqrt{3}$
From ΔBCD
 $\tan 60^{\circ} = \frac{BD}{CD}$
 $\sqrt{3} = \frac{30}{d}$
 $d = \frac{30}{\sqrt{3}} = \frac{30 \times \sqrt{3}}{\sqrt{3} \times \sqrt{3}} = \frac{30 \times \sqrt{3}}{3} = 10\sqrt{3}$

Distance between the persons= $x + d = 30\sqrt{3} + 10\sqrt{3} = 40\sqrt{3} m$

Example-7. A straight highway leads to the foot of a tower. Ramaiah standing at the top of the tower

observes a car at an angle of depression 30^o. The car is approaching the foot of the tower with a uniform speed. Six seconds later, the angle of depression of the car is found to be 60^o. Find the time taken by the car to reach the foot of the tower from this point



Time taken to travel 'x' meters = 6 seconds.

Time taken to travel the distance of 'd' meter $=\frac{x}{2}=\frac{6}{2}=3$ seconds

Exercise - 12.2

- A TV tower stands vertically on the side of a road. From a point on the other side directly
 opposite to the tower, the angle of elevation of the top of tower is 60°. From another point 10 m
 away from this point, on the line joining this point to the foot of the tower, the angle of elevation
 of the top of the tower is 30°. Find the height of the tower and the width of the road.
- Sol: Let the height of the tower=CD=h m



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D

$$\sqrt{3} = \frac{h}{x} \Rightarrow h = \sqrt{3}x \rightarrow (1)$$

From $\triangle ACD$
$$\tan 30^{0} = \frac{CD}{AC}$$

$$\frac{1}{\sqrt{3}} = \frac{h}{10 + x} \Rightarrow h = \frac{10 + x}{\sqrt{3}} \rightarrow (2)$$

From (1) & (2)
$$\sqrt{3}x = \frac{10 + x}{\sqrt{3}}$$

$$\sqrt{3} \times \sqrt{3}x = 10 + x$$

$$3x = 10 + x$$

$$3x - x = 10$$

$$2x = 10$$

$$x = \frac{10}{2} = 5$$

From (1), $h = \sqrt{3}x = \sqrt{3} \times 5 = 5\sqrt{3}$
 \therefore The width of road = 5 m

The height of tower = $5\sqrt{3}m$

A 1.5 m tall boy is looking at the top of a temple which is 30 meter in height from a point at certain distance. The angle of elevation from his eye to the top of the crown of the temple increases from 30° to 60° as he walks towards the temple. Find the distance he walked towards the temple.



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 $\Rightarrow x + d = 28.5\sqrt{3}$ $\Rightarrow 9.5\sqrt{3} + d = 28.5\sqrt{3}$ $\Rightarrow d = 28.5\sqrt{3} - 9.5\sqrt{3} = 19\sqrt{3}$

Distance walked by the boy towards the temple= $19\sqrt{3} = 19 \times 1.732 = 32.908 m$

3. A statue stands on the top of a 2m tall pedestal. From a point on the ground, the angle of elevation of the top of the statue is 60° and from the same point, the angle of elevation of the top of the pedestal is 45°. Find the height of the statue.



- 4. From the top of a building, the angle of elevation of the top of a cell tower is 60° and the angle of depression to its foot is 45°. If distance of the building from the tower is 7m, then find the height of the tower.
- Sol: Let the height of building=AB=CE=x m

The height of tower=CD=h m

Distance between tower and building=AC=BE=7 m

From $\triangle ABC$

$$\tan 45^{\circ} = \frac{AB}{AC}$$
$$1 = \frac{x}{7} \Rightarrow x = 7$$
From $\triangle BDE$
$$\tan 60^{\circ} = \frac{DE}{BE}$$



$$\sqrt{3} = \frac{h - x}{7}$$

$$h - x = 7\sqrt{3}$$

$$h - 7 = 7\sqrt{3}$$

$$h = 7\sqrt{3} + 7 = 7(\sqrt{3} + 1) = 7(1.732 + 1) = 7 \times 2.732 = 19.124 \text{ m}$$

 \therefore The height of the tower = 19.124 *m*

5. A wire of length 18 m had been tied with electric pole at an angle of elevation 30° with the ground. As it is covering a long distance, it was cut and tied at an angle of elevation 60° with the ground. How much length of the wire was cut?



the height of the building.

Sol: Let height of the building=AB=h mThe height of the tower=CD=30mAC=d m

From ∆ADC

$$\tan 60^{\circ} = \frac{30}{d}$$



$$\sqrt{3} = \frac{30}{d} \Rightarrow d = \frac{30}{\sqrt{3}}$$

From ΔBAC
$$\tan 30^{0} = \frac{h}{d}$$
$$\frac{1}{\sqrt{3}} = \frac{h}{d}$$
$$h = d \times \frac{1}{\sqrt{3}} = \frac{30}{\sqrt{3}} \times \frac{1}{\sqrt{3}} = \frac{30}{3} = 10$$

- \therefore The height of the building = 10 m
- 7. Two poles of equal heights are standing opposite to each other on either side of the road, which is 120 feet wide. From a point between them on the road, the angles of elevation of the top of the poles are 60^o and 30^o respectively. Find the height of the poles and the distances of the point from the poles.



The distances of the point from the poles=30 feet, 90 feet

- 8. The angles of elevation of the top of a tower from two points at a distance of 4 m and 9 m, find the height of the tower from the base of the tower and in the same straight line with it are complementary.
- Sol: Let the height of the tower=h

Observation points=C,D AC=4 m, AD=9 m Let $\angle ACB = \theta$ then $\angle ADB = 90 - \theta$ From $\triangle BAC$

$$\tan\theta = \frac{h}{4} \to (1)$$

From ∆BAD

$$\tan(90 - \theta) = \frac{h}{9}$$

$$\cot \theta = \frac{h}{9} \Rightarrow \tan \theta = \frac{9}{h} \rightarrow (2)$$

$$h$$

$$(90 - \theta) / \theta$$

$$D = 0$$

$$C = 4$$

$$A$$



- \therefore The height of the tower = 6 m
- 9. The angle of elevation of a jet plane from a point A on the ground is 60°. After a flight of 15 seconds, the angle of elevation changes to 30°. If the jet plane is flying at a constant height of $1500 \sqrt{3}$ meter, find the speed of the jet plane.($\sqrt{3} = 1.732$)
- Sol: Jet plane flying height from ground=BE=CD=1500 $\sqrt{3} m$

From $\triangle ABE$ *d m*_____∕ D В $\tan 60^0 = \frac{BE}{AE}$ $\sqrt{3} = \frac{1500\sqrt{3}}{r}$ $500\sqrt{3}m$ $1500\sqrt{3}$ x = 150060⁰ From $\triangle ACD$ Ц 30 $\tan 30^{\circ} = \frac{CD}{AC}$ С Е A x mdm $\frac{1}{\sqrt{3}} = \frac{1500\sqrt{3}}{x+d}$ $x + d = 1500\sqrt{3} \times \sqrt{3} = 1500 \times 3 = 4500$ $1500 + d = 4500 \Longrightarrow d = 3000$ The jet plane travels a distance of 3000 m in 15 seconds The speed of the jet plane = $\frac{\text{distance}}{\text{time}} = \frac{3000}{15} = 200 \text{ m/s}$