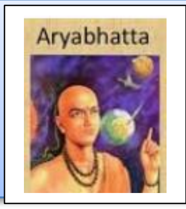
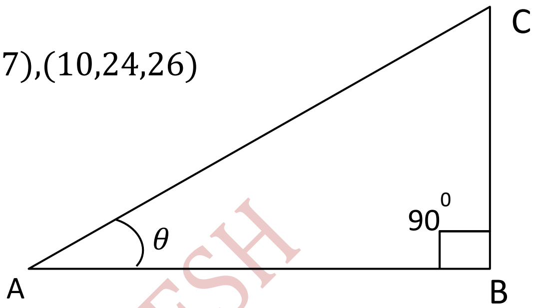
TRIGONOMETRY



1. Father of trigonometry Hipparchus.
2. In India, the father of trigonometry is **Aryabhata .**
3. Pythagoras theorem (Baudhayan theorem): In a right triangle, the square of length of the hypotenuse is equal to the sum of the squares of lengths of the other two sides.
4. $In\ \Delta ABC\ , \angle B = 90^0\ then\ AC^2 = AB^2 + BC^2$
5. Some pythagoran triplets
   (3,4,5),(5,12,13),(6,8,10),(7,24,25),(8,15,17),(10,24,26)
   (20,21,29)



NAMING THE SIDES IN A RIGHT TRIANGLE:

AC = Hypotenuse

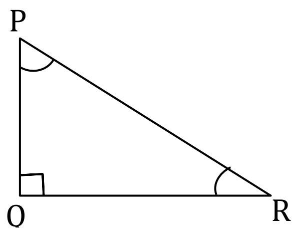
BC = Opposite side of angle A (θ)

AB = Adjacent side of angle A (θ)

Do This $(Page\ No - 271)$

Identify "Hypotenuse", "Opposite side" and "Adjacent side" for the given angles in the given triangles.

1. For angle R

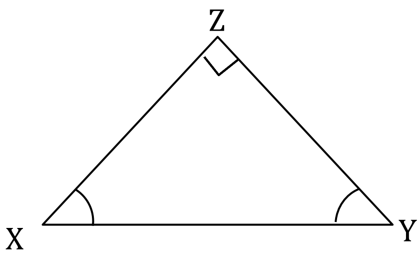


Sol: Hypotenuse=PR

Opposite side of angle R=PQ

Adjacent side of angle R=RQ

2. (i) For angle X



Sol: Hypotenuse=XY

Opposite side of angle X=ZY

Adjacent side of angle X=XZ

(ii) For angle X

Sol: Hypotenuse=XY

Opposite side of angle Y=ZX

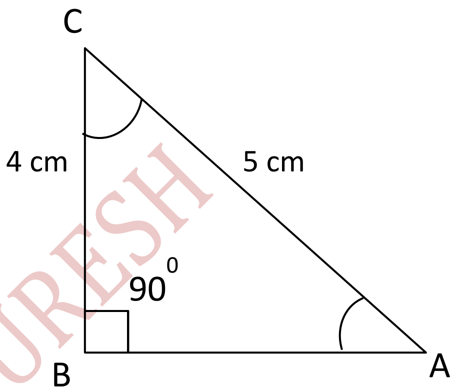
Adjacent side of angle Y=YZ

TRY THIS

Write lengths of "Hypotenuse", "Opposite side" and "Adjacent side" for the given angles in the given triangles.

Sol: $In\ \Delta ABC\ , \angle B\ = 90^0$

$AB^2 + BC^2 = AC^2$ (From Pythagoras theorem)



$AB^2 + 4^2 = 5^2$

$AB^2 + 16 = 25$

$AB^2 + 16 = 25$

$AB^2 = 25 - 16$

$AB^2 = 9 = 3^2$

AB=3 cm

1. **For angle C**

   Hypotenuse=AC=5 cm

   Opposite side of angle C=AB=3 cm

   Adjacent side of angle C=BC=4 cm.

2. **For angle A**

   Hypotenuse=AC=5 cm

   Opposite side of angle A=BC=4 cm

   Adjacent side of angle A=AB=3 cm.

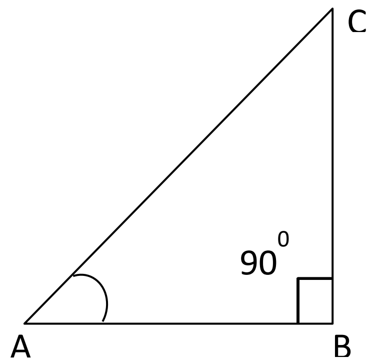
TRIGONOMETRIC RATIOS

(i) $SinA = \frac{(Length\ of\ )the\ side\ opposite\ to\ angle\ A}{(Length\ of)\ hypotenuse} = \frac{BC}{AC}$

(ii) $\cos A = \frac{\text{(Length of) the side adjacent to angle A}}{\text{(Length of) hypotenuse}} = \frac{AB}{AC}$

(iii) $\tan A = \frac{\text{(Length of)the side opposite to angle A}}{\text{Length of the side adjacent toangle A}} = \frac{BC}{AB}$

(iv) $cosec\ A = \frac{(Length\ of)\ hypotenuse}{(Length\ of\ )the\ side\ opposite\ to\ angle\ A} = \frac{AC}{BC}$



(v) $\sec A = \frac{(Length\ of\ ) hypotenuse}{(Length\ of)\ the\ side\ adjacent\ to angle\ A} = \frac{AC}{AB}$

(vi) $\cot A = \frac{(Length\ of\ ) the\ side\ adjacent\ to angle\ A}{(Length\ of\ ) the\ side\ opposite\ to\ angle\ A} = \frac{AB}{BC}$

RELATION SHIP BETWEEN TRIGONOMETERIC RATIOS:

(i) $SinA = \frac{1}{cosec\ A}$, $cosec\ A = \frac{1}{SinA}$, $SinA \times cosec\ A = 1$

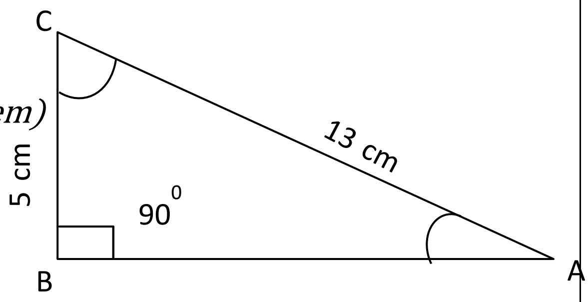
(ii) $cos\ A = \frac{1}{sec\ A}$, $sec\ A = \frac{1}{cos\ A}$, $cos\ A \times sec\ A = 1$

(iii) $tan\ A = \frac{1}{cot\ A}$, $cot\ A = \frac{1}{tan\ A}$, $tan\ A \times cot\ A = 1$

(iv) $\frac{SinA}{cos\ A} = tan\ A$, $\frac{cos\ A}{SinA} = cot\ A$

Do This $(Page\ No - 274)$

1. **Find (i) sin C (ii) cos C and (iii) tan C in the adjacent triangle**



Sol: $In\ \Delta ABC\ , \angle B = 90^0$

$AB^2 + BC^2 = AC^2$ (From Pythagoras theorem)

$AB^2 + 5^2 = 13^2$

$AB^2 + 25 = 169$

$AB^2 = 169 - 25$

$AB^2 = 144 \Rightarrow AB = \sqrt{144} \Rightarrow AB = 12\ cm$

(i) $sin\ C = \frac{Side\ opposite\ to\ \angle C}{Hypotenuse} = \frac{AB}{AC} = \frac{12}{13}$

$\therefore \sin C = \frac{12}{13}$

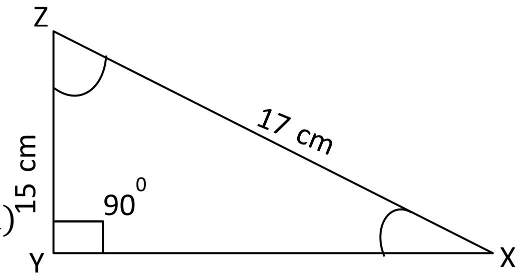
(ii) $cos\ C = \frac{Side\ adjacent\ to\ \angle C}{Hypotenuse} = \frac{BC}{AC} = \frac{5}{13}$

$\therefore \cos C = \frac{5}{13}$

$$(iii)\ tan\ C\ = \frac{Side\ opposite\ to\ \angle C}{Side\ adjacent\ to \angle C} = \frac{AB}{BC} = \frac{12}{5}$$

$\therefore \tan C = \frac{12}{5}$

2. In a triangle XYZ, ∠Y is right angle, XZ = 17cm and YZ = 15 cm, then find (i) sin X (ii) cos Z (iii) tan X



Sol: In $\Delta XYZ, \angle Y = 90^0$

$XY^2 + YZ^2 = XZ^2$ (From Pythagoras theorem)

$XY^2 + 15^2 = 17^2$

$XY^2 + 225 = 289$

$XY^2 = 289 - 225$

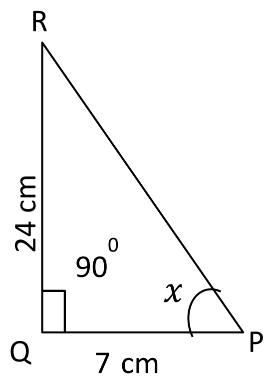
$XY^2 = 64 \Rightarrow XY = \sqrt{64} \Rightarrow XY = 8$ cm

$$(i)\ \sin X = \frac{\text{Side opposite to } \angle X}{\text{Hypotenuse}} = \frac{YZ}{XZ} = \frac{15}{17}$$

$$(ii)\ \cos Z = \frac{\text{Side adjacent to } \angle Z}{\text{Hypotenuse}} = \frac{YZ}{XZ} = \frac{15}{17}$$

$$(iii)\ \tan X\ = \frac{\text{Side opposite to } \angle X}{\text{Side adjacent to } \angle X} = \frac{YZ}{XY} = \frac{17}{8}$$

3. In a triangle PQR with right angle at Q, the value of ∠ P is x, PQ = 7 cm and QR = 24 cm, then find sin x and cos x.



Sol: In $\Delta PQR, \angle Q = 90^0$

$PR^2 = PQ^2 + QR^2$ (From Pythagoras theorem)

$PR^2 = 7^2 + 24^2$

$PR^2 = 49 + 576$

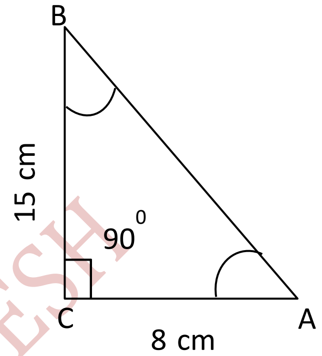
$PR^2 = 625 \Rightarrow PR = \sqrt{625} \Rightarrow PR = 25$ cm

$$\sin x = \frac{\text{Side opposite to angle } x}{\text{Hypotenuse}} = \frac{RQ}{PR} = \frac{24}{25}$$

$$\cos x = \frac{\text{Side adjacent to angle } x}{\text{Hypotenuse}} = \frac{PQ}{PR} = \frac{7}{25}$$

TRY THIS $(Page\ No - 274)$

In a right angle triangle ABC, right angle is at C. BC + CA = 23 cm and BC – CA = 7cm, then find sin A and tan B.



Sol: BC + CA = 23 cm------(1)

BC – CA = 7cm---------(2)

(1)+(2)⇒ 2 BC=23+7=30 cm

$BC = \frac{30}{2} = 15\text{cm}$

$(1) \Rightarrow 15 + CA = 23 \Rightarrow CA = 23 - 15 \Rightarrow CA = 8\ cm$

In ΔABC, $\angle C = 90^0$

$AB^2 = AC^2 + BC^2$ (From Pythagoras theorem)

$AB^2 = 8^2 + 15^2$

$AB^2 = 64 + 225 = 289$

$AB = \sqrt{289} = 17\ cm$

$$\sin A = \frac{\text{Side opposite to } \angle A}{\text{Hypotenuse}} = \frac{BC}{AB} = \frac{15}{17}$$

$$\tan B = \frac{\text{Side opposite to } \angle B}{\text{Side adjacent to } \angle B} = \frac{AC}{BC} = \frac{8}{15}$$

THINK - DISCUSS

(i) $\sin x = \frac{4}{3}$ does exist for some value of angle x ?

Sol: We know that in right angled triangle side is less than hypotenuse.

$$\Rightarrow \quad \frac{\text{side}}{\text{hypotenuse}} < 1$$

Given $\sin x = \frac{4}{3} > 1$

So, $\sin x = \frac{4}{3}$ does exist for some value of angle x .

(ii) The value of sin A and cos A is always less than 1. Why?

Sol: We know that in right angled triangle side is less than hypotenuse.

$$\Rightarrow \quad \frac{\text{side}}{\text{hypotenuse}} < 1$$

So, sin A and cos A is always less than 1 .

(iii) tan A is product of tan and A.

Sol: Which is wrong. Since tan A means the tangent value of angleA.

THINK - DISCUSS *(Page No – 275)*

1. $\frac{\sin A}{\cos A}$ **equal to tan A ?**

Sol: yes

$$\frac{\sin A}{\cos A} = \frac{\frac{\text{Side opposite to } \angle A}{\text{Hypotenuse}}}{\frac{\text{Side adjacent to} \angle A}{\text{Hypotenuse}}} = \frac{\text{Side opposite to } \angle A}{\text{Side adjacent to} \angle A} = \tan A$$

2. Is $\frac{\cos A}{\sin A}$ **equal to cot A ?**

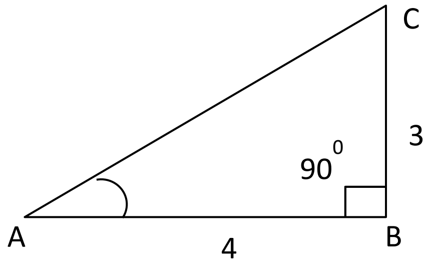
Sol: yes

$$\frac{\cos A}{\sin A} = \frac{\frac{Side\ adjacent\ to\ \angle A}{Hypotenuse}}{\frac{Side\ opposite\ to\ \angle A}{Hypotenuse}} = \frac{Side\ \ adjacent\ to \angle A}{Side\ opposite\ to\ \angle A} = \cot\ A$$

Example-1. If tan A $= \frac{3}{4}$ **, then find the other trigonometric ratio of angle** A.

Sol: $\quad \tan A = \frac{3}{4} = \frac{BC}{AB}$

BC=3, AB=4



In ΔABC , $\angle B = 90^0$

$AC^2 = AB^2 + BC^2$ (From Pythagoras theorem)

$AC^2 = 4^2 + 3^2$

$AC^2 = 16 + 9$

$AC^2 = 25 \Rightarrow AC = \sqrt{25} \Rightarrow AC = 5$

$\text{Sin A} = \frac{BC}{AC} = \frac{3}{5},$

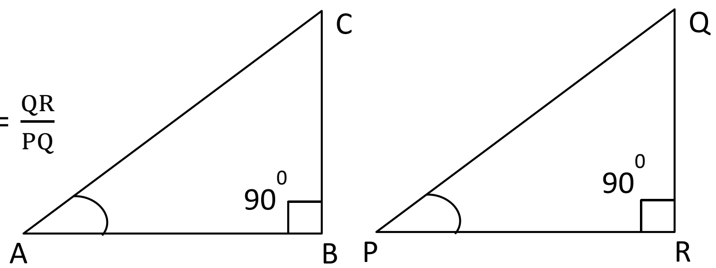
$\cos A = \frac{AB}{AC} = \frac{4}{5}$

$\text{cosec A} = \frac{AC}{BC} = \frac{5}{3}$

$\text{Sec A} = \frac{AC}{AB} = \frac{5}{4}$

$\cot A = \frac{AB}{BC} = \frac{4}{3}$

Example-2. In ΔABC and ΔPQR, if $\angle A$ and $\angle P$ are acute angles such that $\sin A = \sin P$ then prove that $\angle A = \angle P$.



Sol: we have $\sin A = \frac{BC}{AC}$ and $\sin P = \frac{QR}{PQ}$

Given $\sin A = \sin P$

$\frac{BC}{AC} = \frac{QR}{PQ} = k \text{ (say)}$

$\frac{BC}{QR} = \frac{AC}{PQ} \rightarrow (1)$

$BC = k.AC$ and $QR = k.PQ$

By using Pythagoras theorem

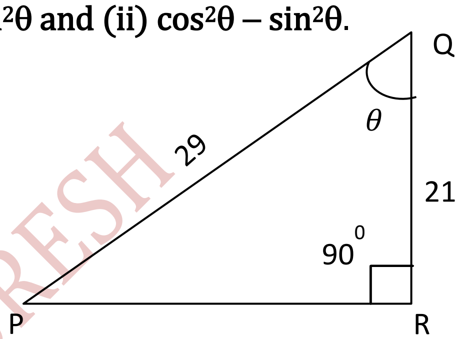
$$\frac{AB}{PR} = \frac{\sqrt{AC^2 - BC^2}}{\sqrt{PQ^2 - QR^2}} = \frac{\sqrt{AC^2 - k^2AC^2}}{\sqrt{PQ^2 - k^2PQ^2}} = \frac{\sqrt{AC^2(1-k^2)}}{\sqrt{PQ^2(1-k^2)}} = \frac{AC}{PQ} \rightarrow (2)$$

Hence, $\frac{AC}{PQ} = \frac{AB}{PR} = \frac{BC}{QR}$ (From (1) *and* (2))

Then $\Delta ABC \sim \Delta PQR$ (S.S.S similarity)

Therefore, $\angle A = \angle P$ (by CPST)

Example-3. Consider a triangle PQR, right angled at R, in which PQ = 29 units, QR = 21 units and $\angle PQR = \theta$, then find the values of (i) $\cos^2\theta + \sin^2\theta$ and (ii) $\cos^2\theta - \sin^2\theta$.



Sol: In ΔPQR, $\angle R = 90^0$

$PR^2 + QR^2 = PQ^2$ (From Pythagoras theorem)

$PR^2 = PQ^2 - QR^2$

$PR^2 = 29^2 - 21^2$

$PR^2 = 841 - 441 = 400 \Rightarrow PR = \sqrt{400} \Rightarrow PR = 20$

$$\sin\theta = \frac{PR}{PQ} = \frac{20}{29}$$

$$\cos\theta = \frac{QR}{PQ} = \frac{21}{29}$$

(i) $\cos^2\theta + \sin^2\theta = \left(\frac{21}{29}\right)^2 + \left(\frac{20}{29}\right)^2 = \frac{441}{841} + \frac{400}{841} = \frac{441+400}{841} = \frac{841}{841} = 1$

(ii) $\cos^2\theta - \sin^2\theta = \left(\frac{21}{29}\right)^2 - \left(\frac{20}{29}\right)^2 = \frac{441}{841} - \frac{400}{841} = \frac{441-400}{841} = \frac{41}{841}$

EXERCISE - 11.1

1. **In right angle triangle ABC, 8 cm, 15 cm and 17 cm are the lengths of AB, BC and CA respectively. Then, find out sin A, cos A and tan A.**

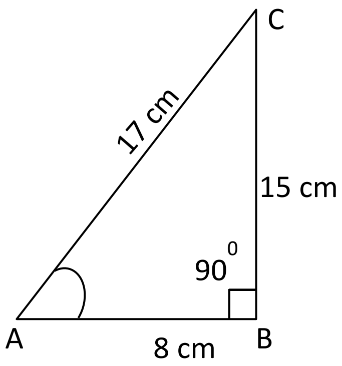
Sol: AB=8 cm, BC=15 cm, CA=17 cm

Here CA is hypotenuse. Therefore $\angle B = 90^0$

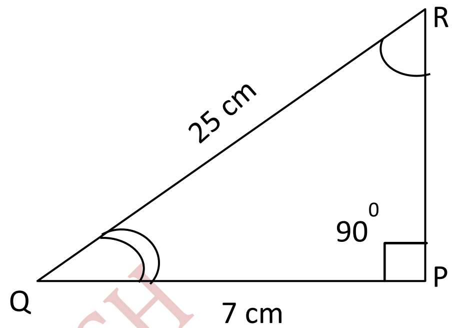
$\sin A = \frac{BC}{AC} = \frac{15}{17}$

$\cos A = \frac{AB}{AC} = \frac{8}{17}$

$\tan A = \frac{BC}{AB} = \frac{15}{8}$



2. The sides of a right angle triangle PQR are PQ = 7 cm, QR = 25cm and $\angle P = 90^0$ respectively .Then find, tan Q − tan R.



Sol: In $\Delta PQR, \angle P = 90^0$

$PR^2 + PQ^2 = RQ^2$ (From Pythagoras theorem)

$PR^2 = RQ^2 - PQ^2$

$PR^2 = 25^2 - 7^2$

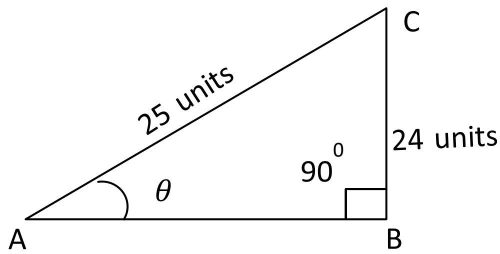
$PR^2 = 625 - 49 = 576 \Rightarrow PR = \sqrt{576} \Rightarrow PR = 24$ cm

$\tan Q = \frac{PR}{PQ} = \frac{24}{7}$

$\tan R = \frac{PQ}{PR} = \frac{7}{24}$

$\tan Q - \tan R = \frac{24}{7} - \frac{7}{24} = \frac{24 \times 24 - 7 \times 7}{7 \times 24} = \frac{576 - 49}{168} = \frac{527}{168}$

3. **In a right angle triangle ABC with right angle at B, in which a = 24 units, b = 25 units and $\angle BAC = \theta$. Then, find cos θ and tan θ.**



Sol: a=BC = 24 units, b =AC= 25 units

In $\Delta ABC, \angle B = 90^0$

$AB^2 + BC^2 = AC^2$ (From Pythagoras theorem)

$AB^2 = AC^2 - BC^2$

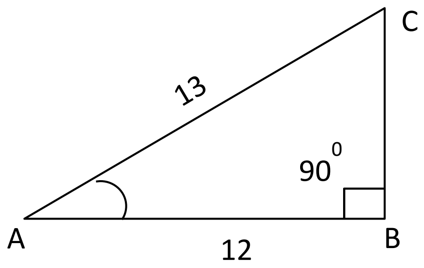
$AB^2 = 25^2 - 24^2$

$AB^2 = 625 - 576 = 49 \Rightarrow AB = \sqrt{49} \Rightarrow AB = 7$ units

$$\cos\theta = \frac{AB}{AC} = \frac{7}{25}$$

$$\tan\theta = \frac{BC}{AB} = \frac{24}{7}$$

4. **If $\cos A = \frac{12}{13}$, then find sin A and tan A.**



Sol: $\cos A = \frac{12}{13} = \frac{AB}{AC}$

AB=12, AC=13

In ΔABC, $\angle B = 90^0$

$AB^2 + BC^2 = AC^2$ (From Pythagoras theorem)

$BC^2 = AC^2 - AB^2$

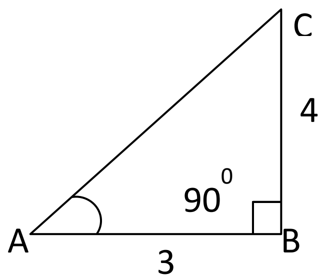
$BC^2 = 13^2 - 12^2$

$BC^2 = 169 - 144 = 25 \Rightarrow BC = \sqrt{25} \Rightarrow BC = 5$ units

$$\sin A = \frac{Opposite\ side\ to\ \angle A}{Hypotenuse} = \frac{BC}{AC} = \frac{5}{13}$$

$$\cos A = \frac{Adjacent\ side\ to\ \angle A}{Hypotenuse} = \frac{BC}{AB} = \frac{5}{12}$$

5. **If 3. tan A=4, then find sin A and cos A.**



Sol: $tan\ A = \frac{4}{3} = \frac{BC}{AB}$

Let $BC = 4$, $AB = 3$

$AC^2 = AB^2 + BC^2$ ($pythagoras\ therem$)

$AC^2 = (3)^2 + (4)^2 = 9 + 16 = 25$

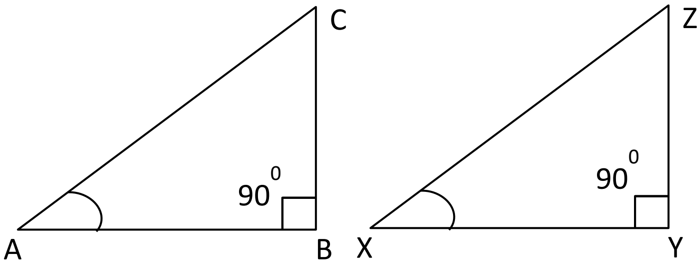
$AC = \sqrt{25} = 5$

$$\sin A = \frac{Opposite\ side\ to\ \angle A}{Hypotenuse} = \frac{BC}{AC} = \frac{4}{5}$$

$$\cos A = \frac{Adjacent\ side\ to\ \angle A}{Hypotenuse} = \frac{AB}{AC} = \frac{3}{5}$$

6. **In ΔABC and ΔXYZ, if ∠A and ∠X are acute angles such that cos A = cos X then show that ∠A = ∠X.**

Sol: we have $\cos A = \frac{AB}{AC}$ and $\cos X = \frac{XY}{XZ}$



Given $\cos A = \cos X$

$$\frac{AB}{AC} = \frac{XY}{XZ} = k \text{ (say)}$$

$AB = k.AC$ and $XY = k.XZ$

By using Pythagoras theorem

$$\frac{BC}{YZ} = \frac{\sqrt{AC^2 - AB^2}}{\sqrt{XZ^2 - XY^2}} = \frac{\sqrt{AC^2 - k^2AC^2}}{\sqrt{XZ^2 - k^2XZ^2}} = \frac{\sqrt{AC^2(1-k^2)}}{\sqrt{XZ^2(1-k^2)}} = \frac{AC}{XZ}$$

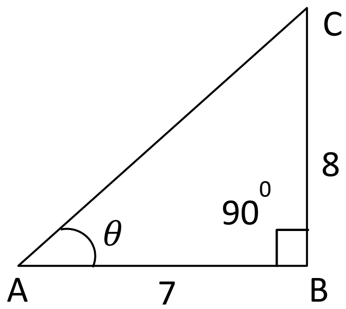
$$Hence, \frac{AC}{XZ} = \frac{AB}{XY} = \frac{BC}{YZ}$$

then $\Delta ABC \sim \Delta XYZ$ (S.S.S similarity)

Therefore, $\angle A = \angle X$ (by CPST)

7. **Given $\cot\theta = \frac{7}{8}$, then evaluate (i) $\frac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)}$ (ii) $\frac{(1+\sin\theta)}{\cos\theta}$**

Sol: $cot\,\theta = \frac{7}{8} = \frac{AB}{BC}$



$$AC^2 = AB^2 + BC^2 = 7^2 + 8^2 = 49 + 64 = 113$$

$$AC = \sqrt{113}$$

$\sin\theta = \frac{BC}{AC} = \frac{8}{\sqrt{113}}$, $\cos\theta = \frac{AB}{AC} = \frac{7}{\sqrt{113}}$

(i) $\frac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)}$

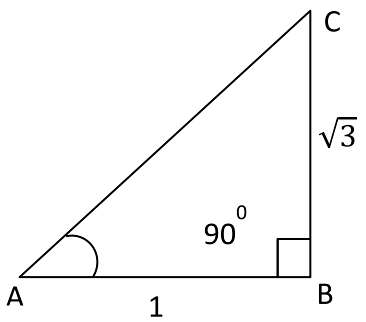
$$= \frac{\left(1+\frac{8}{\sqrt{113}}\right)\left(1-\frac{8}{\sqrt{113}}\right)}{\left(1+\frac{7}{\sqrt{113}}\right)\left(1-\frac{7}{\sqrt{113}}\right)} = \frac{(\sqrt{113}+8)(\sqrt{113}-8)}{(\sqrt{113}+7)(\sqrt{113}-7)} = \frac{113-64}{113-49} = \frac{49}{64}$$

(ii) $$\frac{1+\sin\theta}{\cos\theta} = \frac{1+\frac{8}{\sqrt{113}}}{\frac{7}{\sqrt{113}}} = \frac{\sqrt{113}+8}{7}$$

8. **In a right angle triangle ABC, right angle is at B, if $\tan A = \sqrt{3}$ then find the value of**

(i) $\sin A \cos C + \cos A \sin C$ (ii) $\cos A \cos C - \sin A \sin C$.

Sol: $\tan A = \frac{\sqrt{3}}{1} = \frac{BC}{AB}$



BC=$\sqrt{3}$, AB=1

$AC^2 = AB^2 + BC^2$ (*pythagoras therem*)

$AC^2 = (1)^2 + (\sqrt{3})^2 = 1 + 3 = 4$

$AC = \sqrt{4} = 2$

$$\sin A = \frac{Opposite\ side\ to\ \angle A}{Hypotenuse} = \frac{BC}{AC} = \frac{\sqrt{3}}{2}$$

$$\cos A = \frac{Adjacent\ side\ to\ \angle A}{Hypotenuse} = \frac{AB}{AC} = \frac{1}{2}$$

$$\sin C = \frac{Opposite\ side\ to\ \angle C}{Hypotenuse} = \frac{AB}{AC} = \frac{1}{2}$$

$$\cos C = \frac{Adjacent\ side\ to\ \angle C}{Hypotenuse} = \frac{BC}{AC} = \frac{\sqrt{3}}{2}$$

(i) sin A cos C + cos A sin C

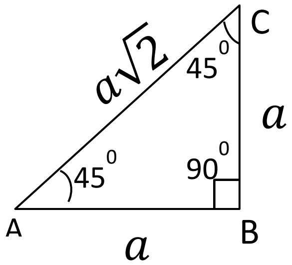
$$= \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} + \frac{1}{2} \times \frac{1}{2} = \frac{3}{4} + \frac{1}{4} = \frac{3+1}{4} = \frac{4}{4} = 1$$

(ii) cos A cos C – sin A sin C

$$= \frac{1}{2} \times \frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{2} \times \frac{1}{2} = \frac{\sqrt{3}}{4} - \frac{\sqrt{3}}{4} = 0$$

TRIGONOMETRIC RATIOS OF SOME SPECIFIC ANGLES:

TRIGONOMETRIC RATIOS OF 45^0 :



In $\Delta ABC, \angle B = 90^0$ *and* $\angle A = \angle C = 45^0$

Therefore AB=BC=a (equal angles opposite sides are equal)

Then $AC^2 = AB^2 + BC^2 = a^2 + a^2 = 2a^2$

$AC = \sqrt{2a^2} = a\sqrt{2}$

$$\sin 45^0 = \frac{BC}{AC} = \frac{a}{a\sqrt{2}} = \frac{1}{\sqrt{2}}$$

$$cosec\ 45^0 = \frac{AC}{BC} = \frac{a\sqrt{2}}{a} = \sqrt{2}$$

$$\cos 45^0 = \frac{AB}{AC} = \frac{a}{a\sqrt{2}} = \frac{1}{\sqrt{2}}$$

$$\sec 45^0 = \frac{AC}{AB} = \frac{a\sqrt{2}}{a} = \sqrt{2}$$

$$\tan 45^0 = \frac{BC}{AC} = \frac{a}{a} = 1$$

$$\cot 45^0 = \frac{AB}{BC} = \frac{a}{a} = 1$$

TRIGONOMETRIC RATIOS OF 30^0 AND 60^0 :

ΔABC *is an equilateral triangle and* $AD \perp BC$

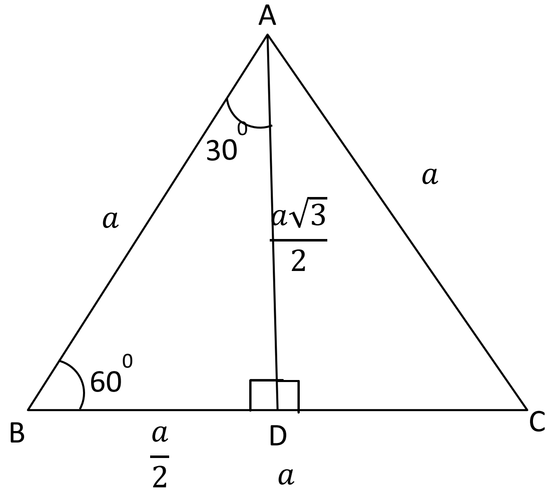
$AB = BC = AC = a \quad and \quad BD = DC = \frac{a}{2}$

$\angle BAD = \angle CAD = 30^0$

In ΔADB , $\angle D = 90^0$

$$AD^2 = AB^2 - BD^2 = a^2 - \frac{a^2}{4} = \frac{3a^2}{4}$$

$$AD = \sqrt{\frac{3a^2}{4}} = \frac{a\sqrt{3}}{2}$$



$$\sin 60^0 = \frac{AD}{AB} = \frac{\frac{a\sqrt{3}}{2}}{a} = \frac{a\sqrt{3}}{2a} = \frac{\sqrt{3}}{2}$$

$$\cos 60^0 = \frac{BD}{AB} = \frac{\frac{a}{2}}{a} = \frac{a}{2a} = \frac{1}{2}$$

$$\tan 60^0 = \frac{AD}{BD} = \frac{\frac{a\sqrt{3}}{2}}{\frac{a}{2}} = \frac{a\sqrt{3}}{a} = \sqrt{3}$$

$$cosec\ 60^0 = \frac{1}{sin\ 60^0} = \frac{2}{\sqrt{3}}$$

$$\sec 60^0 = \frac{1}{cos\ 60^0} = \frac{2}{1} = 2$$

$$\cot 60^0 = \frac{1}{\tan 60^0} = \frac{1}{\sqrt{3}}$$

$$\sin 30^0 = \frac{BD}{AB} = \frac{\frac{a}{2}}{a} = \frac{a}{2a} = \frac{1}{2}$$

$$\cos 30^0 = \frac{AD}{AB} = \frac{\frac{a\sqrt{3}}{2}}{a} = \frac{a\sqrt{3}}{2a} = \frac{\sqrt{3}}{2}$$

$$\tan 30^0 = \frac{BD}{AD} = \frac{\frac{a}{2}}{\frac{a\sqrt{3}}{2}} = \frac{a}{a\sqrt{3}} = \frac{1}{\sqrt{3}}$$

$$\mathrm{cosec}\ 30^0 = \frac{1}{sin\ 30^0} = \frac{2}{1} = 2$$

$$\sec 30^0 = \frac{1}{cos\ 30^0} = \frac{2}{\sqrt{3}}$$

$$\cot 30^0 = \frac{1}{tan\ 30^0} = \frac{\sqrt{3}}{1} = \sqrt{3}$$

TRIGONOMETRIC RATIOS OF 0⁰ AND 90⁰ :

In ΔABC If $\angle A = 0^0$ then $BC = 0\ and\ AB = AC = a$

$$sin 0^0 = \frac{BC}{AC} = \frac{0}{a} = 0$$

$$\cos 0^0 = \frac{AB}{AC} = \frac{a}{a} = 1$$

$$\tan 0^0 = \frac{BC}{AB} = \frac{0}{a} = 0$$

$$cose\ 0^0 = \frac{1}{sin 0^0} = \frac{1}{0} = not\ defined$$

$$\sec 0^0 = \frac{1}{cos\ 0^0} = \frac{1}{1} = 1$$

$$\cot 0^0 = \frac{1}{\tan 0^0} = \frac{1}{0} = not\ define$$

RIGONOMETRIC RATIOS OF 0^0 TO 90^0:

	$\angle A$	0^0	30^0	45^0	60^0	90^0
		$\sqrt{\frac{0}{4}}=0$	$\sqrt{\frac{1}{4}}=\frac{1}{2}$	$\sqrt{\frac{2}{4}}=\sqrt{\frac{1}{2}}=\frac{1}{\sqrt{2}}$	$\sqrt{\frac{3}{4}}=\frac{\sqrt{3}}{2}$	$\sqrt{\frac{4}{4}}=1$
	sin A	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
	cos A	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
$\frac{\sin A}{\cos A}$	$\tan A$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	not defined
$\frac{1}{\tan A}$	$\cot A$	not defined	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0
$\frac{1}{\cos A}$	$\sec A$	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	not defined
$\frac{1}{\sin A}$	$cosec\ A$	not defined	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1

THINK - DISCUSS

What can you say about the values of sin A and cos A, as the value of angle A increases from 0^0 to 90^0 ?

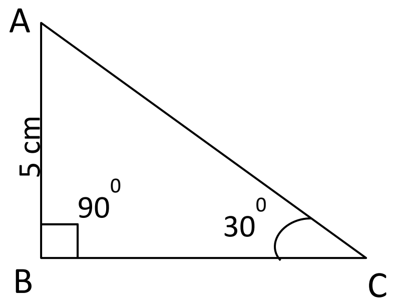
(i) If A $\geq$ B, then sin A $\geq$ sin B. Is it true ?

Sol: (i) Yes, If A $\geq$ B, then sin A $\geq$ sin B is true

(ii) If A $\geq$ B, then cos A $\geq$ cos B. Is it true ? Discuss

Sol: No, If A $\geq$ B, then cos A $\leq$ cos B

Example-4. In ΔABC, right angle is at B, AB = 5 cm and $\angle ACB = 30^0$. Determine the lengths of the sides BC and AC.



Sol: Given AB=5 cm and $\angle ACB=30^0$

$$\tan 30^0 = \frac{AB}{BC}$$

$$\frac{1}{\sqrt{3}} = \frac{5}{BC}$$

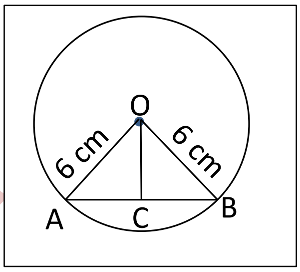
$BC = 5\sqrt{3}\ cm$

$\sin 30^0 = \frac{AB}{AC}$

$\frac{1}{2} = \frac{5}{AC}$

$AC = 5 \times 2 = 10\ cm$

Example-5. A chord of a circle of radius 6cm is making an angle 60^0 at the centre. Find the length of the chord.



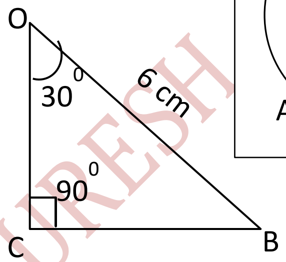
Sol: Radius of circle OA=OB=6cm and $\angle AOB = 60^0$

OC ⊥ AB then OC is angle bisector

$\angle AOC = \angle BOC = 30^0$

From ΔCOB , $sin 30^0 = \frac{BC}{OB}$

$\frac{1}{2} = \frac{BC}{6} \Rightarrow BC = \frac{6}{2} = 3cm$



Length of the chord AB= 2BC= $2 \times 3cm = 6cm$

Example-6. In ΔPQR, right angle is at Q, PQ = 3 cm and PR = 6 cm. Determine ∠QPR and ∠PRQ.

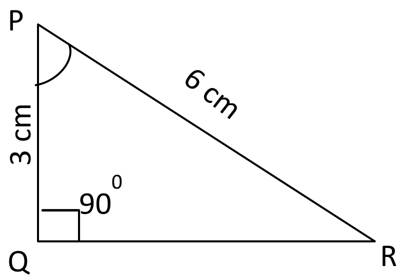
Sol: Given PQ = 3 cm and PR = 6 cm

$\cos P = \frac{PQ}{PR} = \frac{3}{6} = \frac{1}{2} = \cos 60^0$

So, $\angle QPR = 60^0$

$\sin R = \frac{PQ}{PR} = \frac{3}{6} = \frac{1}{2} = \sin 30^0$

So, $\angle PRQ = 30^0$



Example-7. $\boldsymbol{If\ sin\ (A-B) = \frac{1}{2}, cos\ (A+B) = \frac{1}{2}, 0^0 < A+B \leq 90^0, A > B,}$

$\boldsymbol{find\ A\ and\ B.}$

Sol: $sin\ (A - B) = \frac{1}{2} = sin\ 30^0$

$\Rightarrow A - B = 30^0$----------(1)

$cos\ (A\ +\ B) = \frac{1}{2} = cos\ 60^0$

$\Rightarrow \mathrm{A + B = 60^0}$----------(2)

(1)+(2)$\Rightarrow A - B + A + B = 30^0 + 60^0$

$\Rightarrow \mathrm{2A = 90^0 \Rightarrow A = \frac{90^0}{2} = 45^0}$

Substitute $\mathrm{A = 45^0}$ value in (2)

$45^0 + B = 60^0$

$\Rightarrow B = 60^0 - 45^0 = 15^0$

$\therefore A = 45^0 \quad and \quad B = 15^0$

EXERCISE - 11.2

1. Evaluate the following.

(i) $\mathbf{\sin 45^0 + \cos 45^0}$

Sol: $\sin 45^0 + \cos 45^0$

$$= \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} = \frac{2}{\sqrt{2}} = \frac{\sqrt{2} \times \sqrt{2}}{\sqrt{2}} = \sqrt{2}$$

(ii) $\dfrac{\boldsymbol{cos\ 45^0}}{\boldsymbol{sec\ 30^0 + cosec\ 60^0}}$

Sol: $\dfrac{cos\ 45^0}{sec\ 30^0 + cosec\ 60^0}$

$$= \frac{\frac{1}{\sqrt{2}}}{\frac{2}{\sqrt{3}} + \frac{2}{\sqrt{3}}} = \frac{\frac{1}{\sqrt{2}}}{\frac{4}{\sqrt{3}}} = \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{4} = \frac{\sqrt{3}}{4\sqrt{2}}$$

(iii) $\dfrac{\mathbf{\sin 30^0 + \tan 45^0} - \boldsymbol{cosec\ 60^0}}{\mathbf{\cot 45^0 + \cos 60^0 - \sec 30^0}}$

Sol: $\dfrac{sin\ 30^0 + tan\ 45^0 - cosec\ 60^0}{cot\ 45^0 + cos\ 60^0 - sec\ 30^0}$

$$= \frac{\frac{1}{2} + 1 - \frac{2}{\sqrt{3}}}{1 + \frac{1}{2} - \frac{2}{\sqrt{3}}} = 1$$

(iv) 2 tan²45º + cos²30º – sin² 60º.

Sol: 2 tan²45º + cos²30º – sin² 60º.

$$= 2 \times (1)^2 + \left(\frac{\sqrt{3}}{2}\right)^2 - \left(\frac{\sqrt{3}}{2}\right)^2 = 2 \times 1 = 2$$

(v) $\boldsymbol{\frac{sec^2 60^0 - tan^2 60^0}{sin^2 30^0 + cos^2 30^0}}$

Sol: $\frac{sec^2 60^0 - tan^2 60^0}{sin^2 30^0 + cos^2 30^0} = \frac{(2)^2 - (\sqrt{3})^2}{\left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2} = \frac{4-3}{\frac{1}{4} + \frac{3}{4}} = \frac{1}{\frac{4}{4}} = \frac{1}{1} = 1$

2. Choose the right option and justify your choice.

(i) $\boldsymbol{\frac{2 \tan 30^0}{1 + tan^2 45^0}}$

Sol: $\frac{2\, tan\, 30^0}{1 + tan^2 45^0} = \frac{2 \times \frac{1}{\sqrt{3}}}{1 + (1)^2} = \frac{\frac{2}{\sqrt{3}}}{2} = \frac{2}{2 \times \sqrt{3}} = \frac{1}{\sqrt{3}} = tan\, 30^0$

(ii) $\boldsymbol{\frac{1 - tan^2 45^0}{1 + tan^2 45^0}}$

Sol: $\frac{1 - tan^2 45^0}{1 + tan^2 45^0} == \frac{1-1}{1+1} = \frac{0}{2} = 0$

(iii) $\boldsymbol{\frac{2\, tan\, 30^0}{1 - tan^2 30^0}}$

Sol: $\frac{2\, tan\, 30^0}{1 - tan^2 30^0} = \frac{2 \times \frac{1}{\sqrt{3}}}{1 - \left(\frac{1}{\sqrt{3}}\right)^2} = \frac{\frac{2}{\sqrt{3}}}{1 - \frac{1}{3}} = \frac{\frac{2}{\sqrt{3}}}{\frac{2}{3}} = \frac{2}{\sqrt{3}} \times \frac{3}{2} = \frac{3}{\sqrt{3}} = \frac{\sqrt{3} \times \sqrt{3}}{\sqrt{3}} = \sqrt{3}$

3. Evaluate sin 60º cos 30º + sin 30º cos 60º . What is the value of sin (60º + 30º). What can you conclude?

Sol: $\sin 60^0 \cos 30^0 + \sin 30^0 \cos 60^0$

$$= \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} + \frac{1}{2} \times \frac{1}{2} = \frac{3}{4} + \frac{1}{4} = \frac{3+1}{4} = \frac{4}{4} = 1$$

$sin(60^0 + 30^0) = \sin 90^0 = 1$

Conclusion: $sin(60^0 + 30^0) = sin\ 60^0\ cos\ 30^0 + sin\ 30^0\ cos\ 60^0$

$\sin(A + B) = \sin A \cos B + \cos A \sin B$

4. Is it right to say $\cos(60^0 + 30^0) = cos\ 60^0\ cos\ 30^0 - sin\ 60^0\ sin\ 30^0$

Sol: $\cos(60^0 + 30^0) = \cos 90^0 = 0$

$$cos\ 60^0\ cos\ 30^0 - sin\ 60^0\ sin\ 30^0 = \frac{1}{2} \times \frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{2} \times \frac{1}{2} = \frac{\sqrt{3}}{4} - \frac{\sqrt{3}}{4} = 0$$

$\therefore \cos(60^0 + 30^0) = cos\ 60^0\ cos\ 30^0 - sin\ 60^0\ sin\ 30^0$

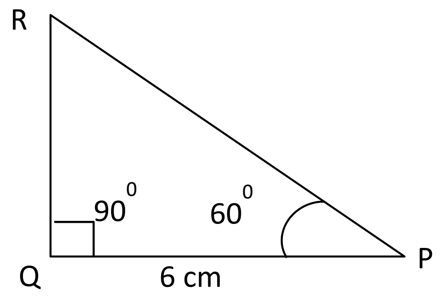
$\cos(A + B) = \cos A \cos B - \sin A \sin B$

5. In right angle triangle ΔPQR, right angle is at Q and PQ = 6cms ∠ RPQ = 60°.

Determine the lengths of QR and PR.

Sol: In ΔPQR , ∠Q=90^0

PQ = 6cms ∠ RPQ = 60°



$cos\ 60^0 = \frac{PQ}{PR}$

$$\frac{1}{2} = \frac{6}{PR}$$

$$PR = \frac{6 \times 2}{1} = 12$$

$\therefore$ PR = 12 cm

$$\tan 60^0 = \frac{QR}{PQ}$$

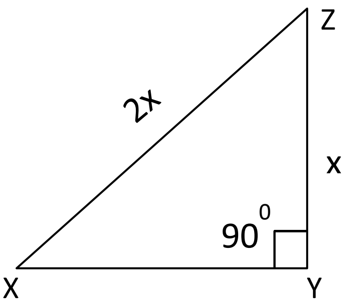
$$\sqrt{3} = \frac{QR}{6}$$

$\therefore QR = 6\sqrt{3}\ cm$

6. **In ΔXYZ, right angle is at Y, YZ = x, and XZ = 2x then determine $\angle YXZ$ and $\angle YZX$.**

Sol: $\angle YXZ = \alpha$ and $\angle YZX = \beta$

$\sin\alpha = \frac{YZ}{XZ} = \frac{x}{2x} = \frac{1}{2} = sin30^0$



$\sin\alpha = sin30^0$

$\therefore \alpha = \angle YXZ = 30^0$

$\alpha + 90^0 + \beta = 180^0$(angle sum property)

$30^0 + 90^0 + \beta = 180^0$

$\beta = 180^0 - 120^0 = 60^0$

$\therefore \beta = \angle YZX = 60^0$

7. **Is it right to say that sin (A + B) = sin A + sin B? Justify your answer.**

Sol: $\sin(A+B) = sin\,A + sin\,B$ is not correct

Justification:

Let $A = 30^0$ *and* $B = 60^0$

$\sin(A+B) = \sin(30^0 + 60^0) = \sin 90^0 = 1$

$sin\,A + sin\,B = \sin 30^0 + sin\,60^0 = \frac{1}{2} + \frac{\sqrt{3}}{2} = \frac{1+\sqrt{3}}{2}$

$\sin(A+B) \neq sin\,A + sin\,B$

THINK - DISCUSS

For which value of acute angle $\frac{\cos\theta}{1-\sin\theta} + \frac{\cos\theta}{1+\sin\theta} = 4$ is true?

For which value of $0^0 \le \theta \le 90^0$, above equation is not defined?

$\frac{\cos\theta}{1-\sin\theta} + \frac{\cos\theta}{1+\sin\theta} = 4$

Sol: $\frac{\cos\theta}{1-\sin\theta} + \frac{\cos\theta}{1+\sin\theta} = 4$

$\frac{\cos\theta(1+\sin\theta)+\cos\theta(1-\sin\theta)}{1-\sin^2\theta} = 4$

$\frac{\cos\theta(1+\sin\theta+1-\sin\theta)}{\cos^2\theta} = 4$

$\frac{COS\,\theta \times 2}{\cos^2\theta} = 4$

$\frac{2}{\cos\theta} = 4$

$\cos\theta = \frac{2}{4} = \frac{1}{2} = cos60^0$

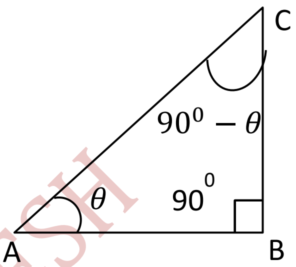
$\therefore\ \theta = 60^0$

For $\theta = 90^0$ the equation is not defined.

Since $\sin 90^0 = 1$ .So, $1 - \sin\theta = 1 - sin\, 90^0 = 1 - 1 = 0$

TRIGONOMETRIC RATIOS OF COMPLEMENTARY ANGLES:

If the sum of two angles is 90^0 then they are called complementary angles.



The complementary angle of θ is $(90^0 - \theta)$

Example: show that $\sin\,(90^0 - \theta) = \cos\theta$.

Sol: In $\Delta ABC, \angle B = 90^0$ and $\angle A = \theta$

Then $\angle C = 90^0 - \theta$

$sin\,(90^0 - \theta) = \frac{AB}{AC} = cos\,\theta$

So,	$\sin(90^o - A) = \cos A$		$\cos(90^o - A) = \sin A$
	$\tan(90^o - A) = \cot A$	and	$\cot(90^o - A) = \tan A$
	$\sec(90^o - A) = \text{cosec}\, A$		$\text{cosec}(90^o - A) = \sec A$

Example $-$ 8. Evaluat $\frac{sec35^0}{cosec\ 55^0}$

Sol: We know that $\sec A = \text{cosec}\,(90^0 - A)$

$sec35^0 = \text{cosec}\,(90^0 - 35^0)$

$\sec 35^0 = \text{cosec}\ 55^0$

$\frac{sec35^0}{cosec\ 55^0} = \frac{cosec\ 55^0}{cosec\ 55^0} = 1$

Example-9. If cos 7A = sin(A – 6^o), where 7A is an acute angle, find the value of A.

Sol: : Given cos 7A = sin(A – 6^o)

$\cos\theta = sin\,(90^0 - \theta)$

$sin\,(90^0 - 7A) = \sin(A - 6^0)$

$(90^0 - 7A)$ and $(A - 6^0)$ are acute angles

Therefore $90^0 - 7A = A - 6^0$

$A + 7A = 90^0 + 6^0$

$8A = 96^0$

$A = \frac{96^0}{8} = 12^0$

Example-10. $\boldsymbol{If \sin A = \cos B}$ **, then prove that** $\mathbf{A + B = 90^0}$.

Sol: : Given that $\sin A = \cos B$

$\cos\theta = sin\,(90^0 - \theta)$

$\sin A = sin\,(90^0 - B)$

Since A and $90^0 - B$ are acute angles.

$A = 90^0 - B$

$\Rightarrow A + B = 90^0$

Example-11: Express $\mathbf{\sin 81^0 + \tan 81^0}$ **in terms of trigonometric ratios of angles between** $\boldsymbol{0^0 \; and \; 45^0}$**.**

Sol: $\sin 81^0 + \tan 81^0$

$\sin\theta = \cos(90^0 - \theta)$

$\tan\theta = \cot(90^0 - \theta)$

$= \cos(90^0 - 81^0) + \cot(90^0 - 81^0)$

$= \cos 9^0 + \cot 9^0$

Example-12. If A, B and C are interior angles of triangle ABC, then show that

$\mathbf{\sin\left(\frac{B+C}{2}\right) = \cos\frac{A}{2}}$

Sol: Given A, B and C are interior angles of a triangle ABC then

$A+B+C= 180^0$

Dividing by 2 on both sides we get

$$\frac{A}{2} + \frac{B+C}{2} = \frac{180^0}{2}$$

$$\frac{B+C}{2} = 90^0 - \frac{A}{2}$$

On taking sin ratio on both sides

$\sin\left(\frac{B+C}{2}\right) = \sin\left(90^0 - \frac{A}{2}\right)$

$\sin\left(\frac{B+C}{2}\right) = \cos\frac{A}{2}$

$\sin(90^0 - \theta) = \cos\theta$

EXERCISE 11.3

1. Evaluate

(i) $\frac{tan36^0}{cot54^0}$

$\tan\theta = \cot(90^0 - \theta)$

Sol: $\frac{tan36^0}{cot54^0} = \frac{cot(90^0 - 36^0)}{cot54^0} = \frac{cot54^0}{cot54^0} = 1$

(ii) $\cos 12^0 - \sin 78^0$

Sol: $\cos 12^0 - \sin 78^0$

$\sin\theta = \cos(90^0 - \theta)$

$= \cos 12^0 - \cos(90^0 - 78^0)$

$= \cos 12^0 - \cos 12^0$

$= 0$

(iii) $cosec\ 31^0 - \sec 59^0$

Sol: $cosec\ 31^0 - \sec 59^0$

$\sec\theta = \mathrm{cosec}(90^0 - \theta)$

$= cosec\ 31^0 - cosec(90^0 - 59^0)$

$= cosec\ 31^0 - cosec\ 31^0$

$= 0$

(iv) $\sin 15^0 \sec 75^0$

Sol: $\sin 15^0 \sec 75^0$

$\sec\theta = \mathrm{cosec}(90^0 - \theta)$

$cose\ \theta = \frac{1}{\sin\theta}$

$= \sin 15^0 \times \mathrm{cosec}(90^0 - 75^0)$

$= sin\ 15^0 \times \mathrm{cosec}\ 15^0$

$= sin\ 15^0 \times \frac{1}{sin\ 15^0} = 1$

(v) $\tan 26^0 \tan 64^0$

Sol: $\tan 26^0 \tan 64^0$

$\tan\theta = \cot(90^0 - \theta)$

$cot\ \theta = \frac{1}{\tan\theta}$

$= tan\ 26^0\ cot(90^0 - 64^0)$

$= \tan 26^0 \times \cot 26^0$

$= \tan 26^0 \times \frac{1}{\tan 26^0} = 1$

2. **Show that**

(i) $\tan 48^0 \tan 16^0 \tan 42^0 \tan 74^0 = 1$

Sol: $\tan 48^0 \tan 16^0 \tan 42^0 \tan 74^0$

$= \tan 48^0 \tan 16^0 \cot(90^0 - 42^0)\cot(90^0 - 74^0)$

$= \tan 48^0 \tan 16^0 \cot 48^0 \cot 16^0$

$= \tan 48^0 \times \tan 16^0 \times \frac{1}{\tan 48^0} \times \frac{1}{\tan 16^0}$

$= 1$

$\tan\theta = \cot(90^0 - \theta)$
$\cot\theta = \frac{1}{\tan\theta}$

(ii) $\boldsymbol{\cos 36^0 . \cos 54^0 - \sin 36^0 . \sin 54^0}$

Sol: $\cos 36^0 . \cos 54^0 - \sin 36^0 . \sin 54^0$

$= \sin(90^0 - 36^0).\sin(90^0 - 54^0) - \sin 36^0 . \sin 54^0$

$= \sin 54^0 . \sin 36^0 - \sin 36^0 . \sin 54^0$

$= 0$

$\cos\theta = \sin(90^0 - \theta)$

3. If $\tan 2A = \cot(A - 18^0)$

Sol: $\tan 2A = \cot(A - 18^0)$

$\cot(90^0 - 2A) = \cot(A - 18^0)$

$90^0 - 2A = A - 18^0$

$A + 2A = 90^0 + 18^0$

$3A = 108^0 \Rightarrow A = \frac{108^0}{3} \Rightarrow A = 36^0.$

$\tan\theta = \cot(90^0 - \theta)$

4. **If tan A = cot B where A and B are acute angles, prove that A + B = 90°.**

Sol: tan A = cot B

tan A = tan(90^0 −B)

A = 90^0 −B (since A and B are acute angles)

A + B = 90°

$\cot\theta = \tan(90^0 - \theta)$

5. **If A, B and C are interior angles of a triangle ABC, then show that $\tan\left(\frac{A+B}{2}\right) = \cot\frac{C}{2}$.**

Sol: Given A,B and C are interior angles of a triangle ABC then

A+B+C= 180°

Dividing by 2 on both sides we get

$$\frac{A+B}{2} + \frac{C}{2} = \frac{180^0}{2}$$

$$\frac{A+B}{2} = 90^0 - \frac{C}{2}$$

On taking tan ratio on both sides

$$\tan\left(\frac{A+B}{2}\right) = \tan\left(90^0 - \frac{C}{2}\right)$$

$$\tan\left(\frac{A+B}{2}\right) = \cot\frac{C}{2}$$

$\tan(90^0 - \theta) = \cot\theta$

6. Express sin 75° + cos 65° in terms of trigonometric ratios of angles between 0° and 45° .

Sol: sin 75° + cos 65°

$= \cos(90^0 - 75^0) + \sin(90^0 - 65^0)$

$= \cos 15^0 + \sin 25^0$

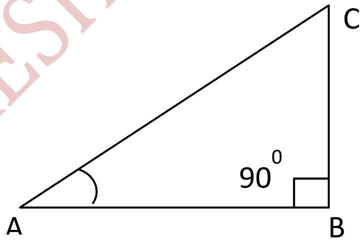
$sin\theta = \cos(90^0 - \theta)$

$cos\theta = \sin(90^0 - \theta)$

TRIGONOMETRIC IDENTITIES:

An identity is a mathematical equation which is true for all the values of the variables in the equation.

Example: Prove the identity $\boldsymbol{sin^2A + cos^2A = 1}$



Sol: In ΔABC , ∠B = 90^0

$AB^2 + BC^2 = AC^2$ (Pythagoras theorem)

Dividing each term by AC^2, we get

$$\frac{AB^2}{AC^2} + \frac{BC^2}{AC^2} = \frac{AC^2}{AC^2}$$

$$\left(\frac{AB}{AC}\right)^2 + \left(\frac{BC}{AC}\right)^2 = 1$$

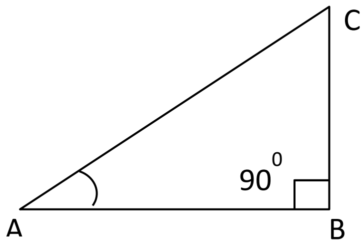
$$\sin^2A + \cos^2A = 1$$

Example: Prove the identity $\boldsymbol{sec^2A - tan^2A = 1}$

Sol: In ΔABC , ∠B = 90^0

$AB^2 + BC^2 = AC^2$ (Pythagoras theorem)

Dividing each term by AB^2, we get



$$\frac{AB^2}{AB^2} + \frac{BC^2}{AB^2} = \frac{AC^2}{AB^2}$$

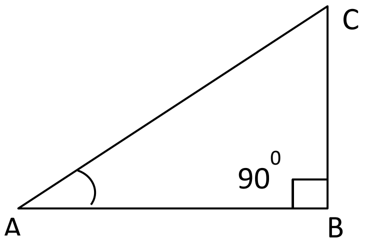
$$1 + \left(\frac{BC}{AB}\right)^2 = \left(\frac{AC}{AB}\right)^2$$

$$1 + \tan^2A = \sec^2A$$

$$\sec^2A - \tan^2A = 1$$

Example: Prove the identity $\boldsymbol{cosec^2A - cot^2A = 1}$

Sol: In ΔABC , $\angle B = 90^0$



$AB^2 + BC^2 = AC^2$ (Pythagoras theorem)

Dividing each term by BC^2, we get

$$\frac{AB^2}{BC^2} + \frac{BC^2}{BC^2} = \frac{AC^2}{BC^2}$$

$$\left(\frac{AB}{BC}\right)^2 + 1 = \left(\frac{AC}{BC}\right)^2$$

$$\cot^2 A + 1 = \text{cosec}^2 A$$

$$\text{cosec}^2 A - \cot^2 A = 1$$

TRIGONOMETRIC IDENTITIES

$sin^2 A + cos^2 A = 1$	$sec^2 A - tan^2 A = 1$	$cosec^2 A - cot^2 A = 1$
$sin^2 A = 1 - cos^2 A$	$sec^2 A = 1 + tan^2 A$	$cosec^2 A = 1 + cot^2 A$
$\sin A = \sqrt{1 - cos^2 A}$	$tan^2 A = sec^2 A - 1$	$cot^2 A = cosec^2 A - 1$
$cos^2 A = 1 - sin^2 A$	$\sec A + \tan A = \frac{1}{secA - tanA}$	$\text{cosec} A + \cot A = \frac{1}{cosecA - cotA}$
$\cos A = \sqrt{1 - sin^2 A}$	$\sec A - \tan A = \frac{1}{secA + tanA}$	$\text{cosec} A - \cot A = \frac{1}{cosecA + cotA}$

THINK - DISCUSS

Are these identities true for $0^0 \leq A \leq 90^0$? If not, for which values of A they are true?

$$\boldsymbol{sec^2 A - tan^2 A = 1} \qquad \boldsymbol{cosec^2 A - cot^2 A = 1}$$

Sol: sec 90^0 and tan 90^0 are not defined.

So, $sec^2 A - tan^2 A = 1$ is not true for 90^0.

Cosec 0^0 and cot 0^0 are not defined.

So, $cosec^2 A - cot^2 A = 1$ is not true for 0^0.

DO THIS

(i) If $\sin A = \frac{15}{17}$, then find cos A

Sol: We know that $sin^2 A + cos^2 A = 1$

$$cos^2 A = 1 - sin^2 A = 1 - \left(\frac{15}{17}\right)^2$$

$$= 1 - \frac{225}{289} = \frac{289 - 225}{289} = \frac{64}{289}$$

$$\cos A = \sqrt{\frac{64}{289}} \Rightarrow \cos A = \frac{8}{17}$$

(ii) **If $\tan x = \frac{5}{12}$, then find $sec\ x$.**

Sol: $sec^2 x - tan^2 x = 1$

$$sec^2 x = 1 + tan^2 x$$

$$= 1 + \left(\frac{5}{12}\right)^2$$

$$= 1 + \frac{25}{144}$$

$$= \frac{169}{144}$$

$$\sec x = \sqrt{\frac{169}{144}} \Rightarrow \sec x = \frac{13}{12}$$

(iii) If $sec\ \theta = \frac{25}{7}$, then find $\cot \theta$.

Sol: $cosec^2\theta - cot^2\theta = 1$

$$cot^2\theta = cosec^2\theta - 1$$

$$= \left(\frac{25}{7}\right)^2 - 1$$

$$= \frac{625}{49} - 1$$

$$= \frac{625 - 49}{49}$$

$$= \frac{576}{49}$$

$$\cot \theta = \sqrt{\frac{576}{49}} \Rightarrow \cot \theta = \frac{24}{7}$$

TRY THIS

Evaluate the following and justify your answer.

(i) $$\frac{sin^2 15^0 + sin^2 75^0}{cos^2 36^0 + cos^2 54^0}$$

Sol : $$\frac{sin^2 15^0 + sin^2 75^0}{cos^2 36^0 + cos^2 54^0}$$

$$= \frac{sin^2 15^0 + cos^2 15^0}{sin^2 54^0 + cos^2 54^0} = \frac{1}{1} = 1$$

$\sin\theta = \cos(90^0 - \theta)$

$\sin 75^0 = \cos(90^0 - 75^0) = \cos 15^0$

$\cos\theta = \sin(90^0 - \theta)$

$\cos 36^0 = \sin(90^0 - 36^0) = \sin 54^0$

(ii) $\sin 5^0 \cos 85^0 + \cos 5^0 \sin 85^0$

Sol: $\sin 5^0 \cos 85^0 + \cos 5^0 \sin 85^0$

$= \sin 5^0 \times \sin 5^0 + \cos 5^0 \times \cos 5^0$

$= \sin^2 5^0 + \cos^2 5^0 = 1$

$\cos 85^0 = \sin 5^0$

$\sin 85^0 = \cos 5^0$

(iii) $\sec 16^0 \ \operatorname{cosec} 74^0 - \cot 74^0 \tan 16^0$

Sol: $\sec 16^0 \ \operatorname{cosec} 74^0 - \cot 74^0 \tan 16^0$

$= \sec 16^0 \times \sec 16^0 - \tan 16^0 \times \tan 16^0$

$= \sec^2 16^0 - \tan^2 16^0$

$= 1$

$\operatorname{cose} \theta = \sec(90^0 - \theta)$
$\operatorname{cosec} 74^0 = \sec(90^0 - 74^0)$
$= \sec 16^0$
$\cot\theta = \tan(90^0 - \theta)$
$\cot 74^0 = \tan(90^0 - 74^0) = \tan 16^0$

Example-13. Show that $\cot\theta + \tan\theta = \sec\theta \operatorname{cosec}\theta$

$\sec^2 A - \tan^2 A = 1$

Sol: LHS $= \cot\theta + \tan\theta$

$= \frac{\cos\theta}{\sin\theta} + \frac{\sin\theta}{\cos\theta}$

$= \frac{\cos^2\theta + \sin^2\theta}{\sin\theta\cos\theta}$

$= \frac{1}{\sin\theta\cos\theta}$

$= \frac{1}{\sin\theta} \times \frac{1}{\cos\theta}$

$= \operatorname{cosec}\theta \, . \sec\theta$

Example-14. Show that $\tan^2\theta + \tan^4\theta = \sec^4\theta - \sec^2\theta$

Sol: L.H.S$= \tan^2\theta + \tan^4\theta$

$= \tan^2\theta(1 + \tan^2\theta)$

$= (\sec^2\theta - 1)\sec^2\theta$

$= \sec^4\theta - \sec^2\theta = R.H.S$

$\sec^2 A - \tan^2 A = 1$

$\Rightarrow \sec^2 A = 1 + \tan^2 A$

$\sec^2 A - 1 = \tan^2 A$

Example – 15: Prove that $\sqrt{\frac{1+\cos\theta}{1-\cos\theta}} = \operatorname{cosec}\theta + \cot\theta.$

Solution: $L.H.S = \sqrt{\frac{1+\cos\theta}{1-\cos\theta}}$

$= \sqrt{\frac{(1+\cos\theta)(1+\cos\theta)}{(1-\cos\theta)(1+\cos\theta)}}$

$$= \sqrt{\frac{(1+\cos\theta)^2}{1-\cos^2\theta}}$$

$$= \sqrt{\frac{(1+\cos\theta)^2}{\sin^2\theta}} \qquad (1-\cos^2\theta = \sin^2\theta)$$

$$= \frac{1+\cos\theta}{\sin\theta}$$

$$= \frac{1}{\sin\theta} + \frac{\cos\theta}{\sin\theta} = \text{cosec}\,\theta + \cot\theta = \text{R.H.S}$$

Exercise 11.4

1. **Evaluate the following :**

(i) $(1 + \tan\theta + \sec\theta)\,(1 + \cot\theta - \text{cosec}\,\theta)$

Sol: $(1 + \tan\theta + \sec\theta)\,(1 + \cot\theta - \text{cosec}\,\theta)$

$$= \left(1 + \frac{\sin\theta}{\cos\theta} + \frac{1}{\cos\theta}\right)\left(1 + \frac{\cos\theta}{\sin\theta} - \frac{1}{\sin\theta}\right)$$

$$= \left(\frac{\cos\theta + \sin\theta + 1}{\cos\theta}\right)\left(\frac{\sin\theta + \cos\theta - 1}{\sin\theta}\right)$$

$$= \left[\frac{(\sin\theta + \cos\theta)^2 - 1^2}{\sin\theta\cos\theta}\right]$$

$$= \left(\frac{\sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta - 1}{\sin\theta\cos\theta}\right)$$

$$= \frac{1 + 2\sin\theta\cos\theta - 1}{\sin\theta\cos\theta}$$

$$= \frac{2\sin\theta\cos\theta}{\sin\theta\cos\theta} = 2$$

(ii) $(\sin\theta + \cos\theta)^2 + (\sin\theta - \cos\theta)^2$

Sol: $(\sin\theta + \cos\theta)^2 + (\sin\theta - \cos\theta)^2$

$$= \sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta + \sin^2\theta + \cos^2\theta - 2\sin\theta\cos\theta$$

$$= 1 + 1 = 2$$

$\sin^2\theta + \cos^2\theta = 1$

(iii) $(\sec^2\theta - 1)(\text{cosec}^2\theta - 1)$

Sol: $(\sec^2\theta - 1)(\text{cosec}^2\theta - 1)$

$$= \tan^2\theta \times \cot^2\theta$$

$$= \tan^2\theta \times \frac{1}{\tan^2\theta} = 1$$

2. ***Show that*** $(\text{cosec}\,\theta - \cot\theta)^2 = \dfrac{1-\cos\theta}{1+\cos\theta}$

Sol: $(cosec\,\theta - cot\,\theta)^2$

$$= \left(\frac{1}{\sin\theta} - \frac{\cos\theta}{\sin\theta}\right)^2$$

$$= \left(\frac{1-\cos\theta}{\sin\theta}\right)^2$$

$$= \frac{(1-\cos\theta)^2}{sin^2\theta}$$

$$= \frac{(1-cos\,\theta)^2}{1-cos^2\theta}$$

$$= \frac{(1-cos\,\theta)(1-cos\,\theta)}{(1+cos\,\theta)(1-cos\,\theta)}$$

$$= \frac{1-cos\,\theta}{1+cos\,\theta}$$

3. Show that $\sqrt{\frac{1+\sin A}{1-\sin A}} = \sec A + \tan A$

sol: $\sqrt{\frac{1+sin\,A}{1-sin\,A}}$

$$= \sqrt{\frac{(1+sin\,A)(1+sin\,A)}{(1-sin\,A)(1+sin\,A)}}$$

$$= \sqrt{\frac{(1+sin\,A)^2}{1-sin^2A}}$$

$$= \frac{\sqrt{(1+sin\,A)^2}}{\sqrt{cos^2A}}$$

$$= \frac{1+sinA}{\cos A}$$

$$= \frac{1}{\cos A} + \frac{\sin A}{\cos A}$$

$$= \sec A + \tan A$$

4. Show that $\frac{1-tan^2A}{cot^2A-1} = tan^2A$

sol: $\frac{1-tan^2A}{cot^2A-1}$

$$= \frac{1 - tan^2 A}{\frac{1}{tan^2 A} - 1}$$

$$= \frac{1 - tan^2 A}{\frac{1 - tan^2 A}{tan^2 A}}$$

$$= (1 - tan^2 A) \times \frac{tan^2 A}{1 - tan^2 A}$$

$$= tan^2 A$$

5. Show that $\frac{1}{\cos\theta} - \cos\theta = \tan\theta . \sin\theta$

Sol: $\frac{1}{cos\,\theta} - cos\,\theta$

$$= \frac{1 - cos^2\theta}{\cos\theta}$$

$$= \frac{sin^2\theta}{\cos\theta}$$

$$= \frac{\sin\theta}{\cos\theta} \times \sin\theta$$

$$= \tan\theta\,.\sin\theta$$

6. Simplify secA (1 – sinA) (secA + tanA)

Sol: $secA\,(1 - sinA)\,(secA + tanA)$

$$= \frac{1}{\cos A} \times (1 - sinA)\left(\frac{1}{\cos A} + \frac{\sin A}{\cos A}\right)$$

$$= \frac{(1 - sinA)}{\cos A} \times \frac{(1 + sinA)}{cos\,A}$$

$$= \frac{1 - sin^2 A}{cos^2 A}$$

$$= \frac{cos^2 A}{cos^2 A} = 1$$

7. Prove that $(\sin A + cosecA)^2 + (\cos A + \sec A)^2 = 7 + tan^2 A + cot^2 A$

Sol: $\text{L.H.S} = (sin\,A + cosecA)^2 + (cos\,A + sec\,A)^2$

$$= sin^2 A + cosec^2 A + 2\sin A . cosec\,A + cos^2 A + sec^2 A + 2. cos\,A . \sec A$$

$$= sin^2 A + cos^2 A + cosec^2 A + sec^2 A + 2\sin A \times \frac{1}{\sin A} + 2cos\,A \times \frac{1}{\cos A}$$

$$= 1 + cot^2 A + 1 + tan^2 A + 1 + 2 + 2$$

$$= 7 + tan^2 A + cot^2 A$$

$$= \text{R.H.S}$$

8. Simplify $(1-\cos\theta)(1+\cos\theta)(1+\cot^2\theta)$

Sol: $(1-cos\ \theta)(1+cos\theta)(1+cot^2\theta)$

$= (1-cos^2\theta)cosec^2\theta$

$= sin^2\theta \times \dfrac{1}{sin^2\theta} = 1$

9. If $\sec\theta + \tan\theta = p$, then what is the value of $\sec\theta - \tan\theta$?

Sol: We know that $sec^2\theta - tan^2\theta = 1$

$(\sec\theta + \tan\theta)(\sec\theta - \tan\theta) = 1$

$\sec\theta - \tan\theta = \dfrac{1}{sec\ \theta + tan\ \theta}$

$\sec\theta - \tan\theta = \dfrac{1}{p}$

10. If $\operatorname{cosec}\theta + \cot\theta = k$ then prove that $\cos\theta = \dfrac{k^2-1}{k^2+1}$

Sol: Given $cosec\ \theta + \cot\theta = K$ --------(1)

$cose^2\theta - cot^2\theta = 1$

$(cose\ \theta + \cot\theta)(cose\ \theta - cot\ \theta) = 1$

$k(cose\ \theta - cot\ \theta) = 1$

$(cose\ \theta - cot\ \theta) = \frac{1}{k}$ ---------(2)

(1)+(2)

$cosec\ \theta + \cot\theta = k$

$(cose\ \theta - cot\ \theta) = \frac{1}{K}$

$2cosec\ \theta = k + \frac{1}{k}$

$cosec\ \theta = \frac{k^2+1}{2k}$

$\sin\theta = \frac{2k}{k^2+1}$

(1)-(2)

$cosec\ \theta + \cot\theta = k$

$(cosec\ \theta - cot\ \theta) = \frac{1}{K}$
(−) (+)

$2cot\ \theta = k - \frac{1}{k}$

$cot\ \theta = \frac{k^2-1}{2k}$

$\frac{\cos\theta}{\sin\theta} = \frac{k^2-1}{2k}$

$\sin\theta \times \dfrac{cos\ \theta}{sin\ \theta} = \dfrac{2k}{k^2+1} \times \dfrac{k^2-1}{2k}$

$\Rightarrow \cos\theta = \dfrac{k^2-1}{k^2+1}$

BALABHADRA SURESH-9866845885