CHAPTER 10

X CLASS-2023-24
MENSURATION (NOTES)

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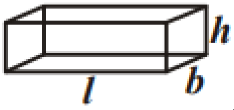
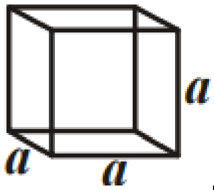
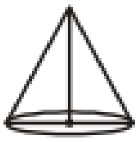
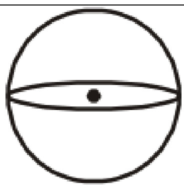
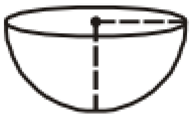
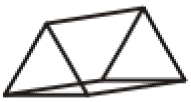
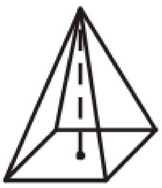
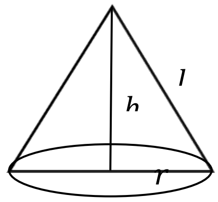
S.N	Figure	Name of the solid and Nomenclature	Lateral/Curved surface area L.S.A/C.S.A	Total surface are T.S.A	Volume
1		**Cuboid** $l = length$ $b = breadth$ $h = height$	$2h(l+b)$	$2(lb+bh+lh)$	lbh
2		**Cube** $a = side$	$4a^2$	$6a^2$	a^3
3		**Regular circular Cylinder** $r = radius$ $h = height$	$2\pi rh$	$2\pi r(h+r)$	$\pi r^2 h$
4		**Cone** $r = radius$ $h = height$ $l = slant\ height$ $l = \sqrt{h^2+r^2}$	πrl	$\pi r(l+r)$	$\frac{1}{3}\pi r^2 h$
5		**Sphere** $r = radius$	$4\pi r^2$	$4\pi r^2$	$\frac{4}{3}\pi r^3$
6		**Hemisphere** $r = radius$	$2\pi r^2$	$3\pi r^2$	$\frac{2}{3}\pi r^3$
7		**Right prism**	$(Perimeter\ of\ base) \times height$	L.S.A +Area of base	$\frac{1}{3}(area\ of\ base) \times height$
8		**Right pyramid**	$\frac{1}{2}(Perimeter\ of\ base) \times slantheight$	L.S.A +Area of base	$\frac{1}{3}(area\ of\ base) \times height$

Example-1. The radius of a conical tent is 7 meters and its height is 10 meters. Calculate the length of canvas used in making the tent if width of canvas is 2m. $\left[\text{Use } \pi = \frac{22}{7}\right]$

Sol:



Radius of the conical tent(r)=7 m

Height (h)=10 m

Slant height of the cone $= l = \sqrt{r^2 + h^2}$

$= \sqrt{7^2 + 10^2}$

$= \sqrt{49 + 100}$

$= \sqrt{149}$

$= 12.2$ m

	12.2
1	$\overline{1}\ \overline{49}.\overline{00}$
	−1
22	49
	− 44
242	500
	−484
	16

L. S. A of tent $= \pi rl$

$= \frac{22}{7} \times 7 \times 12.2$

$= 22 \times 12.2$

$= 268.4 \text{ m}^2$

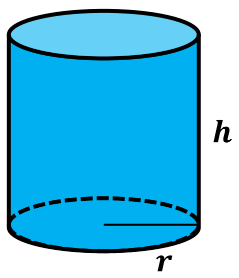
Area of canvas used=268.4 m^2

The width of the canvas=2m

Length of canvas used $= \frac{\text{Area of canvas used}}{\text{Width of canvas}} = \frac{268.4}{2} = 134.2$ m

Example-2. An oil drum is in the shape of a cylinder having the following dimensions: diameter is 2 m. and height is 7 meters. The painter charges ₹3 per m² to paint the drum. Find the total charges to be paid to the painter for 10 drums?

Solution:



The diameter of the (oil drum) cylinder=d=2m

Radius(r) $= \frac{d}{2} = \frac{2}{2} = 1$ m

Height of drum (h)=7 m

T. S. A of oil drum $= 2\pi r(r + h)$

$= 2 \times \frac{22}{7} \times (1 + 7)$

$= \frac{44 \times 8}{7} = \frac{352}{7} = 50.28\ m^2$

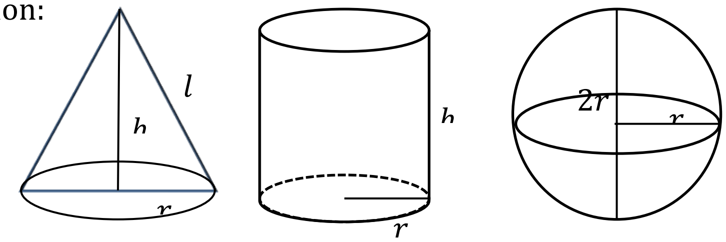
T. S. A of 10 oil drums $= 10 \times 50.28 \text{ m}^2 = 502.8 \text{ m}^2$

Painting charge per 1 m^2= ₹3

Cost of painting of 10 drums = 502.8 ×3 = ₹1508.40

Example-3. A sphere, a cylinder and a cone are of the same radius and same height. Find the ratio of their curved surface areas?

Solution:



Let r be the common radius of a sphere, a cone and cylinder.

Height of sphere = its diameter = 2r.

Then, the height of the cone = height of cylinder = height of sphere (h). = 2r.

Slant height of the cone $= \mathrm{l} = \sqrt{\mathrm{r}^2 + \mathrm{h}^2} = \sqrt{\mathrm{r}^2 + (2\mathrm{r})^2} = \sqrt{\mathrm{r}^2 + 4\mathrm{r}^2} = \sqrt{5\,\mathrm{r}^2} = \sqrt{5}\mathrm{r}$

$\mathrm{S}_1 = \mathrm{C.S.A}$ of sphere $= 4\pi\mathrm{r}^2$

$\mathrm{S}_2 = \mathrm{C.S.A}$ of cylinder $= 2\pi\mathrm{rh} = 2\pi\mathrm{r} \times 2\mathrm{r} = 4\pi\mathrm{r}^2$

$\mathrm{S}_3 = \mathrm{C.S.A}$ of cone $= \pi\mathrm{rl} = \pi\mathrm{r} \times \sqrt{5}\mathrm{r} = \sqrt{5}\pi\mathrm{r}^2$

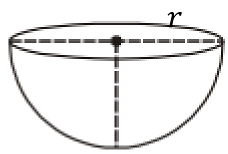
$\mathrm{S}_1 : \mathrm{S}_2 : \mathrm{S}_3 = 4\pi\mathrm{r}^2 : 4\pi\mathrm{r}^2 : \sqrt{5}\pi\mathrm{r}^2 = 4 : 4 : \sqrt{5}$

Ratio of curved surface areas$= 4 : 4 : \sqrt{5}$

Example-4. A company wanted to manufacture 1000 hemispherical basins from a thin steel sheet. If the radius of hemispherical basin is 21 cm., find the required area of steel sheet to manufacture the above hemispherical basins?

Solution : Radius of the hemispherical basin (r) = 21 cm

Surface area of a hemispherical basin $= 2\pi\mathrm{r}^2$



$= 2 \times \frac{22}{7} \times 21 \times 21$

$= 2 \times 22 \times 3 \times 21$

$= 2772\ \mathrm{cm}^2$

Hence, the steel sheet required for one basin = $2772\ \mathrm{cm}^2$

Total area of steel sheet required for 1000 basins = 2772×1000

$= 2772000\ \mathrm{cm}^2$

$= \frac{2772000}{10000}\mathrm{m}^2$

$= 277.2\ \mathrm{m}^2$

$1m = 100\ cm$
$1m^2 = 10000\ cm^2$
$1\ cm^2 = \frac{1}{10000}m^2$

Example-5. A right circular cylinder has base radius 14cm and height 21cm. Find:

(i) Area of base or area of each end (ii) Curved surface area (iii) Total surface area and (iv) Volume of the right circular cylinder.

Solution : Radius of the cylinder (r) = 14cm

Height of the cylinder (h) = 21cm

(i)Area of base or area of each end $= \pi r^2 = \frac{22}{7} \times 14 \times 14 = 22 \times 2 \times 14 = 616\ cm^2$

(ii)Curved surface area $= 2\pi rh = 2 \times \frac{22}{7} \times 14 \times 21 = 44 \times 2 \times 21 = 1848\ cm^2$

(iii)Total surface area $= 2\pi r(r+h) = 2 \times \frac{22}{7} \times 14 \times (14+21) = 44 \times 2 \times 35 = 3080 cm^2$

(iv)Volume of the right circular cylinder $= \pi r^2 h = \frac{22}{7} \times 14 \times 14 \times 21$

$$= 22 \times 2 \times 14 \times 21 = 12936\ cm^2.$$

Example-6. Find the volume and surface area of a sphere of radius 2.1cm ($\pi = \frac{22}{7}$)

Solution : Radius of sphere(r) $= 2.1 = \frac{21}{10}$ cm

Surface area of sphere $= 4\pi r^2$

$$= 4 \times \frac{22}{7} \times \frac{21}{10} \times \frac{21}{10}$$

$$= \frac{4 \times 22 \times 3 \times 21}{100}$$

$$= \frac{5544}{100} = 55.44\ cm^2$$

Volume of sphere $= \frac{4}{3}\pi r^3$

$$= \frac{4}{3} \times \frac{22}{7} \times \frac{21}{10} \times \frac{21}{10} \times \frac{21}{10}$$

$$= \frac{4 \times 22 \times 21 \times 21}{1000}$$

$$= \frac{38808}{1000} = 38.808\ cm^3$$

Example-7. Find the volume and the total surface area of a hemisphere of radius 3.5 cm. ($\pi = \frac{22}{7}$)

Solution : Radius of sphere (r) = 3.5 cm $= \frac{35}{10}$ cm $= \frac{7}{2}$ cm

Volume of hemisphere $= \frac{2}{3}\pi r^3$

$$= \frac{2}{3} \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times \frac{7}{2} = \frac{11 \times 7 \times 7}{6} = \frac{539}{6} = 89.83\ cm^3$$

T.S.A of hemisphere $= 3\pi r^2$

$$= 3 \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2}$$

$$= \frac{231}{2} = 115.5\ cm^2$$

Exercise - 10.1

1. A joker's cap is in the form of right circular cone whose base radius is 7cm and height is 24 cm. Find the area of the sheet required to make 10 such caps.

Sol: Radius of the cap (cone)=r=7 cm

Height of the cap (cone)=h=24 cm

Slant height of the cap $= l = \sqrt{r^2 + h^2}$

$$= \sqrt{7^2 + 24^2} = \sqrt{49 + 576} = \sqrt{625} = 25 \text{ cm}$$

L.S.A of cap $= \pi rl$

$$= \frac{22}{7} \times 7 \times 25 = 22 \times 25 = 550 \text{cm}^2$$

Area of the sheet required to make 1cap $= 550 \text{ cm}^2$.

Area of the sheet required to make 10 such caps $= 10 \times 550 \text{ cm}^2 = 5500 \text{ cm}^2$

2. A sports company was ordered to prepare 100 paper cylinders without caps for shuttle cocks. The required dimensions of the cylinder are 35 cm length /height and its radius is 7 cm. Find the required area of thin paper sheet needed to make 100 cylinders?

Sol: Radius of the cylinder(r) =7 cm

Height of the cylinder (h) =35 cm

L.S.A of cylinder $= 2\pi rh$

$$= 2 \times \frac{22}{7} \times 7 \times 35 = 44 \times 35 = 1540 \text{ cm}^2$$

The required area of thin paper sheet needed to make 100 cylinders

$$=100 \times 1540 \text{ cm}^2$$

$$= 154000 \text{cm}^2 = \frac{154000}{10000} \text{ m}^2 = 15.4 \text{ m}^2$$

3. Find the volume of right circular cone with radius 6 cm. and height 7cm.

Sol: Radius of cone(r)=6 cm

Height of cone(h)=7 cm

Volume of cone $= \frac{1}{3}\pi r^2 h$

$$= \frac{1}{3} \times \frac{22}{7} \times 6 \times 6 \times 7$$

$$= 22 \times 12 = 264 \text{ cm}^3$$

4. The lateral surface area of a cylinder is equal to the curved surface area of a cone. If the radius be the same, find the ratio of the height of the cylinder and slant height of the cone.

Sol: Radius of cylinder =Radius of cone=r

The height of the cylinder =h and slant height of the cone=l

Given L.S.A of cylinder=C.S.A of cone

$$\Rightarrow 2\pi rh = \pi rl$$

$$\Rightarrow \frac{h}{l} = \frac{\pi r}{2\pi r} = \frac{1}{2}$$

$$\therefore h:l = 1:2$$

5. A self-help group wants to manufacture joker's caps (conical caps) of 3cm. radius and 4 cm. height. If the available colour paper sheet is 1000 cm², than how many caps can be manufactured from that paper sheet?

Sol: Radius of the cap(cone)=r=3 cm

Height of the cap(cone)=h=4 cm

Slant height of the cap $= l = \sqrt{r^2 + h^2}$

$$= \sqrt{3^2 + 4^2} = \sqrt{9 + 16} = \sqrt{25} = 5 \text{ cm}$$

C. S. A of cap $= \pi rl$

$$= \frac{22}{7} \times 3 \times 5 = \frac{330}{7} \text{ cm}^2$$

Area of the sheet required to make 1cap $= \frac{330}{7}$ cm².

Available colour paper sheet =1000 cm²

Number of caps manufactured $= \frac{1000}{\frac{330}{7}} = \frac{1000 \times 7}{330} = \frac{700}{33} = 21.21$

Number of caps=21

6. **A cylinder and cone have bases of equal radii and are of equal heights. Show that their volumes are in the ratio of 3:1.**

Sol: Radius of cylinder= Radius of cone=r

Height of cylinder=Height of cone=h

Volume of cylinder: Volume of cone $= \pi r^2 h : \frac{1}{3}\pi r^2 h$

$$= 1:\tfrac{1}{3} = 1 \times 3:\tfrac{1}{3} \times 3 = 3:1$$

7. A solid iron rod has a cylindrical shape. Its height is 11 cm. and base diameter is 7cm. Then find the total volume of 50 rods?

Sol: Diameter of cylinder (d)=7 cm

Radius of cylinder(r) $= \frac{7}{2}$ cm

Height of cylinder (h)=11 cm

Volume of cylinder $= \pi r^2 h$

$$= \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times 11 = \frac{847}{2} = 423.5 \text{ cm}^3$$

The volume of 1 rod=423.5 cm^3

The total volume of 50 rods=50 × 423.5 = 21175cm^3

8. A heap of rice is in the form of a cone of diameter 12 m. and height 8 m. Find its volume? How much canvas cloth is required to cover the heap ? (Use π = 3.14)

Sol: Diameter of heap (cone)=d=12 m

Radius of cone(r) $= \frac{12}{2} = 6$ m

Height of cone (h)=8 m

Volume of cone $= \frac{1}{3}\pi r^2 h$

$$= \frac{1}{3} \times 3.14 \times 6 \times 6 \times 8 = 301.44 \text{ m}^3$$

Slant height of the cone $= l = \sqrt{r^2 + h^2}$

$$= \sqrt{6^2 + 8^2} = \sqrt{36 + 64} = \sqrt{100} = 10 \text{ m}$$

C. S. A of heap $= \pi r l$

$$= 3.14 \times 6 \times 10 = 188.4 \text{ m}^2$$

The required canvas cloth to cover the heap= 188.4 m^2

9. The curved surface area of a cone is 4070 cm² and its diameter is 70 cm. What is its slant height?

Sol: Diameter of cone (d)=70 cm

Radius of cone(r) $= \frac{70}{2} = 35$ cm

Given C.S.A of cone=4070 cm^2

$$\pi r l = 4070$$

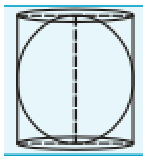
$$\frac{22}{7} \times 35 \times l = 4070$$

$$l = \frac{4070 \times 7}{22 \times 35} = 37 \text{ cm}$$

∴ Slant height of the cone=37 cm

THINK - DISCUSS

A sphere is inscribed in a cylinder. Is the surface of the sphere equal to the curved surface of the cylinder? If yes, explain how?



Sol: Yes

Ler radius of sphere =r

The surface area of sphere = $4\pi r^2$

Radius of cylinder=r

Height of cylinder(h)=2r

The C.S.A of cylinder $= 2\pi rh = 2\pi r \times 2r = 4\pi r^2$

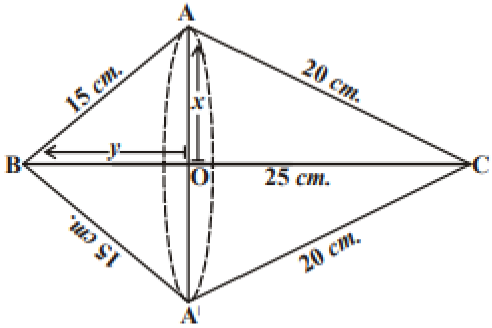
Example-8. A right triangle, whose base and height are 15 cm. and 20 cm. respectively, is made to revolve about its hypotenuse. Find the volume and surface area of the double cone so formed (Use π=3.14).

Sol: Let ABC be the right angled triangle such that

$AB = 15cm$ and $AC = 20$ cm Using Pythagoras theorem in ΔABC we have

$BC^2 = AB^2 + AC^2$

$BC^2 = 15^2 + 20^2 = 225 + 400 = 625$



$BC = \sqrt{625} = 25$ cm

Let $OA = x$ and $OB = y$

$\Delta BOA \sim \Delta BAC$ (A. A similarity)

$$\frac{BO}{BA} = \frac{OA}{AC} = \frac{BA}{BC}$$

$$\Rightarrow \frac{y}{15} = \frac{x}{20} = \frac{15}{25}$$

$$\Rightarrow \frac{y}{15} = \frac{15}{25} \quad \text{and} \quad \frac{x}{20} = \frac{15}{25}$$

$$\Rightarrow y = \frac{15}{25} \times 15 \quad \text{and} \quad x = \frac{15}{25} \times 20$$

$$\Rightarrow y = 9 \quad \text{and} \quad x = 12$$

$$\Rightarrow OB = 9 \text{ cm} \quad \text{and} \quad OA = 12 \text{ cm}$$

Volume of the double cone = volume of the cone CAA^1 + volume of the cone BAA^1

$$= \frac{1}{3}\pi (OA)^2 \times OC + \frac{1}{3}\pi (OA)^2 \times OB$$

$$= \frac{1}{3}\pi (OA)^2[OC + OB]$$

$$= \frac{1}{3}\pi (OA)^2 \times BC$$

$$= \frac{1}{3} \times 3.14 \times 12 \times 12 \times 25$$

$$= 3768 \text{ cm}^3$$

Surface area of the doubled cone

= (Curved surface area of cone CAA^1) + (Curved surface area of cone BAA^1)

= $(\pi \times OA \times AC) + (\pi \times OA \times AB)$

= $(\pi \times 12 \times 20) + (\pi \times 12 \times 15)$ cm²

$= 420\,\pi\ cm^2 = 420 \times 3.14\ cm^2 = 1318.8\ cm^2$

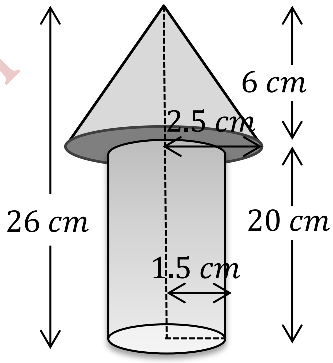
Example-9. A wooden toy rocket is in the shape of a cone mounted on a cylinder as shown in the adjacent figure. The height of the entire rocket is 26 cm, while the height of the conical part is 6cm. The base of the conical position has a diameter of 5cm, while the base diameter of the cylindrical portion is 3cm. If the conical portion is to be painted orange and the cylindrical portion is to be painted yellow, find the area of the rocket painted with each of these colour (Take π = 3.14)

Sol: Conical part:

$\text{Diameter}(d) = 5\ cm\ ; \quad \text{radius}(r) = \frac{5}{2} cm; \quad \text{height}(h) = 6cm$

$l = \sqrt{r^2 + h^2} = \sqrt{(2.5)^2 + 6^2} = \sqrt{6.25 + 36} = \sqrt{42.25} = 6.5 = \frac{65}{10} cm$

$\text{Area to be painted orange} = \text{C.S.A of cone} = \pi rl$



$= 3.14 \times \frac{5}{2} \times \frac{65}{10} = \frac{314}{100} \times \frac{5}{2} \times \frac{65}{10}$

$= \frac{157 \times 5 \times 65}{1000} = \frac{51025}{1000}$

$= 51.025\ cm^2$

Cylindrical part:

$Diameter(d) = 3\ cm\ ; \quad radius(r) = \frac{3}{2} = 1.5cm$

$Height(h) = 26 - 6 = 20cm$

Area to be painted yellow= C.S.A of cylinder+Area of the base of the cylinder

$= 2\pi rh + \pi r^2$

$= \pi r(2h + r)$

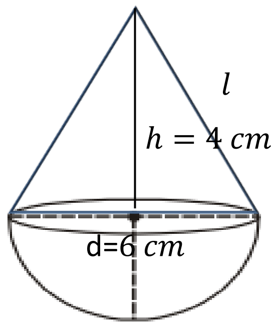
$= 3.14 \times 1.5 \times (2 \times 20 + 1.5)$

$= 3.14 \times 1.5 \times 41.5$

$= 195.465\ cm^2$

Exercise - 10.2

1. **A toy is in the form of a cone mounted on a hemisphere. The diameter of the base and the height of the cone are 6 cm and 4 cm respectively. Determine the surface area of the toy. [use π = 3.14]**



Sol: : Diameter of cone (d)=6 cm

Radius of cone (r) $= \frac{6}{2} = 3$ cm

Height of cone(h)=4 cm

Slant height $= l = \sqrt{r^2 + h^2}$

$$= \sqrt{3^2 + 4^2} = \sqrt{9 + 16} = \sqrt{25} = 5 \text{ cm}$$

Radius of hemisphere (r)=3 cm

Surface area of the toy= C.S.A of cone+ C.S.A of hemisphere

$$= \pi rl + 2\pi r^2$$

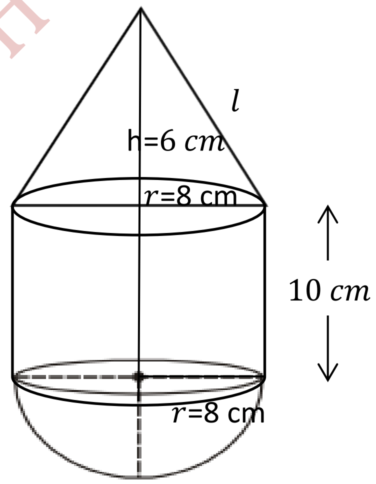
$$= \pi r(l + 2r)$$

$$= 3.14 \times 3 \times (5 + 2 \times 3)$$

$$= 3.14 \times 3 \times 11$$

$$= 103.62 \text{ cm}^2$$

2. A solid is in the form of a right circular cylinder with a hemisphere at one end and a cone at the other end. The radius of the common base is 8 cm. and the heights of the cylindrical and conical portions are 10 cm and 6 cm respectively. Find the total surface area of the solid. [use $\pi = 3.14$]



Sol: Cone:

Radius(r)=8 cm

Height(h)=6 cm

Slant height $= l = \sqrt{r^2 + h^2}$

$$= \sqrt{8^2 + 6^2} = \sqrt{64 + 36} = \sqrt{100} = 10 \text{ cm}$$

$\text{C.S.A of cone} = \pi rl = \pi \times 8 \times 10 = 80\pi \text{ cm}^2$

Cylinder:

Radius(r)=8 cm

Height(h)=10 cm

$\text{C.S.A of cylinder} = 2\pi rh = 2\pi \times 8 \times 10 = 160\pi \text{ cm}^2$

Hemisphere:

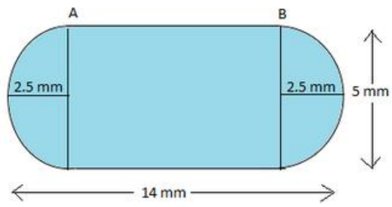
Radius(r)=8 cm

$\text{C.S.A of hemisphere} = 2\pi r^2 = 2\pi \times 8 \times 8 = 128\pi \text{ cm}^2$

Total surface area of solid= $\text{C.S.A of cone} + \text{C.S.A of cylinder} + \text{C.S.A of hemisphere}$

$$= 80\pi + 160\pi + 128\pi = 348\pi = 368 \times 3.14 = 1155.52 \text{ cm}^2$$

3. A medicine capsule is in the shape of a cylinder with two hemispheres stuck to each of its ends. The length of the capsule is 14 mm. and the width is 5 mm. Find its surface area



Sol: Cylinder part:

Radius(r)= 2.5 mm

$\text{Height(h)} = 14 - 2 \times 2.5 = 14 - 5 = 9 \text{ mm}$

Radius of hemisphere(r)= 2.5 mm

Surface area of capsule = C. S. A of cylinder part + 2 × C. S. A of hemesphere

$= 2\pi rh + 2 \times 2\pi r^2$

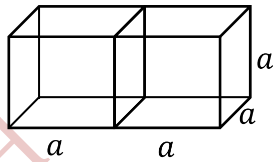
$= 2\pi r(h + 2r)$

$= 2 \times \frac{22}{7} \times 2.5 \times (9 + 2 \times 2.5)$

$= \frac{44}{7} \times 2.5 \times (9 + 5)$

$= \frac{44}{7} \times 2.5 \times 14 = 44 \times 2.5 \times 2 = 44 \times 5 = 220\ mm^2$

4. **Two cubes each of volume 64 cm^3 are joined end to end together. Find the total surface area of the resulting cuboid.**



Sol: Volume of cube=64 cm^3

$a^3 = 64 = 4^3$

side of the cube(a) = 4 cm

Cube has six faces normally when two equal cubes are placed together; two side faces are not visible. We left with 12 − 2 =10 squared faces

∴ Surface area of resulting cuboid. $=10a^2 = 10 \times 4 \times 4 = 160\ cm^2$

5. **A storage tank consists of a circular cylinder with a hemisphere stuck on either end. If the external diameter of the cylinder be 1.4 m. and its length be 8 m. find the cost of painting it on the outside at rate of D20 per m^2 .**

Sol: Cylinder part:

Diameter (d)=1.4 m

Radius(r)= $\frac{1.4}{2} = 0.7 = \frac{7}{10}$ m

Height(h) = 8 m

Radius of hemisphere(r)= $\frac{7}{10}$ m

Surface area of storage tank = C. S. A of cylinder part + 2 × C. S. A of hemesphere

$= 2\pi rh + 2 \times 2\pi r^2$

$= 2\pi r(h + 2r)$

$= 2 \times \frac{22}{7} \times \frac{7}{10} \times \left(8 + 2 \times \frac{7}{10}\right)$

$= \frac{44}{10} \times \left(8 + \frac{14}{10}\right)$

$= \frac{44}{10} \times \frac{94}{10} = \frac{4136}{100}$

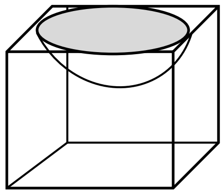
$= 41.36\ m^2$

Cost of painting per 1 m^2=20

Total cost of painting $=41.36 \times 20=₹827.20$

6. **A hemisphere is cut out from one face of a cubical wooden block such that the diameter of the hemisphere is equal to the length of the cube. Determine the surface area of the remaining solid.**

Sol: Side of cube=a units



Diameter of the hemi sphere= a units

Radius of the hemisphere $= r = \frac{a}{2}$ units

Required area of the remaining solid

$= \text{T.S.A of cube} - \text{Area of circle with radius } \frac{a}{2} + \text{L.S.A of hemisphere}$

$= 6a^2 - \pi r^2 + 2\pi r^2$

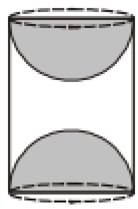
$= 6a^2 + \pi r^2$

$= 6a^2 + \pi \left(\frac{a}{2}\right)^2$

$= 6a^2 + \frac{\pi a^2}{4}$

$= a^2 \left(6 + \frac{\pi}{4}\right)$ sq units

7. **A wooden article was made by scooping out a hemisphere from each end of a solid cylinder, as shown in the figure. If the height of the cylinder is 10 cm. and its base radius is of 3.5 cm, find the total surface area of the article.**



Sol: Radius of cylinder = Radius of hemi sphere $= (r) = 3.5\text{cm} = \frac{35}{10}$ cm

The height of the cylinder (h)= 10 cm

Total surface area of the article = C.S.A of cylinder + 2 × C.S.A of hemi sphere

$= 2\pi rh + 2 \times 2\pi r^2$

$= 2\pi r(h + 2r)$

$= 2 \times \frac{22}{7} \times \frac{35}{10} \times \left(10 + 2 \times \frac{35}{10}\right)$

$= \frac{44 \times 35}{70} \times \frac{170}{10} = 22 \times 17 = 374 \text{ cm}^2$

Volume of Combination of Solids

Try This

1. **If the diameter of the cross - section of a wire is decreased by 5%, by what percentage should the length be increased so that the volume remains the same ?**

Sol: Let radius of cross section=r , diameter= 2r and length of wire=h

Volume of wire $= \pi r^2 h \rightarrow (1)$

If 5% of diameter of cross section is decreased then new diameter = 95% of 2r

$$= \frac{95}{100} \times 2r = \frac{19r}{10} \Rightarrow \text{New radius}(r_1) = \frac{19r}{2 \times 10} = \frac{19r}{20}$$

Let new length $= h_1$

Volume of wire $= \pi {r_1}^2 h_1 = \pi \left(\frac{19r}{20}\right)^2 h_1 \rightarrow (2)$

From (1) and (2)

$$\pi \left(\frac{19r}{20}\right)^2 h_1 = \pi r^2 h \Rightarrow \pi \times \frac{361r^2}{400} \times h_1 = \pi r^2 h$$

$$\Rightarrow h_1 = \frac{\pi r^2 h \times 400}{361r^2\,\pi} = \frac{400}{361} h$$

Increase in length $= h_1 - h = \frac{400}{361} h - h = \frac{400h - 361h}{361} = \frac{39}{361} h$

Percentage increase in length $= \frac{\text{Increase in length}}{\text{Orinal length}} \times 100\%$

$$= \frac{\frac{39}{361} h}{h} \times 100 = \frac{3900}{361} = 10.8\,\%$$

2. **Surface area of a sphere and cube are equal. Then find the ratio of their volumes.**

Sol: Surface area of sphere = surface area of cube

$$4\pi r^2 = 6a^2$$

$$r^2 = \frac{6a^2}{4\pi} \Rightarrow r = \sqrt{\frac{6a^2}{4\pi}} \Rightarrow r = \sqrt{\frac{3}{2\pi}} \times a$$

The ratio of their volumes $= \frac{4}{3} \pi r^3 : a^3$

$$= \frac{4}{3} \pi \left(\sqrt{\frac{3}{2\pi}} \times a\right)^3 : a^3$$

$$= \frac{4}{3} \pi \times \frac{3\sqrt{3}}{2\pi\sqrt{2\pi}} \times a^3 : a^3$$

$$= \frac{2\sqrt{3}}{\sqrt{2\pi}} : 1$$

$$= \sqrt{\frac{6}{\pi}} : 1 \quad \text{or} \quad 1 : \sqrt{\frac{\pi}{6}}$$

Example-10. A solid toy is in the form of a right circular cylinder with hemispherical shape at one end and a cone at the other end. Their common diameter is 4.2 cm and the height of the cylindrical and conical portions are 12cm and 7cm respectively. Find the volume of the solid toy.

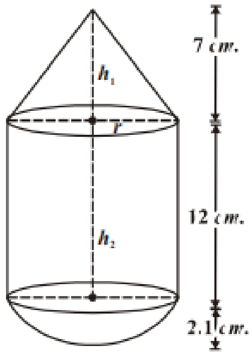
[Use $\pi = \frac{22}{7}$]

Sol: Let height of the conical portion $h_1 = 7$cm

The height of cylindrical portion $h_2 = 12$ cm

Diameter(d)=4.2 cm

Radius (r) $= \frac{4.2}{2} = 2.1 = \frac{21}{10}$ cm



Volume of the solid toy

= Volume of the Cone + Volume of the Cylinder + Volume of the Hemisphere

$$= \frac{1}{3}\pi r^2 h_1 + \pi r^2 h_2 + \frac{2}{3}\pi r^3$$

$$= \pi r^2 \left[\frac{1}{3}h_1 + h_2 + \frac{2}{3}r\right]$$

$$= \frac{22}{7} \times \left(\frac{21}{10}\right)^2 \times \left[\frac{1}{3} \times 7 + 12 + \frac{2}{3} \times \frac{21}{10}\right]$$

$$= \frac{22}{7} \times \frac{441}{100} \times \left[\frac{7}{3} + 12 + \frac{21}{15}\right]$$

$$= \frac{22}{7} \times \frac{441}{100} \times \left[\frac{35 + 180 + 21}{15}\right]$$

$$= \frac{22}{7} \times \frac{441}{100} \times \frac{236}{15} = \frac{27258 \times 8}{125 \times 8} = \frac{218064}{1000} = 218.064 \text{ cm}^3$$

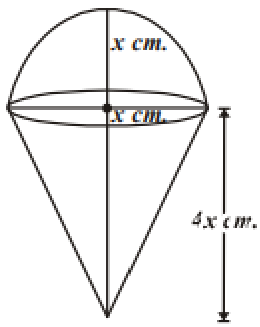
Example-11. A cylindrical container is filled with ice-cream whose diameter is 12 cm. and height is 15 cm. The whole ice-cream is distributed to 10 children in equal cones having hemispherical tops. If the height of the conical portion is twice the diameter of its base, find the diameter of the ice-cream cone.

Sol: Cylinder:

Diameter of cylindrical container = 12 cm . Radius=6cm

Its height (h) = 15 cm

$\therefore$ Volume of cylindrical container $= \pi r^2 h = \pi(6)^2\, 15 = 540\pi \text{ cm}^3 \rightarrow (1)$



Cone:

Let the radius of the base of conical ice cream(r) = x cm

$\therefore$ diameter = 2x cm

Then, the height of the conical ice-cream(h) = 2 (diameter) = 2(2x) = 4x cm

Volume of ice - cream cone

= Volume of conical portion + Volume of hemispherical portion

$$= \frac{1}{3}\pi r^2 h + \frac{2}{3}\pi r^3$$

$$= \frac{1}{3}\pi r^2(h + 2r)$$

$$= \frac{1}{3} \times \pi \times x^2 \times (4x + 2x)$$

$$= \frac{1}{3} \times \pi \times x^2 \times 6x$$

$$= 2\pi x^3 \text{ cm}^3$$

Volume of 10 ice − cream cones $= 10 \times 2\pi x^3 cm^3 = 20\pi x^3 cm^3 \rightarrow (2)$

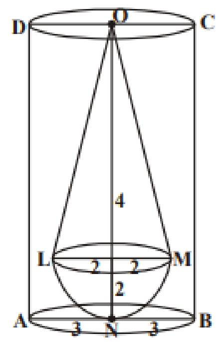
From (1) and (2) :

$20\pi x^3 = 540\pi$

$x^3 = \frac{540\pi}{20\pi} = 27 = 3^3 \Rightarrow x = 3$

∴The diameter of the ice-cream cone=2x $= 2 \times 3 = 6$ cm

Example-12. A solid consisting of a right circular cone standing on a hemisphere, is placed upright in a right circular cylinder full of water and touches the bottom. Find the volume of water left in the cylinder, given that the radius of the cylinder is 3 cm. and its height is 6cm. The radius of the hemisphere is 2 cm. and the height of the cone is 4 cm.(Take $\pi = \frac{22}{7}$)



Sol: Volume of cylinder $= \pi r^2 h = \pi \times 3^2 \times 6 = 54\pi \text{ cm}^3$

Volume of hemi sphere $= \frac{2}{3}\pi r^3 = \frac{2}{3}\pi \times 2^3 = \frac{16}{3}\pi cm^3$

Volume of cone $= \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi \times 2^2 \times 4 = \frac{16}{3}\pi cm^3$

Volume of cone and hemi sphere $= \frac{16}{3}\pi + \frac{16}{3}\pi = \frac{32}{3}\pi cm^3$

Volume of water left in cylinder

= Volume of Cylinder − Volume of Cone and Hemisphere

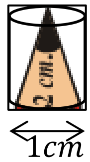
$$= 54\pi - \frac{32}{3}\pi$$

$$= \frac{162\pi - 32\pi}{3} = \frac{130\pi}{3} = \frac{130}{3} \times \frac{22}{7} = \frac{2860}{21} = 136.19 cm^3$$

Example-13. A cylindrical pencil is sharpened to produce a perfect cone at one end with no over all loss of its length. The diameter of the pencil is 1cm and the length of the conical portion is 2cm. Calculate the volume of the shavings. Give your answer correct to two places if it is in decimal.(Take $\pi = \frac{22}{7}$)

Sol: Diameter of the pencil (d)= 1cm

So, radius of the pencil (r) $= 0.5 = \frac{5}{10} = \frac{1}{2}$ cm



Length of the conical portion = h = 2cm

Volume of showings = Volume of cylinder of length 2 cm and base radius 0.5 cm. – volume of the cone formed by this cylinder.

$$= \pi r^2 h - \frac{1}{3}\pi r^2 h = \frac{2}{3}\pi r^2 h = \frac{2}{3} \times \frac{22}{7} \times \frac{1}{2} \times \frac{1}{2} \times 2 = \frac{22}{21} = 1.05\ cm^3$$

EXERCISE-10.3

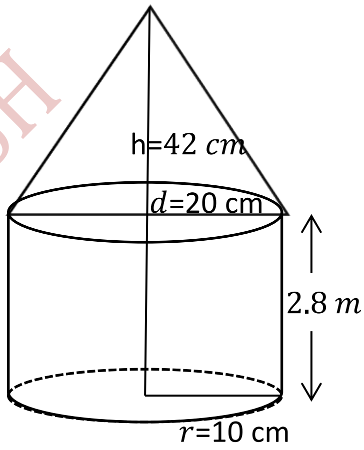
1. An iron pillar consists of a cylindrical portion of 2.8 m. height and 20 cm. in diameter and a cone of 42 cm. height surmounting it. Find the weight of the pillar if 1 cm³ of iron weighs 7.5 g.

Sol: Cone part:

Diameter (d) =20cm

Radius(r) =10cm

Height (h) =42 cm



Volume of the cone $= \frac{1}{3}\pi r^2 h$

$$= \frac{1}{3} \times \frac{22}{7} \times 10 \times 10 \times 42 = 4400 cm^3$$

Cylinder part:

Radius(r) =10cm

Height (h) =2.8 m=280 cm

Volume of cylinder $= \pi r^2 h$

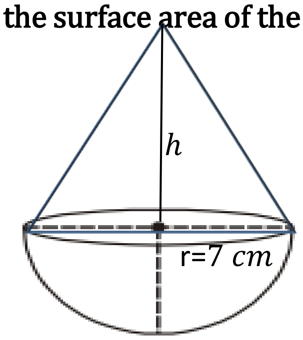
$$= \frac{22}{7} \times 10 \times 10 \times 280 = 88000\ cm^3$$

Volume of iron piller $= 4400 + 88000 = 92400\ cm^3$

Weight of the pillar per $1cm^3$=7.5 g

Total weight of the pillar=92400×7.5 g = 693000 g = 693 kg

2. A toy is made in the form of hemisphere surmounted by a right cone whose circular base is joined with the plane surface of the hemisphere. The radius of the base of the cone is 7 cm. and its volume is $\frac{3}{2}$ of the hemisphere. Calculate the height of the cone and the surface area of the toy correct to 2 places of decimal. $\left(\textbf{Take } \pi = 3\frac{1}{7}\right)$



Sol: Radius of hemi sphere = Radius of cone =(r)= 7cm

Volume of cone $= \frac{3}{2} \times$ Volume of hemisphere

$$\frac{1}{3}\pi r^2 h = \frac{3}{2} \times \frac{2}{3}\pi r^3$$

$$\text{Height} = h = \frac{3\pi r^3}{\pi r^2} = 3r = 3 \times 7 = 21 \text{ cm}$$

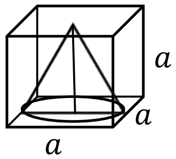
$$\text{Slant height (l)} = \sqrt{r^2 + h^2} = \sqrt{7^2 + 21^2} = \sqrt{49 + 441} = \sqrt{490} = 22.135 \text{ cm}$$

The surface area of toy = C. S. A of cone + C. S. A of hemisphere

$$= \pi rl + 2\pi r^2 = \pi r(l + 2r) = \frac{22}{7} \times 7 \times (22.135 + 14) = 22 \times 36.135 = 794.97 \text{ cm}^2$$

3. **Find the volume of the largest right circular cone that can be cut out of a cube whose edge is 7 cm.**

Sol: Edge of cube (a) =7 cm



$$\text{Radius of cone(r)} = \frac{7}{2} \text{cm}$$

$$\text{Height of cone(h)} = 7\text{cm}$$

$$\text{Volume of cone} = \frac{1}{3}\pi r^2 h$$

$$= \frac{1}{3} \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times 7 = \frac{11 \times 49}{6} = \frac{539}{6} = 89.83 \text{ cm}^3$$

4. **A cylindrical tub of radius 5cm and length 9.8 cm is full of water. A solid in the form of right circular cone mounted on a hemisphere is immersed into the tub. The radius of the hemisphere is 3.5 cm and height of cone outside the hemisphere is 5cm. Find the volume of water left in the tub $\left(\text{Take } \pi = \frac{22}{7}\right)$**

Sol: Cylindrical tub:

Radius(r)=5 cm

$$\text{Length(h)} = 9.8 \text{ cm} = \frac{98}{10} \text{cm}$$

$$\text{Volume of cylindrecal tub} = \pi r^2 h$$

$$= \frac{22}{7} \times 5 \times 5 \times \frac{98}{10} = 55 \times 14 = 770 \text{ cm}^3$$

$$\text{Radius of cone} = \text{Radius of hemisphere} = (r) = 3.5\text{cm} = \frac{35}{10} = \frac{7}{2}\text{cm}$$

$$\text{Height of cone (h)} = 5 \text{ cm}$$

Volume of solid = volume of cone + volume of hemisphere

$$= \frac{1}{3}\pi r^2 h + \frac{2}{3}\pi r^3$$

$$= \frac{1}{3}\pi r^2 (h + 2r)$$

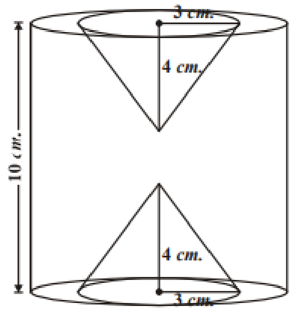
$$= \frac{1}{3} \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times \left(5 + 2 \times \frac{7}{2}\right)$$

$$= \frac{1}{3} \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times 12 = 22 \times 7 = 154 \text{ cm}^3$$

Volume of water left in the tub = Volume of the tub − volume of solid immersed

$$= 770 - 154 = 616 \text{ cm}^3$$

5. **In the adjacent figure, the height of a solid cylinder is 10 cm and diameter is 7cm. Two equal conical holes of radius 3cm and height 4 cm are cut off as shown the figure. Find the volume of the remaining solid**



Sol: Cylinder part:

Diameter(d) = 7 cm

$\text{Radius}(r) = \frac{7}{2}\text{cm}$

Height(h) = 10 cm

$\text{Volume of cylinder} = \pi r^2 h$

$= \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times 10 = 385 \text{ cm}^3$

Cone:

$\text{Radius}(r) = 3\text{cm}$

Heigh(h) = 4 cm

$\text{volume of cone} = \frac{1}{3}\pi r^2 h = \frac{1}{3} \times \frac{22}{7} \times 3 \times 3 \times 4 = \frac{264}{7} \text{ cm}^3$

$\text{Total volume of two conical holes} = 2 \times \frac{264}{7} = \frac{528}{7} \text{ cm}^3$

$\text{The volume of the remaining solid} = 385 - \frac{528}{7} = \frac{7 \times 385 - 528}{7}$

$$= \frac{2695 - 528}{7} = \frac{2167}{7} = 309.57 \text{ cm}^3$$

6. **Spherical Marbles of diameter 1.4 cm. are dropped into a cylindrical beaker of diameter 7 cm., which contains some water. Find the number of marbles that should be dropped in to the beaker, so that water level rises by 5.6 cm.**

Sol: $\text{Radius of cylindrical beaker}(r) = \frac{7}{2} \text{ cm}$

$\text{Height of rise water } (h) = 5.6 = \frac{56}{10}\text{cm}$

$\text{Volume of the water rise} = \pi r^2 h$

$= \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times \frac{56}{10} = \frac{22 \times 7 \times 14}{10} \text{ cm}^3$

$\text{Radius of marble}(r) = \frac{1.4}{2} = 0.7 = \frac{7}{10}\text{cm}$

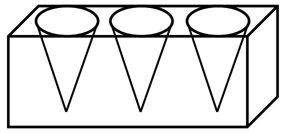
$\text{Volume of spherical marble} = \frac{4}{3}\pi r^3 = \frac{4}{3} \times \frac{22}{7} \times \frac{7}{10} \times \frac{7}{10} \times \frac{7}{10} = \frac{88 \times 49}{3 \times 1000} \text{ cm}^3$

$\text{Required number of marbles} = \frac{\text{Volume of the water rise}}{\text{Volume of marble}}$

$$= \frac{\frac{22 \times 7 \times 14}{10}}{\frac{88 \times 49}{3 \times 1000}} = \frac{22 \times 7 \times 14}{10} \times \frac{3 \times 1000}{88 \times 49} = 3 \times 50 = 150$$

7. **A pen stand is made of wood in the shape of cuboid with three conical depressions to hold the pens. The dimensions of the cuboid are 15cm by 10 cm by 3.5 cm. The radius of each of the depression is 0.5 cm and the depth is 1.4cm. Find the volume of wood in the entire stand**

Sol: Cuboid;



$l = 15$ cm, $b = 10$cm, $h = 3.5$ cm

Volume of cuboid $= lbh = 15 \times 10 \times 3.5 = 525\ cm^3$

Cone:

$\text{Radius}(r) = 0.5\ cm = \frac{5}{10} cm$; $\text{Depth}(h) = 1.4\ cm = \frac{14}{10} cm$

Volume of each depression $= \frac{1}{3}\pi r^2 h = \frac{1}{3} \times \frac{22}{7} \times \frac{5}{10} \times \frac{5}{10} \times \frac{14}{10} = \frac{11}{30}\ cm^3$

Total volume of three depressions $= 3 \times \frac{11}{30} = \frac{11}{10} = 1.1 cm^3$

The volume of wood in the entire stand= $525 - 1.1 = 523.9\ cm^3$

Conversion of Solid from One Shape to Another

The shapes of solids are converted into another shape. In this process, the volume always remains the same.

Example-14. A cone of height 24cm and radius of base 6cm is made up of modelling clay. A child reshapes it in the form of a sphere. Find the radius of the sphere..

Sol: Height of cone (h)=24 cm

Radius of cone(r)=6 cm

Volume of cone $= \frac{1}{3}\pi r^2 h = \frac{1}{3} \times \pi \times 6 \times 6 \times 24\ cm^3$

Radius of sphere=r_1

The shapes of solids are converted into another shape. In this process, the volume always remains the same

Volume of sphere = Volume of cone

$$\frac{4}{3}\pi {r_1}^3 = \frac{1}{3} \times \pi \times 6 \times 6 \times 24$$

$${r_1}^3 = \frac{6 \times 6 \times 24}{4} = 6^3 \Rightarrow r_1 = 6$$

The radius of the sphere=6 cm

Do This

1. **A copper rod of diameter 1 cm. and length 8 cm. is drawn into a wire of length 18m of uniform**

thickness. Find the thickness of the wire.

Sol: Copper rod:

Diameter (d) =1 cm

Radius(r) $= \frac{1}{2}$ cm

Length (h)=8 cm

Volume of copper rod (cylinder) $= \pi r^2 h = \pi \times \frac{1}{2} \times \frac{1}{2} \times 8 = 2\pi \text{ cm}^3$

Copper wire:

Length (h)=18 m=1800 cm

Radius=r

Volume of copper wire (cylinder) $= \pi r^2 h = \pi r^2 \times 1800 \text{cm}^3$

The shapes of solids are converted into another shape. In this process, the volume always remains the same.

Volume of copper wire = Volume of copper rod

$\pi r^2 \times 1800 = 2\pi$

$r^2 = \frac{2}{1800} = \frac{1}{900} = \left(\frac{1}{30}\right)^2 \Rightarrow r = \frac{1}{30}$

The thickness of the wire $= 2r = 2 \times \frac{1}{30} = \frac{1}{15}$ cm

2. **Pravali house has a water tank in the shape of a cylinder on the roof. This is filled by pumping water from a sump (an under ground tank) which is in the shape of a cuboid. The sump has dimensions 1.57 m. ×1.44 m. × 95 cm. The water tank has radius 60 cm. and height 95 cm. Find the height of the water left in the sump after the water tank has been completely filled with water from the sump which had been full of water. Compare the capacity of the tank with that of the sump. ($\pi = 3.14$)**

Sol: Dimensions of sump (cuboid) :

l = 1.57 m = 157 cm; b = 1.44m = 147 cm; h = 95 cm

Volume of sump = lbh = 157 ×144 × 95 = 2147760 cm^3

Dimensions of water tank (Cylinder):

Radius(r) =60 cm

Height (h) =95 cm

Volume of water tank $= \pi r^2 h = \frac{22}{7} \times 60 \times 60 \times 95 = 1074857 \text{ cm}^3$

Volume of water left in sump= Volume of sump − Volume of water tank

$= 2147760 - 1074857 = 1072903 \text{ cm}^3$

Let the height of the water left in the sump=H

$l \times b \times \mathrm{H} = 1072903$

$157 \times 144 \times \mathrm{H} = 1072903$

$$\mathrm{H} = \frac{1072903}{157 \times 144} = 47.5 \text{ cm}$$

Height of water used in sump=95-47.5=47.5

$$\frac{\text{Capacity of the tank}}{\text{Capacity of the sump}} = \frac{47.5}{95} = \frac{1}{2}$$

Example-15. The diameter of the internal and external surfaces of a hollow hemispherical shell are 6 cm. and 10 cm. respectively. It is melted and recast into a solid cylinder of diameter 14 cm. Find the height of the cylinder.

Sol: Hemi spherical shell:

Internal diameter (d)= 6cm

Internal radius(r)=3 cm

External diameter (D)= 10cm

External radius(R)=5 cm

$$\text{Volume of hollow hemi sperical shell} = \frac{2}{3}\pi(\mathrm{R}^3 - \mathrm{r}^3)$$

$$= \frac{2}{3}\pi(5^3 - 3^3)$$

$$= \frac{2}{3}\pi(125 - 27)$$

$$= \frac{2}{3}\pi \times 98 = \frac{196\pi}{3}\text{cm}^3$$

Cylinder:

Diameter (d)=14 cm : Radius=7 cm

Height=h

$\text{Volume of cylinder} = \pi r^2 h = \pi \times 7 \times 7 \times h \text{ cm}^3$

from problem : Volume of cylinder = Volume of hollow hemi sperical shell

$$\pi \times 7 \times 7 \times h = \frac{196\pi}{3}$$

$$h = \frac{196}{7 \times 7 \times 3} = \frac{4}{3} = 1.33 \text{ cm}$$

Hence, height of the cylinder = 1.33 cm

Example-16. A hemispherical bowl of internal radius 15 cm. contains a liquid. The liquid is to be filled into cylindrical bottles of diameter 5 cm. and height 6 cm. How many bottles are necessary to empty the bowl ?

Sol: Hemispherical bowl:

Internal radius(r)=15 cm

Volume of liquid contained in hemispherical bowl $=\frac{2}{3}\pi r^3=\frac{2}{3}\times\pi\times 15^3\ cm^3$

Cylindrical bottle:

Diameter (d)=5 cm ; Radius(r)=$\frac{5}{2}$ cm ; height (h)=6 cm

Volume of 1 bottle $=\pi r^2 h=\pi\times\left(\frac{5}{2}\right)^2\times 6\ cm^3$

Number of bottles required $=\frac{\text{Volume of hemispherical bowl}}{\text{Volume of1 bottle}}$

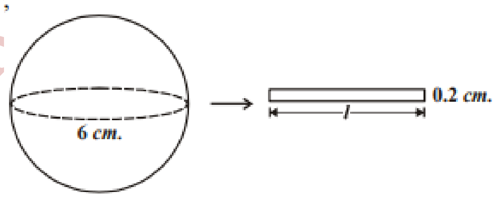
$$=\frac{\frac{2}{3}\times\pi\times 15\times 15\times 15}{\pi\times\frac{5}{2}\times\frac{5}{2}\times 6}=\frac{2\times 15\times 15\times 15\times 2\times 2}{3\times 5\times 5\times 6}=60$$

Example-17. The diameter of a metallic sphere is 6cm. It is melted and drawn into a wire having diameter of the cross section as 0.2 cm. Find the length of the wire.

Sol: Metallic sphere:

Diameter(d)=6 cm

Radius(r)=3cm



Volume of sphere $=\frac{4}{3}\pi r^3=\frac{4}{3}\pi\times 3^3=36\pi\ cm^3$

Cylindrical Wire:

Diameter (d)= 0.2 cm ;Radius(r)=0.1 cm=$\frac{1}{10}$ cm

Volume of the wire $=\pi r^2 h=\pi\times\left(\frac{1}{10}\right)^2\times h=\pi\times\frac{h}{100}\ cm^3$

The shapes of solids are converted into another shape. In this process, the volume always remains the same.

Volume of the wire = Volume of sphere

$$\pi\times\frac{h}{100}=36\pi$$

$$h=3600\ cm=36\ m$$

Required length of wire=36 m.

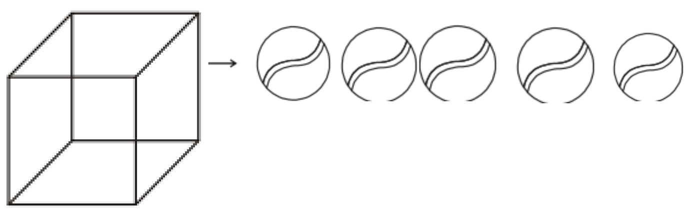
Example-18. How many spherical balls can be made out of a solid cube of lead whose edge measures 44 cm and each ball being 4 cm. in diameter.

Sol: Edge of lead cube(a)=44 cm

Volume of cube $=a^3=44^3\ cm^3$

Spherical ball:

Diameter(d)=4 cm ; Radius(r)=2cm

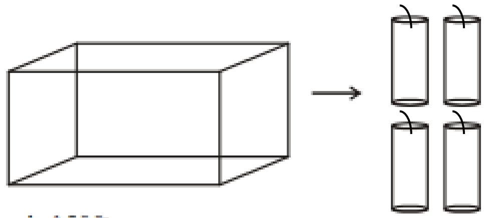


Volume of 1 spherical ball $= \frac{4}{3}\pi r^3 = \frac{4}{3} \times \frac{22}{7} \times 2 \times 2 \times 2 cm^3$

Number of spherical balls $= \frac{\text{Volume of lead cube}}{\text{Volume of 1 spherical ball}}$

$= \frac{44 \times 44 \times 44}{\frac{4}{3} \times \frac{22}{7} \times 2 \times 2 \times 2} = \frac{44 \times 44 \times 44 \times 3 \times 7}{4 \times 22 \times 8} = 11 \times 11 \times 3 \times 7 = 2541$

Example-19. A women self-help group (DWACRA) is supplied a rectangular solid (cuboid shape) of wax with dimensions 66 cm., 42 cm., 21 cm., to prepare cylindrical candles each 4.2 cm. in diameter and 2.8 cm. of height. Find the number of candles.



Sol: Volume of wax in the rectangular solid = lbh

$= (66 \times 42 \times 21)\ cm^3$

Radius of cylindrical candle(r) $= \frac{4.2}{2} = 2.1 = \frac{21}{10}$ cm.

Height of cylindrical candle (h) $= 2.8 = \frac{28}{10}$ cm

Volume of 1 candle $= \pi r^2 h = \frac{22}{7} \times \frac{21}{10} \times \frac{21}{10} \times \frac{28}{10}\ cm^3$

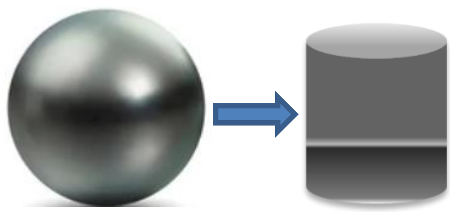
Number of candles $= \frac{\text{Volume of wax in the rectangular solid}}{\text{Volume of 1 candle}}$

$= \frac{66 \times 42 \times 21}{\frac{22}{7} \times \frac{21}{10} \times \frac{21}{10} \times \frac{28}{10}} = \frac{66 \times 42 \times 21 \times 7 \times 10 \times 10 \times 10}{22 \times 21 \times 21 \times 28} = 1500$

Hence, the number of cylindrical wax candles is 1500.

EXERCISE - 10.4

1. **A metallic sphere of radius 4.2 cm. is melted and recast into the shape of a cylinder of radius 6cm. Find the height of the cylinder.**



Sol: Metalic sphere:

radius(r) $= 4.2 = \frac{42}{10}$ cm

Volume of sphere $= \frac{4}{3}\pi r^3$

$= \frac{4}{3} \times \frac{22}{7} \times \frac{42}{10} \times \frac{42}{10} \times \frac{42}{10}\ cm^3$

Cylinder:

Radius=6cm; Height=h

Volume of cylinder $= \pi r^2 h$

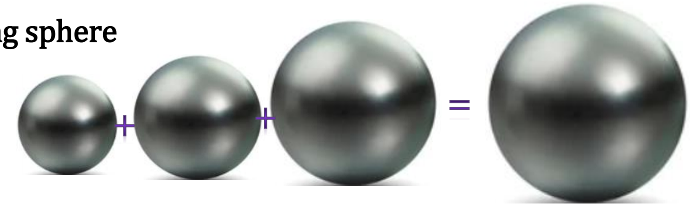
$= \frac{22}{7} \times 6 \times 6 \times h\ cm^3$

Sphere recast into cylinder, their volumes are same

$$\frac{22}{7} \times 6 \times 6 \times h = \frac{4}{3} \times \frac{22}{7} \times \frac{42}{10} \times \frac{42}{10} \times \frac{42}{10}$$

$$h = \frac{4}{3} \times \frac{42}{10} \times \frac{42}{10} \times \frac{42}{10} \times \frac{1}{6 \times 6} = \frac{4 \times 14 \times 7 \times 7}{1000} = \frac{2744}{1000} = 2.744 \text{ cm}$$

2. Metallic spheres of radius 6 cm., 8 cm. and 10 cm. respectively are melted to form a single solid sphere. Find the radius of the resulting sphere



Sol: Radii of three spheres are

$r_1 = 6\text{cm}, r_2 = 8 \text{ cm}, r_3 = 10 \text{ cm}$

Let radius of resulting sphere=r

Volume of resulting sphere =Sum of volumes of the three spheres

$$\frac{4}{3}\pi r^3 = \frac{4}{3}\pi {r_1}^3 + \frac{4}{3}\pi {r_2}^3 + \frac{4}{3}\pi {r_3}^3$$

$$r^3 = {r_1}^3 + {r_2}^3 + {r_3}^3$$

$$= 6^3 + 8^3 + 10^3$$

$$= 216 + 512 + 1000 = 1728 = 12^3$$

$$r = 12$$

Radius of resulting sphere=12 cm

3. A 20m deep well with diameter 7 m. is dug and the earth from digging is evenly spread out to form a platform 22 m. × 14 m. Find the height of the platform.

Sol: well (cylinder):

Diameter (d)=7 m

$$\text{Radius}(r) = \frac{7}{2} \text{m}$$

Depth (h)=20 m

Volume of well $= \pi r^2 h$

$$= \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times 20 \text{ m}^3$$

Platform (cuboid):

$l = 22\text{m}; b = 14 \text{ m}; \ \ h = ?$

Volume of platform $= lbh$

$$= 22 \times 14 \times h \text{ m}^3$$

Volume of platform= Volume of well

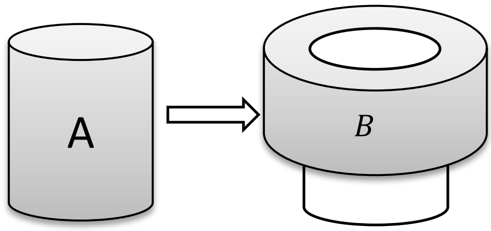
$$22 \times 14 \times h = \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times 20$$

$$h = \frac{22 \times 7 \times 7 \times 20}{7 \times 2 \times 2 \times 22 \times 14} = \frac{5}{2} = 2.5 \text{ m}$$

4. A well of diameter 14 m. is dug 15 m. deep. The earth taken out of it has been spread evenly all

around it in the shape of a circular ring of width 7 m. to form an embankment. Find the height of the embankment.

Sol: well (Cylinder A):



Diameter (d)=14 m

Radius(r)=7 m

Depth(h)=15 m

Volume of well $= \pi r^2 h$

$= \pi \times 7 \times 7 \times 15\ m^3$

Embankment (Hollow cylinder B):

Inner radius(r)=7 m

Outer radius(R) $= 7 + 7 = 14$ m ; height=h

Volume of enbankment $= \pi(R^2 - r^2)h$

$= \pi(14^2 - 7^2) \times h$

$= \pi \times (14 + 7) \times (14 - 7) \times h$

$= \pi \times 21 \times 7 \times h\ m^3$

Volume of enbankment = Volume of well

$\pi \times 21 \times 7 \times h = \pi \times 7 \times 7 \times 15$

$$h = \frac{7 \times 7 \times 15}{21 \times 7} = 5m$$

Height of the enbankment = 5 m

5. **A container shaped like a right circular cylinder having diameter 12 cm. and height 15 cm. is full of ice cream. The ice-cream is to be filled into cones of height 12 cm. and diameter 6 cm., having a hemispherical shape on the top. Find the number of such cones which can be filled with ice cream.**

Sol: Container (cylinder):

Diameter (d)=12cm; Radius(r)=6 cm

Height(h)=15 cm

Volume of ice − cream in container $= \pi r^2 h = \pi \times 6^2 \times 15 = \pi \times 36 \times 15\ cm^3$

Cone:

Diameter (d)=6cm; Radius(r)=3 cm

Height(h)=12 cm

Volume of cone $= \frac{1}{3}\pi r^2 h = \frac{1}{3} \times \pi \times 3^2 \times 12 = 36\pi\ cm^3$

Hemisphere:

Diameter (d)=6cm; Radius(r)=3 cm

Volume of hemisphere $= \frac{2}{3}\pi r^3 = \frac{2}{3} \times \pi \times 3^3 = \frac{2}{3} \times \pi \times 27 = 18\pi cm^3$

The volume of ice-cream in each cone=$36\pi + 18\pi = 54\pi\ cm^3$

The number of cones which can be filled with ice cream

$$= \frac{\text{Volume of icecream in container}}{\text{Volume of icecream in each cone}}$$

$$= \frac{\pi \times 36 \times 15}{54 \times \pi}$$

$$= 10$$

6. **How many silver coins, 1.75 cm. in diameter and thickness 2 mm., need to be melted to form a cuboid of dimensions 5.5 cm. ×10 cm. × 3.5 cm.?**

Sol: Volume of cuboid $= 5.5 \times 10 \times 3.5 = 55 \times \frac{35}{10} = \frac{11 \times 35}{2} cm^3$

Silver coin (cylinder):

$Diameter(d) = 1.75\ cm, \quad Radius(r) = \frac{1.75}{2} = \frac{175}{200} = \frac{7}{8} cm$

$Thickness(h) = 2mm = \frac{2}{10} cm$

Volume of 1 silver coin $= \pi r^2 h = \frac{22}{7} \times \frac{7}{8} \times \frac{7}{8} \times \frac{2}{10} = \frac{11 \times 7}{16 \times 10}\ cm^3$

Number of silver coins $= \frac{\text{Volume of cuboid}}{\text{Volume of 1 silver coin}}$

$$= \frac{\frac{11 \times 35}{2}}{\frac{11 \times 7}{16 \times 10}} = \frac{11 \times 35 \times 16 \times 10}{2 \times 11 \times 7} = 400$$

7. **A vessel is in the form of an inverted cone. Its height is 8 cm. and the radius of its top is 5 cm. It is filled with water up to the rim. When lead shots, each of which is a sphere of radius 0.5cm are dropped into the vessel, $\frac{1}{4}$ of the water flows out. Find the number of lead shots dropped into the vessel.**

Sol: Cone:

Radius(r) =5cm

Height (h)=8cm

The volume of water in cone $= \frac{1}{3}\pi r^2 h = \frac{1}{3} \times \pi \times 5 \times 5 \times 8\ cm^3$

Lead shot (sphere):

$Radius(r) = 0.5cm = \frac{5}{10} = \frac{1}{2} cm$

Volume of 1 lead shot $= \frac{4}{3}\pi r^3 = \frac{4}{3} \times \pi \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{\pi}{6}\ cm^3$

From problem

Volume of lead shots dropped into the vessel $= \frac{1}{4} \times$ volume of water in cone

The number of lead shots dropped into vessel $= \dfrac{\frac{1}{4} \times volume\ of\ water\ in\ cone}{Volume\ of\ 1\ lead\ shot}$

$$= \frac{\frac{1}{4} \times \frac{1}{3} \times \pi \times 5 \times 5 \times 8}{\frac{\pi}{6}} = \frac{\pi \times 5 \times 5 \times 2 \times 6}{3 \times \pi} = 100$$

8. **A solid metallic sphere of diameter 28 cm is melted and recast into a number of smaller cones, each of diameter $4\frac{2}{3}$ cm and height 3cm. Find the number of cones so formed.**

Sol: Metallic sphere:

$Diameter(d) = 28\ cm$; $Radius(r) = 14\ cm$

Volume of metallic sphere $= \frac{4}{3}\pi r^3 = \frac{4}{3} \times \pi \times 14 \times 14 \times 14\ cm^3$

Cone:

Diameter$(d) = 4\frac{2}{3} = \frac{14}{3} cm$; Radius$(r) = \frac{14}{2 \times 3} = \frac{7}{3} cm$

Height (h)=3 cm

$Volume\ of\ 1\ cone = \frac{1}{3}\pi r^2 h = \frac{1}{3} \times \pi \times \frac{7}{3} \times \frac{7}{3} \times 3 = \pi \times \frac{7}{3} \times \frac{7}{3}\ cm^3$

The number of cones formed $= \dfrac{\text{Volume of metallic sphere}}{\text{Volume of 1 cone}}$

$$= \frac{\frac{4}{3} \times \pi \times 14 \times 14 \times 14}{\pi \times \frac{7}{3} \times \frac{7}{3}} = \frac{4 \times 14 \times 14 \times 14 \times 3}{7 \times 7} = 672$$