X CLASS-2023-24
Polynomials (Notes)

1. Algebraic Expression : An algebraic expression is an expression made up of variables and constants along with mathematical operators.
2. Polynomial: A polynomial is an algebraic expression in which the exponent on any variable is a whole number.
   Example: $2x + 5, 3x^2 + 5x + 6, -5y$ ...

Polynomials	Not polynomials
$2x$	$4x^{\frac{1}{2}}$
$\frac{1}{3}x - 4$	$3x^2 + 4x^{-1} + 5$
$x^2 - 2x - 1$	$4 + \frac{1}{x}$

3. Degree of a polynomial: The highest power of x in a polynomial $p(x)$ is called the degree of the polynomial $p(x)$.
4. Linear polynomial: A polynomial of degree 1 is called a linear polynomial.
   Example: $3x + 5, 7x - 8, -9x, \ldots$
   The general form a linear polynomial in variable x is $ax + b$ $(a, b \in R, a \neq 0)$.
5. Quadratic polynomial : A polynomial of degree 2 is called a quadratic polynomial.
   Example: $x^2 - 5x + 6, 2x^2 - 5, 7x^2, \ldots$
   The general form a quadratic polynomial in variable x is $ax^2 + bx + c$ $(a, b, c \in R, a \neq 0)$.
6. Cubic polynomial : A polynomial of degree 3 is called a cubic polynomial.
   Example: $5x^3 - 4x^2 + x - 1, 2x^3 - 3x + 5, -3x^3 - 10, \ldots\ldots$
   The general form a cubic polynomial in variable x is $ax^3 + bx^2 + cx + d$ $(a, b, c, d \in R, a \neq 0)$.
7. The general form of n^{th} degree polynomial in one variable x:
   $p(x) = a_0x^n + a_1x^{n-1} + a_2x^{n-2} + \cdots\cdots + a_{n-1}x + a_n$ is a polynomial of n^{th} degree, where $a_0, a_1, a_2, \ldots\ldots\ldots, a_{n-1}, a_n$ are real coefficients and $a_0 \neq 0$.
8. Value of a polynomial at a given point:
   The value of $p(x)$ at $x = k$ is $p(k)$.(substitute k value in x place)

Do this (page 49)

i) $P(x) = x^2 - 5x - 6$, find the values of $p(1), p(2), p(3), p(0), p(-1), p(-2), p(-3)$.

Sol: $P(x) = x^2 - 5x - 6$

$P(1) = (1)^2 - 5 \times 1 - 6$

$= 1 - 5 - 6$

$= 1 - 11$

$= -10$

$P(2) = (2)^2 - 5 \times 2 - 6$

$= 4 - 10 - 6$

$= 4 - 16$

$= -12$

$P(3) = (3)^2 - 5 \times 3 - 6$

$= 9 - 15 - 6$

$= 9 - 21$

$= -12$

$P(0) = (0)^2 - 5 \times 0 - 6$

$= 0 - 0 - 6$

$= -6$

$P(-1) = (-1)^2 - 5 \times (-1) - 6$

$= 1 + 5 - 6$

$= 6 - 6$

$= 0$

$P(-2) = (-2)^2 - 5 \times (-2) - 6$

$= 4 + 10 - 6$

$= 14 - 6$

$= 8$

$P(-3) = (-3)^2 - 5 \times (-3) - 6$

$= 9 + 15 - 6$

$= 21 - 6$

$= 15$

ii) $P(m) = m^2 - 3m + 1, find\ the\ values\ of\ p(1), p(-1),$

Sol: $P(m) = m^2 - 3m + 1$

$P(1) = (1)^2 - 3 \times 1 + 1$

$= 1 - 3 + 1$

$= 2 - 3$

$= -1$

$P(-1) = (-1)^2 - 3 \times (-1) + 1$

$= 1 + 3 + 1$

$= 5$

Zeroes of a polynomial:

Areal number k is said to be a zero of a polynomial $p(x), if\ p(k) = 0$.

Do this

(i) Let $p(x) = x^2 - 4x + 3$. Find the value of $p(0)$, $p(1)$, $p(2)$, $p(3)$ and obtain zeroes of the polynomial $p(x)$.

(ii) Check whether -3 and 3 are the zeroes of the polynomial $x^2 - 9$.

i)

Sol : $p(x) = x^2 - 4x + 3$

$p(0) = (0)^2 - 4 \times 0 + 3$

$= 0 - 0 + 3$

$= 3$

$p(1) = (1)^2 - 4 \times 1 + 3$

$= 1 - 4 + 3$

$= 4 - 4$

$= 0$

$p(2) = (2)^2 - 4 \times 2 + 3$

$= 4 - 8 + 3$

$= 7 - 8$

$= -1$

$p(3) = (3)^2 - 4 \times 3 + 3$

$= 9 - 12 + 3$

$= 12 - 12$

$= 0$

$p(1) = 0\ and\ p(3) = 0$

$\therefore$ 1,3 *are the zeroes of the polynomial* $p(x) = x^2 - 4x + 3$

ii)

Sol : $p(x) = x^2 - 9$

$p(-3) = (-3)^2 - 9$

$= 9 - 9$

$= 0$

$p(3) = (3)^2 - 9$

$= 9 - 9$

$= 0$

$p(-3) = 0 \quad and \;\; p(3) = 0$

So -3 and 3 are the zeroes of the polynomial $x^2 - 9$

EXERCISE - 3.1

1. If $p(x) = 5x^7 - 6x^5 + 7x - 6$, find
   (i). Coefficient of x^5 Ans: -6
   (ii). Degree of $p(x)$ Ans: 7
   (iii). Constant term. Ans: -6

Degree of the polynomial

Constant term

$p(x) = 5x^{7} - 6x^5 + 7x - 6$

Coefficient of x^5

2. State which of the following statements are true and which are false? Give reasons for your choice.
   (i). The degree of the polynomial $\sqrt{2}x^2 - 3x + 1$ *is* $\sqrt{2}$.

   Sol: false . Because the degree of the given polynomial is 2 .

   (ii). The coefficient of x^2 in the polynomial $3x^3 - 4x^2 + 5x + 7$ is 2 .

   Sol: false. Because the coefficient of $x^2 = -4$.

   (iii). $\frac{1}{x^2-5x+6}$ is a quadratic polynomial.

   Sol: false.

   (iv). The degree of a polynomial is one more than the number of terms in it.

   Sol: false . Because the degree of the polynomial $5x^7 - 6x^5 + 7x - 6$ is 7 and number of terms in it 4.

3. If $p(t) = t^3 - 1$, find the values of $p(1), p(-1), p(0), p(2), p(-2)$.

Sol: $P(t) = t^3 - 1$

$P(1) = 1^3 - 1$
$= 1 - 1$
$= 0$

$P(-1) = (-1)^3 - 1$
$= -1 - 1$
$= -2$

$P(0) = 0^3 - 1$
$= 0 - 1 = -1$

$P(-2) = (-2)^3 - 1$
$= -8 - 1 = -9$

4. Check whether -2 and 2 are the zeroes of the polynomial $x^4 - 16$.

Sol: $p(x) = x^4 - 16$

$p(-2) = (-2)^4 - 16$

$= 16 - 16$

$= 0$

$p(2) = (2)^4 - 16$

$= 16 - 16$

$= 0$

$p(-2) = 0$ *and* $p(2) = 0$

So -2 and 2 are the zeroes of the polynomial $x^4 - 16$.

5. Check whether 3 and -2 are the zeroes of the polynomial $p(x)$ *when* $p(x) = x^2 - x - 6$.

Sol: $\mathrm{P}(x) = x^2 - x - 6$

$\mathrm{P}(3) = 3^2 - 3 - 6$

$= 9 - 3 - 6$

$= 9 - 9$

$= 0$

$\mathrm{P}(-2) = (-2)^2 - (-2) - 6$

$= 4 + 2 - 6$

$= 6 - 6$

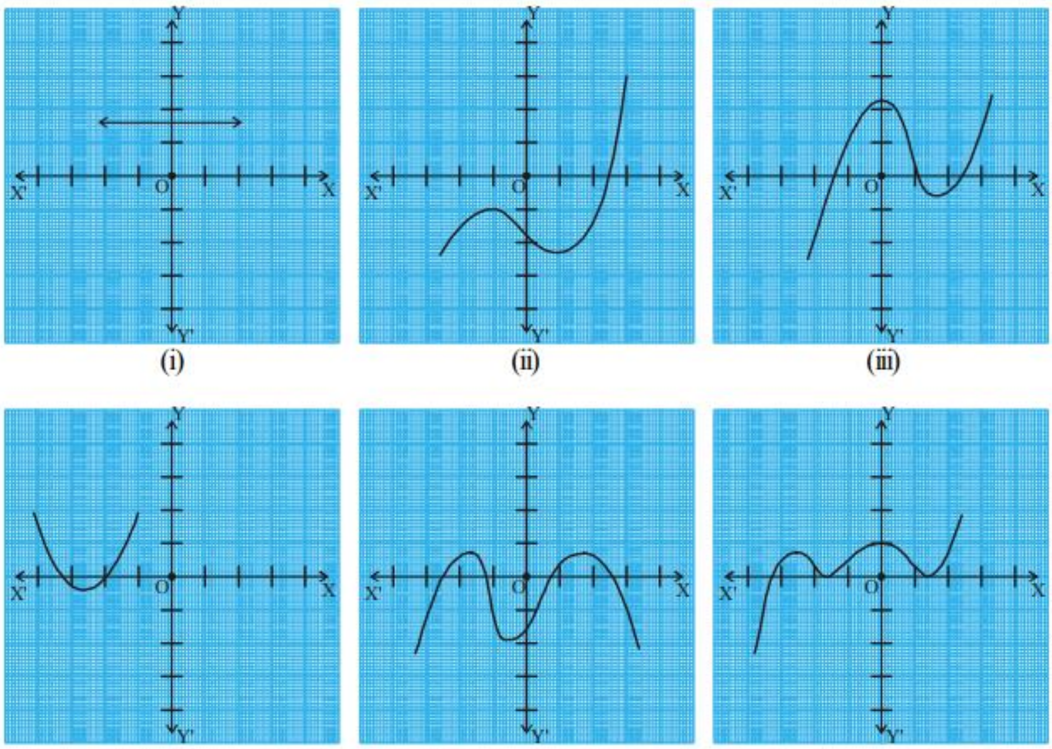
$= 0$

$\mathrm{P}(3) = 0$ and $\mathrm{P}(-2) = 0$.

So 3 and -2 are the zeroes of the polynomial $\mathrm{P}(x) = x^2 - x - 6$.

EXERCISE – 3.2

1. The graphs of $y = p(x)$ are given in the figure below, for some polynomials $p(x)$. In each case, find the number of zeroes of $p(x)$.



(i) (ii) (iii)

(iv) (v) (vi)

Sol: (i) The number of zeroes =0 (no zeroes)

Since the graph does not intersects X-axis

(ii) The number of zeroes = 1

Since the graph intersects X-axis at one point only.

(iii) The number of zeroes = 3

Since the graph intersects X-axis at three points .

(iv) The number of zeroes = 2

Since the graph intersects X-axis at two points.

(v) The number of zeroes = 4

Since the graph intersects X-axis at four points.

(vi) The number of zeroes = 3

Since the graph intersects X-axis at three points.

2. Find the zeroes of the given polynomials.

(i). $P(x) = 3x$

Sol: Let $P(x) = 0$

$3x = 0$

$x = 0$

The zero of the given polynomial $P(x) = 3x$ is 0.

(ii). $\mathrm{P}(x) = x^2 + 5x + 6$

Sol: Let $\mathrm{P}(x) = 0$

$x^2 + 5x + 6 = 0$

$x^2 + 2x + 3x + 6 = 0$

$x(x+2) + 3(x+2) = 0$

$(x+2)(x+3) = 0$

$(x+2) = 0$ or $(x+3) = 0$

$x = -2$ or $x = -3$

$\therefore$ The zeroes of the polynomial $\mathrm{P}(x) = x^2 + 5x + 6$ are -2 and -3.

(iii). $p(x) = (x+2)(x+3)$

Sol: Let $\mathrm{P}(x) = 0$

$(x+2)(x+3) = 0$

$(x+2) = 0$ or $(x+3) = 0$

$x = -2$ or $x = -3$

$\therefore$ The zeroes of the polynomial $\mathrm{P}(x) = (x+2)(x+3)$ are -2 and -3.

(iv). $p(x) = x^4 - 16$

Sol: Let $\mathrm{P}(x) = 0$

$x^4 - 16 = 0$

$(x^2)^2 - 4^2 = 0$

$a^2 - b^2 = (a+b)(a-b)$

$(x^2+4)(x^2-4) = 0$

$(x^2+4) = 0 \;\; or \; (x^2-4) = 0$

$x^2 = -4 \;\; or \;\; x^2 = 4$

$x = \sqrt{-4} \, or \; x = \sqrt{4} = \pm 2$

$x = 2, -2 \quad (\sqrt{-4} \; is \; not \; a \; real \; number)$

∴ The zeroes of the polynomial $p(x) = x^4 - 16$ are $2, -2$

4. Why are $\frac{1}{4}$ and -1 zeroes of the polynomial $p(x) = 4x^2 + 3x - 1$

Sol: $p(x) = 4x^2 + 3x - 1$

$p\left(\frac{1}{4}\right) = 4\left(\frac{1}{4}\right)^2 + 3 \times \frac{1}{4} - 1$

$= 4 \times \frac{1}{16} + \frac{3}{4} - 1$

$= \frac{1}{4} + \frac{3}{4} - 1$

$= \frac{4}{4} - 1 = 1 - 1 = 0$

$p(-1) = 4(-1)^2 + 3 \times (-1) - 1$

$= 4 \times 1 - 3 - 1$

$= 4 - 3 - 1$

$= 4 - 4$

$= 0$

$p\left(\frac{1}{4}\right) = 0 \; and \; p(-1) = 0$

so $\frac{1}{4}$ and -1 are zeroes of the polynomial $p(x) = 4x^2 + 3x - 1$.

3.5 Relationship between Zeroes and Coefficients of a Polynomial

1. If $\alpha \; and \; \beta$ are the zeroes of the quadratic polynomial $ax^2 + bx + c \; (a, b, c \in R, a \neq 0)$ then

i) Sum of zeroes $= \alpha + \beta = \frac{-(coefficient \; of \; x)}{coefficient \; of x^2} = \frac{-b}{a}$

ii) Product of zeroes $= \alpha\beta = \frac{constant \; term}{coefficient \; of x^2} = \frac{c}{a}$

Coefficient of $x^2 = a$
Coefficient of $x = b$
Constant term $= c$

2. If $\alpha \; and \; \beta$ are the zeroes of the quadratic polynomial then the quadratic polynomial

$= \text{k}[x^2 - (\alpha+\beta)x + \alpha\beta] = k[x^2 - (sum \; of \; zeroes)x + product \; of \; zeroes]$(k is constant)

Do This

Find the zeroes of the quadratic polynomials given below. Find the sum and product of the zeroes and verify relationship to the coefficients of terms in the polynomial.

(i) $p(x) = x^2 - x - 6$ (ii) $p(x) = x^2 - 4x + 3$

(iii) $p(x) = x^2 - 4$ (iv) $p(x) = x^2 + 2x + 1$

(i). $\boldsymbol{p(x) = x^2 - x - 6}$

Coefficient of $x^2 = a$=1
Coefficient of $x = b$=-1
Constant term = c= -6

Sol: $p(x) = x^2 - x - 6$

$= x^2 - 3x + 2x - 6$

$= x(x-3) + 2(x-3)$

$= (x-3)(x+2)$

To find zeroes let $p(x) = 0$

$(x-3)(x+2) = 0$

$x - 3 = 0 \quad or \quad x + 2 = 0$

$x = 3 \quad or \quad x = -2$

The zeroes of the polynomial $p(x) = x^2 - x - 6$ are 3 and -2

$\alpha = 3 \;\; and \;\; \beta = -2$

Sum of the zeroes = $(3) + (-2) = 1 = \frac{-(-1)}{1} = \frac{-(coefficient\ of\ x)}{coefficient\ of x^2} = \frac{-b}{a}$

Product of the zeroes = $(3) \times (-2) = -6 = \frac{-6}{1} = \frac{constant\ term}{coefficient\ of x^2} = \frac{c}{a}$

(ii). $p(x) = x^2 - 4x + 3$

Coefficient of $x^2 = a = 1$
Coefficient of $x = b = -4$
Constant term = $c = 3$

Sol: $p(x) = x^2 - 4x + 3$

$= x^2 - 3x - x + 3$

$= x(x-3) - 1(x-3)$

$= (x-3)(x-1)$

To find zeroes let $p(x) = 0$

$(x-3)(x-1) = 0$

$x - 3 = 0 \quad or \quad x - 1 = 0$

$x = 3 \quad or \quad x = 1$

The zeroes of the polynomial $p(x) = x^2 - 4x + 3$ are 3 and 1

$\alpha = 3 \;\; and \;\; \beta = 1$

Sum of the zeroes = $(3) + (1) = 4 = \frac{-(-4)}{1} = \frac{-(coefficient\ of\ x)}{coefficient\ of x^2} = \frac{-b}{a}$

Product of the zeroes = (3) × (1)= 3 = $\frac{3}{1} = \frac{constant\ term}{coefficient\ of x^2} = \frac{c}{a}$

(iii). $p(x) = x^2 - 4$

Coefficient of $x^2 = a = 1$
Coefficient of $x = b = 0$
Constant term = $c = -4$

Sol: $p(x) = x^2 - 4$

$= x^2 - 2^2$

$= (x + 2)(x - 2)$

To find zeroes let $p(x) = 0$

$(x + 2)(x - 2) = 0$

$x + 2 = 0 \quad or \quad x - 2 = 0$

$x = -2 \quad or \quad x = 2$

The zeroes of the polynomial $p(x) = x^2 - 4$ are -2 and 2

$\alpha = -2\ and\ \beta = 2$

Sum of the zeroes = (−2) + 2= 0 =$\frac{-(0)}{1} = \frac{-(coefficient\ of\ x)}{coefficient\ of x^2} = \frac{-b}{a}$

Product of the zeroes = (−2) × 2= −4 = $\frac{-4}{1} = \frac{constant\ term}{coefficient\ of x^2} = \frac{c}{a}$

(iv). $p(x) = x^2 + 2x + 1$

Coefficient of $x^2 = a = 1$
Coefficient of $x = b = 2$
Constant term = $c = 1$

Sol: $p(x) = x^2 + 2x + 1$

$= x^2 + x + x + 1$

$= x(x + 1) + 1(x + 1)$

$= (x + 1)(x + 1)$

To find zeroes let $p(x) = 0$

$(x + 1)(x + 1) = 0$

$x + 1 = 0 \quad or \quad x + 1 = 0$

$x = -1 \quad or \quad x = -1$

The zeroes of the polynomial $p(x) = x^2 + 2x + 1$ are -1 and -1

$\alpha = -1\ and\ \beta = -1$

Sum of the zeroes $= (-1) + (-1) = -2 = \frac{-(2)}{1} = \frac{-(coefficient\ of\ x)}{coefficient\ of x^2} = \frac{-b}{a}$

Product of the zeroes $= (-1) \times (-1) = 1 = \frac{1}{1} = \frac{constant\ term}{coefficient\ of x^2} = \frac{c}{a}$

Example-3. Find the zeroes of the quadratic polynomial $x^2 + 7x + 10$, and verify the relationship between the zeroes and the coefficients.

Sol: $p(x) = x^2 + 7x + 10$

$= x^2 + 5x + 2x + 10$

$= x(x+5) + 2(x+5)$

$= (x+5)(x+2)$

Coefficient of $x^2 = a = 1$
Coefficient of $x = b = 7$
Constant term $= c = 10$

To find zeroes let $p(x) = 0$

$(x+5)(x+2) = 0$

$x + 5 = 0 \quad or \quad x + 2 = 0$

$x = -5 \quad or \quad x = -2$

The zeroes of the polynomial $p(x) = x^2 + 7x + 10$ are -5 and -2

$\alpha = -5\ and\ \beta = -2$

Sum of the zeroes $= (-5) + (-2) = -7 = \frac{-(7)}{1} = \frac{-(coefficient\ of\ x)}{coefficient\ of x^2} = \frac{-b}{a}$

Product of the zeroes $= (-5) \times (-2) = 10 = \frac{10}{1} = \frac{constant\ term}{coefficient\ of x^2} = \frac{c}{a}$

Example-4. Find the zeroes of the polynomial $x^2 - 3$ and verify the relationship between the zeroes and the coefficients.

Sol: $(x) = x^2 - 3$

$= x^2 - \sqrt{3}^2$

$= (x+\sqrt{3})(x-\sqrt{3})$

Coefficient of $x^2 = a = 1$
Coefficient of $x = b = 0$
Constant term $= c = -3$

To find zeroes let $p(x) = 0$

$(x+\sqrt{3})(x-\sqrt{3}) = 0$

$x+\sqrt{3}=0 \quad or \quad x-\sqrt{3}=0$

$x=-\sqrt{3} \quad or \quad x=\sqrt{3}$

The zeroes of the polynomial $p(x)=x^2-3$ are $-\sqrt{3}$ and $\sqrt{3}$

$\alpha=-2 \ and \ \beta=\ 2$

Sum of the zeroes = $(-\sqrt{3})+\sqrt{3}=0=\frac{-(0)}{1}=\frac{-(coefficient\ of\ x)}{coefficient\ of x^2}=\frac{-b}{a}$

Product of the zeroes = $(-\sqrt{3})\times\sqrt{3}=-3=\frac{-3}{1}=\frac{constant\ term}{coefficient\ of x^2}=\frac{c}{a}$

Example-5. Find the quadratic polynomial, whose sum and product of the zeroes are – 3 and 2, respectively.

Sol: Sum of the zeroes = $\alpha+\beta=-3$

Product of the zeroes= $\alpha\beta=2$

quadratic polynomial = $k[x^2-(\alpha+\beta)x+\alpha\beta]$

$=k[x^2-(-3)x+2]$

$=k[x^2+3x+2]$

When k=1

Required quadratic polynomial $=[x^2+3x+2]$

Example-6. Find the quadratic polynomial whose zeroes are 2 and $\frac{-1}{3}$.

Sol: Zeroes of the polynomial are $\alpha=2$ and $\beta=-\frac{1}{3}$

$\alpha+\beta=\ 2+\left(-\frac{1}{3}\right)=\frac{6-1}{3}=\frac{5}{3}$

$\alpha\times\beta=\ 2\times\left(-\frac{1}{3}\right)=-\frac{2}{3}$

The quadratic polynomial = $k[x^2-(\alpha+\beta)x+\alpha\beta]$

$=k[x^2-\left(\frac{5}{3}\right)x+\left(-\frac{2}{3}\right)]$

$=k[\ x^2-\frac{5}{3}x-\frac{2}{3}]$

When k=3

The quadratic polynomial = $3[\ x^2-\frac{5}{3}x-\frac{2}{3}]$

$=3\times\ x^2-3\times\frac{5}{3}x-3\times\frac{2}{3}$

$=3x^2-5x-2.$

TRY THIS (page- 64)

(i). Find a quadratic polynomial with zeroes -2 and $\frac{1}{3}$

Sol: Zeroes of the polynomial are $\alpha = -2$ and $\beta = \frac{1}{3}$

$$\alpha + \beta = (-2) + \left(\frac{1}{3}\right) = \frac{-6+1}{3} = \frac{-5}{3}$$

$$\alpha \times \beta = (-2) \times \left(\frac{1}{3}\right) = -\frac{2}{3}$$

The quadratic polynomial is= $k[x^2 - (\alpha + \beta)x + \alpha\beta]$

$$= k[x^2 - \left(-\frac{5}{3}\right)x + \left(-\frac{2}{3}\right)]$$

$$= k[\, x^2 + \frac{5}{3}x - \frac{2}{3}\,]$$

When k=3

The quadratic polynomial $= 3[\, x^2 + \frac{5}{3}x - \frac{2}{3}\,]$

$$= 3 \times\ x^2 + 3 \times \frac{5}{3}x - 3 \times \frac{2}{3}$$

$$= 3x^2 + 5x - 2.$$

(ii). What is the quadratic polynomial whose sum of zeroes is $\frac{-3}{2}$ and the product of zeroes is -1.

Sol: Sum of the zeroes $= \alpha + \beta = -\frac{3}{2}$

Product of the zeroes= $\alpha\beta = -1$

quadratic polynomial $= k[x^2 - (\alpha + \beta)x + \alpha\beta]$

$$= k\left[x^2 - \left(-\frac{3}{2}\right)x + (-1)\right]$$

$$= k\left[x^2 + \frac{3}{2}x - 1\right]$$

When k=2

Required quadratic polynomial $= 2 \times \left[x^2 + \frac{3}{2}x - 1\right]$

$$= 2x^2 + 3x - 2$$

EXERCISE – 3.3

1. Find the zeroes of the following quadratic polynomials and verify the relashion ship between the zeroes and the coefficients.

(i). $x^2 - 2x - 8$

Sol: $p(x) = x^2 - 2x - 8$

$= x^2 - 4x + 2x - 8$

$= x(x - 4) + 2(x - 4)$

$= (x-4)(x+2)$

To find zeroes let $p(x) = 0$

$(x-4)(x+2) = 0$

$x - 4 = 0 \quad or \quad x + 2 = 0$

$x = 4 \quad or \quad x = -2$

The zeroes of the polynomial $p(x) = x^2 - 2x - 8$ are 4 and -2

$\alpha = 4 \; and \; \beta = -2$

Coefficient of $x^2 = a = 1$

Coefficient of $x = b = -2$

Constant term $= c = -8$

Sum of the zeroes = $(4) + (-2)$= 2 =$\frac{-(-2)}{1} = \frac{-(coefficient\ of\ x)}{coefficient\ of x^2}$ =$\frac{-b}{a}$

Product of the zeroes = $(4) \times (-2)$= $-8 = \frac{-8}{1} = \frac{constant\ term}{coefficient\ of x^2}$ = $\frac{c}{a}$

(ii). $4s^2 - 4s + 1$

Coefficient of $s^2 = a = 4$

Coefficient of $s = b = -4$

Constant term $= c = 1$

Sol: $p(s) = 4s^2 - 4s + 1$

$= 4s^2 - 2s - 2s + 1$

$= 2s(2s-1) - 1(2s-1)$

$= (2s-1)(2s-1)$

To find zeroes let $p(s) = 0$

$(2s-1)(2s-1) = 0$

$2s - 1 = 0 \quad or \quad 2s - 1 = 0$

$2s = 1 \quad or \quad 2s = 1$

$s = \frac{1}{2} \quad or \quad s = \frac{1}{2}$

The zeroes of the polynomial $p(s) = 4s^2 - 4s + 1$are $\frac{1}{2}$ and $\frac{1}{2}$

Sum of the zeroes = $\frac{1}{2} + \frac{1}{2} = \frac{2}{2}$= 1 =$\frac{-(-4)}{4} = \frac{-(coefficient\ of\ s)}{coefficient\ of s^2}$ =$\frac{-b}{a}$

Product of the zeroes = $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4} = \frac{constant\ term}{coefficient\ of s^2} = \frac{c}{a}$

(iii). $6x^2 - 3 - 7x$

Coefficient of $x^2 = a = 6$

Coefficient of $x = b = -7$

Constant term $= c = -3$

Sol: $p(x) = 6x^2 - 7x - 3$

$= 6x^2 - 9x + 2x - 3$

$= 3x(2x-3) + 1(2x-3)$

$= (2x-3)(3x+1)$

To find zeroes let $p(x) = 0$

$(2x - 3)(3x + 1) = 0$

$2x - 3 = 0 \quad or \quad 3x + 1 = 0$

$2x = 3 \quad or \quad 3x = -1$

$x = \frac{3}{2} \quad or \quad x = \frac{-1}{3}$

The zeroes of the polynomial $p(x) = 6x^2 - 7x - 3$ are $\frac{3}{2}$ and $\frac{-1}{3}$

$\alpha = 4 \ and \ \beta = -2$

Sum of the zeroes = $(\frac{3}{2}) + (\frac{-1}{3}) = \frac{9-2}{6} = \frac{7}{6} = \frac{-(-7)}{6} = \frac{-(coefficient\ of\ x)}{coefficient\ of x^2} = \frac{-b}{a}$

Product of the zeroes = $(\frac{3}{2}) \times (\frac{-1}{3}) = \frac{-3}{6} = = \frac{constant\ term}{coefficient\ of x^2} = \frac{c}{a}$

(iv). $4u^2 + 8u$

Coefficient of $u^2 = a = 4$

Coefficient of $u = b = 8$

Constant term $= c = 0$

Sol: $P(u) = 4u^2 + 8u$

$= 4u(u + 2)$

To find zeroes let $P(u) = 0$

$4u(u + 2) = 0$

$4u = 0 \quad or \quad u + 2 = 0$

$u = 0 \quad or \quad u = -2$

The zeroes of the polynomial $p(u) = 4u^2 + 8u$ are 0 and -2

Sum of the zeroes = $0 + (-2) = -2 = \frac{-8}{4} = \frac{-(coefficient\ of\ u)}{coefficient\ of u^2} = \frac{-b}{a}$

Product of the zeroes = $0 \times (-2) = 0 = \frac{0}{4} = \frac{constant\ term}{coefficient\ of u^2} = \frac{c}{a}$

(v). $t^2 - 15$

Coefficient of $t^2 = a = 1$

Coefficient of $t = b = 0$

Constant term $= c = -15$

Sol: $p(t) = t^2 - 15$

$= t^2 - (\sqrt{15})^2$

$= (t + \sqrt{15})(t - \sqrt{15})$

To find zeroes let $P(t) = 0$

$(t + \sqrt{15})(t - \sqrt{15}) = 0$

$t + \sqrt{15} = 0 \quad or \quad t - \sqrt{15} = 0$

$t = \sqrt{15}$ *or* $t = -\sqrt{15}$

The zeroes of the polynomial $p(t) = t^2 - 15$ are $\sqrt{15}$ and $-\sqrt{15}$

Sum of the zeroes $= \sqrt{15} + (-\sqrt{15}) = 0 = \frac{-0}{1} = \frac{-(coefficient\ of\ t)}{coefficient\ of t^2} = \frac{-b}{a}$

Product of the zeroes $= \sqrt{15} \times (-\sqrt{15}) = -15 = \frac{-15}{1} = \frac{constant\ term}{coefficient\ of t^2} = \frac{c}{a}$

(vi). $3x^2 - x - 4$

Coefficient of $x^2 = a = 3$
Coefficient of $x = b = -1$
Constant term $= c = -4$

Sol: $p(x) = 3x^2 - x - 4$

$= 3x^2 - 4x + 3x - 4$

$= x(3x - 4) + 1(3x - 4)$

$= (3x - 4)(x + 1)$

To find zeroes let $p(x) = 0$

$(3x - 4)(x + 1) = 0$

$3x - 4 = 0$ *or* $x + 1 = 0$

$3x = 4$ *or* $x = -1$

$x = \frac{4}{3}$ *or* $x = -1$

The zeroes of the polynomial $p(x) = 3x^2 - x - 4$ are $\frac{4}{3}$ and -1

$\alpha = 4$ *and* $\beta = -2$

Sum of the zeroes $= (\frac{4}{3}) + (-1) = \frac{4-3}{3} = \frac{1}{3} = \frac{-(-1)}{3} = \frac{-(coefficient\ of\ x)}{coefficient\ of x^2} = \frac{-b}{a}$

Product of the zeroes $= (\frac{4}{3}) \times (-1) = \frac{-4}{3} = \frac{constant\ term}{coefficient\ of x^2} = \frac{c}{a}$

2. Find the quadratic polynomial in each case, with the given numbers as the sum and product of its zeroes respectively.

(i). $\frac{1}{4}, -1$

Sol: sum of the zeroes $= \alpha + \beta = \frac{1}{4}$

Product of zeroes $= \alpha\beta = -1$

The quadratic polynomial is= $k[x^2 - (\alpha + \beta)x + \alpha\beta]$

$= k[x^2 - \left(\frac{1}{4}\right)x + (-1)]$

$= k[\, x^2 - \frac{1}{4}x - 1\,]$

When k=4

Quadratic polynomial= $4 \times [\, x^2 - \frac{1}{4}x - 1 \,]$

$= 4x^2 - x - 4$

(ii). $\sqrt{2}, \frac{1}{3}$

Sol: sum of the zeroes $= \alpha + \beta = \sqrt{2}$

Product of zeroes $= \alpha\beta = \frac{1}{3}$

The quadratic polynomial is= k$[x^2 - (\alpha + \beta)x + \alpha\beta]$

$= k[x^2 - \sqrt{2}x + \left(\frac{1}{3}\right)]$

When k=3

Quadratic polynomial= $3 \times [x^2 - \sqrt{2}x + \left(\frac{1}{3}\right)]$

$= 3x^2 - 3\sqrt{2}x + 1$

(iii). 0,$\sqrt{5}$

Sol: sum of the zeroes $= \alpha + \beta = 0$

Product of zeroes $= \alpha\beta = \sqrt{5}$

The quadratic polynomial is= k$[x^2 - (0)x + \sqrt{5}]$

$= k[x^2 + \sqrt{5}]$

When k=1

Quadratic polynomial= $x^2 + \sqrt{5}$

(iv). 1,1

Sol: Sum of the zeroes $= \alpha + \beta = 1$

Product of the zeroes= $\alpha\beta = 1$

quadratic polynomial $= k[x^2 - (\alpha + \beta)x + \alpha\beta]$

$= k[x^2 - (1)x + 1]$

$= k[x^2 - x + 1]$

When k=1

Required quadratic polynomial $= [x^2 - x - 1]$

(v). $-\frac{1}{4}, \frac{1}{4}$

Sol: Sum of the zeroes $= \alpha + \beta = -\frac{1}{4}$

Product of the zeroes$= \alpha\beta = \frac{1}{4}$

quadratic polynomial $= k[x^2 - (\alpha + \beta)x + \alpha\beta]$

$$= k\left[x^2 - \left(-\frac{1}{4}\right)x + \frac{1}{4}\right]$$

$$= k\left[x^2 + \frac{1}{4}x + \frac{1}{4}\right]$$

When k=4

Required quadratic polynomial $= 4 \times \left[x^2 + \frac{1}{4}x + \frac{1}{4}\right]$

$$= 4x^2 + x + 1$$

(vi). 4,1

Sol: Sum of the zeroes $= \alpha + \beta = 4$

Product of the zeroes$= \alpha\beta = 1$

quadratic polynomial $= k[x^2 - (\alpha + \beta)x + \alpha\beta]$

$$= k[x^2 - (4)x + 1]$$

$$= k[x^2 - 4x + 1]$$

When k=1

Required quadratic polynomial $= [x^2 - 4x + 1]$

3. Find the quadratic polynomial, for the zeroes α, β given in each case.

(i). 2,-1

Sol: $\alpha = 2, \beta = -1$

Sum of the zeroes $= \alpha + \beta = 2 + (-1) = 1$

Product of the zeroes$= \alpha\beta = 2 \times (-1) = -2$

quadratic polynomial $= k[x^2 - (\alpha + \beta)x + \alpha\beta]$

$$= k[x^2 - (1)x + (-2)]$$

$= k[x^2 - x - 2]$

When k=1

Required quadratic polynomial $= x^2 - x - 2$

(ii). $\sqrt{3}, -\sqrt{3}$

Sol: $\alpha = \sqrt{3}, \beta = -\sqrt{3}$

Sum of the zeroes $= \alpha + \beta = \sqrt{3} + (-\sqrt{3}) = 0$

Product of the zeroes$= \alpha\beta = \sqrt{3} \times (-\sqrt{3}) = -3$

quadratic polynomial $= k[x^2 - (\alpha + \beta)x + \alpha\beta]$

$= k[x^2 - (0)x + (-3)]$

$= k[x^2 - 3]$

When k=1

Required quadratic polynomial $= x^2 - 3$

(iii). $\frac{1}{4}, -1$

Sol: $\alpha = \frac{1}{4}, \beta = -1$

Sum of the zeroes $= \alpha + \beta = \frac{1}{4} + (-1) = \frac{1-4}{4} = \frac{-3}{4}$

Product of the zeroes$= \alpha\beta = \frac{1}{4} \times (-1) = -\frac{1}{4}$

quadratic polynomial $= k[x^2 - (\alpha + \beta)x + \alpha\beta]$

$= k\left[x^2 - \left(\frac{-3}{4}\right)x + (-\frac{1}{4})\right]$

$= k\left[x^2 + \frac{3}{4}x - \frac{1}{4}\right]$

When k=4

Required quadratic polynomial $= 4 \times \left[x^2 + \frac{3}{4}x - \frac{1}{4}\right]$

$$= 4 \times x^2 + 4 \times \frac{3}{4}x - 4 \times \frac{1}{4}$$

$= 4x^2 + 3x - 1$

(iv). $\frac{1}{2}, \frac{3}{2}$

Sol: $\alpha = \frac{1}{2}, \beta = \frac{3}{2}$

Sum of the zeroes $= \alpha + \beta = \frac{1}{2} + \frac{3}{2} = \frac{1+3}{2} = \frac{4}{2} = 2$

Product of the zeroes$= \alpha\beta = \frac{1}{2} \times \frac{3}{2} = \frac{3}{4}$

quadratic polynomial $= k[x^2 - (\alpha + \beta)x + \alpha\beta]$

$= k\left[x^2 - (2)x + \frac{3}{4}\right]$

$= k\left[x^2 - 2x + \frac{3}{4}\right]$

When k=4

Required quadratic polynomial $= 4 \times \left[x^2 - 2x + \frac{3}{4}\right]$

$= 4 \times x^2 - 4 \times 2x - 4 \times \frac{3}{4}$

$= 4x^2 - 8x - 3$

CUBIC POLYNOMIALS:

1. If α, β, γ are the zeroes of the cubic polynomial $ax^3 + bx^2 + cx + d$ $(a, b, c, d \in R, a \neq 0)$ then

   (i). $\alpha + \beta + \gamma = \frac{-(\text{coefficient of } x^2)}{\text{coefficient of } x^3} = \frac{-b}{a}$

   (ii). $\alpha\beta + \beta\gamma + \gamma\alpha = \frac{\text{coefficient of } x}{\text{coefficient of} x^3} = \frac{c}{a}$

   (iii). $\alpha\beta\gamma = \frac{-(\text{constant term})}{\text{coefficient of} x^3} = \frac{-d}{a}$

2. If α, β, γ are the zeroes of the cubic polynomial then the cubic polynomial is

   $= k[x^3 - (\alpha + \beta + \gamma)x^2 + (\alpha\beta + \beta\gamma + \gamma\alpha)x - \alpha\beta\gamma]$

DO THIS (page- 66)

1. If α, β, γ are the zeroes of the cubic polynomial , find the values $\alpha + \beta + \gamma, \alpha\beta + \beta\gamma + \gamma\alpha, \alpha\beta\gamma$

   1) $x^3 - 3x^2 - x - 2$ $[a = 1, b = -3, c = -1, d = -2]$

   Sol: $\alpha + \beta + \gamma = \frac{-b}{a} = \frac{-(-3)}{1} = \frac{3}{1} = 3$

   $\alpha\beta + \beta\gamma + \gamma\alpha = \frac{c}{a} = \frac{-1}{1} = -1$

$$\alpha\beta\gamma = \frac{-d}{a} = \frac{-(-2)}{1} = \frac{2}{1} = 2$$

2) $4x^3 + 8x^2 - 6x - 2$ $\quad [a = 4, b = 8, c = -6, d = -2]$

Sol: $\alpha + \beta + \gamma = \frac{-b}{a} = \frac{-8}{4} = -2$

$$\alpha\beta + \beta\gamma + \gamma\alpha = \frac{c}{a} = \frac{-6}{4} = \frac{-3}{2}$$

$$\alpha\beta\gamma = \frac{-d}{a} = \frac{-(-2)}{4} = \frac{2}{4} = \frac{1}{2}$$

3) $x^3 + 4x^2 - 5x - 2$ $\quad [a = 1, b = 4, c = -5, d = -2]$

Sol: $\alpha + \beta + \gamma = \frac{-b}{a} = \frac{-4}{1} = -4$

$$\alpha\beta + \beta\gamma + \gamma\alpha = \frac{c}{a} = \frac{-5}{1} = -5$$

$$\alpha\beta\gamma = \frac{-d}{a} = \frac{-(-2)}{1} = \frac{2}{1} = 2$$

4) $x^3 + 5x^2 + 4$ $\quad [a = 1, b = 5, c = 0, d = 4]$

Sol: $\alpha + \beta + \gamma = \frac{-b}{a} = \frac{-5}{1} = -5$

$$\alpha\beta + \beta\gamma + \gamma\alpha = \frac{c}{a} = \frac{0}{1} = 0$$

$$\alpha\beta\gamma = \frac{-d}{a} = \frac{-4}{1} = -4$$

Example -7: Verify that $3, -1, -\frac{1}{3}$ are the zeroes of the cubic polynomial $p(x) = 3x^3 - 5x^2 - 11x - 3$, and then verify the relationship between the zeroes and the coefficients.

Sol: $P(x) = 3x^3 - 5x^2 - 11x - 3$

$\text{P}(3) = 3(3)^3 - 5(3)^2 - 11(3) - 3$

$= 3 \times 27 - 5 \times 9 - 33 - 3$

$= 81 - 45 - 33 - 3$

$= 81 - 81 = 0$

$\text{P}(-1) = 3(-1)^3 - 5(-1)^2 - 11(-1) - 3$

$= 3 \times (-1) - 5 \times 1 + 11 - 3$

$= -3 - 5 + 11 - 3$

$= -11 + 11 = 0$

$P\left(-\frac{1}{3}\right) = 3(-\frac{1}{3})^3 - 5\left(-\frac{1}{3}\right)^2 - 11(-\frac{1}{3}) - 3$

$= 3\left(-\frac{1}{27}\right) - 5\left(\frac{1}{9}\right) + \frac{11}{3} - 3$

$= -\frac{1}{9} - \frac{5}{9} + \frac{11}{3} - 3$

$= \frac{-1-5+33-27}{9}$

$= \frac{-33+33}{9} \quad = 0$

$P(3) = 0$, $P(-1) = 0$, and $P\left(-\frac{1}{3}\right) = 0$

So $3, -1, -\frac{1}{3}$ are the zeroes of the cubic polynomial $P(x) = 3x^3 - 5x^2 - 11x - 3$

$we\ take\ \alpha = 3, \beta = -1, \gamma = -\frac{1}{3}$

$$\alpha + \beta + \gamma = 3 + (-1) + \left(\frac{-1}{3}\right) = 2 - \frac{1}{3} = \frac{5}{3} = \frac{-(-5)}{3} = \frac{-b}{a}$$

$$\alpha\beta + \beta\gamma + \gamma\alpha = 3 \times (-1) + (-1) \times \left(\frac{-1}{3}\right) + \left(\frac{-1}{3}\right) \times 3 = -3 + \frac{1}{3} - 1 = \frac{-11}{3} = \frac{c}{a}$$

$$\alpha\beta\gamma = 3 \times (-1) \times \left(\frac{-1}{3}\right) = 1 = \frac{-(-3)}{3} = \frac{-d}{a}$$

Exercise – 3.3

4. Verify 1,-1 and -3 are the zeroes of the cubic polynomial $x^3 + 3x^2 - x - 3$ and check the relation between the zeroes and the coefficients.

Sol: $p(x) = x^3 + 3x^2 - x - 3$

Coefficient of $x^3 = a = 1$

Coefficient of $x^2 = b = 3$

Coefficient of $x = c = -1$

Constant term $= d = -3$

$p(1) = (1)^3 + 3(1)^2 - 1 - 3$

$= 1 + 3 - 1 - 3 = 4 - 4 = 0$

$p(-1) = (-1)^3 + 3(-1)^2 - (-1) - 3$

$= -1 + 3 + 1 - 3 = 4 - 4 = 0$

$p(-3) = (-3)^3 + 3(-3)^2 - (-3) - 3$

$= -27 + 27 + 3 - 3 = 30 - 30 = 0$

$p(1) = 0, p(-1) = 0\ \ and\ \ p(-3) = 0$

$\therefore$ $1, -1$ and -3 are the zeroes of the cubic polynomial $x^3 + 3x^2 - x - 3$.

Now $\alpha = 1,\ \ \beta = -1\ \ and\ \ \gamma = -3$

$\alpha + \beta + \gamma = 1 + (-1) + (-3) = -3 = \frac{-3}{1} = \frac{-b}{a}$

$\alpha\beta + \beta\gamma + \gamma\alpha = 1 \times (-1) + (-1) \times (-3) + (-3) \times 1 = -1 + 3 - 3 = -1 = \frac{-1}{1} = \frac{c}{a}$

$\alpha\beta\gamma = 1 \times (-1) \times (-3) = 3 = \frac{-(-3)}{1} = \frac{-d}{a}$

Division Algorithm for Polynomials

Division fact:

$$\begin{array}{r} Quotient \\ Divisor\overline{)\,Dividend} \\ \underline{\qquad\qquad} \\ \underline{Remainder} \end{array}$$

Dividend = (Divisor × Quotient) + Remainder

Example-8. Divide $2x^2 + 3x + 1$ by $x + 2$.

Solution:

$$\begin{array}{r} 2x - 1 \\ x + 2\overline{)\,2x^2 + 3x + 1} \\ 2x^2 + 4x \\ (-)\quad(-)\quad \\ \hline -x + 1 \\ -x - 2 \\ (+)\ (+) \\ \hline 3 \\ \hline \end{array}$$

$Divisor = x + 2$

$Dividend = 2x^2 + 3x + 1$

$Quotient = 2x - 1$
$Remainder = 3$
(Divisor × Quotient) + Remainder

$= (x + 2)(2x - 1) + 3$

$= 2x^2 - x + 4x - 2 + 3$

$= 2x^2 + 3x + 1 = Dividend$

Example-9. Divide $3x^3 + x^2 + 2x + 5$ by $1 + 2x + x^2$.

Solution:

$$\begin{array}{r} 3x - 5 \\ x^2 + 2x + 1\overline{)\,3x^3 + x^2 + 2x + 5} \\ 3x^3 + 6x^2 + 3x \\ (-)\quad(-)\quad(-) \\ \hline -5x^2 - x + 5 \\ -5x^2 - 10x - 5 \\ (+)\quad(+)\quad(+) \\ \hline 9x + 10 \\ \hline \end{array}$$

$Divisor = x^2 + 2x + 1$

$Dividend = 3x^3 + x^2 + 2x + 5$

$Quotient = 3x - 5$

$Remainder = 9x + 10$

(Divisor × Quotient) + Remainder

$= (x^2 + 2x + 1)(3x - 5) + (9x + 10)$

$= 3x^3 + 6x^2 + 3x - 5x^2 - 10x - 5 + 9x + 10$

$= 3x^3 + x^2 + 2x + 5$

Division Algorithm for polynomials:

11. Any polynomial p(x) and any non-zero polynomial g(x), there are polynomials q(x) and r(x) such that p(x) = g(x) q(x) + r(x), where either r(x) = 0 or degree r(x) < degree g(x) if r(x) ≠ 0

(i) If g(x) is linear polynomial then r(x) = r is a constant.

(ii) If degree of q(x) = 1, then degree of p(x) = 1 + degree of g(x).

(iii) If p(x) is divided by (x – a), then the remainder is p(a).

(iv) If r = 0, we say q(x) divides p(x) exactly or q(x) is a factor of p(x)

Example-10. Divide $3x^2 - x^3 - 3x + 5$ by $-1 - x^2$, and verify the division algorithm.

Solution:

$Dividend = -x^3 + 3x^2 - 3x + 5$

$Divisor = -x^2 + x - 1$

$Quotient = x - 2$

$Remainder = 3$

Dividend = Divisor × Quotient + Remainder

$= (-x^2 + x - 1) \times (x - 2) + 3$

$= -x^3 - 2x^2 + x^2 - 2x - x + 2 + 3$

$= -x^3 + 3x^2 - 3x + 5$

Hence the division algorithm is verified.

$x - 2$

$-x^2 + x - 1 \,\big)\, -x^3 + 3x^2 - 3x$

$-x^3 + x^2 - x$

(+) (−) (+)

$2x^2 - 2x + 5$

$2x^2$

(−) (+) (−)

3

Example-11. Find all the zeroes of $2x^4 - 3x^3 - 3x^2 + 6x - 2$, if you know that two of its zeroes are $\sqrt{2}$ and $-\sqrt{2}$.

Solution: Since two of the zeroes are $\sqrt{2}$ and $-\sqrt{2}$.

We can divide by $(x - \sqrt{2}) \times (x + \sqrt{2})$

$= x^2 - (\sqrt{2})^2 = x^2 - 2$

$2x^2 - 3x + 1$

$x^2 - 2 \,\big)\, 2x^4 - 3x^3 - 3x^2 + 6x - 2$

$2x^4 \quad - 4x^2$

(−) (+)

$-3x^3 + x^2 + 6x - 2$

$-3x^3 \quad + 6x$

(+) (−)

$x^2 - 2$

$x^2 - 2$

(−) (+)

0

$Quotient = 2x^2 - 3x + 1$

$= 2x^2 - 2x - x + 1$

$= 2x(x-1) - 1(x-1)$

$= (x-1)(2x-1)$

$For\ zeroes\ \ x-1=0\ \ and\ 2x-1=0$

$\Rightarrow x = 1\ and\ x = \frac{1}{2}$

Therefore, the zeroes of the given polynomial are $\sqrt{2}, -\sqrt{2}, 1\ and\ \frac{1}{2}$.

EXERCISE – 3.4

1. Divide the polynomial p(x) by the polynomial g(x) and find the quotient and remainder in each of the following :

$(i)\ p(x) = x^3 - 3x^2 + 5x - 3, g(x) = x^2 - 2$

$Quotient = x - 3$

$Remainder = 7x - 9$

$$
\begin{array}{r|l}
 & x \quad -3 \\
x^2 - 2 & x^3 - 3x^2 + 5x - 3 \\
 & \underset{(-)}{x^3} \qquad\quad \underset{(+)}{-2x} \\
\hline
 & -3x^2 + 7x - 3 \\
 & \underset{(+)}{-3x^2} \qquad \underset{(-)}{+6} \\
\hline
 & 7x - 9 \\
\hline
\end{array}
$$

$(ii)\ p(x) = x^4 - 3x^2 + 4x + 5, g(x) = 2 - x^2$

Sol:

$Quotient = x^2 + x - 3$

$Remainder = 8$

$$
\begin{array}{r|l}
 & x^2 + x - 3 \\
x^2 - x + 1 & x^4 \qquad - 3x^2 + 4x + 5 \\
 & \underset{(-)}{x^4} \ \underset{(+)}{- x^3} \ \underset{(-)}{+ x^2} \\
\hline
 & x^3 - 4x^2 + 4x \\
 & \underset{(-)}{x^3} \ \underset{(+)}{- x^2} \ \underset{(-)}{+ x} \\
\hline
 & -3x^2 + 3x + 5 \\
 & \underset{(-)}{-3x^2} \ \underset{(-)}{+ 3x} \ \underset{(+)}{- 3} \\
\hline
 & 8 \\
\hline
\end{array}
$$

$(iii)\ p(x) = x^4 - 5x + 6, g(x) = 2 - x^2$

Sol:

$Quotient = -x^2 - 2$

$Remainder = -5x + 10$

$$
\begin{array}{r|l}
 & -x^2 \ -2 \\
-x^2 + 2 & x^4 \qquad - 5x + 6 \\
 & \underset{(-)}{x^4} \ \underset{(+)}{- 2x^2} \\
\hline
 & 2x^2 - 5x + 6 \\
 & \underset{(-)}{2x^2} \qquad \underset{(+)}{- 4} \\
\hline
 & -5x + 10 \\
\hline
\end{array}
$$

2. Check in which case the first polynomial is a factor of the second polynomial by dividing the second polynomial by the first polynomial :

$(i)\ t^2 - 3, 2t^4 + 3t^3 - 2t^2 - 9t - 12$

Sol:

Here remainder=0

$t^2 - 3$ *is a factor of* $2t^4 + 3t^3 - 2t^2 - 9t - 12$

$$
\begin{array}{r}
2t^2 + 3t \\
t^2 - 3 \,\big)\, 2t^4 + 3t^3 - 2t^2 - 9t \\
2t^4 \qquad\quad - 6t^2 \\
(-) \qquad\quad (+) \\
\hline
3t^3 + 4t^2 - 9t - 12 \\
3t^3 \qquad - 9t \\
(-) \qquad\quad (+) \\
\hline
4t^2 \qquad - 12 \\
4t^2 \qquad - 12 \\
(-) \qquad\quad (+) \\
\hline
0 \\
\hline
\end{array}
$$

$(ii)\ x^2 + 3x + 1, 3x^4 + 5x^3 - 7x^2 + 2x + 2$

$$
\begin{array}{r}
3x^2 - 4x \\
x^2 + 3x + 1 \,\big)\, 3x^4 + 5x^3 - 7x^2 + 2x + 2 \\
3x^4 + 9x^3 + 3x^2 \qquad\qquad \\
(-) \quad (-) \quad (-) \qquad\qquad \\
\hline
-4x^3 - 10x^2 + 2x + 2 \\
-4x^3 - 12x^2 - 4x \qquad \\
(+) \quad (+) \qquad\qquad \\
\hline
2x^2 + 6x + 2 \\
2x^2 + 6x + 2 \\
(-) \quad (-) \quad (-) \\
\hline
0 \\
\hline
\end{array}
$$

Here remainder =0

So, $x^2 + 3x + 1$ *is a factor of* $3x^4 + 5x^3 - 7x^2 + 2x + 2$

$(iii) x^3 - 3x + 1, x^5 - 4x^3 + x^2 + 3x + 1$

Sol:

$$
\begin{array}{r}
x^2 - 1 \\
x^3 - 3x + 1 \,\big)\, x^5 - 4x^3 + x^2 + 3x + 1 \\
x^5 - 3x^3 + x^2 \qquad\qquad \\
(-) \quad (+) \quad (-) \qquad\qquad \\
\hline
-x^3 \qquad + 3x + 1 \\
-x^3 \qquad + 3x - 1 \\
(+) \qquad (-) \quad (+) \\
\hline
2 \\
\hline
\end{array}
$$

Here remainder=2

$x^3 - 3x + 1$ *is not a factor of* $x^5 - 4x^3 + x^2 + 3x + 1$

3. Obtain all other zeroes of $3x^4 + 6x^3 - 2x^2 - 10x - 5$, if two of its zeroes are

$\sqrt{\frac{5}{3}}$ *and* $-\sqrt{\frac{5}{3}}$

Sol:

Given two zeroes are $\sqrt{\frac{5}{3}}$ *and* $-\sqrt{\frac{5}{3}}$

$i.e. x = \sqrt{\frac{5}{3}}$ and $x = -\sqrt{\frac{5}{3}}$

$x - \sqrt{\frac{5}{3}} = 0$ and $x + \sqrt{\frac{5}{3}} = 0 \Rightarrow \left(x - \sqrt{\frac{5}{3}}\right) \times \left(x + \sqrt{\frac{5}{3}}\right) = 0 \Rightarrow x^2 - \frac{5}{3} = 0 \Rightarrow 3x^2 - 5 = 0$

$3x^2 - 5$ is a factor of $3x^4 + 6x^3 - 2x^2 - 10x - 5$

$$
\begin{array}{r|l}
 & x^2 + 2x + 1 \\
3x^2 - 5 & 3x^4 + 6x^3 - 2x^2 - 10x - 5 \\
 & 3x^4 \qquad - 5x^2 \\
 & (-) \qquad (+) \\
\hline
 & 6x^3 + 3x^2 - 10x - 5 \\
 & 6x^3 \qquad - 10x \\
 & (-) \qquad (+) \\
\hline
 & 3x^2 \qquad - 5 \\
 & 3x^2 \qquad - 5 \\
 & (-) \qquad (+) \\
\hline
 & 0 \\
\hline
\end{array}
$$

$Quotient = x^2 + 2x + 1 = (x+1)(x+1)$

$For\ zeroes\ \ x + 1 = 0$ and $x + 1 = 0 \Rightarrow x = -1$ and $x = -1$

The other zeroes of given polynomial are $-1, -1$

4. On dividing $x^3 - 3x^2 + x + 2$ by a polynomial g(x), the quotient and remainder were x – 2 and – 2x + 4, respectively. Find g(x).

Sol: $Dividend = p(x) = x^3 - 3x^2 + x + 2$

$Quotien = q(x) = x - 2$

$Remainder = r(x) = -2x + 4$

$Divisor = g(x) = ?$

By division algorithm

p(x) = g(x) q(x) + r(x)

$x^3 - 3x^2 + x + 2 = g(x) \times (x - 2) + (-2x + 4)$

$x^3 - 3x^2 + x + 2 + 2x - 4 = g(x) \times (x - 2)$

$x^3 - 3x^2 + 3x - 2 = g(x) \times (x - 2)$

$g(x) = \frac{x^3 - 3x^2 + 3x - 2}{x - 2}$

$$
\begin{array}{r|l}
 & x^2 - x + 1 \\
x - 2 & x^3 - 3x^2 + 3x - 2 \\
 & x^3 - 2x^2 \\
 & (-) \quad (+) \\
\hline
 & -x^2 \\
 & -x^2 + 2x \\
 & (+) \quad (-) \\
\hline
 & x - 2 \\
 & x - 2 \\
 & (-) \quad (+) \\
\hline
 & 0 \\
\hline
\end{array}
$$

$g(x) = x^2 - x + 1$

5. Give examples of polynomials p(x), g(x), q(x) and r(x), which satisfy the division algorithm and

(i) deg p(x) = deg q(x)

Sol: $p(x) = 4x^2 + 8x + 4$

$q(x) = x^2 + 2x + 1$

deg p(x) = deg q(x)=2

$g(x) = 4 \quad and\ r(x) = 0$

(ii) deg q(x) = deg r(x)

Sol: $q(x) = x^2 + 2x + 1$

$r(x) = 2x^2 + 3$

$g(x) = x - 2$

$p(x) = (x - 2) \times (x^2 + 2x + 1) + 2x^2 + 3$

$= x^3 + 2x^2 + x - 2x^2 - 4x - 2 + 2x^2 + 3$

$= x^3 + 2x^2 - 3x + 1$

(iii) deg r(x) = 0

Sol: $p(x) = x^2 + 5x + 7$

$g(x) = x + 2$

$q(x) = x + 3$

$r(x) = 1$

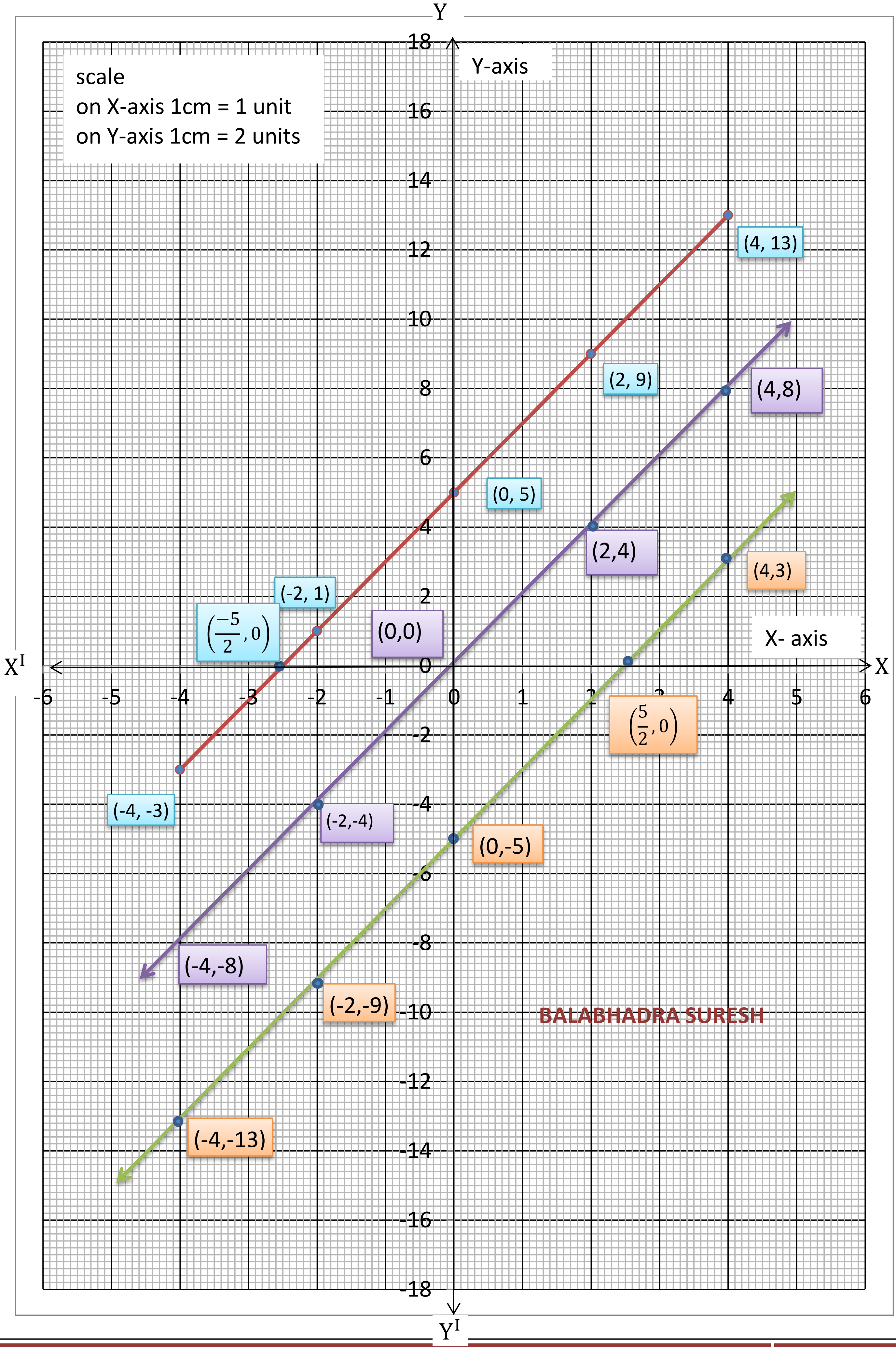
Deg r(x)=0

CHAPTER 3

POLYNOMIALS –GRAPHS

Prepared by : BALABHADRA SURESH, SA(MATHS),E.G.Dt

1. The graph of $y = ax + b(a \neq 0)$ is a straight line which intersects the X-axis at exactly one point ,namely $\left(\frac{-b}{a}, 0\right)$.
2. The linear polynomial $ax + b$ $(a \neq 0)$ has exactly one zero $= \frac{-b}{a}$ (the x-coordinate of the point where the graph of $y = ax + b$ intersects the X-axis).
3. The graph of $y = ax^2 + bx + c(a \neq 0)$ either opens upwards like $\cup$ (if $a > 0$) or opens downwards like $\cap$ (if $a < 0$). The shape of these curves are called **parabolas.**
4. The graph of $y = ax^2 + bx + c(a \neq 0)$ intersects X-axis at two points $(\alpha, 0)$ and $(\beta, 0)$ then $\boldsymbol{\alpha}, \boldsymbol{\beta}$ are the zeroes of the polynomial $ax^2 + bx + c$.
5. The graph of $y = ax^2 + bx + c(a \neq 0)$ touches X-axis one point at $(\alpha, 0)$ then $'\boldsymbol{\alpha}'$ is only one zero of the polynomial $ax^2 + bx + c$.
6. The graph of $y = ax^2 + bx + c(a \neq 0)$ does not intersects X-axis then the polynomial $ax^2 + bx + c$ has no zeroes.
7. Every linear polynomial have at most one zero.
8. Every quadratic polynomial have at most two zeroes.
9. Every cubic polynomial have at most three zeroes


scale
on X-axis 1cm = 1 unit
on Y-axis 1cm = 2 units
Y
Y-axis
X- axis
X
X^I
Y^I
(4, 13)
(2, 9)
(0, 5)
(-2, 1)
$\left(\frac{-5}{2},0\right)$
(-4, -3)
(4,8)
(2,4)
(0,0)
(-2,-4)
(-4,-8)
(4,3)
$\left(\frac{5}{2},0\right)$
(0,-5)
(-2,-9)
(-4,-13)
BALABHADRA SURESH

Do This

Draw the graph of (i) $y = 2x + 5$, (ii) $y = 2x - 5$, (iii) $y = 2x$ and find the point of intersection on X-axis. Is the x-coordinate of these points also the zeroes of the polynomial?

(i). $y = 2x + 5 = p(x)$

x	-4	-2	0	2	4
$2x$	-8	-4	0	4	8
5	5	5	5	5	5
$y = 2x + 5$	-3	1	5	9	13
(x, y)	$(-4, -3)$	$(-2,1)$	$(0,5)$	$(2,9)$	$(4,13)$

The graph of $y = 2x + 5$ intersects X- axis at $\left(\frac{-5}{2}, 0\right)$.

$x - cordinate\ of\ the\ point = \frac{-5}{2}$

$p\left(\frac{-5}{2}\right) = 2 \times \left(\frac{-5}{2}\right) + 5 = -5 + 5 = 0$

The zero of the polynomial $y = 2x + 5$ is $\frac{-5}{2}$.

(ii). $y = 2x - 5 = p(x)$

x	-4	-2	0	2	4
$2x$	-8	-4	0	4	8
-5	-5	-5	-5	-5	-5
$y = 2x + 5$	-13	-9	-5	-1	3
(x, y)	$(-4, -13)$	$(-2, -9)$	$(0, -5)$	$(2, -1)$	$(4,3)$

The graph of $y = 2x - 5$ intersects X- axis at $\left(\frac{5}{2}, 0\right)$. $x - cordinate\ of\ the\ point = \frac{5}{2}$

$p\left(\frac{5}{2}\right) = 2 \times \left(\frac{5}{2}\right) - 5 = 5 - 5 = 0$

The zero of the polynomial $y = 2x - 5$ is $\frac{5}{2}$.

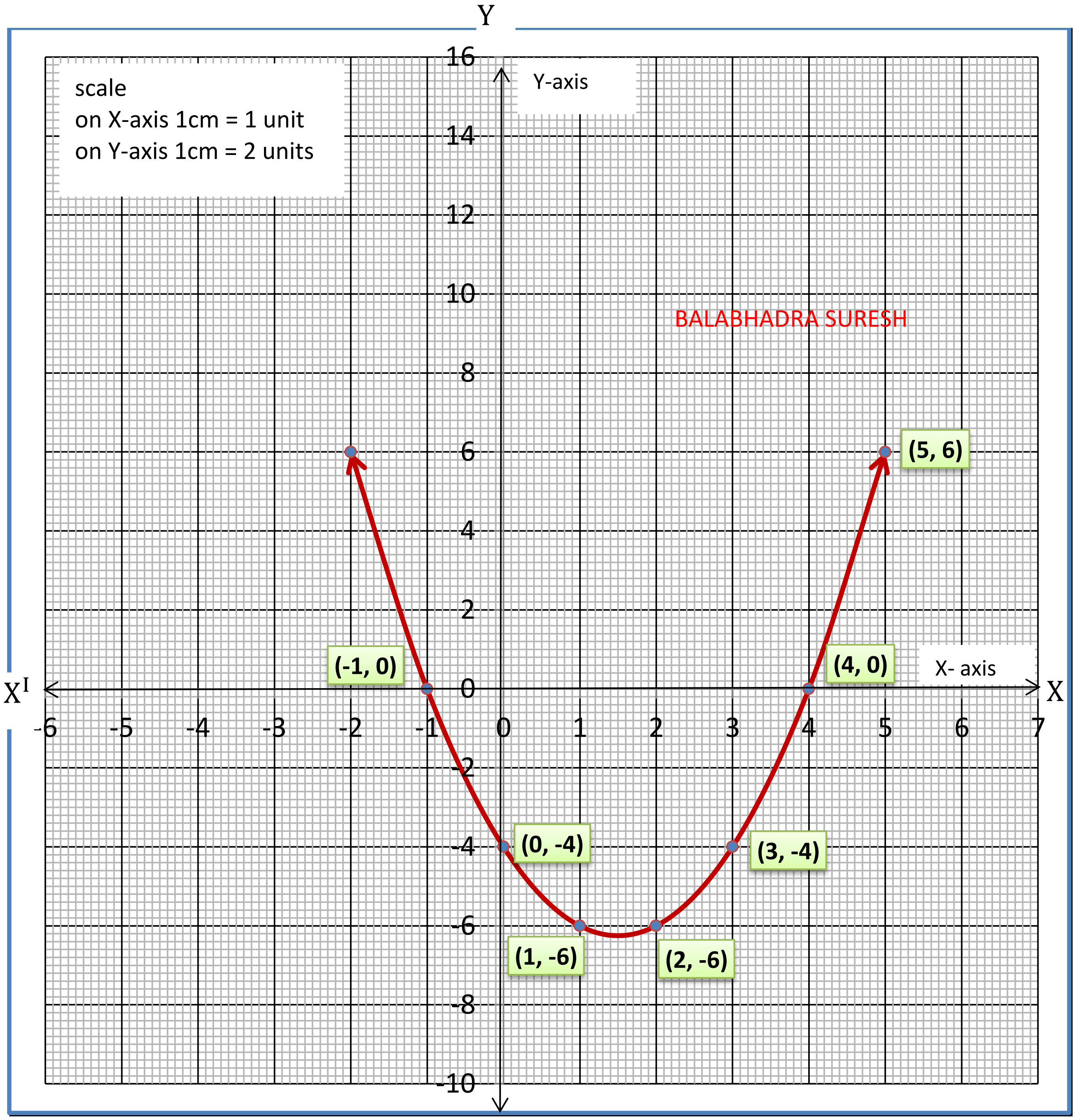
(iii). $y = 2x = p(x)$

x	-4	-2	0	2	4
$y = 2x$	-8	-4	0	4	8
(x, y)	$(-4, -8)$	$(-2, -4)$	$(0,0)$	$(2,4)$	$(4,8)$

The graph of $y = 2x$ intersects X- axis at $(0,0)$. $x - cordinate\ of\ the\ point = 0$

$p(0) = 2 \times (0) = 0$

The zero of the polynomial $y = 2x$ is 0


Y
scale
on X-axis 1cm = 1 unit
on Y-axis 1cm = 2 units
Y-axis
BALABHADRA SURESH
(5, 6)
(-1, 0)
(4, 0)
X- axis
X^I
X
(0, -4)
(3, -4)
(1, -6)
(2, -6)
16
14
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-2
-4
-6
-8
-10
-6
-5
-4
-3
-2
-1
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7

Example: Draw the graph of polynomial $P(\mathrm{x}) = x^2 - 3x - 4$ and find the zeroes . Justify the answers.

Sol: $P(\mathrm{x}) = x^2 - 3x - 4 = y$

x	-2	-1	0	1	2	3	4	5
x^2	4	1	0	1	4	9	16	25
$-3x$	6	3	0	-3	-6	-9	-12	-15
-4	-4	-4	-4	-4	-4	-4	-4	-4
$y = x^2 - 3x - 4$	-6	0	-4	-6	-6	-4	0	6
(x, y)	$(-2,6)$	$(-1,0)$	$(0,-4)$	$(1,-6)$	$(2,-6)$	$(3,-4)$	$(4,0)$	$(5,6)$

Graph of $y = x^2 - 3x - 4$ intersects X-axis at (-1,0) and (4,0).

Zeroes of the given polynomial $P(\mathrm{x}) = x^2 - 3x - 4$ are -1 *and* 4.

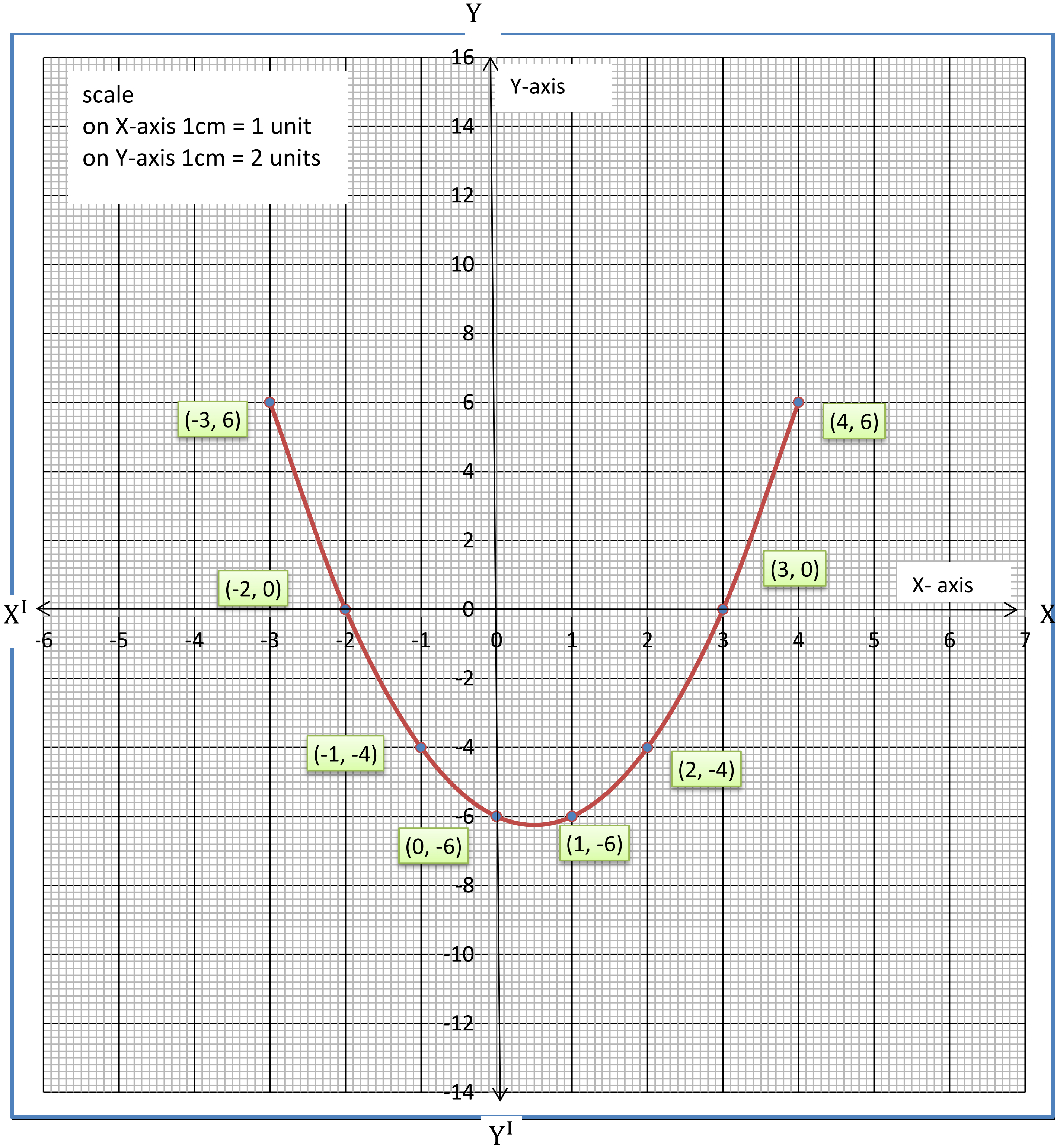
Justification: $P(\mathrm{x}) = x^2 - 3x - 4$

$$P(-1) = (-1)^2 - 3(-1) - 4$$
$$= 1 + 3 - 4$$
$$= 4 - 4$$
$$= 0$$

$$P(4) = 4^2 - 3 \times 4 - 4$$
$$= 16 - 12 - 4$$
$$= 16 - 16$$
$$= 0$$

$$P(-1) = 0 \;\; and \;\; P(4) = 0$$

$$\therefore \; -1 \; and \; 4 \; are \; the \; zeroes \; of \; the \; polynomial \; P(x) = x^2 - 3x - 4$$


Y
scale
on X-axis 1cm = 1 unit
on Y-axis 1cm = 2 units
Y-axis
X- axis
X
X^I
Y^I
(-3, 6)
(4, 6)
(-2, 0)
(3, 0)
(-1, -4)
(2, -4)
(0, -6)
(1, -6)
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-2
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-10
-12
-14
-6
-5
-4
-3
-2
-1
0
1
2
3
4
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7

TRY THIS:(**Page-53**):

(i) Draw the graph of $y = x^2 - x - 6$ and find zeroes in each case. What do you notice?

Sol: $p(\text{x}) = x^2 - x - 6 = y$

x	-3	-2	-1	-0	1	2	3	4
x^2	9	4	1	0	1	4	9	16
$-x$	3	2	1	0	-1	-2	-3	-4
-6	-6	-6	-6	-6	-6	-6	-6	-6
$y = x^2 - x - 6$	6	0	-4	-6	-6	-4	0	6
(x, y)	$(-3,6)$	$(-2,0)$	$(-1,-4)$	$(0,-6)$	$(1,-6)$	$(2,-4)$	$(3,0)$	$(4,6)$

Graph of $y = x^2 - x - 6$ intersects X-axis at (-2,0) and (3,0).

Zeroes of the polynomial $p(\text{x}) = x^2 - x - 6$ are -2 *and* 3

Justification: $P(\text{x}) = x^2 - x - 6$

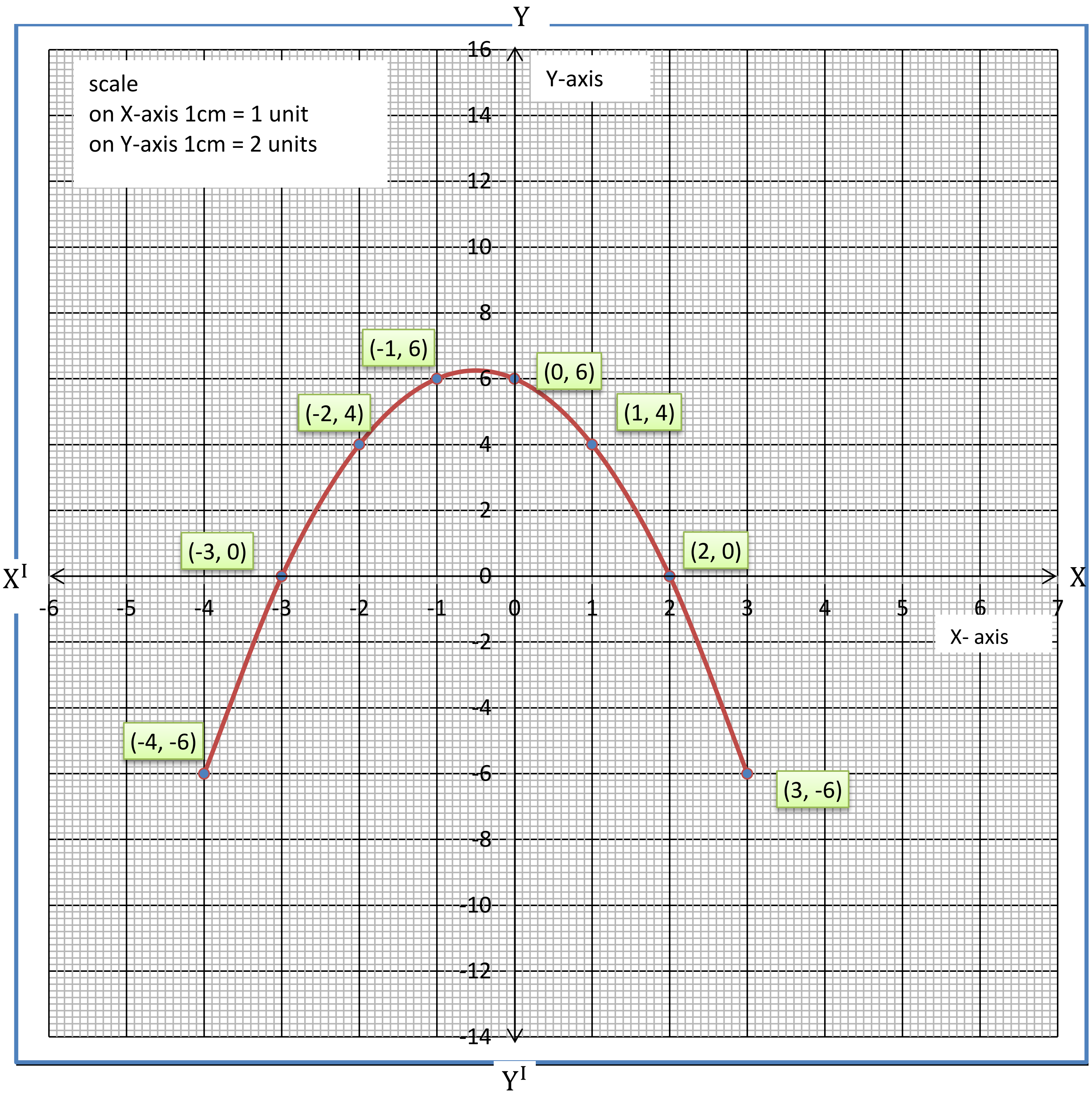
$$P(-2) = (-2)^2 - (-2) - 6 = 4 + 2 - 6 = 6 - 6 = 0$$

$$P(3) = 3^2 - 3 - 6 = 9 - 3 - 6 = 9 - 9 = 0$$

$$P(-2) = 0 \text{ and } P(3) = 0$$

$$\therefore -2 \text{ and } 3 \text{ are zeroes of polynomial } P(x) = x^2 - x - 6$$

We notice that in polynomial $p(x) = ax^2 + bx + c$ if $a > 0$ then the graph (parabola) opens upwards like $\cup$.



Y
scale
on X-axis 1cm = 1 unit
on Y-axis 1cm = 2 units
Y-axis
16
14
12
10
8
6
4
2
0
-2
-4
-6
-8
-10
-12
-14
(-1, 6)
(0, 6)
(-2, 4)
(1, 4)
(-3, 0)
(2, 0)
(-4, -6)
(3, -6)
X
-6
-5
-4
-3
-2
-1
0
1
2
3
4
5
6
7
X- axis

(ii) Draw the graph of $y = 6 - x - x^2$ and find zeroes in each case .What do you notice?

Sol: $p(\text{x}) = 6 - x - x^2 = -x^2 - x + 6 = y$

x	-4	-3	-2	-1	-0	1	2	3
$-x^2$	-16	-9	-4	-1	0	-1	-4	-9
$-x$	4	3	2	1	0	-1	-2	-3
6	6	6	6	6	6	6	6	6
$y = 6 - x - x^2$	-6	0	4	6	6	4	0	-6
(x, y)	$(-4, -6)$	$(-3,0)$	$(-2,4)$	$(-1,6)$	$(0,6)$	$(1,4)$	$(2,0)$	$(3, -6)$

Graph of $y = 6 - x - x^2$ intersects X-axis at (-3,0) and (2,0).

Zeroes of the polynomial $p(\text{x}) = 6 - x - x^2$ are -3 *and* 2

Justification: $P(\text{x}) = 6 - x - x^2$

$P(-3) = 6 - (-3) - (-3)^2$

$= 6 + 3 - 9$

$= 9 - 9$

$= 0$

$P(2) = 6 - 2 - 2^2$

$= 6 - 2 - 4$

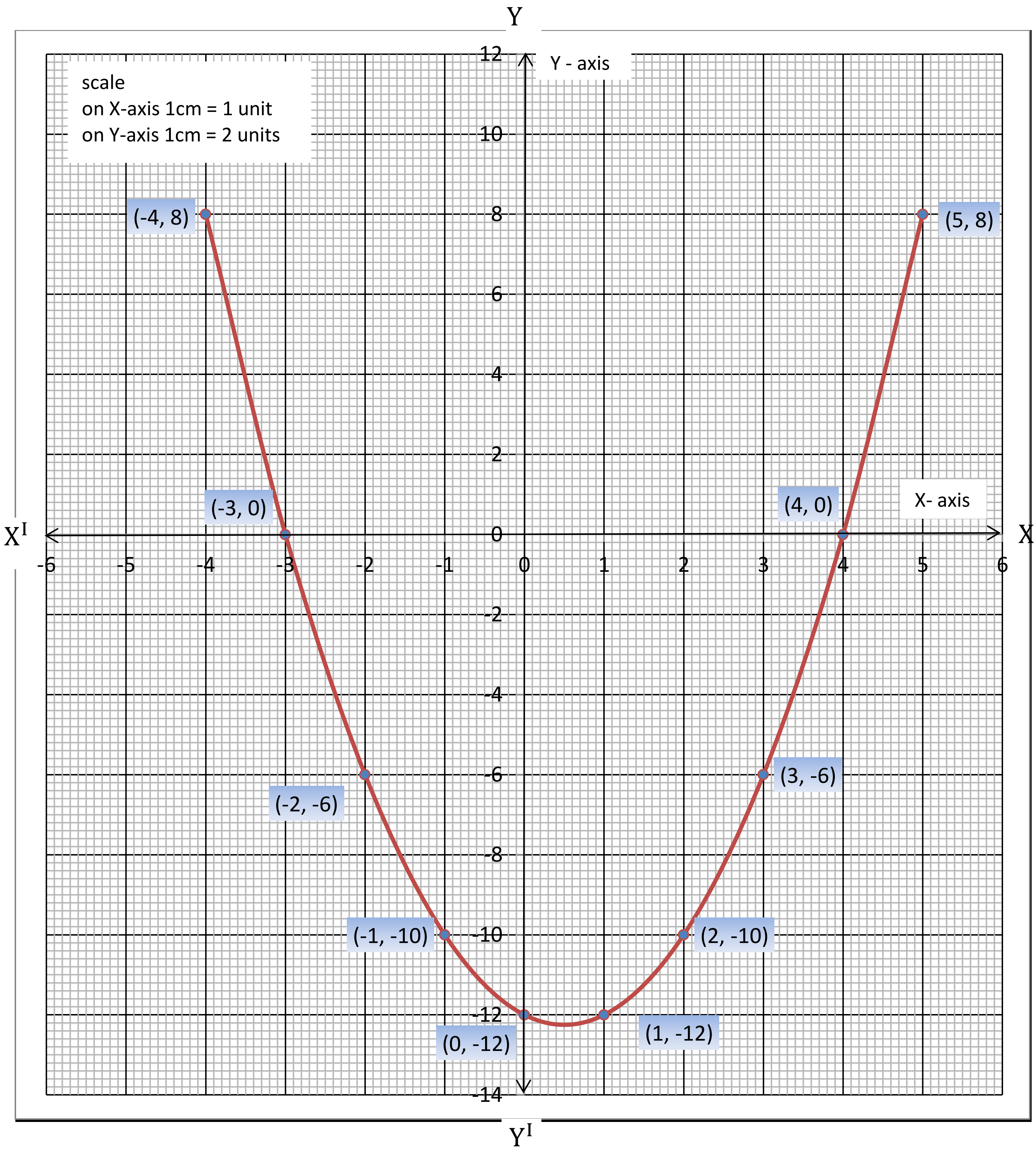
$= 6 - 6$

$= 0$

$P(-3) = 0 \ \ and \ P(2) = 0$

$\therefore \ -3 \ and \ 2 \ are \ the \ zeroes \ of \ the \ polynomial \ P(x) = 6 - x - x^2$

We notice that in polynomial $p(x) = ax^2 + bx + c$ if $a < 0$ then the graph (parabola) opens downwards like ∩.


Y
Y - axis
scale
on X-axis 1cm = 1 unit
on Y-axis 1cm = 2 units
(-4, 8)
(5, 8)
(-3, 0)
(4, 0)
X- axis
X^I
X
(-2, -6)
(3, -6)
(-1, -10)
(2, -10)
(0, -12)
(1, -12)
Y^I

EXERCISE – 3.2

3. (i) Draw the graph of polynomial $p(x) = x^2 - x - 12$and find the zeroes in each case . Justify the answers .

Sol: $P(x) = x^2 - x - 12 = y$

x	-4	-3	-2	-1	0	1	2	3	4
x^2	16	9	4	1	0	1	4	9	16
$-x$	4	3	2	1	0	-1	-2	-3	-4
-12	-12	-12	-12	-12	-12	-12	-12	-12	-12
$y = x^2 - x - 12$	0	0	-6	-10	-12	-12	-10	-6	0
(x, y)	(-4,8)	(-3,0)	(-2,-6)	(-1,-10)	(0,-12)	(1,-12)	(2,-10)	(3,-6)	(4,0)

Graph of $y = x^2 - x - 12$ intersects the X- axis at (-3,0) and (4,0).

The zeroes of the polynomial $P(x) = x^2 - x - 12$ are **-3** and **4.**

Justification:- $P(x) = x^2 - x - 12$

P(-3) $= (-3)^2 - (-3) - 12$

$= 9 + 3 - 12$

$= 12 - 12$

$= 0$

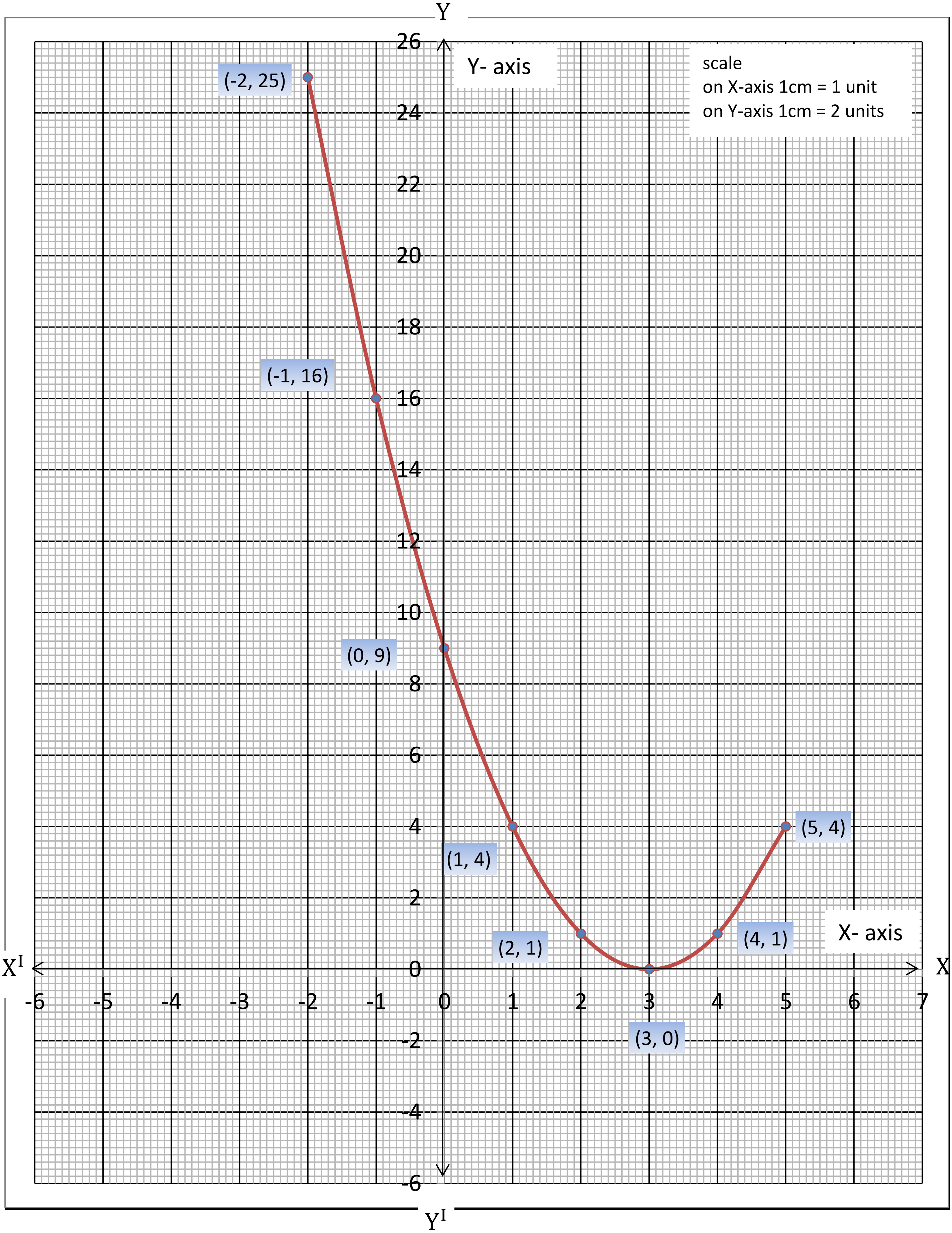
P(4) $= (4)^2 - 4 - 12$

$=16-16$

$= 0$

P(-3) =0 and P(4) = 0

∴ -3 and 4 are the zeroes of the polynomial $P(x) = x^2 - x - 12$



Y
Y- axis
scale
on X-axis 1cm = 1 unit
on Y-axis 1cm = 2 units
(-2, 25)
(-1, 16)
(0, 9)
(1, 4)
(2, 1)
(3, 0)
(4, 1)
(5, 4)
X- axis
X
X^I
Y^I
26
24
22
20
18
16
14
12
10
8
6
4
2
0
-2
-4
-6
-6 -5 -4 -3 -2 -1 0 1 2 3 4 5 6 7

(ii) Draw the graph of polynomial $p(x) = x^2 - 6x + 9$and find the zeroes in each case . Justify the answers .

Sol: $P(\mathrm{x}) = x^2 - 6x + 9 = y$

x	-2	-1	0	1	2	3	4	5
x^2	4	1	0	1	4	9	16	25
$-6x$	12	6	0	-6	-12	-18	-24	-30
9	9	9	9	9	9	9	9	9
$y = x^2 - 6x + 9$	25	16	9	4	1	0	1	4
(x, y)	(-2,25)	(-1,16)	(0,9)	(1,4)	(2,1)	(3,0)	(4,1)	(5,4)

Graph of $y = x^2 - 6x + 9$ touches X-axis at (3,0)

Zeroes of the polynomial $P(\mathrm{x}) = x^2 - 6x + 9$ are 3,3.

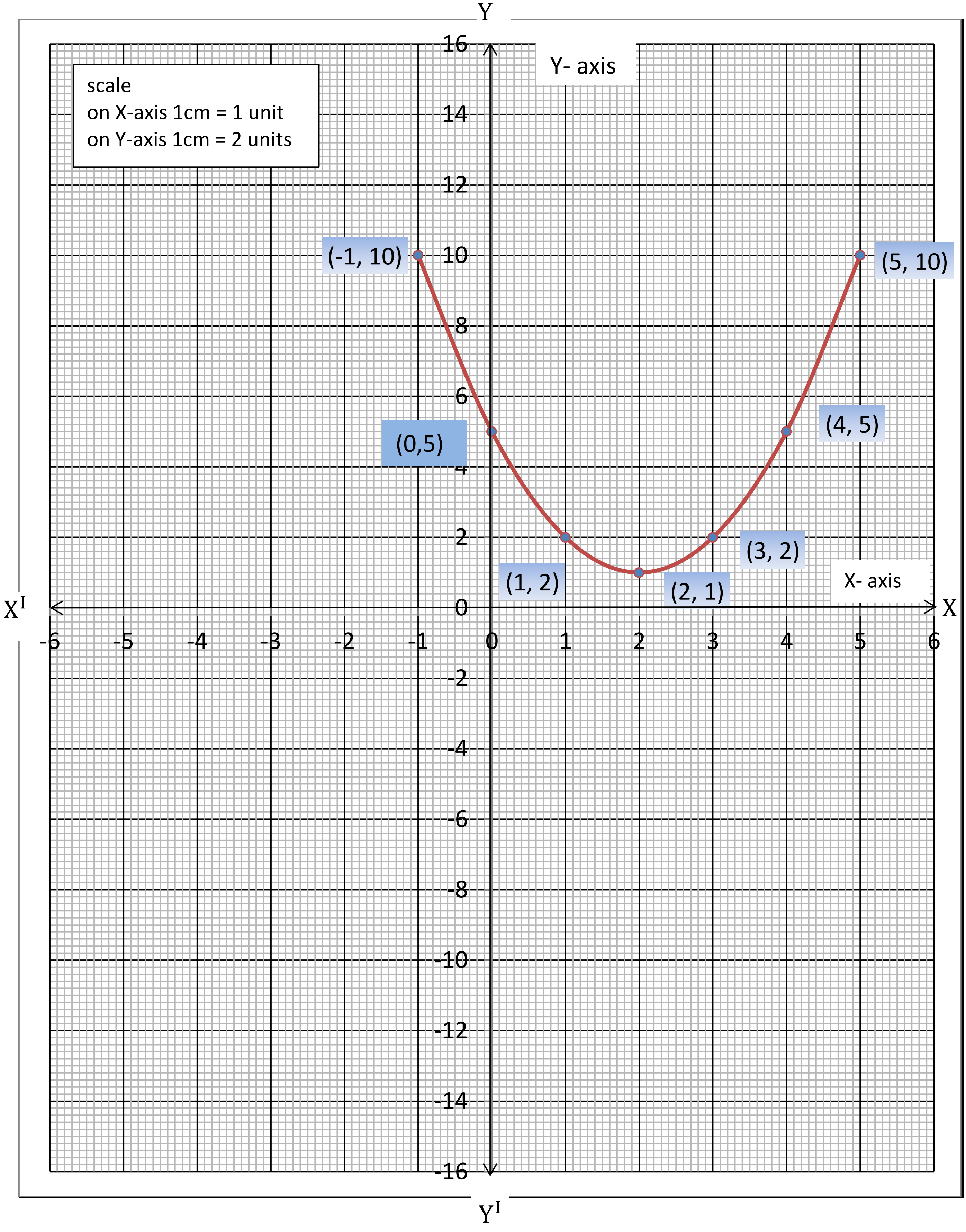
Justification: $P(\mathrm{x}) = x^2 - 6x + 9$

$$P(3) = 3^2 - 6 \times 3 + 9$$

$$= 9 - 18 + 9$$

$$= 18 - 18 = 0$$

$\therefore 3$ *is the zero of the polynomial* $P(x) = x^2 - 6x + 9$


Y
Y- axis
scale
on X-axis 1cm = 1 unit
on Y-axis 1cm = 2 units
(-1, 10)
(5, 10)
(0,5)
(4, 5)
(1, 2)
(3, 2)
(2, 1)
X- axis
X^{I}
X
Y^{I}

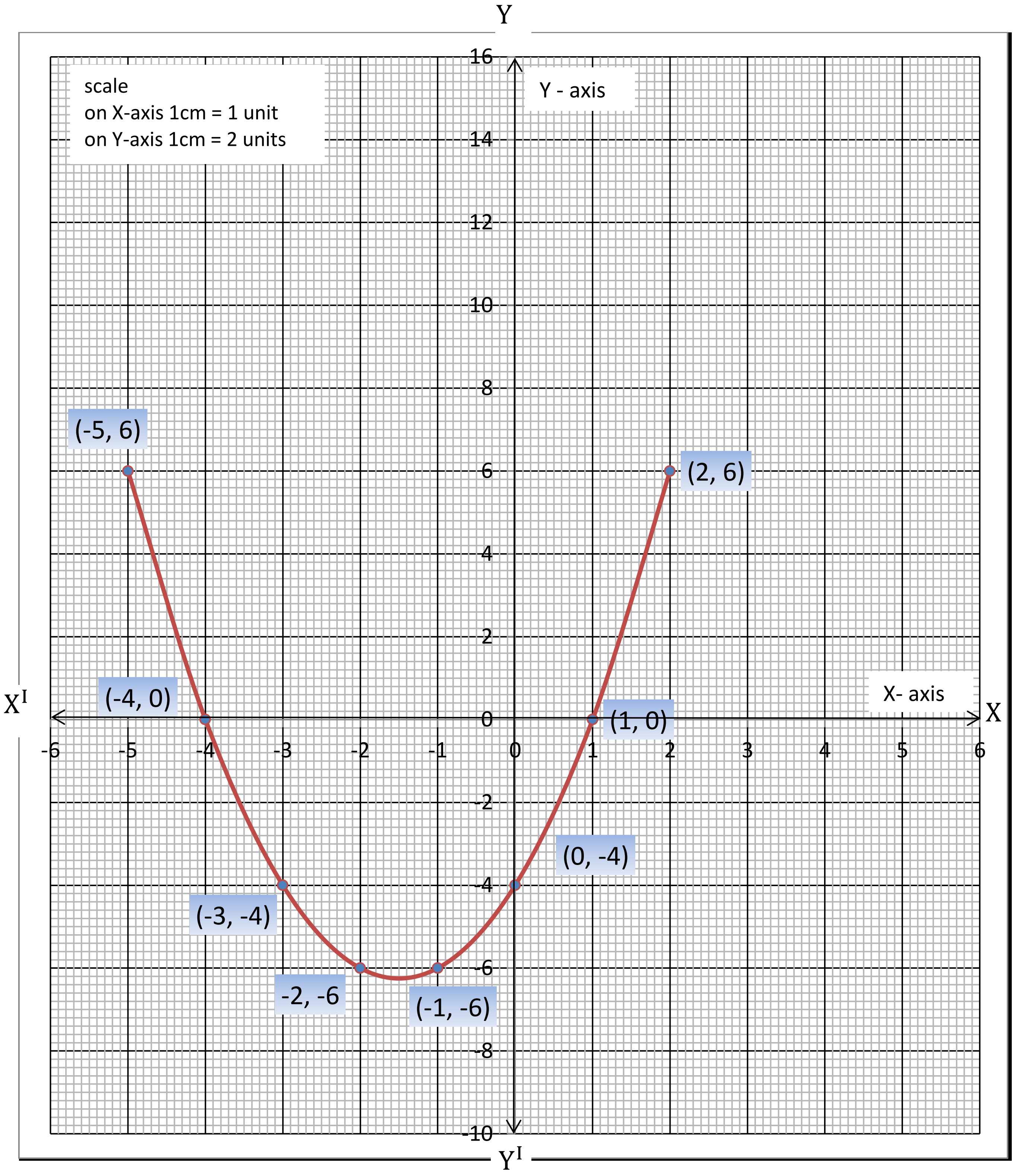
(iii) Draw the graph of polynomial $p(x) = x^2 - 4x + 5$and find the zeroes . Justify the answers

Sol: $\mathrm{P}(x) = x^2 - 4x + 5 = \mathrm{y}$

x	-2	-1	0	1	2	3	4
x^2	4	1	0	1	4	9	16
$-4x$	8	4	0	-4	-8	-12	-16
5	5	5	5	5	5	5	5
$y = x^2 - 4x + 5$	17	10	5	2	1	2	5
(x, y)	(-2,17)	(-1,10)	(0,5)	(1,2)	(2,1)	(3,3)	(4,5)

Graph of $y = x^2 - 4x + 5$ does not intersects the X- axis .

So there are no real zeroes for the given polynomial $\mathrm{P}(x) = x^2 - 4x + 5$.


Y
scale
on X-axis 1cm = 1 unit
on Y-axis 1cm = 2 units
Y - axis
16
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(-5, 6)
(2, 6)
(-4, 0)
(1, 0)
(0, -4)
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-2, -6
(-1, -6)
X- axis
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6

(iv) Draw the graph of polynomial $p(x) = x^2 + 3x - 4$ and find the zeroes . Justify the answers

Sol: $P(x) = x^2 + 3x - 4 = y$

x	-5	-4	-3	-2	-1	0	1	2
x^2	25	16	9	4	1	0	1	4
$3x$	-15	-12	-9	-6	-3	0	3	6
-4	-4	-4	-4	-4	-4	-4	-4	-4
$y = x^2 + 3x - 4$	6	0	-4	-6	-6	-4	0	6
(x, y)	(-5,6)	(-4,0)	(-3,-4)	(-2,-6)	(-1,-6)	(0,-4)	(1,0)	(2,6)

Graph of $y = x^2 + 3x - 4$ intersects the X- axis at (-4,0) and (1,0).
So the zeroes of the polynomial $P(x) = x^2 + 3x - 4$are **-4** and **1.**

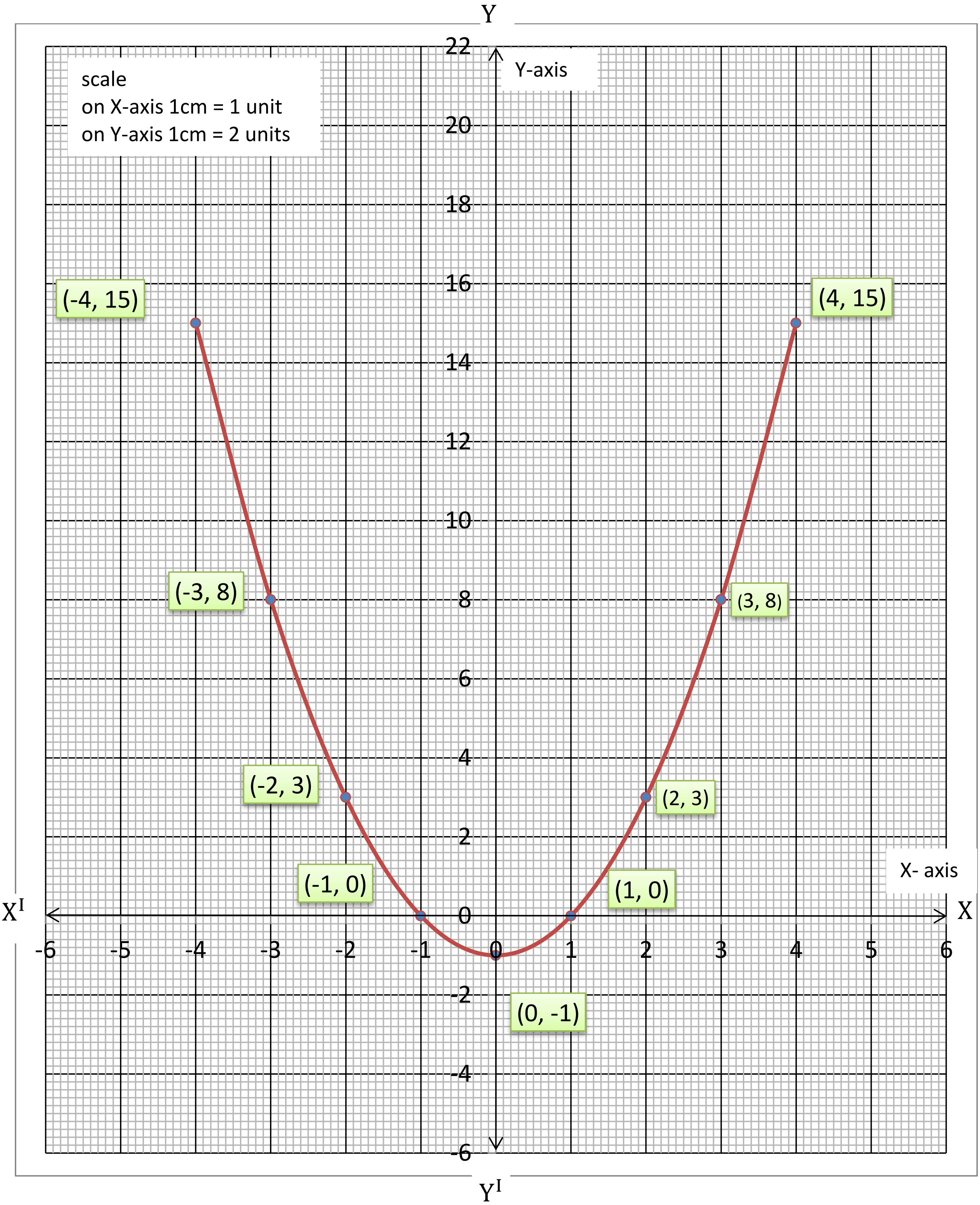
Justification:- $P(x) = x^2 + 3x - 4$

P(-4) = $(-4)^2 + 3(-4) - 4$
$= 16 - 12 - 4$
$=$ 16-16
$= 0$

P(1) = $(1)^2 + 3 \times 1 - 4$
$=1+3 - 4$
$= 4 - 4$
$= 0$

P(-4) =0 and P(1) = 0

$\therefore\ -4$ *and* 1 *are the zeroes of the polynomial* $P(x) = x^2 + 3x - 4$


Y
Y-axis
scale
on X-axis 1cm = 1 unit
on Y-axis 1cm = 2 units
22
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(-4, 15)
(4, 15)
(-3, 8)
(3, 8)
(-2, 3)
(2, 3)
(-1, 0)
(1, 0)
(0, -1)
X- axis
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X^I
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-1
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5
6
Y^I

(v) Draw the graph of polynomial $P(x) = x^2 - 1$ and find the zeroes . Justify the answers

Sol: $P(x) = x^2 - 1 = y$

x	-4	-3	-2	-1	0	1	2	3	4
x^2	16	9	4	1	0	1	4	9	16
-1	-1	-1	-1	-1	-1	-1	-1	-1	-1
$y = x^2 - 1$	15	8	3	0	-1	0	3	8	15
(x, y)	(-4,15)	(-3,8)	(-2,3)	(-1,0)	(0,-1)	(1,0)	(2,3)	(3,8)	(4,15)

The graph of $y = x^2 - 1$ intersects the X- axis at (-1,0) and (1,0).

So the zeroes of the polynomial $P(x) = x^2 - 1$are **-1** and **1.**

Justification: $P(x) = x^2 - 1$

$P(1) = 1^2 - 1$ $\quad$ $P(-1) = (-1)^2 - 1$

$= 1 - 1$ $\quad$ $= 1 - 1$

$= 0$ $\quad$ $= 0$

P(1) =0 and P(-1) = 0

$\therefore$ 1 and -1 are the zeroes of the polynomial $P(x) = x^2 - 1$

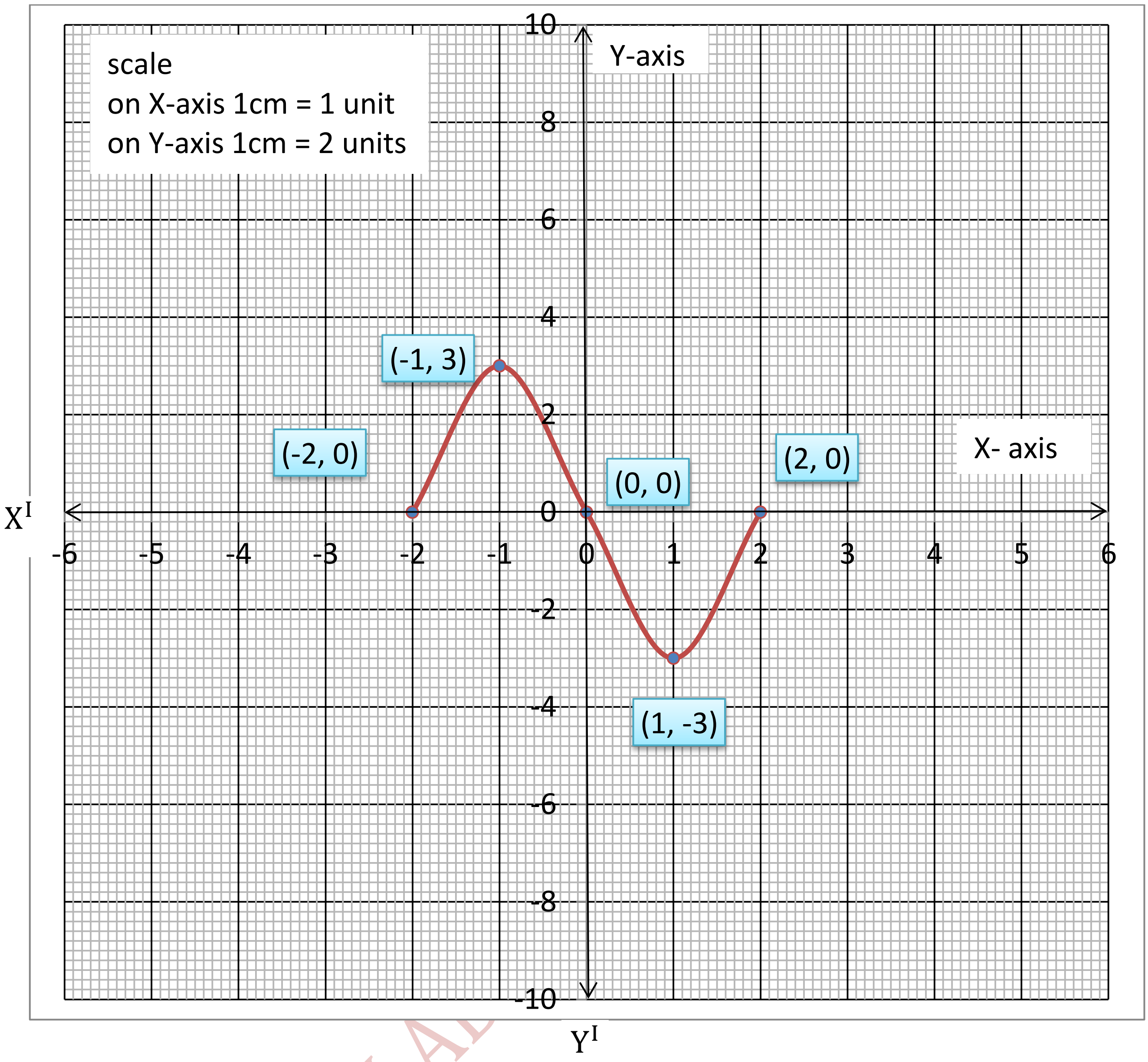
1. Draw the graph of cubic polynomial $p(x) = x^3 - 4x$ and find the zeroes of the polynomial.

Sol: $p(x) = x^3 - 4x = y$

x	-2	-1	0	1	2
x^3	-8	-1	0	1	8
$-4x$	8	4	0	-4	-8
$y = x^3 - 4x$	0	3	0	-3	0
(x, y)	$(-2,0)$	$(-1,3)$	$(0,0)$	$(1,-3)$	$(2,0)$

The graph of $y = x^3 - 4x$ intersects the x-axis at $(-2,0)$, $(0,0)$ and $(2,0)$
The zeroes of the cubic polynomial $p(x) = x^3 - 4x$ are $-2, 0$ *and* 2

Y



2. .Draw the graph of cubic polynomial $p(x) = x^3$ and find the zeroes of the polynomial.

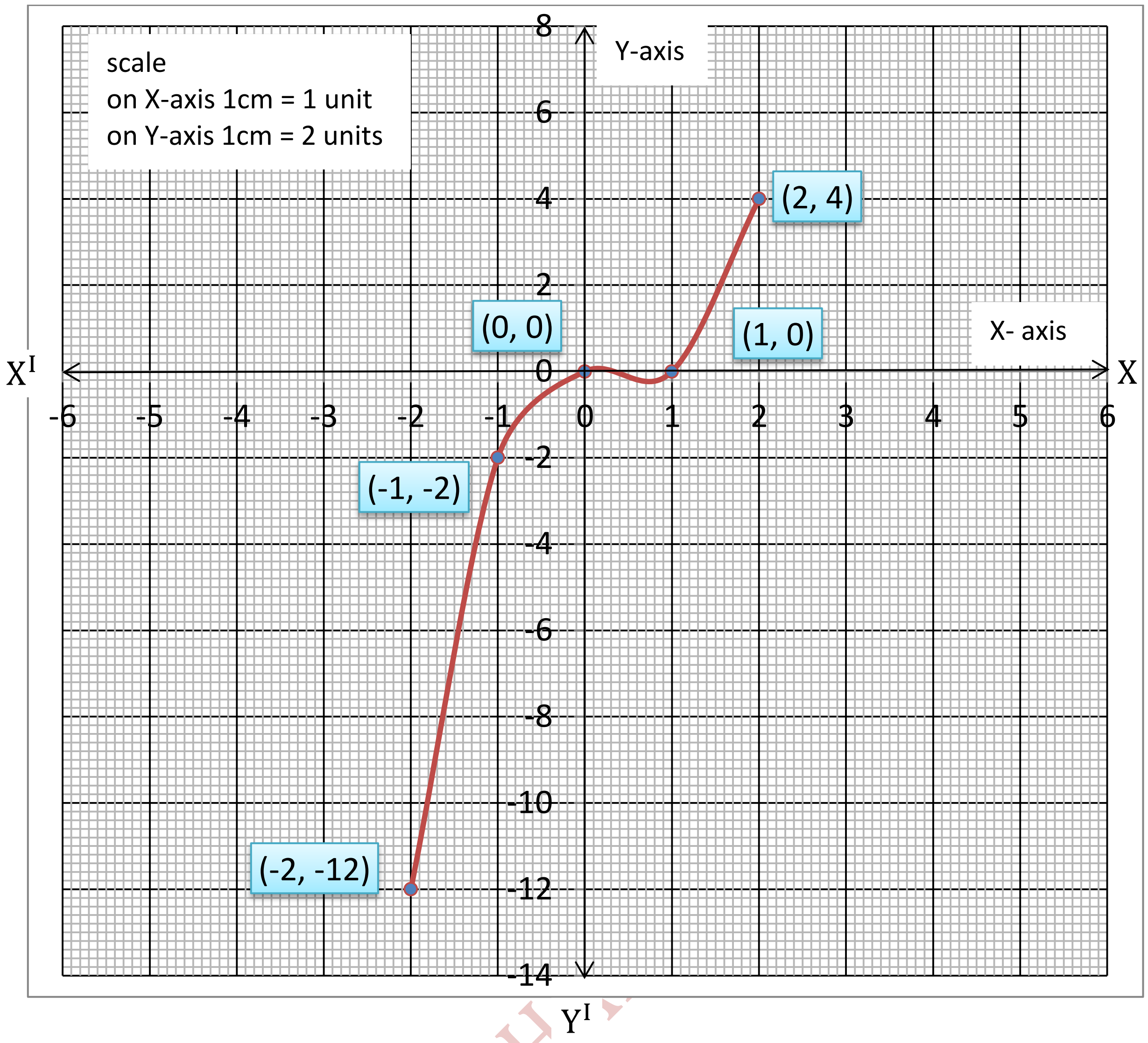
Sol: $p(x) = x^3 = y$

x	-2	-1	0	1	2
$y = x^3$	-8	-1	0	1	8
(x, y)	$(-2, -8)$	$(-1, -1)$	$(0,0)$	$(1,1)$	$(2,8)$

The graph of $y = x^3$ intersects the x-axis at (0,0)

The zeroes of the cubic polynomial $p(x) = x^3$ is 0 .

Y



3. Draw the graph of cubic polynomial $p(x) = x^3 - x^2$ and find the zeroes of the polynomial.

Sol: $p(x) = x^3 - 4x = y$

x	-2	-1	0	1	2
x^3	-8	-1	0	1	8
$-x^2$	-4	-1	0	-1	-4
$y = x^3 - x^2$	-12	-2	0	0	4
(x, y)	$(-2, -12)$	$(-1, -2)$	$(0,0)$	$(1,0)$	$(2,4)$

The graph of $y = x^3 - x^2$ intersects the x-axis at (0,0) and (1,0)

The zeroes of the cubic polynomial $p(x) = x^3 - 4x$ are 0 *and* 1.