CHAPTER 1

X CLASS-2023-24

1. Real Numbers

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Do This

Find q and r for the following pairs of positive integers a and b, satisfying $a = bq + r$.

(i) $a = 13,\quad b = 3$ (ii) $a = 8,\ b = 80$ (iii) $a = 125,\ b = 5$

(iv) $a = 132,\ b = 11$

(i) $13 = 3 \times 4 + 1$

$q = 4$ and $r = 0$

(ii) $8 = 80 \times 0 + 8$

$q = 0$ and $r = 8$

(iii) $125 = 5 \times 25 + 0$

$q = 25$ and $r = 0$

(iv) $132 = 11 \times 12 + 0$

$q = 12$ and $r = 0$

(Euclid's division lemma): Given positive integers a and b, there exist unique pair of integers q and r satisfying $a = bq + r, 0 \le r < b$

Do This

Find the HCF of the following by using Euclid division lemma.

(i) 50 and 70 (ii) 96 and 72 (iii) 300 and 550

(iv) 1860 and 2015

(i) $70 = 50 \times 1 + 20$
$50 = 20 \times 2 + 10$
$20 = 10 \times 2 + 0$

$HCF\ of\ 50\ and\ 70 = 10$

(ii) $96 = 72 \times 1 + 24$
$72 = 24 \times 3 + 0$

$HCF\ of\ 96\ and\ 72 = 24$

(iii) $550 = 300 \times 1 + 250$

$300 = 250 \times 1 + 50$
$250 = 50 \times 5 + 0$

$HCF\ of\ 300\ and\ 550 = 50$

(i) $2015 = 1860 \times 1 + 155$
$1860 = 155 \times 12 + 0$

$HCF\ of\ 1860\ and\ 2015 = 155$

Exercise - 1.1

1. Use Euclid's division algorithm to find the HCF of

(i) 900 and 270

Sol: $900 = 270 \times 3 + 90$

$270 = 90 \times 3 + 0$

$HCF\ of\ 900\ and\ 270\ \ = 90$

(ii) 196 and 38220

Sol: $38220 = 196 \times 195 + 0$

$HCF\ of\ 196\ and\ 38220\ \ = 195$

196) 38220 (195
38220
0

(iii) 1651 and 2032

$Sol: 2032 = 1651 \times 1 + 381$

$1651 = 381 \times 4 + 127$

$381 = 127 \times 3 + 0$

$HCF\ of\ 1651\ and\ 2032\ \ = 127$

2. **Use Euclid division lemma to show that any positive odd integer is of the form 6q + 1, or 6q + 3 or 6q + 5, where q is some integers.**

Sol : Let a be any positive integer and b=6

By Euclid division lemma $a = 6q + r,\ \ for\ some\ integer\ q \geq 0\ \ and\ \ 0 \leq r < 6$

So $r = 0$ or 1 or2 or 3 or 4 or 5

$a = 6q\ \ or\ \ (6q + 1)\ \ or\ \ (6q + 2)\ \ or\ \ (6q + 3)\ or\ \ (6q + 4)$ or $(6q + 5)$

$a = 6q = 2(3q) = 2m$

$\Longrightarrow 6q$ is an even integer

$a = 6q + 1 = 2(3q) + 1 = 2m + 1$

$\Rightarrow 6q + 1$ is an odd integer

$a = 6q + 2 = 2(3q + 1) = 2n$

$\Rightarrow 6q + 2$ is an even integer

$a = 6q + 3 = 6q + 2 + 1 = 2(3q + 1) + 1 = 2n + 1$

$\Rightarrow 6q + 3\ is\ an\ odd\ integer$

$a = 6q + 4 = 2(3q + 2) = 2z$

$\Rightarrow 6q + 4\ is\ an\ even\ integer$

$a = 6q + 5 = 6q + 4 + 1 = 2(3q + 2) + 1 = 2z + 1$

$\Rightarrow 6q + 5\ is\ an\ odd\ integer$

$\therefore The$ odd integer is of the form $6q + 1\ or\ 6q + 3 or\ 6q + 5$

3. Use Euclid's division lemma to show that the square of any positive integer is of the form 3p, 3p + 1

Sol : Let a be any positive integer and b=3

By Euclid division lemma $a = 6q + r,\ \ for\ some\ integer\ q \geq 0\ \ and\ \ 0 \leq r < 3$

So r = 0 or 1 or 2

$a = 3q$ or $3q + 1$ or $3q + 2$

$a = 3q$

$\Rightarrow a^2 = (3q)^2 = 9q^2 = 3(3q^2) = 3p$

$a = (3q + 1)$

$\Rightarrow a^2 = (3q + 1)^2 = 9q^2 + 6q + 1 = 3(3q^2 + 2q) + 1 = 3p + 1$

$a = (3q + 2)$

$\Rightarrow a^2 = (3q + 2)^2 = 9q^2 + 12q + 4 = 9q^2 + 12q + 3 + 1 = 3(3q^2 + 4q + 1) + 1 = 3p + 1$

$\therefore$ The square of any positive integer is of the form $3p$ or $3p + 1$.

4. **Use Euclid's division lemma to show that the cube of any positive integer is of the form . $9m$ or $9m + 1$ or $9m + 8$**

Sol : Let a be any positive integer and b=3

By Euclid division lemma $a = 3q + r$, *for some integer* $q \geq 0$ *and* $0 \leq r < 3$

So r=0 or 1 or 2

$a = 3q$ or $3q + 1$ or $3q + 2$

$a = 3q$

$\Rightarrow a^3 = (3q)^3 = 27q^3 = 9(3q^3) = 9\,m$

$a = (3q + 1)$

$\Rightarrow a^3 = (3q + 1)^3 = 27q^3 + 27q^2 + 9q + 1 = 9(3q^3 + 3q^2 + q) + 1 = 9m + 1$

$a = (3q + 2)$

$\Rightarrow a^3 = (3q + 2)^3 = 27q^3 + 54q^2 + 36q + 8 = 9(3q^3 + 3q^2 + 4q) + 8 = 9m + 8$

$\therefore$the cube of any positive integer is of the form . $9m$ or $9m + 1$ or $9m + 8$.

5. **Show that one and only one out of n, n + 2 or n + 4 is divisible by 3, where n is any positive integer.**

Sol: Let n be any positive integer and b=3

By Euclid division lemma $n = 6q + r$, *for some integer* $q \geq 0$ *and* $0 \leq r < 3$

So r=0 or 1 or 2

$n = 3q$ or $3q + 1$ or $3q + 2$

case 1: $n = 3q$ (it is divisible by 3)

$n + 2 = 3q + 2$(*it is not divisible by* 3)

$n + 4 = 3q + 4 = 3q + 3 + 1 = 3(q + 1) + 1 = 3m + 1$(*it is not divisible by* 3)

case 2: $n = 3q + 1$(*it is not divisible by* 3)

$n + 2 = 3q + 1 + 2 = 3q + 3 = 3(q + 1) = 3m$(*it is divisible by* 3)

$n + 4 = 3q + 1 + 4 = 3q + 5 = 3q + 3 + 2 = 3(q + 1) + 2$

$= 3m + 2 (it\ is\ not\ divisible\ by\ 3\ )$

case 3: $n = 3q + 2 (it\ is\ not\ divisible\ by\ 3\ )$

$n + 2 = 3q + 2 + 2 = 3q + 4 = 3q + 3 + 1 = 3(q + 1) + 1$

$= 3m + 1 (it\ is\ not\ divisible\ by\ 3\ )$

$n + 4 = 3q + 2 + 4 = 3q + 6 = 3(q + 2) = 3m (it\ is\ divisible\ by\ 3\ )$

$\therefore$ *one and only one out of* $n, n + 2$ *or* $n + 4$ *is divisible by* 3

Example 3. Consider the numbers of the form 4^n where n is a natural number. Check whether there is any value of n for which 4^n ends with zero?

Sol: If the prime factorisation of a number contains 2 and 5 then the number ends with the digit zero.

$4^n = (2^2)^n = 2^{2n}$

5 is not in prime factorisation of 4^n

So 4^n does not end with zero for any natural number n

Fundamental Theorem of Arithmetic : : Every composite number can be expressed (factorised) as a product of primes, and this factorization is unique, apart from the order in which the prime factors occur.

Prime numbers $= \{2,3,5,7,11,13,17,19,23,29,..\}$

Composite numbers $= \{4,6,8,9,10,12,14,15,16,18,20,21,22,24,25,26,27,28,30....\}$

'1' is neither a prime nor a composite

HCF (Highest Common Factor):

Product of the smallest power of each common prime factor of the numbers.

LCM(Lowest Common Multiple):

Product of the greatest power of each prime factor of the numbers.

Do This

Find the HCF and LCM of the following given pairs of numbers by prime factorisation method.

(i) 120, 90 (ii) 50, 60 (iii) 37, 49

(i) $120 = 2^3 \times 3^1 \times 5^1$

$90 = 2^1 \times 3^2 \times 5^1$

H. C. F of 120,90 $= 2^1 \times 3^1 \times 5^1 = 30$

L. C. M of 120,90 $= 2^3 \times 3^2 \times 5^1$

$= 8 \times 9 \times 5 = 360$

$$\begin{array}{r|l} 2 & 120 \\ \hline 2 & 60 \\ \hline 2 & 30 \\ \hline 3 & 15 \\ \hline & 5 \end{array} \qquad \begin{array}{r|l} 2 & 90 \\ \hline 3 & 45 \\ \hline 3 & 15 \\ \hline & 5 \end{array}$$

(ii) $50 = 2^1 \times 5^2$

$60 = 2^2 \times 3^1 \times 5^1$

H.C.F of 50,60 $= 2^1 \times 5^1 = 10$

L.C.M of 50,60 $= 2^2 \times 3^1 \times 5^2 = 4 \times 3 \times 25 = 300$

$$\begin{array}{r|l} 2 & 50 \\ \hline 5 & 25 \\ \hline & 5 \end{array} \qquad \begin{array}{r|l} 2 & 60 \\ \hline 2 & 30 \\ \hline 3 & 15 \\ \hline & 5 \end{array}$$

(iii) $37 = 37^1$

$49 = 7^2$

$L.C.M\ of\ 37,49 = 37^1 \times 7^2 = 37 \times 49 = 1813$

$H.C.F\ of\ 37,49 = 1\ (H.C.F\ of\ co-primes = 1)$

$$\begin{array}{r|l} 37 & 37 \\ \hline & 1 \end{array} \qquad \begin{array}{r|l} 7 & 49 \\ \hline & 7 \end{array}$$

TRY THIS

Show that $3^n \times 4^m$ cannot end with the digit 0 or 5 for any natural numbers 'n' and 'm'

Sol : If the prime factorisation of a number contain 2 and 5 then the number ends with the digit zero.

The prime factorisation of a number not contain 2 and contain 5 then the number ends with the digit 5.

$3^n \times 4^m = 3^m \times (2^2)^m = 3^m \times 2^{2m}$

The primefactorisation of $3^n \times 4^m$ does not contain 5 .

So $3^n \times 4^m$ cannot end with the digit 0 or 5 for any natural numbers 'n' and 'm' .

EXERCISE - 1.2

1. Express each of the following numbers as a product of its prime factors.

(i) 140 (ii) 156 (iii) 3825 (iv) 5005 (v) 7429

(i)

$$\begin{array}{r|l} 2 & 140 \\ \hline 2 & 70 \\ \hline 5 & 35 \\ \hline 7 & 7 \\ \hline & 1 \end{array}$$

$140 = 2 \times 2 \times 5 \times 7$

$140 = 2^2 \times 5 \times 7$

(ii)

$$\begin{array}{r|l} 2 & 156 \\ \hline 2 & 78 \\ \hline 3 & 39 \\ \hline & 13 \end{array}$$

$156 = 2 \times 2 \times 3 \times 13$

$156 = 2^2 \times 3 \times 13$

(iii)

$$\begin{array}{r|l} 3 & 3825 \\ \hline 3 & 1275 \\ \hline 5 & 425 \\ \hline 5 & 85 \\ \hline 17 & 17 \\ \hline & 1 \end{array}$$

$3825 = 3 \times 3 \times 5 \times 5 \times 17$

$3825 = 3^2 \times 5^2 \times 17$

(iv)

$$\begin{array}{r|l} 5 & 5005 \\ \hline 7 & 1001 \\ \hline 11 & 143 \\ \hline & 13 \end{array}$$

$5005 = 5 \times 7 \times 11 \times 13$

(v)

$$\begin{array}{r|l} 17 & 7429 \\ \hline 19 & 437 \\ \hline & 23 \end{array}$$

$7429 = 17 \times 19 \times 23$

2. Find the LCM and HCF of the following integers by the prime factorization method.

(i) $12,15\ and\ \ 21$

$$\begin{array}{r|l} 2 & 12 \\ \hline 2 & 6 \\ \hline 3 & 3 \\ \hline & 1 \end{array} \qquad \begin{array}{r|l} 3 & 15 \\ \hline 5 & 5 \\ \hline & 1 \end{array} \qquad \begin{array}{r|l} 3 & 21 \\ \hline 7 & 7 \\ \hline & 1 \end{array}$$

$12 = 2^2 \times 3^1$

$15 = 3^1 \times 5^1$

$21 = 3^1 \times 7^1$

$\text{L. C. M of } 12{,}15 \text{ and } 21 = 2^2 \times 3^1 \times 5^1 \times 7^1 = 4 \times 3 \times 5 \times 7 = 420$

$\text{H. C. F of } \textit{12,15 and 21} = 3^1 = 3$

(ii) 17,23 and 29

$17 = 17^1$

$23 = 23^1$

$29 = 29^1$

$L.C.M\ of$ 17,23 and $29 = 17 \times 23 \times 29 = 11339$

$H.C.F\ of$ 17,23 and $29 = 1\ (H.C.F\ of\ co - primes = 1)$

(iv) 8,9 and 25

$$\begin{array}{r|l} 2 & 8 \\ \hline 2 & 4 \\ \hline 2 & 2 \\ \hline & 1 \end{array} \qquad \begin{array}{r|l} 3 & 9 \\ \hline 3 & 3 \\ \hline & 1 \end{array} \qquad \begin{array}{r|l} 5 & 25 \\ \hline 5 & 5 \\ \hline & 1 \end{array}$$

$8 = 2^3$

$9 = 3^2$

$25 = 5^2$

$L.C.M\ of$ 8,9 and $25 = 2^3 \times 3^2 \times 5^2 = 8 \times 9 \times 25 = 1800$

$H.C.F\ of$ 8,9 and $25 = 1\ (H.C.F\ of\ co - primes = 1)$

(v) 72 and 108

$$\begin{array}{r|l} 2 & 72 \\ \hline 2 & 36 \\ \hline 2 & 18 \\ \hline 3 & 9 \\ \hline & 3 \end{array} \qquad \begin{array}{r|l} 2 & 108 \\ \hline 2 & 54 \\ \hline 3 & 27 \\ \hline 3 & 9 \\ \hline & 3 \end{array}$$

$L.C.M\ of$ 72 and $108 = 2^3 \times 3^3 = 8 \times 27 = 216$

$H.C.F\ of$ 72 and $108 = 2^2 \times 3^2 = 4 \times 9 = 36$

(vi) 306 and 657

$$\begin{array}{r|l} 2 & 306 \\ \hline 3 & 153 \\ \hline 3 & 51 \\ \hline 17 & 17 \\ \hline & 1 \end{array} \qquad \begin{array}{r|l} 3 & 657 \\ \hline 3 & 219 \\ \hline 73 & 73 \\ \hline & 1 \end{array}$$

$306 = 2 \times 3^2 \times 17^1$

$657 = 3^2 \times 73$

$L.C.M\ of$ 306 and 657 $= 2 \times 3^2 \times 17^1 \times 73 = 22338$

$H.C.F\ of$ 306 and 657 $= 3^2 = 9.$

3. Check whether 6^n can end with the digit 0 for any natural number n.

Sol : The prime factorisation of a number contain 2 and 5 then the number ends with the digit zero.

$6^n = (2 \times 3)^n = 2^n \times 3^n$

$since\ 5\ is\ not\ present\ in\ prime\ factorisation\ of\ 6^n.$

So 6^n cannot end with the digit zero for any natural number n.

4. Explain why $7 \times 11 \times 13 + 13$ and $7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 + 5$ are composite numbers.

Sol : $7 \times 11 \times 13 + 13$

= 13x (7x11 + 1)

= 13x (77 +1)

= 13x 78 =2x3x13^2 .

2,3 and 13 are the factors of 7x11x13+13 .So 7x11x13+13 is a composite number.

$7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 + 5$

$=5 \times (7 \times 6 \times 4 \times 3 \times 2 \times 1 + 1)$

$=5 \times (1008 + 1)$

$=5 \times 1009$

$5, 1009 are\ factories\ of\ 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 + 5.$

So $7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 + 5 is\ a\ composite\ number$

5. How will you show that $(17 \times 11 \times 2) + (17 \times 11 \times 5)$ is a composite number? Explain.

సాధన : $(17 \times 11 \times 2) + (17 \times 11 \times 5)$

$= 17 \times 11 \times (2 + 5)$

$= 17 \times 11 \times 7$

$(17 \times 11 \times 2) + (17 \times 11 \times 5)$ *canbe written as product of primes .*

$\therefore (17 \times 11 \times 2) + (17 \times 11 \times 5)$ *is a composite number.*

6. What is the last digit of 6^{100}?

సాధన : $6^1 = 6$ For every natural number n , last digit of 6^n = 6

$6^2 = 36$ *So, the last digit of* $6^{100} = 6.$

$6^3 = 216$

1.2.1 RATIONAL NUMBERS AND THEIR DECIMAL EXPANSIONS

DO THIS

Write the following terminating decimals in the form of $\frac{p}{q}$, $q \neq 0$ and p, q are co-primes

(i) 15.265   (ii) 0.1255   (iii) 0.4   (iv) 23.34   (v) 1215.8

What can you conclude about the denominators through this process?

(i). $15.265 = \frac{15265}{1000} = \frac{3053}{200} = \frac{3053}{2^3 \times 5^2}$

(ii). $0.1255 = \frac{1255}{10000} = \frac{251}{2000} = \frac{251}{2^4 \times 5^3}$

(iii). $0.4 = \frac{4}{10} = \frac{2}{5}$

(iv). $23.34 = \frac{2334}{100} = \frac{1167}{50} = \frac{1167}{2^1 \times 5^2}$

(v). $1215.8 = \frac{12158}{10} = \frac{6079}{5}$

We conclude that the denominators are a power of 2 *or* 5 *or both*. ie the denominators are in the form of $2^n \times 5^m$ (n,m are whole numbers)

Theorem-1.3 : Let x be a rational number whose decimal expansion terminates. Then x can be expressed in the form of $\frac{p}{q}$, where p and q are coprime, and the prime factorization of q is of the form $2^n \times 5^m$, where n, m are non-negative integers.

Theorem 1.4 : Let $x = \frac{p}{q}$ be a rational number, such that the prime factorization of q is of the form $2^n \times 5^m$, where n *and* m are non-negative integers. Then x has a decimal expansion which terminates.

Do This

Write the following rational numbers in the form of $\frac{p}{q}$, where q is of the form 2^n5^m where n and m are non-negative integers and then write the numbers in their decimal form

(i) $\frac{3}{4}$ (ii) $\frac{7}{25}$ (iii) $\frac{51}{64}$ (iv) $\frac{14}{25}$ (v) $\frac{80}{100}$

(i). $\frac{3}{4} = \frac{3}{2^2 \times 5^0} = 0.75$

(ii). $\frac{7}{25} = \frac{7}{2^0 \times 5^2} = 0.28$

(iii). $\frac{51}{64} = \frac{51}{2^6 \times 5^0} = 0.796875$

(iv). $\frac{14}{25} = \frac{14}{2^0 \times 5^2} = 0.56$

(v). $\frac{80}{100} = \frac{4}{5} = \frac{4}{2^0 \times 5^1} = 0.8$

Do This

Write the following rational numbers as decimal form and find out the block of repeating digits in the quotient.

(i) $\frac{1}{3}$ (ii) $\frac{2}{7}$ (iii) $\frac{5}{11}$ (iv) $\frac{10}{13}$

(i). $\frac{1}{3} = 0.333\ldots\ldots = 0.\overline{3}$

The block of digit '3'is repeating in the quotient.

(ii). $\frac{2}{7} = 0.285714285\ldots\ldots = 0.\overline{285714}$

The block of digits '285714' is repeating in the quotient.

(iii). $\frac{5}{11} = 0.4545\ldots. = 0.\overline{45}$

The block of digits '45' is repeating in the quotient.

(iv). $\frac{10}{13} = 0.7692307\ldots = 0.\overline{769230}$

The block of digits '769230' is repeating in the quotient.

Theorem-1.5 : Let $x = \frac{p}{q}$ be a rational number, such that the prime factorization of q is not of the form $2^n \times 5^m$, where n and m are non-negative integers. Then, x has a decimal expansion which is non-terminating repeating (recurring).

Example-5. without actual division, state whether the following rational numbers are terminating or non-terminating, repeating decimals.

(i). $\frac{16}{125} = \frac{16}{5^3}$ is terminating decimal.

(ii). $\frac{25}{32} = \frac{25}{2^5}$ is terminating decimal.

(iii). $\frac{100}{81} = \frac{100}{3^4}$ is non-terminating, repeating decimal.

(iv). $\frac{41}{75} = \frac{41}{3^1 \times 5^2}$ is non-terminating, repeating decimal.

Example-6. Write the decimal expansion of the following rational numbers without actual division.

(i). $\frac{35}{50} = \frac{7}{10} = 0.7$

(ii). $\frac{21}{25} = \frac{21}{5^2} = \frac{21\times 2^2}{5^2\times 2^2} = \frac{21\times 4}{(5\times 2)^2} = \frac{84}{100} = 0.84$

(iii). $\frac{7}{8} = \frac{7}{2^3} = \frac{7\times 5^3}{2^3\times 5^3} = \frac{7\times 125}{(2\times 5)^3} = \frac{875}{1000} = 0.875$

EXERCISE - 1.3

1. **Write the following rational numbers in their decimal form and also state which are terminating and which are non-terminating, repeating decimal.**

(i). $\frac{3}{8} = \frac{3}{2^3} = \frac{3\times 5^3}{2^3\times 5^3} = \frac{3\times 125}{(2\times 5)^3} = \frac{375}{1000} = 0.375$ is terminating decimal.

(ii). $\frac{229}{400} = \frac{229}{2^4\times 5^2} = \frac{229\times 5^2}{2^4\times 5^2\times 5^2} = \frac{229\times 25}{(2\times 5)^4} = \frac{5725}{10000} = 0.5725$ is terminating decimal

(iii). $4\frac{1}{5} = \frac{21}{5} = \frac{21\times 2}{5\times 2} = \frac{42}{10} = 4.2$ is terminating decimal.

(iv). $\frac{2}{11} = 0.1818\ldots = 0.\overline{18}$ is non-terminating, repeating decimal

(v). $\frac{8}{125} = \frac{8}{5^3} = \frac{8\times 2^3}{5^3\times 2^3} = \frac{8\times 8}{(5\times 2)^3} = \frac{64}{1000} = 0.064$ is terminating decimal

2. **Without performing division, state whether the following rational numbers will have a terminating decimal form or a non-terminating, repeating decimal form .**

(i) $\frac{13}{3125} = \frac{13}{5^5}$ is terminating decimal.

(ii) $\frac{11}{12} = \frac{11}{2^2\times 3}$ is non-terminating, repeating decimal

(iii) $\frac{64}{455} = \frac{64}{5\times 7\times 13}$ is non-terminating, repeating decimal

(iv) $\frac{15}{600} = \frac{1}{40} = \frac{1}{2^3\times 5}$ is terminating decimal

(v) $\frac{29}{343} = \frac{29}{7^3}$ is non-terminating, repeating decimal

(vi) $\frac{23}{2^3\times 5^2}$ is terminating decimal

(vii) $\frac{129}{2^2 . 5^7 . 7^5}$ $is\ non-terminating, repeating\ decimal$

(viii) $\frac{9}{15} = \frac{3}{5}$ is terminating decimal

(ix) $\frac{36}{100} = \frac{9}{25} = \frac{9}{5^2}$ is terminating decimal

(x) $\frac{77}{210} = \frac{11}{30} = \frac{11}{2 \times 3 \times 5}$

3. Write the following rationales in decimal form using Theorem 1.2.

(i). $\frac{13}{25} = \frac{13}{5^2} = \frac{13 \times 2^2}{5^2 \times 2^2} = \frac{13 \times 4}{(5 \times 2)^2} = \frac{52}{100} = 0.52$

(ii). $\frac{15}{16} = \frac{15}{2^4} = \frac{15 \times 5^4}{2^4 \times 5^4} = \frac{15 \times 625}{(2 \times 5)^4} = \frac{9375}{10000} = 0.9375$

(iii). $\frac{23}{2^3 \times 5^2} = \frac{23 \times 5}{2^3 \times 5^3} = \frac{115}{(2 \times 5)^3} = \frac{115}{1000} = 0.115$

(iv). $\frac{7218}{3^2 \times 5^2} = \frac{7218}{9 \times 5^2} = \frac{802}{5^2} = \frac{802 \times 2^2}{5^2 \times 2^2} = \frac{3208}{(5 \times 2)^2} = \frac{3208}{100} = 32.08$

(v). $\frac{143}{110} = \frac{143}{2 \times 5 \times 11} = \frac{13}{2 \times 5} = \frac{13}{10} = 1.3$

4. The decimal form of some real numbers are given below. In each case, decide whether the number is rational or not. If it is rational, and expressed in form p q , what can you say about the prime factors of q?

(i) 43.123456789 - it is rational

Sol: $43.123456789 = \frac{43123456789}{1000000000} = \frac{43123456789}{2^9 \times 5^9}$

the prime factors of q is in the form of $2^n \times 5^m$

(ii) 0.120120012000120000... - it is not rational.

(iii) $43.\overline{123456789}$ - it is rational.

Sol: $43.\overline{123456789} = \frac{43123456789-43}{999999999} = \frac{43123456746}{3^2 \times 41 \times 271}$.

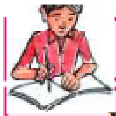
The prime factors of q is not in the form of $2^n \times 5^m$

1.3 Irrational numbers

A real number is called **irrational** ("Q^1" or "S") if it cannot be written in the form $\frac{p}{q}$, where p and q are integers and q≠ 0

Examples: $\sqrt{2}, \sqrt{3}, \sqrt{5}$, 0.1231452148 ……,0.1011011101111 ……

Theorem-1.6 : Let p be a prime number. If p divides a^2, (where a is a positive integer), then p divides a.



Do This

Verify the statement proved above for p= 2, p = 5 and for a^2= 1, 4, 9, 25, 36, 49, 64 and 81.

	a^2	a	p $divides$ a^2	p $divides$ a
For p=2	1	1	No	No
	4	2	Yes	Yes
	9	3	No	No
	16	4	Yes	Yes
	25	5	No	No
	36	6	Yes	Yes
	49	7	No	No
	64	8	Yes	Yes
	81	9	No	No

	a^2	a	p $divides$ a^2	p $divides$ a
For p=5	1	1	No	No
	4	2	No	No
	9	3	No	No
	16	4	No	No
	25	5	Yes	Yes
	36	6	No	No
	49	7	No	No
	64	8	No	No
	81	9	No	No

Example 7. Show that $\sqrt{2}$ is irrational

Solution: Let us assume $\sqrt{2}$ is rational.

Then $\sqrt{2} = \frac{a}{b}$ $(\,a, b\ are\ coprimes)$

Squaring on both sides we get

$$2 = \frac{a^2}{b^2} \Rightarrow 2b^2 = a^2 \rightarrow (1)$$

$$\Rightarrow\ b^2 = \frac{a^2}{2}$$

$$\Rightarrow 2\ divides\ a^2$$

p *be a prime number* .

If p *divides* a^2 *then* p *divides* a

$\Rightarrow$ 2 *divides* a

We can write $a = 2c$ *for some integer* c

$\Rightarrow a^2 = 4c^2$

$\Rightarrow 2b^2 = 4c^2$ (from (1))

$\Rightarrow b^2 = 2c^2$

$\Rightarrow c^2 = \frac{b^2}{2}$

$\Rightarrow$ 2 *divides* b^2

$\Rightarrow$ 2 *divides* b

Therefore , both a *and* b have 2 as a common factor.

But this contradicts the fact that a *and* b are co-prime.

Thus our assumption is false.

So,we conclude that $\sqrt{2}$ is irrational.

Example-8. Show that 5 $-\sqrt{3}$ is irrational.

Solution: : Let us assume that $5 - \sqrt{3}$ is rational .

$5 - \sqrt{3} = \frac{a}{b}$ (a, b *are coprimes*)

$5 - \frac{a}{b} = \sqrt{3}$

$\sqrt{3} = 5 - \frac{a}{b} = \frac{5b - a}{b}$ ……………(1)

Since 5, a *and* b *are integers the R.H.S of* (1) *i.e* $\frac{5b-a}{b}$ *is rational*

so the L.H.S $= \sqrt{3}$ *also rational*

But this contradicts the fact that $\sqrt{3}$ is irretional.

Thus our assumption is false.

So,we conclude that $5 - \sqrt{3}$ is irrational .

Example-9. Show that $3\sqrt{2}$ is irrational.

Solution: Let us assume that $\mathbf{3\sqrt{2}}$ is rational.

Let $3\sqrt{2} = \frac{a}{b}$ (a, b *are coprimes*)

$\sqrt{2} = \frac{a}{3b}$ ……………(1)

Since 3, a *and* b *are integers the R.H.S of* (1)*ie* $\frac{a}{3b}$ *is rational*

$so\ the\ L.H.S\ \sqrt{2}\ also\ rational$

But this contradicts the fact that $\sqrt{2}$ is irretional.

Thus our assumption is false.

So,we conclude that $3\sqrt{2}$ is irrational .

Example-10. Prove that $\sqrt{2}+\sqrt{3}$ is irrational.

Solution: Let us assume that $\sqrt{2}+\sqrt{3}$ is rational.

Let $\sqrt{2}+\sqrt{3} \quad = \frac{a}{b} \quad (\ a, b\ are\ coprimes)$

$$\sqrt{2} \quad = \frac{a}{b} - \sqrt{3}$$

Squaring on both sides, we get

$$(\sqrt{\mathbf{2}})^{\mathbf{2}} \quad = \left(\frac{a}{b} - \sqrt{\mathbf{3}}\right)^2$$

$$2 = \left(\frac{a}{b}\right)^{\mathbf{2}} + (\sqrt{3})^{\mathbf{2}} - 2 \times \frac{a}{b} \times \sqrt{3}$$

$$2 = \frac{a^2}{b^2} + 3 - \frac{2a}{b}\sqrt{3}$$

$$\frac{2a}{b}\sqrt{3} = \frac{a^2}{b^2} + 3 - 2 = \frac{a^2}{b^2} + 1 = \frac{a^2+b^2}{b^2}$$

$$\sqrt{3} = \frac{a^2+b^2}{b^2} \times \frac{b}{2a}$$

$$\sqrt{3} = \frac{a^2+b^2}{2ab}$$

Since $2, a\ and\ b$ are integers, $\frac{a^2+b^2}{2ab}$ is rational ,and so $\sqrt{3}$ is rational .

But this contradicts the fact that $\sqrt{3}$ is irretional.

Thus our assumption is false.

So,we conclude that $\sqrt{2}+\sqrt{3}$ is irrational .

EXERCISE - 1.4

1. Prove that the following are irrational.

(i) $\frac{1}{\sqrt{2}}$

Sol: $Let\ us\ assume\ that\ \frac{1}{\sqrt{2}}\ is\ rational.$

Let $\frac{1}{\sqrt{2}} = \frac{a}{b}$ $(a, b\ are\ coprimes)$

$\sqrt{2} = \frac{b}{a}$

$Since\ a\ and\ b\ are\ integers, \frac{b}{a}\ \ is\ rational\ , and\ so\ \sqrt{2}\ \ is\ rational.$

But this contradicts the fact that $\sqrt{2}$ is irretional.

Thus our assumption is false.

So,we conclude that $\frac{1}{\sqrt{2}}$ is irrational .

(ii) $\boldsymbol{\sqrt{3} + \sqrt{5}}$

Sol: Let us assume that $\sqrt{3} + \sqrt{5}$ is rational.

Let $\sqrt{3} + \sqrt{5} = \frac{a}{b}$ $(a, b\ are\ coprimes)$

$\sqrt{5} = \frac{a}{b} - \sqrt{3}$

Squaring on both sides, we get

$(\sqrt{\mathbf{3}})^{\mathbf{2}} = \left(\frac{a}{b} - \sqrt{\mathbf{5}}\right)^2$

$3 = \left(\frac{a}{b}\right)^{\mathbf{2}} + (\sqrt{5})^{\mathbf{2}} - 2 \times \frac{a}{b} \times \sqrt{5}$

$3 = \frac{a^2}{b^2} + 5 - \frac{2a}{b}\sqrt{5}$

$\frac{2a}{b}\sqrt{5} = \frac{a^2}{b^2} + 5 - 3 = \frac{a^2}{b^2} + 2 = \frac{a^2 + 2b^2}{b^2}$

$\sqrt{5} = \frac{a^2 + 2b^2}{b^2} \times \frac{b}{2a}$

$\sqrt{5} = \frac{a^2 + 2b^2}{2ab}$

Since $2, a\ and\ b$ are integers, $\frac{a^2+2b^2}{2ab}$ is rational ,and so $\sqrt{5}$ is rational .

But this contradicts the fact that $\sqrt{5}$ is irretional.

Thus our assumption is false.

So,we conclude that $\sqrt{3} + \sqrt{5}$ is irrational .

(OR)

Let us assume that $\sqrt{3}+\sqrt{5}$ is rational.

Let $\sqrt{3}+\sqrt{5} \quad = \frac{a}{b}$ $\;(a, b \text{ are coprimes})$

Squaring on both sides, we get

$$\left(\sqrt{3}+\sqrt{5}\right)^{\mathbf{2}} \quad = \left(\frac{a}{b}\right)^2$$

$$\left(\sqrt{3}\right)^2+\left(\sqrt{5}\right)^2+2\times\sqrt{3}\times\sqrt{\mathbf{5}}=\frac{a^2}{b^2}$$

$$3+5+2\sqrt{15}=\frac{a^2}{b^2}$$

$$2\sqrt{15}=\frac{a^2}{b^2}-8=\frac{a^2-8b^2}{b^2}$$

$$\sqrt{15}=\frac{a^2-8b^2}{2b^2}$$

Since 2,8, a and b are integers, $\frac{a^2-8b^2}{2b^2}$ is rational ,and so $\sqrt{15}$ is rational .

But this contradicts the fact that $\sqrt{15}$ is irretional.

Thus our assumption is false.

So,we conclude that $\sqrt{3}+\sqrt{5}$ is irrational .

(iii) $\mathbf{6+\sqrt{2}.}$

Sol: Let us assume that $6+\sqrt{2}$ is rational .

$$6+\sqrt{2}=\frac{a}{b} \;\; (a, b \text{ are coprimes})$$

$$\sqrt{2}=\frac{a}{b}-6=\frac{a-6b}{b}$$

Since 6, a and b are integers, $\frac{a-6b}{b}$ is rational ,and so $\sqrt{2}$ is rational .

But this contradicts the fact that $\sqrt{2}$ is irretional.

Thus our assumption is false.

So,we conclude that $6+\sqrt{2}$ is irrational .

(iv) $\mathbf{\sqrt{5}}$

Sol: Let us assume $\sqrt{5}$ is rational.

Then $\sqrt{5}=\frac{a}{b}$ $\;(a, b \text{ are coprimes})$

Squaring on both sides we get

$$5 = \frac{a^2}{b^2} \Rightarrow 5b^2 = a^2 \rightarrow (1)$$

$p\ be\ a\ prime\ number$.
$If\ p\ divides\ a^2\ \ then\ p\ divides\ a$

$\Rightarrow\ b^2 = \frac{a^2}{5}$

$\Rightarrow 5\ divides\ a^2$

$\Rightarrow\ 5\ divides\ a$

We can write $a = 5c\ \ for\ some\ integer\ c$

$\Rightarrow a^2 = 25c^2$

$\Rightarrow\ 5b^2 = 25c^2$ (from (1))

$\Rightarrow\ b^2 = 5c^2$

$\Rightarrow\ c^2 = \frac{b^2}{5}$

$\Rightarrow 5\ divides\ b^2$

$\Rightarrow\ 5\ divides\ b$

Therefore , both $a\ and\ b$ have 5 as a common factor.

But this contradicts the fact that $a\ and\ b$ are co-prime.

Thus our assumption is false.

So,we conclude that $\sqrt{5}$ is irrational.

(v) $3 + 2\sqrt{5}$

Sol: Let us assume that $3 + 2\sqrt{5}\ is\ rational$.

$$3 + 2\sqrt{5} = \frac{a}{b}\ \ (\ a, b\ are\ coprimes)$$

$$2\sqrt{5} = \frac{a}{b} - 3 = \frac{a - 3b}{b}$$

$$\sqrt{5} = \frac{a-3b}{2b}$$

Since 2,3, $a\ and\ b$ are integers, $\frac{a-3b}{2b}$ is rational ,and so $\sqrt{5}$ is rational .

But this contradicts the fact that $\sqrt{5}$ is irretional.

Thus our assumption is false.

So,we conclude that $3 + 2\sqrt{5}\ \ is\ irrational$.

2. Prove that $\sqrt{p} + \sqrt{q}$ is irrational, where p, q are primes.

Solution: Let us assume that $\sqrt{p}+\sqrt{q}$ is rational.

Let $\sqrt{p}+\sqrt{q} \quad = \frac{a}{b}$ $(\, a, b \; are \; coprimes)$

$$\sqrt{p} \quad = \frac{a}{b} - \sqrt{q}$$

Squaring on both sides, we get

$$\left(\sqrt{p}\right)^{2} \quad = \left(\frac{a}{b} - \sqrt{q}\right)^{2}$$

$$p = \left(\frac{a}{b}\right)^{2} + \left(\sqrt{q}\right)^{2} - 2 \times \frac{a}{b} \times \sqrt{q}$$

$$p = \frac{a^2}{b^2} + q - \frac{2a}{b}\sqrt{q}$$

$$\frac{2a}{b}\sqrt{q} = \frac{a^2}{b^2} + q - p = \frac{a^2 + (q-p)b^2}{b^2}$$

$$\sqrt{q} = \frac{a^2 + (q-p)b^2}{b^2} \times \frac{b}{2a}$$

$$\sqrt{q} = \frac{a^2 + (q-p)b^2}{2ab}$$

Since $2, p, q, a \; and \; b$ are integers, $\frac{a^2+(q-p)b^2}{2ab}$ is rational ,and so $\sqrt{q}$ is rational .

But this contradicts the fact that $\sqrt{q}$ is irretional.(since q is prime so $\sqrt{q}$ is irretional)

Thus our assumption is false.

So,we conclude that $\sqrt{p}+\sqrt{q}$ is irrational .

CHAPTER – 1

(PART -2)

Real Numbers - LOGARITHMS

Prepared by : BALABHADRA SURESH , SA(MATHS).

1. Logarithms are introduced by John Napier.
2. We define $\log_a x = \text{n}$, if $a^n = \text{x}$ $where\ a\ and\ x$ are positive numbers and $a \neq 1$.
3. If $a^x = \text{N}$ then $x = \log_a N$ $(a > 0\ a \neq 1, N > 0\ a, N \in R)$
4. If $5 = 10^x$ then $x = \log_{10} 5$ $(\ logarithm\ of\ 5\ to\ the\ base\ 10)$.

   In this $5 = 10^x$ is called Exponential form and $x = \log_{10} 5$ is called logarithmic form.

Exponential form	logarithmic form
i) $3^5 = 243$	$\log_3 243 = 5$
ii) $64 = 2^x$	$\log_2 64 = x$
iii) $2^7 = 128$	$\log_2 128 = 7$

5. Common logarithm: Logarithm of a number to the base 10 as common logarithm.
6. Laws of logarithms $(a, x, y, \text{N}$ are positive real numbers $a \neq 1)$

i) $\log_a xy = \log_a x + \log_a y$

ii) $\log_a \frac{x}{y} = log_a x - log_a y$

iii) $\log_a x^m = m \times log_a x$

iv) $\log_{a^n} x = \frac{1}{n} \times log_a x$

v) $\log_{a^n} x^m = \frac{m}{n} \times log_a x$

vi) $a^{\log_a x} = x$

vii) $\log_a a = 1$

viii) $\log_a 1 = 0$

Do this

Write the powers to which the bases to be raised in the following

(i) $64 = 2^x$ (ii) $100 = 5^b$ (iii) $\frac{1}{81} = 3^c$ (iv) $100 = 10^z$ (v) $\frac{1}{256} = 4^a$

(i) $64 = 2^x$ $2^6 = 2^x$ $\therefore x = 6$	(ii) $100 = 5^b$ There is no b	(iii) $\frac{1}{81} = 3^c$ $3^{-4} = 3^c$ $\therefore c = -4$	(iv) $100 = 10^z$ $10^2 = 10^z$ $\therefore z = 2$	(v) $\frac{1}{256} = 4^a$ $4^{-4} = 4^a$ $\therefore a = -4$

Try This

Write the following relation in expontial form and find the values of respective variables

(i) $\log_2 32 = x$ (ii) $\log_5 625 = y$ (iii) $\log_{10} 10000 = z$ (iv) $\log_7 \frac{1}{343} = -a$

(i) $\log_2 32 = x$

Exponential form is $2^x = 32$

$\Rightarrow \; 2^x = 2^5$

$\Rightarrow \; x = 5$

$$\begin{array}{r|l} 2 & 32 \\ \hline 2 & 16 \\ \hline 2 & 8 \\ \hline 2 & 4 \\ \hline & 2 \end{array}$$

(ii) $\log_5 625 = y$

Exponential form is $5^y = 625$

$\Rightarrow \; 5^y = 5^4$

$\Rightarrow \; x = 5$

$$\begin{array}{r|l} 5 & 625 \\ \hline 5 & 125 \\ \hline 5 & 25 \\ \hline & 5 \end{array}$$

(iii) $\log_{10} 10000 = z$

Exponential form is $10^z = 10000$

$\Rightarrow \; 10^z = 10^4$

$\Rightarrow \; z = 10$

(iv) $\log_7 \frac{1}{343} = -a$

Exponential form is $7^{-a} = \frac{1}{343}$

$\Rightarrow 7^{-a} = \frac{1}{7^3}$

$$\begin{array}{r|l} 7 & 343 \\ \hline 7 & 49 \\ \hline & 7 \end{array}$$

$\Rightarrow \; 7^{-a} = 7^{-3} \Rightarrow a = 3$

Do This

Express the logarithms of the following as the sum of the logarithm

(i) 35×46 (ii) 235×437 (iii) 2437×3568

$$\log_a xy = \log_a x + \log_a y$$

(i) $\log_{10}(35 \times 46) = \log_{10} 35 + \log_{10} 46$

(ii) $\log_{10}(235 \times 437) = \log_{10} 235 + \log_{10} 437$

(iii) $\log_{10}(2437 \times 3568) = \log_{10} 2437 + \log_{10} 3568$

Do This

Express the logarithms of the following as the difference of logarithms

(i) $\frac{23}{34}$ (ii) $\frac{373}{275}$ (iii) $4325 \div 3734$ (iv) $5055 \div 3303$

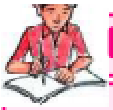
$$\log_a \frac{x}{y} = log_a\, x - log_a\, y$$

(i) $\log_{10} \frac{23}{34} = \log_{10} 23 - \log_{10} 34$

(ii) $\log_{10} \frac{373}{275} = \log_{10} 373 - \log_{10} 275$

(iii) $\log_{10}(4325 \div 3734) = \log_{10} \frac{4325}{3734} = \log_{10} 4325 - \log_{10} 3734$

(iv) $\log_{10}(5055 \div 3303) = \log_{10} \frac{5055}{3303} = \log_{10} 5505 - \log_{10} 3303$



Do THIS

By using the formula $\log_a x^n = n\log_a x$, Convert the following

(i) $\log_2 7^{25}$ (ii) $\log_5 8^{50}$ (iii) $\log 5^{23}$ (iv) $\log 1024$

(i) $\log_2 7^{25} = 25 \times \log_2 7$

(ii) $\log_5 8^{50} = 50 \times \log_5 8$

(iii) $\log 5^{23} = 23 \times \log 5$

(iv) $\log 1024 = \log 2^{10} = 10 \times \log 2$

Try This

(i) Find the value $\log_2 32$ (ii) Find the value of $\log_c \sqrt{c}$

(iii) Find the value $\log_{10} 0.001$ (iv) Find the value of $\log_{\frac{2}{3}} \frac{8}{27}$

(i) $\log_2 32 = \log_2 2^5 = 5 \times \log_2 2 = 5 \times 1 = 5$

(ii) $\log_c \sqrt{c} = \log_c c^{\frac{1}{2}} = \frac{1}{2} \times \log_c c = \frac{1}{2} \times 1 = \frac{1}{2}$

(iii) $\log_{10} 0.001 = \log_{10} \frac{1}{1000} = \log_{10} \frac{1}{10^3} = \log_{10} 10^{-3} = -3 \times \log_{10} 10 = -3 \times 1 = -3$

(iv) $\log_{\frac{2}{3}} \frac{8}{27} = \log_{\frac{2}{3}} \left(\frac{2}{3}\right)^3 = 3 \times \log_{\frac{2}{3}} \frac{2}{3} = 3 \times 1 = 3$

Think - Discuss

We know that, if $7 = 2^x$ then $x = \log_2 7$. Then, what is the value of $2^{\log_2 7}$? Justify your answer. Generalise the above by taking some more examples for $a^{\log_a N}$

Sol : $7 = 2^x \Rightarrow \log_2 7 = x$

$\Rightarrow \; 2^{\log_2 7} = 2^x$

$\Rightarrow \; 2^{\log_2 7} = 7$

$a^{\log_a N} = N$

Example-11: .Expand $\log \frac{343}{125}$.

Sol : $\log \frac{343}{125}$ $= \log 343 - \log 125$

$= \log 7^3 - \log 5^3$

$= 3 \times \log 7 - 3 \times \log 5$

$= 3(\log 7 - \log 5)$

$$\log \frac{x}{y} = \log x - \log y$$

$$\log x^m = m.\log x$$

Example -12: Write $2 \log 3 + 3 \log 5 - 5 \log 2$ as a single logarithm.

Sol : $2 \log 3 + 3 \log 5 - 5 \log 2$

$= \log 3^2 + \log 5^3 - \log 2^5$

$= \log 9 + \log 125 - \log 32$

$= \log \frac{9 \times 125}{32}$

$= \log \frac{1125}{32}$

$$\text{m} \log x = \log x^m$$

$$\log x + \log y - \log z = \log \frac{x \times y}{z}$$

Example-13: Solve $3^x = 5^{x-2}$.

సాధన : $3^x = 5^{x-2}$

Take logarithm on both sides

$\log 3^x = \log 5^{x-2}$

$x \log 3 = (x-2) \log 5$

$$\log x^m = m.\log x$$

$$x \log 3 = x \log 5 - 2 \log 5$$

$$x \log 5 - x \log 3 = 2 \log 5$$

$$x (\log 5 - \log 3) = 2 \log 5$$

$$x = \frac{2 \log 5}{(\log 5 - \log 3)}$$

Example-14: ***Find x if*** $\mathbf{2.\log 5 + \frac{1}{2} \log 9 - \log 3 = \log x}$

సాధన : $\log x = 2.\log 5 + 1/2 \, . \, \log 9 - \log 3$

$= \log 5^2 + \log 9^{\frac{1}{2}} - \log 3$

$= \log 25 + \cancel{\log 3} - \cancel{\log 3}$

$\log x = \log 25 \Rightarrow x = 25$

$$\boxed{\begin{array}{c} m.\log x = \log x^m \\ 9^{\frac{1}{2}} = (3^2)^{\frac{1}{2}} = 3^{2\times\frac{1}{2}} = 3 \end{array}}$$

Exercise – 1.5

1. **Determine the value of the following.**

(i) $\mathbf{\log_{25} 5} = \log_{5^2} 5 = \frac{1}{2} \times \log_5 5 = \frac{1}{2} \times 1 = \frac{1}{2}.$

(ii) $\mathbf{\log_{81} 3} = \log_{3^4} 3 = \frac{1}{4} \times \log_3 3 = \frac{1}{4} \times 1 = \frac{1}{4}.$

(iii) $\mathbf{\log_2 \left(\frac{1}{16}\right)} = \log_2 \left(\frac{1}{2^4}\right) = \log_2 2^{-4} = -4 \times \log_2 2 = -4 \times 1 = -4$

(iv) $\mathbf{\log_7 1} = 0$

(v) $\mathbf{\log_x \sqrt{x}} = \log_x x^{\frac{1}{2}} = \frac{1}{2} \times \log_x x = \frac{1}{2} \times 1 = \frac{1}{2}$

(vi) $\mathbf{\log_2 512} = \log_2 2^9 = 9 \times log_2 2 = 9 \times 1 = 9.$

(vii) $\mathbf{\log_{10} 0.01} = \log_{10} \frac{1}{100} = \log_{10} \frac{1}{10^2} = \log_{10} 10^{-2} = -2 \times \log_{10} 10 = -2 \times 1 = -2$

(viii) $\mathbf{\log_{\frac{2}{3}} \frac{8}{27}} = \log_{\frac{2}{3}} \left(\frac{2}{3}\right)^3 = 3 \times \log_{\frac{2}{3}} \frac{2}{3} = 3 \times 1 = 3$

(ix) $\mathbf{2^{2+\log_2 3}} = 2^2 \times 2^{\log_2 3} = 4 \times 3 = 12$

2. Write the following expressions as log N and find their values.

(i) $\mathbf{\log 2 + \log 5}$

Sol: $\log 2 + \log 5$

$= \log 2 \times 5 \quad (\log_a x + \log_a y = log_a xy)$

$= \log 10$

$= 1$

(ii) $\log_2 16 - \log_2 2$

Sol: $\log_2 16 - \log_2 2$

$= \log_2 \frac{16}{2} \quad (log_a x - log_a y = \log_a \frac{x}{y})$

$= \log_2 8$

$= \log_2 2^3$

$= 3 \times \log_2 2 \ (\log_a x^m = m \times log_a x)$

$= 3 \times 1 = 3 \ (\log_a a = 1)$

(iii) $3 \ \log_{64} 4$

Sol: $3 \ \log_{64} 4$

$= \log_{64} 4^3 \quad (m \times log_a x = log_a x^m)$

$= \log_{64} 64$

$= 1 \quad (\log_a a = 1)$

(iv) $2 \ \log 3 - 3 \ \log 2$

Sol: $2 \ \log 3 - 3 \ \log 2$

$= \log 3^2 - \log 2^3 \quad (m \times log_a x = log_a x^m)$

$= \log 9 - \log 8$

$= \log \frac{9}{8} \quad (log_a x - log_a y = \log_a \frac{x}{y})$

(v) $\log 10 + 2 \log 3 - \log 2$

Sol: $\log 10 + 2 \log 3 - \log 2$

$= \log 10 + \log 3^2 - \log 2 \quad (m \times log_a x = log_a x^m)$

$= \log 10 + \log 9 - \log 2$

$= \log 10 \times 9 - \log 2 \quad (\log_a x + \log_a y = log_a xy)$

$= \log 90 - \log 2$

$= \log \frac{90}{2} \quad (log_a x - log_a y = \log_a \frac{x}{y})$

$= \log 45$

3. Evaluate each of the following in terms of x and y, if it is given $x = \log_2 3$ **and** $y = \log_2 5$

(i) $\log_2 15$

Sol: $\log_2 15$

$= \log_2(3 \times 5)$

$= \log_2 3 + \log_2 5 \qquad (\log_a xy = \log_a x + \log_a y)$

$= x + y$

(ii) $\log_2 7.5$

Sol: $\log_2 7.5$

$= \log_2 \frac{15}{2} \qquad (7.5 = \frac{75}{10} = \frac{15}{2})$

$= \log_2 15 - \log_2 2 \qquad (\log_a \frac{x}{y} = \log_a x - \log_a y)$

$= \log_2(3 \times 5) - \log_2 2$

$= \log_2 3 + \log_2 5 - \log_2 2 \qquad (\log_a xy = \log_a x + \log_a y)$

$= x + y - 1 \qquad (\log_a a = 1)$

(iii) $\log_2 60$

Sol: $\log_2 60$

$$\begin{array}{r|l} 2 & 60 \\ \hline 2 & 30 \\ \hline 3 & 15 \\ \hline & 5 \end{array}$$

$= \log_2(2^2 \times 3 \times 5)$

$= \log_2 2^2 + \log_2 3 + \log_2 5 \qquad (\because \log_a xyz = \log_a x + \log_a y + \log_a z)$

$= 2 \times \log_2 2 + x + y \qquad (\because \log_a x^m = m \times \log_a x)$

$= 2 \times 1 + x + y \qquad (\because \log_a a = 1)$

$= 2 + x + y$

(iv) $\log_2 6750$

Sol: $\log_2 6750$

$$\begin{array}{r|l} 2 & 6750 \\ \hline 3 & 3375 \\ \hline 3 & 1125 \\ \hline 3 & 375 \\ \hline 5 & 125 \\ \hline 5 & 25 \\ \hline & 5 \end{array}$$

$= \log_2 2 \times 3^3 \times 5^3$

$= \log_2 2 + \log_2 3^3 + \log_2 5^3 \qquad (\because \log_a xy = \log_a x + \log_a y)$

$= 1 + 3 \times \log_2 3 + 3 \times \log_2 5 \qquad (\because \log_a x^m = m \times log_a x)$

$= 1 + 3\,x + 3\,\mathrm{y}$

4. Expand the following.

(i) $\log 1000$

Solution: $\log 1000$

$= \log 2^3 \times 5^3$

$= \log 2^3 + \log 5^3 \qquad (\because \log_a xy = \log_a x + \log_a y)$

$= 3 \log 2 + 3 \log 5 \qquad (\because \log_a x^m = m \times \log_a x)$

(ii) $\log \frac{128}{625}$

$$\begin{array}{r|l} 2 & 128 \\ \hline 2 & 64 \\ \hline 2 & 32 \\ \hline 2 & 16 \\ \hline 2 & 8 \\ \hline 2 & 4 \\ \hline & 2 \end{array} \qquad \begin{array}{r|l} 5 & 625 \\ \hline 5 & 125 \\ \hline 5 & 25 \\ \hline & 5 \end{array}$$

Solution: $\log \frac{128}{625}$

$= \log 128 - \log 625$

$= \log 2^7 - \log 5^4$

$= 7 \log 2 - 4 \log 5 \qquad (\because \log_a x^m = m \times log_a x)$

(iii) $\log x^2 y^3 z^4$

Solution: $\log x^2 y^3 z^4$

$= \log x^2 + \log y^3 + \log z^4 \qquad (\because \log_a xy = \log_a x + \log_a y)$

$= 2 \log x + 3 \log y + 4 \log z \qquad (\because \log_a x^m = m \times log_a x)$

(iv) **$\log \frac{p^2 q^3}{r}$**

Solution: $\log \frac{p^2 q^3}{r}$

$= \log p^2 q^3 - \log r \qquad (\because \log_a \frac{x}{y} = log_a x - log_a y)$

$= \log p^2 + \log q^3 - \log r \qquad (\because \log_a xy = \log_a x + \log_a y)$

$= 2 \log p + 3 \log q - \log r \qquad (\because \log_a x^m = m \times log_a x)$

(iv) $\log \sqrt{\frac{x^3}{y^2}}$

Solution: $\log\sqrt{\frac{x^3}{y^2}}$

$= \log\frac{\sqrt{x^3}}{\sqrt{y^2}}$

$= \log\sqrt{x^3} - \log\sqrt{y^2}$ $\quad (\log_a\frac{x}{y} = log_a x - log_a y)$

$= \log x^{\frac{3}{2}} - \log y$ $\quad (\sqrt{a^m} = a^{\frac{m}{2}}$ మరియు $\sqrt{a^2} = a)$

$= \frac{3}{2}\log x - log\, y$ $\quad (\log_a x^m = m \times log_a x)$

5. **If $x^2 + y^2 = 25xy$ then prove that $2.\log(x+y) = 3.\log 3 + \log x + \log y$.**

Solution: $2.\log(x+y) = \log(x+y)^2$ $\quad (\because m \times log_a x = log_a x^m)$

$= \log(x^2 + y^2 + 2xy)$

$= \log\ (25xy + 2xy)$ $\quad (\because x^2 + y^2 = 25xy)$

$= \log(27xy)$

$= \log 27 + \log x + \log y$ $\quad (\because \log(abc) = \log a + \log b + \log c)$

$= \log 3^3 + \log x + \log y$

$= 3.\log 3 + \log x + \log y$ $\quad (\because \log_a x^m = m \times log_a x)$

6. **If $\log(\frac{x+y}{3}) = \frac{1}{2}(\log x + \log y)$ then find the value of $\frac{x}{y} + \frac{y}{x}$.**

సాధన : $\log(\frac{x+y}{3}) = \frac{1}{2}(\log x + \log y)$

$2 \times \log(\frac{x+y}{3}) = (\log x + \log y)$

$\log(\frac{x+y}{3})^2 = \log xy$ $\quad (\because m \times \log_a x = \log_a x^m)$

$(\frac{x+y}{3})^2 = xy$

$x^2 + y^2 + 2xy = 9xy$

$x^2 + y^2 = 9xy - 2xy$

$x^2 + y^2 = 7xy$

$\frac{x^2+y^2}{xy} = 7$

$\frac{x^2}{xy} + \frac{y^2}{xy} = 7$

$\frac{X}{Y} + \frac{Y}{X} = 7$

7. $(\mathbf{2.3})^{x} = (\mathbf{0.23})^{y}$ **= 1000 then find the value of** $\frac{1}{x} - \frac{1}{y}$.

Solution : $(2.3)^x = (0.23)^y = 1000 = 10^3$

$(2.3)^x = 10^3$ and $(0.23)^y = 10^3$

Take logarithm (log) on both sides

$\log(2.3)^x = \log 10^3$ $\log(0.23)^y = \log 10^3$

$\boxed{\log a^m = m.\log a}$

$x \log(2.3) = 3 \log 10$ $y \log(0.23) = 3 \log$

$x \log(2.3) = 3$ $y \log(0.23) = 3$

$\boxed{\log 10 = 1}$

$\log(2.3) = \frac{3}{x}$ $\log(0.23) = \frac{3}{y}$

$\frac{3}{x} - \frac{3}{y} = \log(2.3) - \log(0.23) = log\frac{2.3}{0.23} = log\, 10 = 1$

$\frac{3}{x} - \frac{3}{y} = 1 \Rightarrow 3\left(\frac{1}{x} - \frac{1}{y}\right) = 1$

$\therefore \frac{1}{x} - \frac{1}{y} = \frac{1}{3}$

8. *If* $\mathbf{2^{x+1}} = \mathbf{3^{1-x}}$ **then find the value of** $\boldsymbol{x}$ **.**

Solution : $2^{x+1} = 3^{1-x}$

Take logarithm (log) on both sides

$\log 2^{x+1} = \log 3^{1-x}$

$(x+1) \log 2 = (1-x) \log 3$ $(\log_a x^m = m \times log_a x)$

$$x.\log 2 + \log 2 = \log 3 - x.\log 3$$

$$x.\log 2 + x.\log 3 = \log 3 - \log 2$$

$$x.(\log 2 + \log 3) = \log 3 - \log 2$$

$$x = \frac{\log 3 - \log 2}{\log 3 + \log 2}$$

9. (i) Is $\log 2$ rational or irrational? Justify your answer.

Solution : $log\ 2$ is an irrational number .

Justification : let $\log 2$ is a rational number

Then $\log 2 = \frac{p}{q}$ (p, q are coprimes)

The exponential form is

$10^{\frac{p}{q}} = 2 \Rightarrow \left(10^{\frac{p}{q}}\right)^q = 2^q \Rightarrow 10^p = 2^q$ Which is a contradiction .

Since 10^p ends with '0' but 2^q does not ends with '0'.

Our assumption is wrong

So $\log 2$ is an irrational number

9. (ii) Is $\log 100$ rational or irrational? Justify your answer.

Solution : $log\ 100$ is a rational number

Justification : $\log 100 = \log 10^2 = 2 \times \log 10 = 2 \times 1 = 2$

We know that $2 = \frac{2}{1}$ is a rational number

$\therefore \log 100$ *is a rational number*