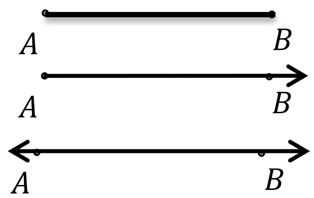
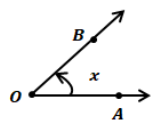
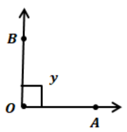
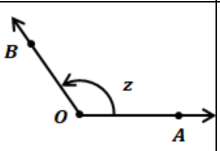
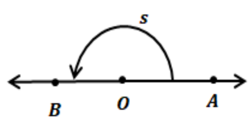
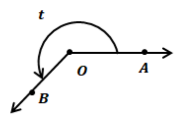
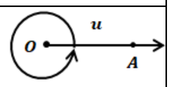
CHAPTER 6

6. LINES AND ANGLES (NOTES)

PREPARED BY: BALABHADRA SURESH



1. A part (or portion) of a line with two end points is called a line-segment
2. The line segment AB is denoted by $\overline{AB}$
3. A part of a line with one end point is called a ray.
4. The ray AB is denoted by $\overrightarrow{AB}$.
5. The line AB is denoted by $\overleftrightarrow{AB}$
6. Sometimes small letters l, m, n, etc. will be used to denote lines.
7. If three or more points lie on the same line, they are called collinear points; otherwise they are called non-collinear points.
8. An angle is formed when two rays originate from the same end point. The rays making an angle are called the arms of the angle and the end point is called the vertex of the angle
9. The angle between 0^0 *and* 90^0 is called acute angle.
10. The angle 90^0 is called right angle
11. The angle between 90^0 *and* 180^0 is called obtuse angle.
12. The angle 180^0 is called straight angle.
13. The angle between 180^0 *and* 360^0 is called reflex angle.

Name	Acute angle	Right angle	Obtuse angle	Straight angle	Reflex angle	Complete angle
Measure	$0^0 < x < 90^0$	$y = 90^0$	$90^0 < z < 180^0$	$s = 180^0$	$180^0 < t < 360^0$	$u = 360^0$
Illustration						

14. **Complementary angles**: Two angles whose sum is 90° are called complementary angles.
15. **Supplementary angles**: Two angles whose sum is 180° are called supplementary angles.
16. **Adjacent angles**: Two angles are adjacent, if they have a common vertex, a common arm and their non-common arms are on different sides of the common arm.

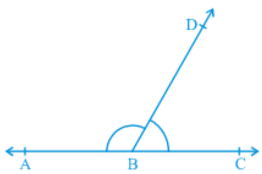


Fig. 6.3 : Linear pair of angles

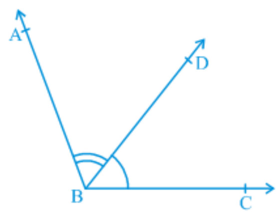


Fig. 6.2 : Adjacent angles

17. **Linear pair of angles**: the sum of two adjacent angles is 180°, then they are called a linear pair of angles.

18. **vertically opposite angles** : If $\overleftrightarrow{AB}$ *and* $\overleftrightarrow{CD}$ intersect at O then

$\angle AOC$ is vertically opposite to $\angle BOD$ *and*

$\angle AOD$ is vertically opposite to $\angle BOC$

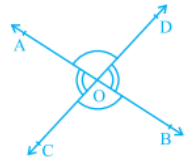
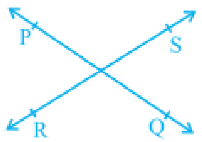
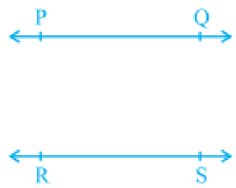


Fig. 6.4 : Vertically opposite angles

19. Intersecting Lines and Non-intersecting Lines:



(i) Intersecting lines



(ii) Non-intersecting (parallel) lines

20. Axiom 6.1 : If a ray stands on a line, then the sum of two adjacent angles so formed is 180°

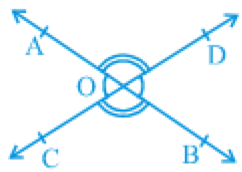
21. Axiom 6.2 : If the sum of two adjacent angles is 180°, then the non-common arms of the angles form a line.

Theorem 6.1 : If two lines intersect each other, then the vertically opposite angles are equal

Proof: let AB and CD be two lines intersecting at O.

Two pairs of vertically opposite angles are

(i) ∠ AOC and ∠ BOD (ii) ∠ AOD and ∠ BOC.



Now, ray OA stands on line CD

∠ AOC + ∠ AOD = 180°(Linear pair axiom)→ (1)

Now, ray OD stands on line AB

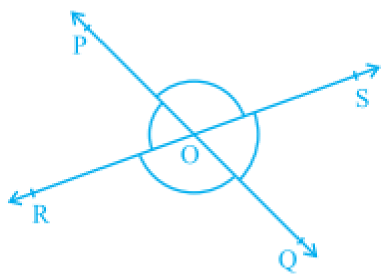
∠ AOD + ∠ BOD = 180° (Linear pair axiom))→ (2)

From (1) and (2): ∠ AOC + ∠ AOD=∠ AOD + ∠ BOD

∠ AOC=∠ BOD

Similarly, we can prove ∠AOD = ∠BOC.

Example 1 : In Fig. 6.9, lines PQ and RS intersect each other at point O. If ∠ POR : ∠ ROQ = 5 : 7, find all the angles



Sol: Given ∠ POR: ∠ ROQ = 5 : 7

$Let\ \angle POR = 5x\ \ and\ \ \angle ROQ = 7x$

∠ POR + ∠ ROQ = 180°(Linear pair of angles)

5x+7x=180°

12x=180°

$$x = \frac{180^0}{12} = 15^0$$

$$\angle POR = 5x = 5 \times 15^0 = 75^0$$

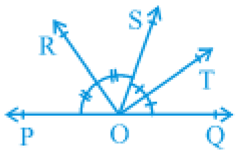
$\angle ROQ = 7x = 7 \times 15^0 = 105^0$

Now ∠ POS = ∠ROQ = 105° (Vertically opposite angles)

∠ SOQ = ∠POR = 75° (Vertically opposite angles)

Example 2 : In Fig. 6.10, ray OS stands on a line POQ. Ray OR and ray OT are angle bisectors of ∠ POS and ∠ SOQ, respectively. If ∠ POS = x, find ∠ ROT

Sol: Given ∠ POS = x



∠ POS + ∠ SOQ = 180° (Linear pair of angles)

x + ∠ SOQ = 180°

∠ SOQ = 180° – x

Ray OR and ray OT are angle bisectors of ∠ POS and ∠ SOQ

$\angle ROS = \frac{1}{2} \times \angle POS$ and $\angle SOT = \frac{1}{2} \times \angle SOQ$

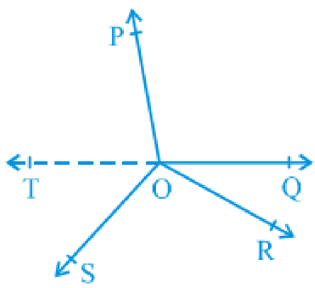
$\angle ROS = \frac{1}{2} \times x$ and $\angle SOT = \frac{1}{2} \times (180° - x)$

$\angle ROS = \frac{x}{2}$ and $\angle SOT = 90^0 - \frac{x}{2}$

$\angle ROT = \angle ROS + \angle SOT = \frac{x}{2} + 90^0 - \frac{x}{2} = 90^0$

Example 3 : In Fig. 6.11, OP, OQ, OR and OS are four rays. Prove that ∠ POQ + ∠ QOR + ∠ SOR + ∠ POS = 360°

Sol: Let us produce ray OQ backwards to a point T



TOQ is a line. So, ∠ TOP + ∠ POQ = 180° (Linear pair axiom)→ (1)

Similarly ∠ TOS + ∠ SOQ = 180° (Linear pair axiom)→ (2)

But ∠ SOQ = ∠ SOR + ∠ QOR

So, (2) becomes ∠TOS + ∠SOR + ∠QOR = 180° → (3)

(1)+(3) we get

∠TOP + ∠POQ+∠TOS + ∠SOR + ∠QOR=180°+180° =360°→ (4)

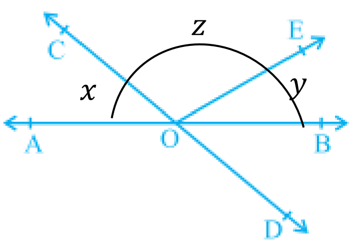
But ∠ TOP + ∠ TOS = ∠ POS

Therefore, (4) becomes ∠ POQ + ∠ QOR + ∠ SOR + ∠ POS = 360°

EXERCISE 6.1

1. **In Fig. 6.13, lines AB and CD intersect at O. If ∠ AOC + ∠ BOE = 70° and ∠ BOD = 40°, find ∠ BOE and reflex ∠ COE.**

Sol: $\angle AOC = \angle BOD$ (Vertically opposite angles)



$x = 40^0$

∠ AOC + ∠ BOE = 70°

$x + y = 70^0$

$40^0 + y = 70^0$

$y = 70^0 - 40^0$

$y = 30^0$

$x + y + z = 180^0$ (Linear angles)

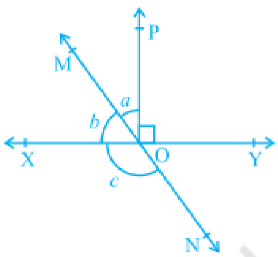
$70^0 + z = 180^0$

$z = 180^0 - 70^0$

$z = 110^0 \Rightarrow \angle COE = 110^0$

Reflex $\angle$ COE = $360^0 - \angle \text{COE} = 360^0 - 110^0 = 250^0$

2. In Fig. 6.14, lines XY and MN intersect at O. If ∠ POY = 90° and a : b = 2 : 3, find c.



Sol: a : b = 2 : 3

$Let\ a = 2x\ and\ b = 3x$

$a + b = 90^0 \Rightarrow 2x + 3x = 90^0 \Rightarrow 5x = 90^0 \Rightarrow x = \frac{90^0}{5} = 18^0$

$a = 2x = 2 \times 18^0 = 36^0\ and\ b = 3x = 3 \times 18^0 = 54^0$

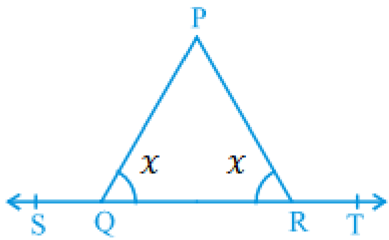
$b + c = 180^0\ (Linear\ pair)$

$54^0 + c = 180^0$

$c = 180^0 - 54^0$

$c = 126^0$

3. In Fig. 6.15, ∠ PQR = ∠ PRQ, then prove that ∠ PQS = ∠ PRT.



Sol: $\angle$ PQR = $\angle$ PRQ = x

$\angle$ PQS + $\angle$ PQR = 180^0 ($Linear\ pair$)

$\angle$ PQS + $x = 180^0 \rightarrow (1)$

$\angle PRT + \angle$ PRQ = 180^0 ($Linear\ pair$)

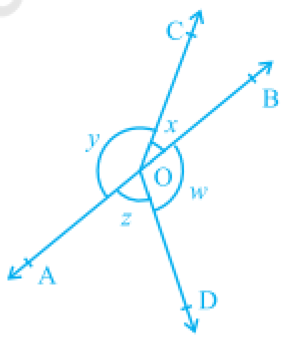
$\angle PRT + x = 180^0 \rightarrow (2)$

From (1) and (2)

$\angle$ PQS + x = $\angle PRT$ + x

$\angle$ PQS = $\angle$ PRT

4. In Fig. 6.16, if x + y = w + z, then prove that AOB is a line



Sol: $(x + y) + (z + w) = 360^0 (Complete\ angle)$

$if\ x\ +\ y\ =\ w\ +\ z\ then$

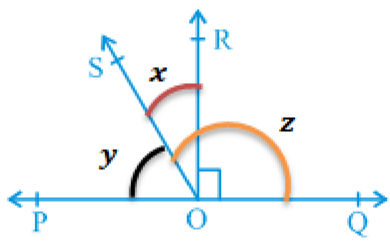
$(x + y) + (x + y) = 360^0$

$2(x + y) = 360^0$

$$(x+y) = \frac{360^0}{2} = 180^0$$

$\therefore$ AOB is a line

5. In Fig. 6.17, POQ is a line. Ray OR is perpendicular to line PQ. OS is another ray lying between rays OP and OR. Prove that $\angle ROS = \frac{1}{2}(\angle QOS - \angle POS)$.



Sol: $\angle ROQ + \angle ROP = 180^0 (Linear\ pair)$

But $\angle ROQ = 90^0$

So, $\angle ROP = 90^0$

$x + y = 90^0$

$2(x+y) = 2 \times 90^0$

$2x + 2y = 180^0 \rightarrow (1)$

$y + z = 180^0 \rightarrow (2)(Linear\ pair)$

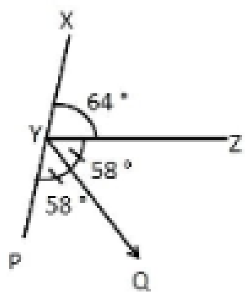
$2x + 2y = y + z$

$2x = y + z - 2y$

$2x = z - y \Rightarrow x = \frac{1}{2}(z-y)$

$\angle ROS = \frac{1}{2}(\angle QOS - \angle POS)$

6. **It is given that $\angle XYZ = 64°$ and XY is produced to point P. Draw a figure from the given information. If ray YQ bisects $\angle ZYP$, find $\angle XYQ$ and reflex $\angle QYP$.**



Sol: $\angle XYZ + \angle ZYP = 180^0$ (linear pair)

$64^0 + \angle ZYP = 180^0$

$\angle ZYP = 180^0 - 64^0$

$\angle ZYP = 116^0$

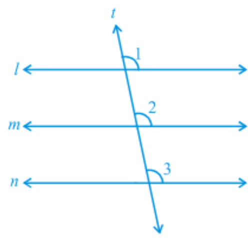
If ray YQ bisects $\angle ZYP$ then

$\angle ZYQ = \angle QYP = \frac{1}{2} \times \angle ZYP = \frac{1}{2} \times 116^0 = 58^0$

$\angle XYQ = \angle XYZ + \angle ZYQ = 64^0 + 58^0 = 122^0$

Reflex $\angle QYP = 360^0 - \angle QYP = 360^0 - 58^0 = 302^0$

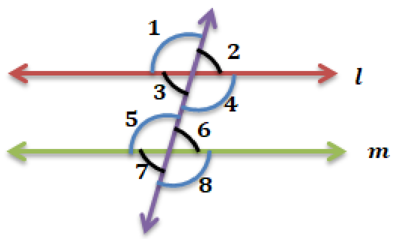
Lines Parallel to the Same Line



Lines which are parallel to the same line are parallel to each other.

If $l \parallel m$ and $l \parallel n$ then $m \parallel n$

If f two parallel lines l and m are cut by a transversal then



(i) Each pair of corresponding angles are equal in measure.

$\angle 1 = \angle 5;\ \ \angle 2 = \angle 6;\ \ \angle 3 = \angle 7;\ \ \angle 4 = \angle 8$

(ii) Each pair of alternate interior angles are equal.

$\angle 3 = \angle 6$; $\angle 4 = \angle 5$

(iii) Each pair of interior angles on the same side of the transversal are supplementary.

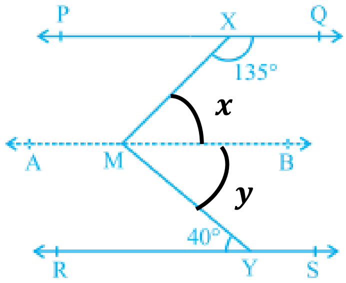
$\angle 3 + \angle 5 = 180^0$; $\angle 4 + \angle 6 = 180^0$

(iv) Each pair of exterior angles on the same side of the transversal are supplementary

$\angle 1 + \angle 7 = 180^0$; $\angle 2 + \angle 8 = 180^0$

Example 4 : In Fig. 6.19, if PQ || RS, ∠ MXQ = 135° and ∠ MYR = 40°, find ∠ XMY

Sol: Draw a line AB parallel to line PQ



$PQ \parallel AB \text{ and } XM \text{ is transversal}$

$x + 135^0 = 180^0 (Co - interior\ angles\ are\ supplementary)$

$x = 180^0 - 135^0$

$x = 45^0$

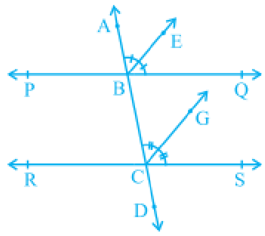
$AB \parallel RS \text{ and } MY \text{ is transversal}$

$y = 40^0 (Alternate\ interior\ angles)$

$\angle XMY = x + y = 45^0 + 40^0 = 85^0$

Example 5 : If a transversal intersects two lines such that the bisectors of a pair of corresponding angles are parallel, then prove that the two lines are parallel.

Sol: A transversal AD intersects PQ and RS at points B and C respectively.



$\overrightarrow{BE}$ is the bisector of ∠ ABQ and $\overrightarrow{CG}$ is the bisector of ∠ BCS; and

$\overrightarrow{BE} \parallel \overrightarrow{CG}$.

$\angle ABE = \frac{1}{2} \angle ABQ$ and $\angle BCG = \frac{1}{2} \angle BCS$

But BE || CG and AD is the transversal.

Therefore, ∠ ABE = ∠ BCG (Corresponding angles axiom)

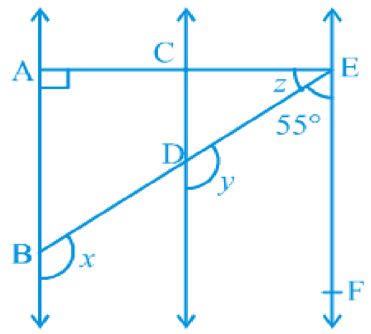
$\frac{1}{2} \angle ABQ = \frac{1}{2} \angle BCS$

$\angle ABQ = \angle BCS$

Corresponding angles are equal. From Converse of corresponding angles axiom

PQ || RS

Example 6 : In Fig. 6.22, AB || CD and CD || EF. Also EA ⊥ AB. If ∠ BEF = 55°, find the values of x, y and z.



Sol: CD || EF and DE is transversal

y + 55° = 180° (Co-interior angles are supplementary)

y = 180º – 55º = 125º

AB || CD and BD is transversal

x = y (corresponding angles)

$x = 125^{\circ}$

AB || EF and AE is transversal.

∠ EAB + ∠ FEA = 180°(Co-interior angles are supplementary)

$90° + z + 55° = 180°$

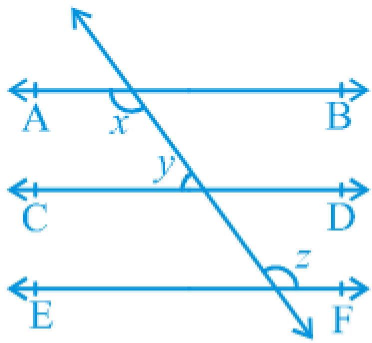
$z + 145° = 180°$

$z = 180° - 145°$

$z = 35°$

EXERCISE 6.2

1. In Fig. 6.23, if AB || CD, CD || EF and y : z = 3 : 7, find x.



$Sol: y : z = 3 : 7$

Let $y = 3a$ and $z = 7a$

$But\ x = z$ (Alternate interior angles)

$x = 7a$

$x + y = 180^0$(co – interior angles are supplementary)

$7a + 3a = 180^0$

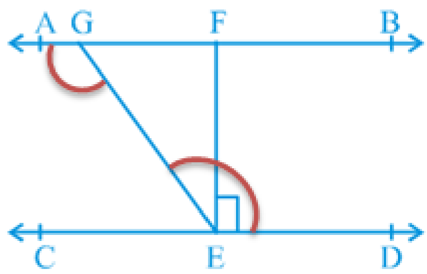
$10a = 180^0$

$$a = \frac{180^0}{10} = 18^0$$

$x = z = 7a = 7 \times 18^0 = 126^0$

$y = 3a = 3 \times 18^0 = 54^0$

2. In Fig. 6.24, if AB || CD, EF ⊥ CD and ∠ GED = 126°, find ∠ AGE, ∠ GEF and ∠ FGE.



Sol: Given EF ⊥ CD

$\angle FED = \angle FEC = 90^0$

AB || CD and GE transversal

$\angle AGE = \angle GED$ (*Alternate interior angles*)

But ∠ GED = 126°

$\angle AGE = 126°$

$\angle GEF = \angle GED - \angle FED = 126° - 90° = 36°$

$\angle FGE + \angle AGE = 180^0$(*Linear pair*)

$\angle FGE + 126° = 180^0$

$\angle FGE = 180^0 - 126° = 54°$

3. In Fig. 6.25, if PQ || ST, ∠ PQR = 110° and ∠ RST = 130°, find ∠ QRS.

Sol: PQ || ST and QR is transversal

$x + 110^0 = 180^0$(*Co interior angles are supplementary*)

$x = 180^0 - 110^0 = 70^0$

$similarly\ \ y + 130^0 = 180^0$

$y + 130^0 = 180^0$

$y = 180^0 - 130^0 = 50^0$

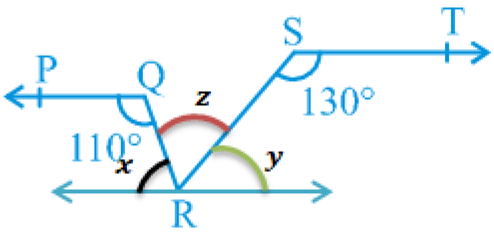
$x + y + z = 180^0 (\ Linear\ angles)$

$70^0 + 50^0 + z = 180^0$

$120^0 + z = 180^0$

$z = 180^0 - 120^0 = 60^0$

$\angle QRS = 60^0$



4. In Fig. 6.26, if AB || CD, ∠ APQ = 50° and ∠ PRD = 127°, find x and y.

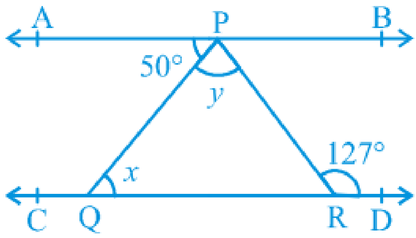
Sol: AB || CD and PQ is transversal

$x = 50^0$(Alternate interior angles)

AB || CD and PR is transversal

$y + 50^0\ = 127^0$(Alternate interior angles)

$y = 127^0 - 50^0\ = 77^0$



5. In Fig. 6.27, PQ and RS are two mirrors placed parallel to each other. An incident ray AB strikes the mirror PQ at B, the reflected ray moves along the path BC and strikes the mirror RS at C and again reflects back along CD. Prove that AB || CD

Sol: Draw BL and CM perpendicular to PQ

$BL \parallel CM\ and\ BC$ is transversal

$\angle LBC\ = \angle MCB\ \ \rightarrow (1)($ Alternate interior angles)

But Angle of incidence = Angle of reflection

$\angle$ABL = $\angle$LBC *and* $\angle$MCB = $\angle$MCD $\rightarrow (2)$

From (1) and (2)

$\angle$ABL = $\angle$MCD $\rightarrow (3)$

$(1) + (3) \Rightarrow \angle$LBC + $\angle$ABL = $\angle$MCB + $\angle$MCD

$\angle$ABC = $\angle$BCD

Pair of alternate interior angles are equal

$\therefore AB \parallel CD$