CHAPTER 5

IX-MATHEMATICS-NCERT(2023-24)

5. INTRODUCTION TO EUCLID'S GEOMETRY (NOTES)

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1. The word 'geometry' comes from the Greek words 'geo', meaning the 'earth', and 'metrein', meaning 'to measure'
2. In the Indian subcontinent, the excavations at Harappa and Mohenjo-Daro, etc. show that the Indus Valley Civilisation (about 3000 BCE) made extensive use of geometry
3. In ancient India, the **Sulbasutras** (800 BCE to 500 BCE) were the manuals of geometrical constructions
4. The **sriyantra** (given in the Atharvaveda) consists of nine interwoven isosceles triangles.
5. Euclid, a teacher of mathematics at Alexandria in Egypt, collected all the known work and arranged it in his famous treatise MATHEMATICS called 'Elements'.
6. Euclid divided the 'Elements' into thirteen chapters, each called a book
7. Euclid listing 23 definitions in Book 1 of the 'Elements'
8. Though Euclid defined a point, a line, and a plane, these definitions are not accepted by mathematicians. Therefore, these terms are now taken as undefined
9. A point is that which has no part.
10. A line is breadthless length.
11. The ends of a line are points.
12. A straight line is a line which lies evenly with the points on itself.
13. A surface is that which has length and breadth only.
14. The edges of a surface are lines.
15. A plane surface is a surface which lies evenly with the straight lines on itself.
16. A system of axioms is called consistent.
17. The statements that were proved are called propositions or theorems.
18. Euclid deduced 465 propositions.
19. Axioms or postulates are the assumptions which are obvious universal truths. They are not proved
20. Theorems are statements which are proved, using definitions, axioms, previously proved statements and deductive reasoning.
21. **Euclid's axioms**
    (1) Things which are equal to the same thing are equal to one another.
    (2) If equals are added to equals, the wholes are equal.
    (3) If equals are subtracted from equals, the remainders are equal.
    (4) Things which coincide with one another are equal to one another.
    (5) The whole is greater than the part.
    (6) Things which are double of the same things are equal to one another.
    (7) Things which are halves of the same things are equal to one another.
22. **Euclid's five postulates**
    (i) A straight line may be drawn from any one point to any other point
    (ii) A terminated line can be produced indefinitely.
    (iii) A circle can be drawn with any centre and any radius.
    (iv) All right angles are equal to one another.

(v) If a straight line falling on two straight lines makes the interior angles on the same side of it taken together less than two right angles, then the two straight lines, if produced indefinitely, meet on that side on which the sum of angles is less than two right angles.

Example 1 : If A, B and C are three points on a line, and B lies between A and C (see Fig. 5.7), then prove that AB + BC = AC.

Sol: AC coincides with AB + BC

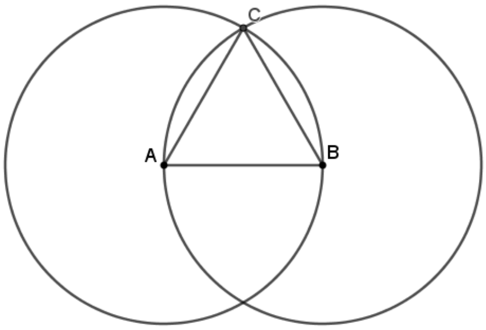
From Euclid's Axiom (4) : things which coincide with one another are equal to one another

$\therefore AB + BC = AC$

Example 2 : Prove that an equilateral triangle can be constructed on any given line segment.

Sol: 1. Using Euclid's Postulate 3, you can draw a circle with point A as the centre and AB as the radius.

2. Draw another circle with point B as the centre and BA as the radius.



3. The two circles meet at a point, say C. Now, draw the line segments AC and BC to form ΔABC.

Proof: $AB = AC$(radii of the same circle)$\rightarrow$(1)

$AB = BC$ (Radii of the same circle)$\rightarrow$(2)

Euclid's axiom that things which are equal to the same thing are equal to one another,

From (1) and (2) : $AB = BC = AC$

So, Δ ABC is an equilateral triangle.

Theorem 5.1 : Two distinct lines cannot have more than one point in common.

Sol: let us suppose that the two lines l and m intersect in two distinct points, say P and Q .

Two lines passing through two distinct points P and Q

But this assumption clashes with the axiom that only one line can pass through two distinct points.

Our assumption is wrong

So, we conclude that two distinct lines cannot have more than one point in common.

EXERCISE 5.1

1. Which of the following statements are true and which are false? Give reasons for your answers.

(i) Only one line can pass through a single point.

Sol: False. We can draw infinite number of lines passing through a single point.

(ii) There are an infinite number of lines which pass through two distinct points.

Sol: False. We can draw only one line pass through two distinct points .

(iii) A terminated line can be produced indefinitely on both the sides.

Sol: True. According to Postulate 2 ,A terminated line can be produced indefinitely.

(iv) If two circles are equal, then their radii are equal.

Sol: True.

(v) In Fig. 5.9, if AB = PQ and PQ = XY, then AB = XY.

Sol: True. Things which are equal to the same thing are equal to one another.

2. Give a definition for each of the following terms. Are there other terms that need to be defined first? What are they, and how might you define them?
(i) parallel lines (ii) perpendicular lines (iii) line segment (iv) radius of a circle (v) square

Sol: First we define (a) Point (b) Ray (c) Line

(a) Point: A small dot has no dimensions.

(b) Line: A line is breadthless length.

(c) Ray: A part of a line it has one end point.

(i) Parallel lines: If two lines have no common points, they are called parallel lines.

(ii) Perpendicular lines: If the angle between two lines is equal to 90º, then these lines are perpendicular to each other.

(iii) Line segment: A terminated line is called a line segment. It has two endpoints.

(iv)Radius of a circle: The distance from the centre to any point on the circle is called the radius of the circle.

(v) Square: A square is a regular quadrilateral which means that it has four equal sides and four right angles

3. Consider two 'postulates' given below: Are these postulates consistent? Do they follow from Euclid's postulates? Explain
(i) Given any two distinct points A and B, there exists a third point C which is in between A and B.
(ii)There exist at least three points that are not on the same line. Do these postulates contain any undefined terms?.

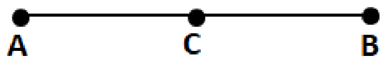
Sol: They are consistent, because they deal with two different situations –

(i) says that given two points A and B, there is a point C lying on the line in between them;

(ii) says that given A and B, you can take C not lying on the line through A and B. These 'postulates' do not follow from Euclid's postulates.

4. **If a point C lies between two points A and B such that AC = BC, then prove that AC = $\frac{1}{2}$AB. Explain by drawing the figure.**

vSol: $AC = BC$ (Given)



$AC + AC = BC + AC$(Equals are added to equals)

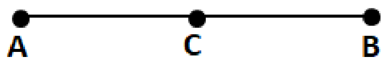
$2AC = AB$ ($BC + AC$ coincides with AB)

$AC = \frac{1}{2}AB$

5. In Question 4, point C is called a mid-point of line segment AB. Prove that every line segment has one and only one mid-point.

Sol: point C is mid-point of line segment AB

$AC=BC$



$AC+AC=BC+AC$(Equals are added to equals)

$2AC=AB$

$AC = \frac{1}{2}AB \rightarrow (1)$

Let's assume that D is another mid-point of AB

$AD=BD \rightarrow (1)$

$AD+AD=BD+AD$(Equals are added to equals)

$2AD=AB$

$AD = \frac{1}{2}AB \rightarrow (2)$

From (1) and (2)

$AC=AD$ (Things which are equal to the same thing are equal to one another)

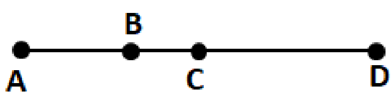
C coincide with D

Thus, a line segment has only one midpoint.

6. **In Fig. 5.10, if $AC = BD$, then prove that $AB = CD$.**

Sol: If $AC=BD$ then

$AB+BC=BC+CD$ (B,C are lies between AC and BD)



$AB+BC-BC=BC+CD-BC$ (Subtracting both sides BC)

$AB=CD$

7. Why is Axiom 5, in the list of Euclid's axioms, considered a 'universal truth'? (Note that the question is not about the fifth postulate.)

Sol: Since this is true for anything in any part of the world, this is a universal truth.