4. LINEAR EQUATIONS IN TWO VARIABLES (Notes)

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1. If a linear equation has two variables then it is called a linear equation in two variables.
2. The general form of linear equation in two variables x, y is $ax + by + c = 0$. Where a, b, c are real numbers, and a, b are not both zero.
3. The process of finding solution(s) is called solving an equation .
4. A linear equation in two variables has infinitely many solutions. Every solution of the linear equation can be represented by a unique point on the graph of the equation.
5. The graphs of x = a and y = a are lines parallel to the y-axis and x-axis, respectively

Example 1 : Write each of the following equations in the form ax + by + c = 0 and indicate the values of a, b and c in each case:

(i) $\mathbf{2x + 3y = 4.37}$

Sol: $2x + 3y = 4.37 \Rightarrow 2x + 3y - 4.37 = 0$

$a = 2, b = 3, c = -4.37$

$(\boldsymbol{ii})$ $\boldsymbol{x - 4 = \sqrt{3}y}$

$Sol: x - 4 = \sqrt{3}y \Rightarrow x - \sqrt{3}y - 4 = 0$

$a = 1, b = -\sqrt{3}, c = -4$

$(\boldsymbol{iii})$ $\mathbf{4 = 5x - 3y}$

Sol: $5x - 3y - 4 = 0$

$a = 5, b = -3, c = -4$

$(\boldsymbol{iv})$ $\mathbf{2x = y}$

Sol: $2x - y = 0$

$a = 2, b = -1, c = 0$

Example 2 : Write each of the following as an equation in two variables:

(i) $x = -5$

$Sol: 1.x + 0.y + 5 = 0$

(ii) y = 2

Sol: $0.x + 1.y - 2 = 0$

(iii) $2x = 3$

Sol: $2.x + 0.y - 3 = 0$

(iv) $5y = 2$

Sol: $0.x + 5.y - 2 = 0$

EXERCISE 4.1

1. The cost of a notebook is twice the cost of a pen. Write a linear equation in two variables to represent this statement.

Sol: Let the cost of a notebook = ₹ x and cost of a pen = ₹ y

The cost of a notebook = 2× the cost of a pen

$x = 2y$

$x - 2y = 0$

2. Express the following linear equations in the form ax + by + c = 0 and indicate the values of a, b and c in each case

(i) $\mathbf{2x + 3y = 9.3\overline{5}}$

$Sol: 2x + 3y - 9.3\overline{5} = 0$

$a = 2, b = 3, c = -9.3\overline{5}$

(ii) $\mathbf{x - 5y - 10 = 0}$

$Sol: 1.x - 5.y - 10 = 0$

$a = 1, b = -5, c = -10$

(iii) $\mathbf{-2x + 3y = 6}$

$Sol: -2.x + 3.y - 6 = 0$

$a = -2, b = 3, c = 6$

(iv) $\mathbf{x = 3y}$

$Sol: x - 3y = 0$

$a = 1, b = -3, c = 0$

(v) $\mathbf{2x = -5y}$

$Sol: 2x + 5y + 0 = 0$

$a = 2, b = 5, c = 0$

(vi) $\mathbf{3x + 2 = 0}$

$Sol: 3x + 0.y + 2 = 0$

$a = 3, b = 0, c = 2$

(vii) $\mathbf{y - 2 = 0}$

$Sol: 0.x + 1.y - 2 = 0$

$a = 0, b = 1, c = -2$

(viii) $\mathbf{5 = 2x}$

$Sol: 2x + 0.y - 5 = 0$

$a = 2, b = 0, c = -5$

Solution of a Linear Equation

(i) Any pair of values of 'x' and 'y' which satisfy the linear equation in two variables $ax + by + c = 0$ is called its solution.

(ii) A linear equation in two variables has infinitely many solutions.

Example 3 : Find four different solutions of the equation x + 2y = 6.

Sol: Given equation $x + 2y = 6$.

(i) $Let\ x = 0 \Rightarrow 0 + 2y = 6$

$\Rightarrow 2y = 6$

$\Rightarrow y = \frac{6}{2} = 3$

$Solution: (0,3)$

$(ii)\ Let\ x = 2 \Rightarrow 2 + 2y = 6$

$\Rightarrow 2y = 6 - 2$

$\Rightarrow 2y = 4$

$\Rightarrow y = \frac{4}{2} = 2$

$Solution: (2,2)$

(iii) $Let\ x = 4 \ \Rightarrow 4 + 2y = 6$

$\Rightarrow 2y = 6 - 4$

$\Rightarrow 2y = 2$

$\Rightarrow y = \frac{2}{2} = 1$

$Solution: (4,1)$

(vi) $Let\ x = 6 \ \Rightarrow 6 + 2y = 6$

$\Rightarrow 2y = 6 - 6$

$\Rightarrow 2y = 0$

$\Rightarrow y = \frac{0}{2} = 0$

$Solution: (6,0)$

Example 4 : Find two solutions for each of the following equations:

$(\boldsymbol{i})$ $\boldsymbol{4x + 3y = 12}$

$Sol: Let\ x = o \Rightarrow 4 \times 0 + 3y = 12$

$\Rightarrow 3y = 12$

$\Rightarrow y = \frac{12}{3} = 4$

$Solution: (0,4)$

$Let\ y = 0 \Rightarrow 4x + 3 \times 0 = 12$

$\Rightarrow 4x = 12$

$\Rightarrow x = \frac{12}{4} = 3$

$Solution: (3,0)$

$(\boldsymbol{ii})$ $\boldsymbol{2x + 5y = 0}$

$Sol: Let\ x = o \Rightarrow 2 \times 0 + 5y = 0$

$\Rightarrow 5y = 0$

$\Rightarrow y = 0$

$Solution: (0,0)$

$Let\ x = 1 \Rightarrow 2 \times 1 + 5y = 0$

$\Rightarrow 2 + 5y = 0$

$\Rightarrow 5y = -2$

$\Rightarrow y = \frac{-2}{5}$

$Solution: \left(1, \frac{-2}{5}\right)$

$(\boldsymbol{iii})\quad \boldsymbol{3y + 4 = 0}$

$Sol: 3y + 4 = 0$

$\Rightarrow 3y = -4$

$\Rightarrow y = \frac{-4}{3}$

$Solutions: \left(0, \frac{-4}{3}\right), \left(1, \frac{-4}{3}\right)$

EXERCISE 4.2

1. Which one of the following options is true, and why? y = 3x + 5 has

(i) a unique solution, (ii) only two solutions, (iii) infinitely many solutions

Sol: (iii) infinitely many solutions is true

2. Write four solutions for each of the following equations:

$(\boldsymbol{i})\boldsymbol{2x + y = 7}$

$(a)\ Let\ x = 0 \ \Rightarrow 2 \times 0 + y = 7$

$\Rightarrow y = 7$

$Solution: (0,7)$

$(b)\ Let\ x = 2 \ \Rightarrow 2 \times 2 + y = 7$

$\Rightarrow 4 + y = 7$

$\Rightarrow y = 7 - 4$

$\Rightarrow y = 3$

$Solution: (2,3)$

$(c)\ Let\ x = 4 \ \Rightarrow 2 \times 4 + y = 7$

$\Rightarrow 8 + y = 7$

$\Rightarrow y = 7 - 8$

$\Rightarrow y = -1$

$Solution: (4, -1)$

$(d)\ Let\ y = 0 \ \Rightarrow 2x + 0 = 7$

$\Rightarrow 2x = 7$

$\Rightarrow x = \frac{7}{2}$

$Solution: \left(\frac{7}{2}, 0\right)$

$(\boldsymbol{ii})\boldsymbol{\pi x + y = 9}$

$(a)\ Let\ x = 0 \ \Rightarrow \pi \times 0 + y = 9$

$\Rightarrow 0 + y = 9$

$\Rightarrow y = 9$

$Solution: (0,9)$

$(b)\ Let\ x = 1 \Rightarrow \pi \times 1 + y = 9$

$\Rightarrow \pi + y = 69$

$\Rightarrow y = 9 - \pi$

$Solution: (1,9 - \pi)$

$(c)\ Let\ x = -1\ \Rightarrow \pi \times (-1) + y = 9$

$\Rightarrow -\pi + y = 69$

$\Rightarrow y = 9 + \pi$

$Solution: (-1,9 + \pi)$

$(d)\ Let\ \ y = 0\ \Rightarrow \pi x + 0 = 9$

$\Rightarrow \pi x = 9$

$\Rightarrow x = \frac{9}{\pi}$

$Solution: \left(\frac{9}{\pi}, 0\right)$

$\boldsymbol{(iii) x\ =\ 4y}$

$(a)\ Let\ x = 0\ \Rightarrow 0 - 4y = 0$

$\Rightarrow -4y = 0$

$\Rightarrow y = 0$

$Solution: (0,0)$

$(b)\ Let\ x = 4\ \Rightarrow 4 = 4y$

$\Rightarrow y = \frac{4}{4} = 1$

$Solution: (4,1)$

$(c)\ Let\ x = 2\ \Rightarrow 2 = 4y$

$\Rightarrow y = \frac{2}{4}$

$\Rightarrow y = \frac{1}{2}$

$Solution: \left(2, \frac{1}{2}\right)$

$(d)\ Let\ y = -1\ \Rightarrow x = 4 \times (-1)$

$\Rightarrow x = -4$

$Solution: (-4, -1)$

3. Check which of the following are solutions of the equation x – 2y = 4 and which are not:

$(i)(0,2)(ii)(2,0)(iii)(4,0)(iv)(\sqrt{2}, 4\sqrt{2})(v)(1,1)$

Sol: $(i)(0,2)$

$LHS = x - 2y = 0 - 2 \times 2 = 0 - 4 = -4 \neq RHS$

$\therefore (0,2)$ is not a solution to the equation.

(ii) $(2,0)$

$LHS = x - 2y = 2 - 2 \times 0 = 2 - 0 = 2 \neq RHS$

$\therefore (2,0)$ is not a solution to the equation.

(iii) $(4,0)$

$LHS = x - 2y = 4 - 2 \times 0 = 4 - 0 = 4 = RHS$

$\therefore (4,0)$ is a solution to the equation

(iv) $(\sqrt{2}, 4\sqrt{2})$

$LHS = x - 2y = \sqrt{2} - 2 \times 4\sqrt{2} = \sqrt{2} + 8\sqrt{2} = 9\sqrt{2} \neq RHS$

$\therefore (\sqrt{2}, 4\sqrt{2})$ is not a solution to the equation.

(v) $(1,1)$

$LHS = x - 2y = 1 - 2 \times 1 = 1 - 2 = -1 \neq RHS$

$\therefore (1,1)$ is not a solution to the equation.

4. **Find the value of k, if $x = 2, y = 1$ is a solution of the equation $2x + 3y = k$.**

Sol: Given equation: $2x + 3y = k$

if $x = 2, y = 1$ is a solution of the given equation then

$2 \times 2 + 3 \times 1 = k$

$4 + 3 = k$

$k = 7$