1.NUMBER SYSTEMS (NOTES)

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1. **Natural numbers**: The numbers which are used for counting are called Natural numbers and represented with letter N
2. Natural numbers N={1,2,3,4,5,……}
3. **Whole numbers**: If '0' is added to Natural numbers then they are called Whole numbers. And is denoted by 'W'
4. Whole numbers W={0,1,2,3,4,5,……}
5. **Integers**: Combination of positive and negative numbers including 0 are called Integers and represented by 'Z' or 'I'.
6. Integers Z=$\{\dots\dots -4, -3, -2, -1, 0, 1, 2, 3, 4, \dots\dots\dots\}$
7. Z comes from the German word "zahlen", which means "to count"
8. **Rational numbers:**

   A number which can be written in the form $\frac{p}{q}$, where p and q are integers and q $\neq$ 0 is called a rational number. Example: $-\frac{2}{3}, \frac{6}{7}, \frac{9}{-5}$ are all rational numbers. Since the numbers 0, –2, 4 can be written in the form $\frac{p}{q}$, they are also rational numbers.

Exp 1 : Are the following statements true or false? Give reasons for your answers.

(i) Every whole number is a natural number.

Sol: False, because zero is a whole number but not a natural number.

(ii) Every integer is a rational number.

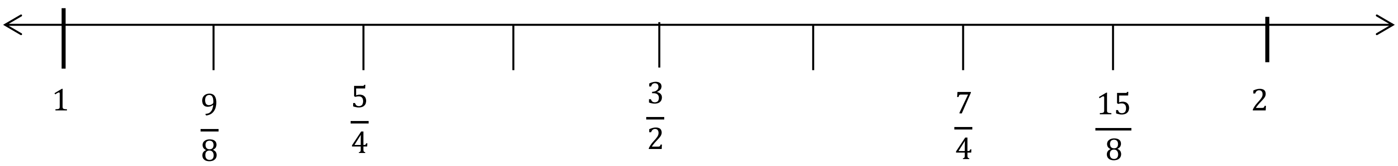
Sol: True, because every integer m can be expressed in the form $\frac{m}{1}$, and so it is a rational number

(iii) Every rational number is an integer.

Sol: False, because $\frac{3}{5}$ is a rational number but not an integer.

Exp 2 : Find five rational numbers between 1 and 2.

Sol 1: If a and b are two rational numbers then a rational number between a and $b = \frac{1}{2}(a+b)$



S.No	Two rational numbers	Between Rational number
1	1 and 2	$\frac{1}{2}(1+2) = \frac{1}{2}(3) = \frac{3}{2}$

2	1 and $\frac{3}{2}$	$\frac{1}{2}\left(1+\frac{3}{2}\right)=\frac{1}{2}\left(\frac{2+3}{2}\right)=\frac{1}{2}\times\frac{5}{2}=\frac{5}{4}$
3	$\frac{3}{2}$ and 2	$\frac{1}{2}\left(\frac{3}{2}+2\right)=\frac{1}{2}\left(\frac{3+4}{2}\right)=\frac{1}{2}\times\frac{7}{2}=\frac{7}{4}$
4	1 and $\frac{5}{4}$	$\frac{1}{2}\left(1+\frac{5}{4}\right)=\frac{1}{2}\left(\frac{4+5}{4}\right)=\frac{1}{2}\times\frac{9}{4}=\frac{9}{8}$
5	$\frac{7}{4}$ and 2	$\frac{1}{2}\left(\frac{7}{4}+2\right)=\frac{1}{2}\left(\frac{7+8}{4}\right)=\frac{1}{2}\times\frac{15}{4}=\frac{15}{8}$

So, the five rational numbers between 1 and 2 are $\frac{9}{8},\frac{5}{4},\frac{3}{2},\frac{7}{4},\frac{15}{8}$

Sol 2:

$$1 < 2 \Rightarrow \frac{1\times 6}{1\times 6} < \frac{2\times 6}{1\times 6} \Rightarrow \frac{6}{6} < \frac{12}{6}$$

$$\Rightarrow \frac{6}{6} < \frac{7}{6} < \frac{8}{6} < \frac{9}{6} < \frac{10}{6} < \frac{11}{6} < \frac{12}{6}$$

So, the five rational numbers are $\frac{7}{6},\frac{8}{6},\frac{9}{6},\frac{10}{6},\frac{11}{6} \Rightarrow \frac{7}{6},\frac{4}{3},\frac{3}{2},\frac{5}{3},\frac{11}{6}$

There are infinitely many rational numbers between any two given rational numbers

EXERCISE 1.1

1. Is zero a rational number? Can you write it in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$?

Sol: yes, zero is a rational number. $0 = \frac{0}{1}$

2. Find six rational numbers between 3 and 4.

Sol: $3 < 4 \Rightarrow \frac{3\times7}{1\times7} < \frac{4\times7}{1\times7} \Rightarrow \frac{21}{7} < \frac{28}{7}$

$$\frac{21}{7} < \frac{22}{7} < \frac{23}{7} < \frac{24}{7} < \frac{25}{7} < \frac{26}{7} < \frac{27}{7} < \frac{28}{7}$$

So, the six rational numbers are $\frac{22}{7},\frac{23}{7},\frac{24}{7},\frac{25}{7},\frac{26}{7},\frac{27}{7}$

3. Find five rational numbers between $\frac{3}{5}$ and $\frac{4}{5}$.

Sol: $\frac{3}{5} < \frac{4}{5}$

$$\Rightarrow \frac{3\times 6}{5\times 6} < \frac{4\times 6}{5\times 6}$$

$$\Rightarrow \frac{18}{30} < \frac{24}{30}$$

$$\Rightarrow \frac{18}{30} < \frac{19}{30} < \frac{20}{30} < \frac{21}{30} < \frac{22}{30} < \frac{23}{30} < \frac{24}{30}$$

So, the five rational numbers between $\frac{3}{5}$ and $\frac{4}{5}$ are $\frac{19}{30},\frac{20}{30},\frac{21}{30},\frac{22}{30},\frac{23}{30}$

4. State whether the following statements are true or false. Give reasons for your answers.

(i) Every natural number is a whole number.

Sol: True.

(ii) Every integer is a whole number.

Sol: False, -5 is an integer but not a whole number

(iii) Every rational number is a whole number.

Sol: False, because $\frac{4}{5}$ is a rational number but not a whole number.

Irrational Numbers

The Pythagoreans in Greece were the first to discover the numbers which were not rationals. These numbers are called irrational numbers

A number cannot be written in the form $\frac{p}{q}$, where p and q are integers and q ≠ 0 is called irrational.

Examples: $\sqrt{2}, \sqrt{5}, \pi, 0.101001000 \ldots$ ***etc***

Real numbers (R) : Collection of both rational (Q) and irrational numbers (Q^1)

Every real number is represented by a unique point on the number line. Also, every point on the number line represents a unique real number.

Exp 3 : Locate $\sqrt{2}$ on the number line.

Sol: 1.Draw number line. Point O at 0 and Point A at 1.

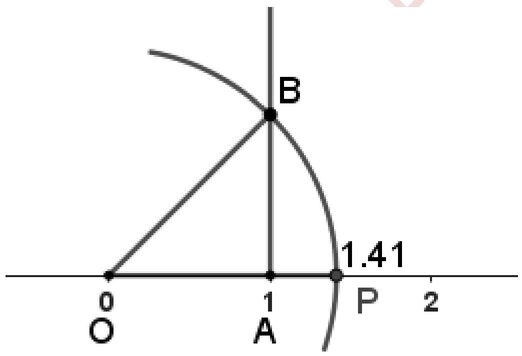
2. Construct AB= 1 unit perpendicular to number line at A

3. Join OB

4. From Pythagoras theorem OB=$\sqrt{1^2+1^2} = \sqrt{1+1} = \sqrt{2}$

4. Draw an arc with centre O and radius OB, intersects number line at P.

5. The point P corresponds to $\sqrt{2}$ on the number line.



Exp 4 : Locate $\sqrt{3}$ on the number line.

Sol: 1. Draw number line. Point O at 0 and Point A at 1.

2. Construct AB= 1 unit perpendicular to number line at A

3. Join OB

4. From Pythagoras theorem OB=$\sqrt{1^2+1^2} = \sqrt{1+1} = \sqrt{2}$

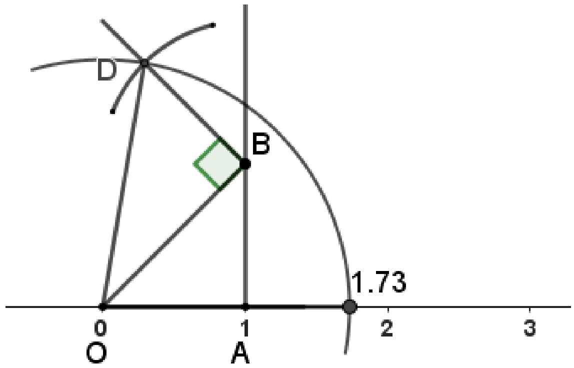
5. Construct BD of unit length perpendicular to OB.

6. Join OD.

7. From Pythagoras theorem OD=$\sqrt{(\sqrt{2})^2 + 1^2} = \sqrt{2+1} = \sqrt{3}$

8. Draw an arc with centre O and radius OD, intersects number line at Q.

9. The point Q corresponds to $\sqrt{3}$ on the number line.



EXERCISE 1.2

1. State whether the following statements are true or false. Justify your answers.

(i) Every irrational number is a real number.

Sol: yes

(ii) Every point on the number line is of the form $\sqrt{m}$, where m is a natural number.

Sol: False , all negative numbers on the number line but it not express as of the form $\sqrt{m}$, where m is a natural number

(iii) Every real number is an irrational number.

Sol: False, real numbers are Collection of both rational (Q) and irrational numbers (Q^1)

2. Are the square roots of all positive integers irrational? If not, give an example of the square root of a number that is a rational number.

Sol: False, because 4 is a positive integer and $\sqrt{4} = \pm 2$ are rational numbers.

3. Show how $\sqrt{5}$ can be represented on the number line.

Sol: 1.Draw number line. Point O at 0 and Point A at 2.

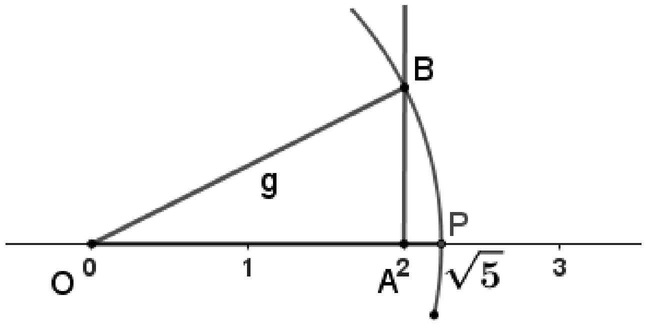
2. Construct AB= 1 unit perpendicular to number line at A

3. Join OB

4. From Pythagoras theorem OB=$\sqrt{2^2 + 1^2} = \sqrt{4+1} = \sqrt{5}$

4. Draw an arc with centre O and radius OB, intersects number line at P.

5. The point P corresponds to $\sqrt{5}$ on the number line..



Real Numbers and their Decimal Expansions

Exp 5 : Find the decimal expansions of $\frac{10}{3}, \frac{7}{8}$ *and* $\frac{1}{7}$

$$3\overline{)10.000\ldots}\;=3.333\ldots:\quad 9,\ 10,\ 9,\ 10,\ 9,\ 1$$

$$8\overline{)7.000\ldots}\;=0.875:\quad 64,\ 60,\ 56,\ 40,\ 40,\ 0$$

$$7\overline{)1.000000}\;=0.142857\ldots:\quad 7,\ 30,\ 28,\ 20,\ 14,\ 60,\ 56,\ 40,\ 35,\ 50,\ 49,\ 1$$

$\frac{10}{3} = 3.333.. = 3.\overline{3}$; $\frac{7}{8} = 0.785$

$\frac{1}{7} = 0.142857142\ldots = 0.\overline{142857}$

Terminating decimal: A decimal number that contains a finite number of digits next to the decimal point is called a Terminating decimal

Non terminating recurring decimal: A Non terminating recurring decimal is a decimal in which some digits after the decimal point repeat without terminating.

Example 6 : Show that 3.142678 is a rational number. In other words, express 3.142678 in the form $\frac{p}{q}$, where p and q are integers and q ≠ 0.

Sol: $3.142678 = \frac{3142678}{1000000}$, and hence it is a rational numbers

Example 7 : Show that $0.3333\ldots = 0.\overline{3}$ can be expressed in the form $\frac{p}{q}$, where p and q are integers and q ≠ 0

Sol: $Let\ x = 0.\overline{3}$

$10x = 3.333\ldots.$

$10x = 3 + 0.3333\ldots.$

$10x = 3 + x$

$10x - x = 3$

$Let\ x = 0.\overline{3} = 0.333\ldots\ldots.. \rightarrow (1)$

$10x = 3.333\ldots. \rightarrow (2)$

From (2)-(1)

$10x = 3.333\ldots\ldots.. \rightarrow (2)$

$x = 0.333\ldots\ldots.. \rightarrow (1)$

$9x = 3$

$x = \frac{3}{9} = \frac{1}{3} \Rightarrow 0.\overline{3} = \frac{1}{3}$

$9x = 3$

$x = \frac{3}{9} = \frac{1}{3} \Rightarrow 0.\overline{3} = \frac{1}{3}$

Example 8 : Show that $1.272727... = 1.\overline{27}$ can be expressed in the form $\frac{p}{q}$, where p and q are integers and q ≠ 0.

Sol: $Let\ x = 1.\overline{27}$

$x = 1.272727 ....$

$100x = 127.272727 ....$

$100x = 126 + 1.272727 ...$

$100x = 126 + x$

$100x - x = 126$

$99x = 126$

$x = \frac{126}{99} = \frac{14}{11}$

$1.\overline{27} = \frac{14}{11}$

$Let\ x = 1.\overline{27} = 1.272727 .... \rightarrow (1)$

$100x = 127.272727 .... \rightarrow (2)$

From (2)-(1)

$100x = 127.272727 .... \rightarrow (2)$

$x = \quad 1.272727 .... \rightarrow (1)$

$99x = 126$

$x = \frac{126}{99} = \frac{14}{11} \Rightarrow 1.\overline{27} = \frac{14}{11}$

Example 9 : Show that $0.2353535... = 0.2\overline{35}$. can be expressed in the form p q , where p and q are integers and q ≠ 0.

Sol: $Let\ x = 0.2\overline{35}$

$x = 0.2353535 ...$

$100x = 23.53535 .....$

$100x = 23.3 + 0.23535 ...$

$1000x = 23.3 + x$

$100x - x = 23.3$

$99x = 23.3$

$x = \frac{23.3}{99} = \frac{233}{990}$

$0.2\overline{35} = \frac{233}{990}$

$Let\ x = 0.2\overline{35} = 0.2353535 ... \rightarrow (1)$

$100x = 235.3535 ..... \rightarrow (2)$

From (2)-(1)

$100x = 23.53535 ..... \rightarrow (2)$

$x = 0.2353535 .... \rightarrow (1)$

$99x = 23.3$

$x = \frac{23.3}{99} = \frac{233}{990} \Rightarrow 0.2\overline{35} = \frac{233}{990}$

Irrational: A number whose decimal expansion is non-terminating non-recurring is irrational.

Examples: $\sqrt{2}, \sqrt{5}, \pi, 0.101001000 ...\ \ etc$

Exp10 : Find an irrational number between $\frac{1}{7}$ *and* $\frac{2}{7}$.

Sol: $\frac{1}{7} = 0.142857 ... .$

$\frac{2}{7} = 0.285714 \ldots.$

Irrational number is non-terminating non-recurring decimal

An irrational number between $\frac{1}{7}$ *and* $\frac{2}{7}$ *is* $0.152000200002 0000 \ldots..$

EXERCISE 1.3

1. **Write the following in decimal form and say what kind of decimal expansion each has**

(i) $\frac{36}{100} = 0.36$

Terminating decimal

(ii) $\frac{1}{11} = 0.090909.. = 0.\overline{09}$

Non terminating recurring decimal

(iii) $4\frac{1}{8} = \frac{33}{8} = 4.125$

Terminating decimal.

(iv) $\frac{3}{13} = 0.23076923 \ldots = 0.\overline{230769}$

Non terminating recurring decimal

(v) $\frac{2}{11} = 0.1818.. = 0.\overline{18}$

Non terminating recurring decimal

(vi) $\frac{329}{400} = 0.8225$

Terminating decimal

```
      0.090909 ...
11 ) 1.000000
      99
      100
       99
       100
        99
         1
```

```
     4.125
8 ) 33.000
    32
    10
     8
     20
     16
      40
      40
       0
```

```
      0.2307692..
13 ) 3.00000000
     26
      40
      39
      10
      00
      100
       91
        90
        78
        120
        117
         30
         26
          4
```

```
      0.1818 ...
11 ) 2.00000..
     11
      90
      88
      20
      11
       90
       88
        0
```

```
     0.8225
4 ) 3.29000
    32
     09
      8
      10
       8
       20
       20
        0
```

2. **You know that** $\frac{1}{7} = \mathbf{0.\overline{142857}}$**.. Can you predict what the decimal expansions of** $\frac{2}{7}, \frac{3}{7}, \frac{4}{7}, \frac{5}{7}, \frac{6}{7}$ **are, without actually doing the long division? If so, how?**

Sol: When divided 1 by 7 remainders are 3,2,6,4,5.

$\frac{1}{7} = 0.\overline{142857}$

2 is a remainder after the second step. So, we write the quatient after the second decimal place

$\frac{2}{7} = 0.\overline{285714}$

$\frac{3}{7} = 0.\overline{428571}$

$\frac{4}{7} = 0.\overline{571428}$

$\frac{5}{7} = 0.\overline{714285}$

```
     0.1428571..
7 ) 1.00000000
     7
     30
     28
      20
      14
       60
       56
        40
        35
         50
         35
          50
          49
           10
            7
            3
```

3. Express the following in the form $\frac{p}{q}$, where p and q are integers and q ≠ 0

(i) $0.\bar{6}$

$sol: Let\ x = 0.\bar{6}$

$x = 0.66666 \ldots..$

$10x = 6.6666 \ldots.$

$10x = 6 + 0.6666 \ldots.$

$10x = 6 + x$

$10x - x = 6$

$9x = 6$

$x = \frac{6}{9} = \frac{2}{3} \Rightarrow 0.\bar{6} = \frac{2}{3}$

$Let\ x = 0.\bar{6} = 0.66666 \ldots.. \rightarrow (1)$

$10x = 6.6666 \ldots. \rightarrow (2)$

From (2)-(1)

$10x = 6.6666 \ldots. \rightarrow (2)$

$x = 0.66666 \ldots.. \rightarrow (1)$

$9x = 6$

$x = \frac{6}{9} = \frac{2}{3} \Rightarrow 0.\bar{6} = \frac{2}{3}$

(ii) $0.4\bar{7}$

$sol: Let\ x = 0.4\bar{7}$

$x = 0.477777 \ldots.$

$10x = 4.777777 \ldots.$

$10x = 4.3 + 0.4777777 \ldots$

$10x = 4.3 + x$

$10x - x = 4.3$

$9x = 4.3$

$x = \frac{4.3}{9} = \frac{43}{90}$

$0.4\bar{7} = \frac{43}{90}$

$Let\ x = 0.4\bar{7} = 0.477777 \ldots. \rightarrow (1)$

$10x = 4.777777 \ldots \rightarrow (2)$

From (2)-(1)

$10x = 4.777777 \ldots \rightarrow (2)$

$x = 0.477777 \ldots.. \rightarrow (1)$

$9x = 4.3$

$x = \frac{4.3}{9} = \frac{43}{90} \Rightarrow 0.4\bar{7} = \frac{43}{90}$

(iii) $0.\overline{001}$

$Sol: Let\ x = 0.\overline{001}$

$x = 0.001001001 \ldots.$

$1000x = 1.001001001 \ldots$

$1000x = 1 + 0.001001001 \ldots$

$1000x = 1 + x$

$1000x - x = 1$

$999x = 1$

$x = \frac{1}{999}$

$0.\overline{001} = \frac{1}{999}$

4. Express 0.99999 .... in the form p q . Are you surprised by your answer? With your teacher and classmates discuss why the answer makes sense.

$sol: Let\ x = 0.999999 \ldots.$

$10x = 9.9999 \ldots$

$10x = 9 + 0.999 \ldots$

$10x = 9 + x$

$10x - x = 9$

$9x = 9$

$x = \frac{9}{9} = 1$

$0.9999 \ldots = 1$

5. **What can the maximum number of digits be in the repeating block of digits in the decimal expansion of $\frac{1}{17}$? Perform the division to check your answer.**

```
    0.0588235294117647..
17)1.00000000
    85
    150
    136
      40
      34
       60
       51
        90
        85
        50
        34
        160
        153
          70
          68
          20
          17
          130
          119
           110
           102
             80
             68
             120
             119
               1
```

6. **Look at several examples of rational numbers in the form $\frac{p}{q}$ ($q \neq 0$), where p and q are integers with no common factors other than 1 and having terminating decimal representations (expansions). Can you guess what property q must satisfy?**

$Sol: \frac{1}{2} = 0.5 \quad \frac{1}{10} = 0.1 \quad \frac{32}{5} = 6.4 \quad \frac{5}{8} = 0.625 \quad \frac{27}{25} = 1.08 \quad \frac{3}{50} = 0.06 \quad \frac{7}{20} = 0.35$

$The\ q\ (denominator) is\ in\ the\ form\ of\ \ \mathbf{2^a \times 5^b}\ where\ a, b\ are\ whole\ numbers.$

7. **Write three numbers whose decimal expansions are non-terminating non-recurring**

Sol: (i) 0.51250535420062101254.....

(ii) 1.20200200020000....

(iii) 0.2012011201112310....

8. **Find three different irrational numbers between the rational numbers $\frac{5}{7}$ *and* $\frac{9}{11}$**

Sol: $\frac{5}{7} = 0.714285 \ldots \ldots ..$

$\frac{9}{11} = 0.8181 \ldots ..$

Three irrational numbers between the rational numbers $\frac{5}{7}$ *and* $\frac{9}{11}$ *are*

(i) 0.722020020002000 …

(ii) 0.73030030003000 ….

(iii) 0.7515115111511125 ….

9. Classify the following numbers as rational or irrational :

(i) $\sqrt{23} \rightarrow$ Irrational number

(ii) $\sqrt{225} = 15 \rightarrow$ Rational number

(iii) $0.3796 = \frac{3796}{10000} \rightarrow$ Rational number

(iv) $7.478478 \ldots = 7.\overline{478} \rightarrow$ Rational number

(v)$1.101001000100001\ldots \rightarrow$ Irrational numbe

Operations on Real Numbers

Example 11 : Check whether $7\sqrt{5}$, $\frac{7}{\sqrt{5}}$, $\sqrt{2}+21$, $\pi - 2$ are irrational numbers or not

Sol: $\sqrt{5} = 2.2360679 \ldots$

$$\frac{7}{\sqrt{5}} = \frac{7 \times \sqrt{5}}{\sqrt{5} \times \sqrt{5}} = \frac{7\sqrt{5}}{5} = \frac{15.656753..}{5} = 3.1304 \ldots$$

$\sqrt{2} + 21 = 1.414213.. + 21 = 22.414213 \ldots$

$\pi - 2 = 3.1415 \ldots - 2 = 1.1415 \ldots.$

All these are non-terminating, non-recurring decimals. Thus they are irrational numbers.

If q is rational and s is irrational then $q + s, q - s, qs$ *and* $\frac{q}{s} (s \neq 0)$ are irrational numbers.

Example 12 : Add $2\sqrt{2} + 5\sqrt{3}$ *and* $\sqrt{2} - 3\sqrt{3}$

Sol: $(2\sqrt{2} + 5\sqrt{3}) + (\sqrt{2} - 3\sqrt{3}) = 2\sqrt{2} + \sqrt{2} + 5\sqrt{3} - 3\sqrt{3} = 3\sqrt{2} + 2\sqrt{3}$

Example 13 : Multiply $6\sqrt{5}$ by $2\sqrt{5}$.

Sol: $6\sqrt{5} \times 2\sqrt{5} = 6 \times 2 \times \sqrt{5} \times \sqrt{5} = 12 \times 5 = 60 \quad (\sqrt{a} \times \sqrt{a} = a)$

Example 14 : Divide $8\sqrt{15}$ by $2\sqrt{3}$

Sol: $$\frac{8\sqrt{15}}{2\sqrt{3}} = \frac{4 \times 2 \times \sqrt{3} \times \sqrt{5}}{2 \times \sqrt{3}} = 4\sqrt{5}$$

Note: (i) The sum or difference of a rational number and an irrational number is irrational. (ii) The product or quotient of a non-zero rational number with an irrational number is irrational. (iii) If we add, subtract, multiply or divide two irrationals, the result may be rational or irrational.

List some properties relating to square roots

Let a and b be positive real numbers. Then

$(i)\ \sqrt{ab} = \sqrt{a} \times \sqrt{b} \quad ; \sqrt{a} \times \sqrt{a} = \sqrt{a^2} = \left(\sqrt{a}\right)^2 = a$

$(ii) \sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}} : if\ b \neq 0$

$(iii)\ \left(\sqrt{a} + \sqrt{b}\right)\left(\sqrt{a} - \sqrt{b}\right) = a - b$

$(iv)\ \left(a + \sqrt{b}\right)\left(a - \sqrt{b}\right) = a^2 - b$

$(v)\ \left(\sqrt{a} + \sqrt{b}\right)\left(\sqrt{c} + \sqrt{d}\right) = \sqrt{a} \times \sqrt{c} + \sqrt{a} \times \sqrt{d} + \sqrt{b} \times \sqrt{c} + \sqrt{b} \times \sqrt{d}$

$= \sqrt{ac} + \sqrt{ad} + \sqrt{bc} + \sqrt{bd}$

$(vi)\ \left(\sqrt{a} + \sqrt{b}\right)^2 = a + 2\sqrt{ab} + b$

$(vii)\ \left(\sqrt{a} - \sqrt{b}\right)^2 = a - 2\sqrt{ab} + b$

Example 15 : Simplify the following expressions:

$(\boldsymbol{i})\quad \left(\mathbf{5} + \sqrt{\mathbf{7}}\right)\left(\mathbf{2} + \sqrt{\mathbf{5}}\right)$

sol: $\left(5 + \sqrt{7}\right)\left(2 + \sqrt{5}\right) = 5 \times 2 + 5 \times \sqrt{5} + \sqrt{7} \times 2 + \sqrt{7} \times \sqrt{5}$

$= 10 + 5\sqrt{5} + 2\sqrt{7} + \sqrt{35}$

$(\boldsymbol{ii})\quad \left(\mathbf{5} + \sqrt{\mathbf{5}}\right)\left(\mathbf{5} - \sqrt{\mathbf{5}}\right)$

Sol: $(x + y)(x - y) = x^2 - y^2$

$\left(5 + \sqrt{5}\right)\left(5 - \sqrt{5}\right) = 5^2 - \left(\sqrt{5}\right)^2 = 25 - 5 = 20$

$(\boldsymbol{iii})\quad \left(\sqrt{\mathbf{3}} + \sqrt{\mathbf{7}}\right)^{\mathbf{2}}$

Sol: $(x + y)^2 = x^2 + 2xy + y^2$

$\left(\sqrt{3} + \sqrt{7}\right)^2 = \left(\sqrt{3}\right)^2 + 2 \times \sqrt{3} \times \sqrt{7} + \left(\sqrt{7}\right)^2$

$= 3 + 2\sqrt{21} + 7 = 10 + 2\sqrt{21}$

$(\boldsymbol{iv})\quad \left(\sqrt{\mathbf{11}} - \sqrt{\mathbf{7}}\right)\left(\sqrt{\mathbf{11}} + \sqrt{\mathbf{7}}\right)$

Sol: $(x - y)(x + y) = x^2 - y^2$

$\left(\sqrt{11} - \sqrt{7}\right)\left(\sqrt{11} + \sqrt{7}\right) = \left(\sqrt{11}\right)^2 - \left(\sqrt{7}\right)^2 = 11 - 7 = 4$

Example* 16: *Rationalise the denominator of $\frac{1}{\sqrt{2}}$

Sol: Rationalise factor of $\sqrt{2} = \sqrt{2}$

$$\frac{1}{\sqrt{2}} = \frac{1 \times \sqrt{2}}{\sqrt{2} \times \sqrt{2}} = \frac{\sqrt{2}}{2}$$

Example 17: Rationalise the denominator of $\frac{1}{2+\sqrt{3}}$

Sol: Rationalise factor of $2+\sqrt{3} = 2-\sqrt{3}$

$$\frac{1}{2+\sqrt{3}} = \frac{1}{2+\sqrt{3}} \times \frac{2-\sqrt{3}}{2-\sqrt{3}} = \frac{2-\sqrt{3}}{2^2-(\sqrt{3})^2} = \frac{2-\sqrt{3}}{4-3} = \frac{2-\sqrt{3}}{1} = 2-\sqrt{3}$$

Example 18 : Rationalise the denominator of $\frac{5}{\sqrt{3}-\sqrt{5}}$

Sol: Rationalise factor of $\sqrt{3}-\sqrt{5} = \sqrt{3}+\sqrt{5}$

$$\frac{5}{\sqrt{3}-\sqrt{5}} = \frac{5}{\sqrt{3}-\sqrt{5}} \times \frac{\sqrt{3}+\sqrt{5}}{\sqrt{3}+\sqrt{5}} = \frac{5(\sqrt{3}+\sqrt{5})}{(\sqrt{3})^2-(\sqrt{5})^2}$$

$$= \frac{5(\sqrt{3}+\sqrt{5})}{3-5} = \frac{5(\sqrt{3}+\sqrt{5})}{-2} = \frac{-5(\sqrt{3}+\sqrt{5})}{2}$$

Example 19 : Rationalise the denominator of $\frac{1}{7+3\sqrt{2}}$

Sol: Rationalise factor of $7+3\sqrt{2} = 7-3\sqrt{2}$

$$\frac{1}{7+3\sqrt{2}} = \frac{1}{7+3\sqrt{2}} \times \frac{7-3\sqrt{2}}{7-3\sqrt{2}} = \frac{7-3\sqrt{2}}{(7)^2-(3\sqrt{2})^2} = \frac{7-3\sqrt{2}}{49-9\times 2} = \frac{7-3\sqrt{2}}{49-18} = \frac{7-3\sqrt{2}}{31}$$

EXERCISE 1.4

1. Classify the following numbers as rational or irrational:

(i) $\mathbf{2-\sqrt{5}}$ → Irrational number

(ii) $\mathbf{(3+\sqrt{23})-\sqrt{23}} = 3+\sqrt{23}-\sqrt{23} = 3$ → Rational number

(iii) $\frac{2\sqrt{7}}{7\sqrt{7}} = \frac{2}{7}$ → Rational number

(iv) $\frac{\mathbf{1}}{\sqrt{\mathbf{2}}} = \frac{1\times\sqrt{2}}{\sqrt{2}\times\sqrt{2}} = \frac{\sqrt{2}}{2}$ → *Irrational number*

(v) 2π → *Irrational number*

2. Simplify each of the following expressions:

(i) $(3+\sqrt{3})(2+\sqrt{2})$

Sol: $(3+\sqrt{3})(2+\sqrt{2}) = 3\times 2 + 3\times\sqrt{2} + \sqrt{3}\times 2 + \sqrt{3}\times\sqrt{2}$

$$= 6 + 3\sqrt{2} + 2\sqrt{3} + \sqrt{6}$$

$(ii)\ \ (\mathbf{3+\sqrt{3}})(\mathbf{3-\sqrt{3}})$

$Sol: (a+b)(a-b) = a^2 - b^2$

$$(3+\sqrt{3})(3-\sqrt{3}) = 3^2 - (\sqrt{3})^2 = 9 - 3 = 6$$

$(iii)\ \ (\sqrt{\mathbf{5}}+\sqrt{\mathbf{2}})^{\mathbf{2}}$

$Sol:\ (a+b)^2 = a^2 + 2ab + b^2$

$$(\sqrt{5}+\sqrt{2})^2 == (\sqrt{5})^2 + 2\times\sqrt{5}\times\sqrt{2} + (\sqrt{2})^2 = 5 + 2\sqrt{10} + 2 = 7 + 2\sqrt{10}$$

$(iv)\ \ (\sqrt{\mathbf{5}}-\sqrt{\mathbf{2}})(\sqrt{\mathbf{5}}+\sqrt{\mathbf{2}})$

$Sol:\ (x-y)(x+y) = x^2 - y^2$

$$(\sqrt{5}-\sqrt{2})(\sqrt{5}+\sqrt{2}) = (\sqrt{5})^2 - (\sqrt{2})^2 = 5 - 2 = 3$$

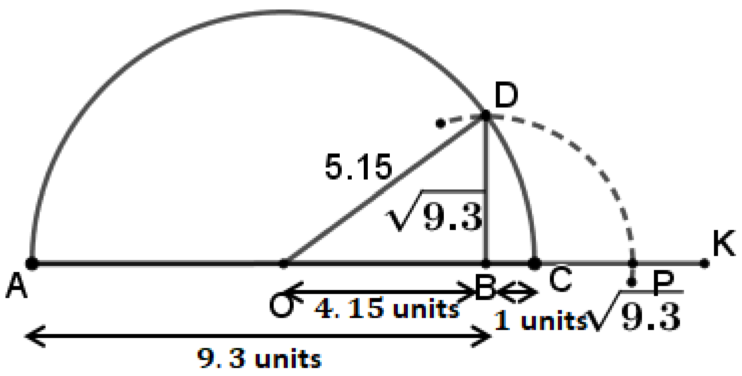
3. **Recall, π is defined as the ratio of the circumference (say c) of a circle to its diameter (say d). That is, π = $\frac{c}{d}$· This seems to contradict the fact that π is irrational. How will you resolve this contradiction?**

$Sol:\ we\ use\ \pi = \frac{22}{7}\ or\ 3.14\ these\ are\ approximate\ values.$

The actual value of π is 3.141592653589….which is non-terminating non-recurring. Hence π is an irrational number.

4. Represent $\sqrt{9.3}$ on the number line.

Sol:



5. **Rationalise the denominators of the following**:

$(i)\ \frac{1}{\sqrt{7}}$

Sol: Rationalise factor of $\sqrt{7} = \sqrt{7}$

$$\frac{1}{\sqrt{7}} = \frac{1\times\sqrt{7}}{\sqrt{7}\times\sqrt{7}} = \frac{\sqrt{7}}{7}$$

$(ii)\ \frac{\mathbf{1}}{\sqrt{\mathbf{7}}-\sqrt{\mathbf{6}}}$

Sol: Rationalise factor of $\sqrt{7}-\sqrt{6}=\sqrt{7}+\sqrt{6}$

$$\frac{1}{\sqrt{7}-\sqrt{6}}=\frac{1}{\sqrt{7}-\sqrt{6}}\times\frac{\sqrt{7}+\sqrt{6}}{\sqrt{7}+\sqrt{6}}=\frac{\sqrt{7}+\sqrt{6}}{(\sqrt{7})^2-(\sqrt{6})^2}=\frac{\sqrt{7}+\sqrt{6}}{7-6}=\frac{\sqrt{7}+\sqrt{6}}{1}=\sqrt{7}+\sqrt{6}$$

$(\boldsymbol{iii})\ \dfrac{\mathbf{1}}{\sqrt{\mathbf{5}}+\sqrt{\mathbf{2}}}$

Sol: Rationalise factor of $\sqrt{5}+\sqrt{2}=\sqrt{5}-\sqrt{2}$

$$\frac{1}{\sqrt{5}+\sqrt{2}}=\frac{1}{\sqrt{5}+\sqrt{2}}\times\frac{\sqrt{5}-\sqrt{2}}{\sqrt{5}-\sqrt{2}}=\frac{\sqrt{5}-\sqrt{2}}{(\sqrt{5})^2-(\sqrt{2})^2}=\frac{\sqrt{5}-\sqrt{2}}{5-2}=\frac{\sqrt{5}-\sqrt{2}}{3}$$

$(\boldsymbol{iv})\dfrac{\mathbf{1}}{\sqrt{\mathbf{7}}-\mathbf{2}}$

Sol: Rationalise factor of $\sqrt{7}-2=\sqrt{7}+2$

$$\frac{1}{\sqrt{7}-2}=\frac{1}{\sqrt{7}-2}\times\frac{\sqrt{7}+2}{\sqrt{7}+2}=\frac{\sqrt{7}+2}{(\sqrt{7})^2-(2)^2}=\frac{\sqrt{7}+2}{7-2}=\frac{\sqrt{7}+2}{5}$$

Laws of Exponents for Real Numbers

(i) $a^m\times a^n=a^{m+n}$

(ii) $\dfrac{a^m}{a^n}=a^{m-n}$

(iii) $(a^m)^n=a^{mn}$

(iv) $a^m\times b^m=(ab)^m$

(v) $\dfrac{a^m}{b^m}=\left(\dfrac{a}{b}\right)^m$

(vi) $\dfrac{1}{a^m}=a^{-m}$

(vii) $\dfrac{1}{a^{-m}}=a^m$

$(viii)$ $a^0=1$

(ix) $\left(\dfrac{a}{b}\right)^{-m}=\left(\dfrac{b}{a}\right)^m$

Example 20 : Simplify

$(\boldsymbol{i})$ $\mathbf{2^{\frac{2}{3}}.2^{\frac{1}{3}}}$

Sol: $2^{\frac{2}{3}}.2^{\frac{1}{3}}=2^{\frac{2}{3}+\frac{1}{3}}=2^{\frac{2+1}{3}}=2^{\frac{3}{3}}=2^1=2$

$(\boldsymbol{ii})$ $\left(\mathbf{3^{\frac{1}{5}}}\right)^{\mathbf{4}}$

Sol: $\left(3^{\frac{1}{5}}\right)^4=3^{\frac{1}{5}\times4}=3^{\frac{4}{5}}$

$(\boldsymbol{iii})$ $\dfrac{\mathbf{7^{\frac{1}{5}}}}{\mathbf{7^{\frac{1}{3}}}}$

Sol: $\dfrac{7^{\frac{1}{5}}}{7^{\frac{1}{3}}}=7^{\frac{1}{5}-\frac{1}{3}}=7^{\frac{3-5}{15}}=7^{\frac{-2}{15}}$

(iv) $\mathbf{13^{\frac{1}{5}}.17^{\frac{1}{5}}}$

Sol: $13^{\frac{1}{5}}.17^{\frac{1}{5}} = (13 \times 17)^{\frac{1}{5}} = 221^{\frac{1}{5}}$

EXERCISE 1.5

1. **Find**

   (i) $64^{\frac{1}{2}} = (8^2)^{\frac{1}{2}} = 8^{2\times\frac{1}{2}} = 8$

   (ii) $32^{\frac{1}{5}} = (2^5)^{\frac{1}{5}} = 2^{5\times\frac{1}{5}} = 2$

   (iii) $125^{\frac{1}{3}} = (5^3)^{\frac{1}{3}} = 5^{3\times\frac{1}{3}} = 5$

2. **Find**

   (i) $9^{\frac{3}{2}} = (3^2)^{\frac{3}{2}} = 3^{2\times\frac{3}{2}} = 3^3 = 27$

   (ii) $32^{\frac{2}{5}} = (2^5)^{\frac{2}{5}} = 2^{5\times\frac{2}{5}} = 2^2 = 4$

   (iii) $16^{\frac{3}{4}} = (2^4)^{\frac{3}{4}} = 2^{4\times\frac{3}{4}} = 2^3 = 8$

   (iv) $125^{\frac{-1}{3}} = (5^3)^{\frac{-1}{3}} = 5^{3\times\frac{-1}{3}} = 5^{-1} = \frac{1}{5}$

3. **Simplify**

   (i) $2^{\frac{2}{3}}.2^{\frac{1}{5}} = 2^{\frac{2}{3}+\frac{1}{5}} = 2^{\frac{10+3}{15}} = 2^{\frac{13}{15}}$

   (ii) $\left(\frac{1}{3^3}\right)^7 = \frac{1^7}{(3^3)^7} = \frac{1}{3^{21}} =$

   (iii) $\frac{11^{\frac{1}{2}}}{11^{\frac{1}{4}}} = 11^{\frac{1}{2}-\frac{1}{4}} = 11^{\frac{2-1}{4}} = 11^{\frac{1}{4}}$

   (iv) $7^{\frac{1}{2}}.8^{\frac{1}{2}} = (7 \times 8)^{\frac{1}{2}} = 56^{\frac{1}{2}}$