CHAPTER 7

IX-MATHEMATICS-NCERT-2023-24

7. TRIANGLES (notes)

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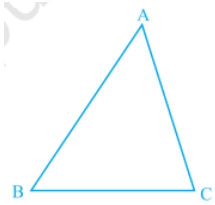
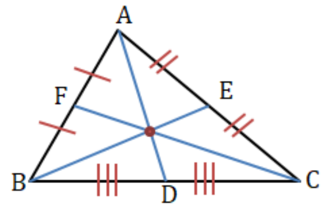


1. A closed figure formed by three intersecting lines is called a triangle.
2. A triangle has three sides, three angles and three vertices.
3. AB, BC, CA are the three sides, ∠ A, ∠ B, ∠ C are the three angles and A, B, C are three vertices
4. Triangle ABC, denoted as Δ ABC



5. **Median**:A median connects a vertex of a triangle to the mid-point of the opposite side.
6. **congruent figures**: The figures that have the same shape and size are called congruent figures

   Ex: (i) Two circles of the same radii are congruent

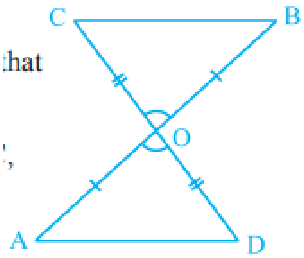
   (ii) Two squares of the same sides are congruent.
7. The two triangles are congruent If the sides and angles of one triangle are equal to the corresponding sides and angles of the other triangle.
8. If Δ PQR is congruent to Δ ABC, we write Δ PQR ≅ Δ ABC.
9. FD ↔ AB, DE ↔ BC and EF ↔ CA and F ↔ A, D ↔ B and E ↔ C .So, Δ FDE ≅ Δ ABC.
10. Congruent triangles corresponding parts are equal and we write in short 'CPCT' for corresponding parts of congruent triangles.

Criteria for Congruence of Triangles

(SAS congruence rule) : Two triangles are congruent if two sides and the included angle of one triangle are equal to the two sides and the included angle of the other triangle

SAS congruence rule holds but not ASS or SSA rule.

Exp 1 : In Fig. 7.8, OA = OB and OD = OC. Show that (i) Δ AOD ≅ Δ BOC and (ii) AD || BC



Sol: (i) In Δ AOD and Δ BOC

OA = OB (Given)

OD = OC (Given)

∠ AOD = ∠ BOC (Vertically opposite angles)

Δ AOD ≅ Δ BOC (by the SAS congruence rule)

(ii) Since Δ AOD ≅ Δ BOC

∠ OAD = ∠ OBC (CPCT)

Alternate interior angles are equal

∴ AD || BC

Example 2 : AB is a line segment and line l is its perpendicular bisector. If a point P lies on l, show that P is equidistant from A and B.

Sol: l is perpendicular bisector of AB

Δ PCA and Δ PCB.

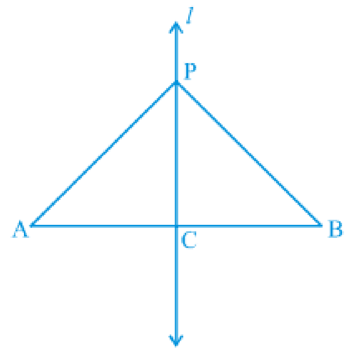
AC = BC (C is midpoint of AB)

$\angle$ PCA = $\angle$ PCB = 90° ($l \perp$ AB)

PC = PC (Common)

$So, \Delta\, PCA \cong \Delta\, PCB$ (SAS $congruence$ $rule$)

$PA = PB$ ($CPCT$)



ASA congruence rule : Two triangles are congruent if two angles and the included side of one triangle are equal to two angles and the included side of other triangle.

AAS and SAA are same as ASA congruence rule.

Example 3 : Line-segment AB is parallel to another line-segment CD. O is the mid-point of AD (see Fig. 7.15). Show that (i) ΔAOB ≅ ΔDOC (ii) O is also the mid-point of BC.

Sol: In Δ AOB and Δ DOC.

$\angle$ ABO = $\angle$ DCO (Alternate interior angles)

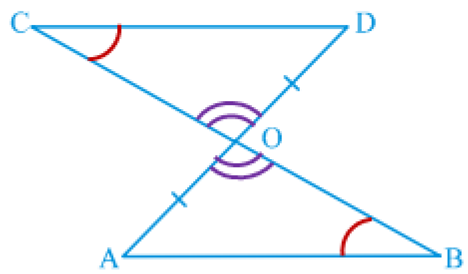
OA = OD (Given)

$\angle$ AOB = $\angle$ DOC (Vertically opposite angles)

$\therefore \Delta AOB \cong \Delta DOC$ (ASA rule)

OB = OC (CPCT)

So, O is the mid-point of BC.



EXERCISE 7.1

1. **In quadrilateral ACBD, AC = AD and AB bisects $\angle$ A (see Fig. 7.16). Show that Δ ABC ≅ Δ ABD. What can you say about BC and BD?**

Sol: In ΔABC and ΔABD

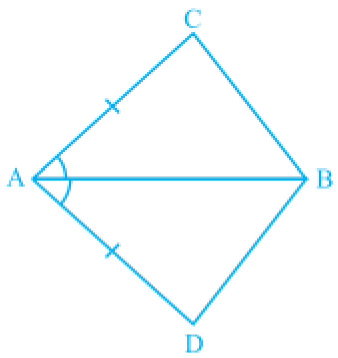
AC = AD (Given)

$\angle$ BAC = $\angle$ BAD (AB bisects $\angle$ A)

AB=AB (Common)

Δ ABC ≅ Δ ABD (SAS congruency rule)

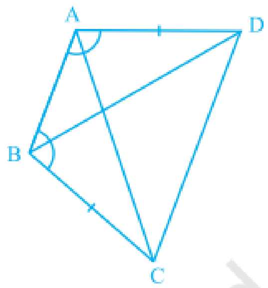
BC=BD(CPCT)



2. **ABCD is a quadrilateral in which AD = BC and $\angle$ DAB = $\angle$ CBA (see Fig. 7.17). Prove that**

   (i) Δ ABD ≅ Δ BAC (ii) BD = AC (iii) $\angle$ ABD = $\angle$ BAC.

Sol: (i) In Δ ABD and Δ BAC



AD = BC (Given)

∠ DAB = ∠ CBA (Given)

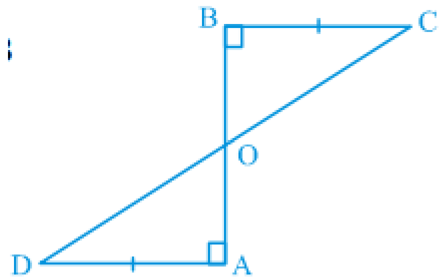
AB=AB (Common)

Δ ABD ≅ Δ BAC (SAS congruence rule)

(ii) Δ ABD ≅ Δ BAC ⇒ BD = AC (CPCT)

(iii) Δ ABD ≅ Δ BAC ⇒ ∠ ABD = ∠ BAC (CPCT)

3. AD and BC are equal perpendiculars to a line segment AB (see Fig. 7.18). Show that CD bisects AB.



Sol: In ΔOAD, ΔOBC

$\angle OAD = \angle OBC = 90^0$ (*Given*)

$\angle AOD = \angle BOC$ (*Vertically opposite angles*)

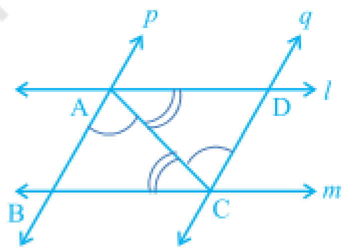
AD=BC (Given)

ΔOAD≅ΔOBC (AAS congruence rule)

OA=OB

∴ CD bisects AB.

4. *l* and *m* are two parallel lines intersected by another pair of parallel lines p and q (see Fig. 7.19). Show that Δ ABC ≅ Δ CDA.



Sol: In ΔABC and ΔCDA

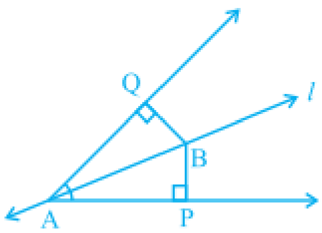
$\angle BAC = \angle DCA$ ($p||q$, *Alternate interior angles*)

AC=AC(Common)

$\angle BCA = \angle DAC$ ($l||m$, *Alternate interior angles*)

ΔABC ≅ΔCDA (By ASA congruence rule)

5. Line *l* is the bisector of an angle ∠ A and B is any point on *l*. BP and BQ are perpendiculars from B to the arms of ∠ A (see Fig. 7.20). Show that: (i) Δ APB ≅ Δ AQB (ii) BP = BQ or B is equidistant from the arms of ∠ A.



Sol: (i) In ΔAPB and ΔAQB

$\angle BAP = \angle BAQ$ (*l is the angle bisector of* ∠*A*)

AB=AB(Common)

$\angle APB = \angle AQB = 90^0$

Δ APB ≅Δ AQB (By ASA congruence rule)

(ii) Δ APB ≅Δ AQB

BP=BQ (By CPCT)

B is equidistant from the arms of ∠ A

6. In Fig. 7.21, AC = AE, AB = AD and ∠ BAD = ∠ EAC. Show that BC = DE.

Sol: ∠ BAD = ∠ EAC (Given)

∠BAD + ∠DAC = ∠EAC +∠∠DAC

$\angle$ BAC = $\angle$ DAE→(1)

In ΔBAC and ΔDAE

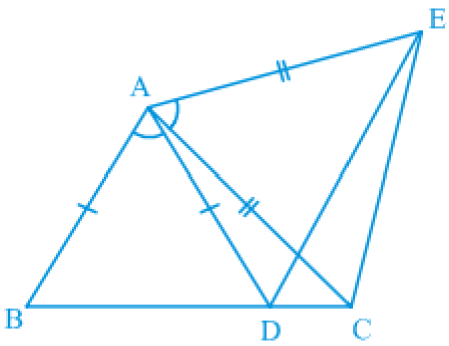
AB = AD (Given)

$\angle$ BAC = $\angle$ DAE (From (1))

AC = AE (Given)

$\Delta BAC \cong \Delta DAE$ (*By SAS congruence rule*)

∴ BC = DE (By CPCT)



7. AB is a line segment and P is its mid-point. D and E are points on the same side of AB such that ∠ BAD = ∠ ABE and ∠ EPA = ∠ DPB (see Fig. 7.22). Show that (i) Δ DAP ≅ Δ EBP (ii) AD = BE

Sol: ∠ EPA = ∠ DPB (given)

∠ EPA+∠DPE = ∠ DPB+∠DPE

∴∠APD=∠BPE →(1)

In ΔAPD and ΔBPE

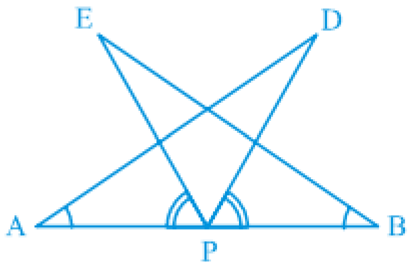
∠ BAD = ∠ ABE (Given)

AP=BP (P is midpoint of AB)

∠APD=∠BPE (From (1))

Δ APD ≅ ΔBPE (*By ASA congruence rule*)

∴ AD = BE (By CPCT)



8. In right triangle ABC, right angled at C, M is the mid-point of hypotenuse AB. C is joined to M and produced to a point D such that DM = CM. Point D is joined to point B (see Fig. 7.23). Show that: (i) Δ AMC ≅ Δ BMD (ii) ∠ DBC is a right angle. (iii) $\Delta\, \boldsymbol{DBC} \cong \Delta\, \boldsymbol{ACB}$ (iv) $\boldsymbol{CM} = \frac{1}{2}\boldsymbol{AB}$

Sol: (i) In Δ AMC and Δ BMD

AM=BM (M is midpoint of AB)

∠ AMC=∠ BMD (Vertically opposite angles)

DM = CM (Given)

∴Δ AMC ≅ Δ BMD (By SAS congruence rule)

(ii) Δ AMC ≅ Δ BMD

∠ACM=∠BDM (By CPCT)

Alternate interior angles are equal

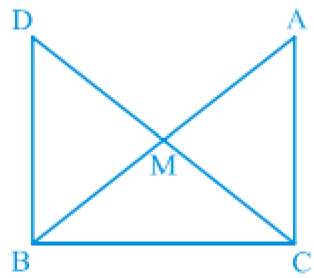
∴ DB || AC

∠DBC +∠ACB=180^0 (co-interior angles are supplementary)

∠DBC+90^0 =180^0(Given ∠ACB=90^0)

∴∠DBC=90^0

(iii) *In* Δ *DBC and* Δ *ACB*



DB=AC (Δ AMC ≅ Δ BMD)

∠DBC=∠ACB = 90^0

BC=CB(common)

$\Delta\, DBC \;\cong\; \Delta\, ACB\; (By\; SAS\; congruence\; rule)$

(iv) $\Delta\, DBC \;\cong\; \Delta\, ACB$

AB=DC (by CPCT)

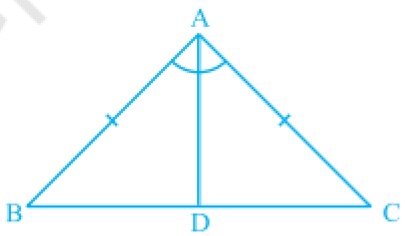
AB=2 CM (CM=DM)

$$CM = \frac{1}{2}AB$$

Some Properties of a Triangle

Theorem 7.2 : Angles opposite to equal sides of an isosceles triangle are equal.

Sol: ΔABC is an isosceles triangle in which AB=AC



Draw AD is angle bisector of ∠A

In Δ BAD and Δ CAD

AB = AC (Given)

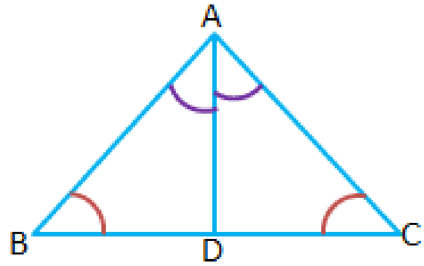
∠ BAD = ∠ CAD (By construction)

AD = AD (Common)

So, Δ BAD ≅ Δ CAD (By SAS rule)

∠B = ∠C (CPCT)

Theorem 7.3 : The sides opposite to equal angles of a triangle are equal.



Proof: In ΔABC, ∠B=∠C

Draw AD is angle bisector of ∠A

In Δ BAD and Δ CAD

∠B=∠C (given)

∠ BAD = ∠ CAD (By construction)

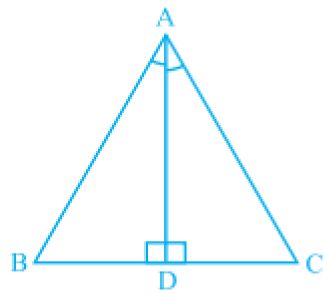
AD = AD (Common)

So, Δ BAD ≅ Δ CAD (By AAS congruence rule)

AB = AC (by CPCT)

Example 4 : In Δ ABC, the bisector AD of ∠ A is perpendicular to side BC (see Fig. 7.27). Show that AB = AC and Δ ABC is isosceles.

Sol: In ΔABD and ΔACD,



∠ BAD = ∠ CAD (Given)

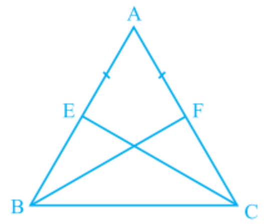
AD = AD (Common)

∠ ADB = ∠ ADC = 90° (Given)

So, Δ ABD ≅ Δ ACD (ASA rule)

So, AB = AC (CPCT) or, Δ ABC is an isosceles triangle.

Example 5 : E and F are respectively the mid-points of equal sides AB and AC of Δ ABC (see Fig. 7.28). Show that BF = CE.



Sol: In Δ ABF and Δ ACE,

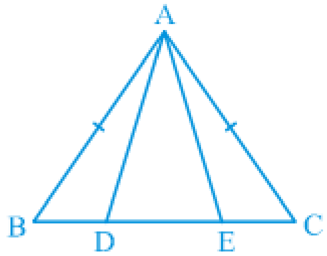
AB = AC (Given)

∠ A = ∠ A (Common)

AF = AE (Halves of equal sides)

So, Δ ABF ≅ Δ ACE (SAS rule) Therefore, BF = CE (CPCT)

Example 6 : In an isosceles triangle ABC with AB = AC, D and E are points on BC such that BE = CD (see Fig. 7.29). Show that AD = AE.



Sol: In Δ ABE and Δ ACD,

AB = AC (Given)

∠ B = ∠ C (Angles opposite to equal sides)

BE = CD (Given)

So, Δ ABE ≅ Δ ACD (SAS congruence rule)

⇒AE = AD (CPCT)

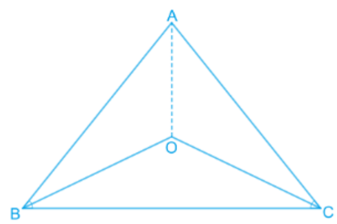
EXERCISE 7.2

1. In an isosceles triangle ABC, with AB = AC, the bisectors of ∠ B and ∠ C intersect each other at O. Join A to O. Show that : (i) OB = OC (ii) AO bisects ∠A

Sol: (i) In ΔABC, The bisectors of ∠ B and ∠ C intersect each other at O

$\angle OBA = \angle OBC = \frac{1}{2}\angle ABC \text{ and } \angle OCA = \angle OCB = \frac{1}{2}\angle ACB$

Given AB = AC



$\Rightarrow \angle ABC = \angle ACB$ (Angles opposite to equal sides)

$\Rightarrow \frac{1}{2}\angle ABC = \frac{1}{2}\angle ACB \Rightarrow \angle OBC = \angle OCB$

$In\ \Delta OBC, \angle OBC = \angle OCB$

$\Rightarrow OB = OC$ (Sides opposite to equal angles) $\rightarrow (i)$

$(ii) \Delta OAB \text{ and } \Delta OAC$

OA=OA(Common)

AB=AC (Given)

OB = OC $(From\ (i))$

ΔOAB ≅ ΔOAC $(SSS\ congruence\ rule)$

⇒ ∠OAB = ∠OAC $(CPCT)$

⇒ AO bisects ∠A

2. In Δ ABC, AD is the perpendicular bisector of BC (see Fig. 7.30). Show that Δ ABC is an isosceles triangle in which AB = AC.

Sol: In ΔADB and ΔADC

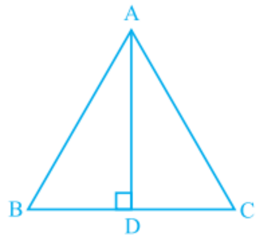
AD = AD (Common)

∠ADB = ∠ADC = 90^0(AD ⊥ BC)

BD = DC(AD is bisector of BC)

ΔADB ≅ ΔADC (SAS congruence rule)

⇒ AB = AC (CPCT)



3. ABC is an isosceles triangle in which altitudes BE and CF are drawn to equal sides AC and AB respectively (see Fig. 7.31). Show that these altitudes are equal.

Sol: Δ ABC is an isosceles triangle. AB=AC

In ΔAEB and ΔAFC

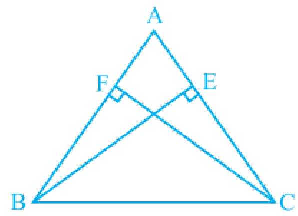
∠A = ∠A (Common angle)

∠AEB = ∠AFC = 90^0 (BE ⊥ AC and CF ⊥ AB)

AB = AC(Given)

ΔAEB ≅ ΔAFC (AAS congruence rule)

⇒ BE = CF (CPCT)



4. ABC is a triangle in which altitudes BE and CF to sides AC and AB are equal (see Fig. 7.32). Show that

(i) Δ ABE ≅ Δ ACF (ii) AB = AC, i.e., ABC is an isosceles triangle.

Sol: In Δ ABE and Δ ACF

∠A = ∠A (Common angle)

∠AEB = ∠AFC = 90^0 (BE ⊥ AC and CF ⊥ AB)

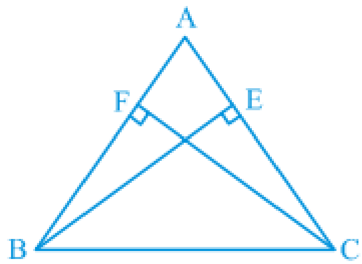
BE = CF(Given)

Δ ABE ≅ Δ ACF (AAS Congruence rule)

(ii) Δ ABE ≅ Δ ACF

⇒ AB = AC (CPCT)

ΔABC is an isosceles triangle.



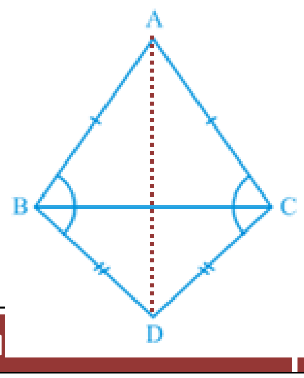
5. ABC and DBC are two isosceles triangles on the same base BC (see Fig. 7.33). Show that ∠ ABD = ∠ACD

Sol: In Δ ABD and Δ ACD

AB = AC (Given)

BD = CD(Given)

AD = AD (Common side)



Δ ABD $\cong$ Δ ACD (SSS Congruence rule)

$\Rightarrow \angle ABD = \angle ACD$(CPCT)

6. ΔABC is an isosceles triangle in which AB = AC. Side BA is produced to D such that AD = AB (see Fig. 7.34). Show that ∠ BCD is a right angle.

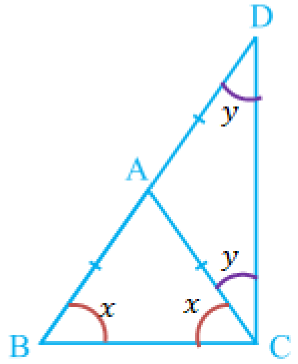
Sol: $In\ \Delta ABC\ , AB = AC$

$\Rightarrow \angle ABC = \angle ACB = x$ (Angles opposite to equal sides are equal)

In Δ ADC, AD = AC

$\Rightarrow \angle ACD = \angle ADC = y$ (Angles opposite to equal sides are equal)

$\angle BCD = \angle ACB + \angle ACD = x + y$



In ΔBDC,

$\angle ABC + \angle ADC + \angle BCD = 180^0$ (Angle sum property of a triangle)

$x + y + (x + y) = 180^0$

$2(x + y) = 180^0$

$(x + y) = \frac{180^0}{2} = 90^0$

$\angle BCD = 90^0$

7. ABC is a right angled triangle in which ∠ A = 90° and AB = AC. Find ∠ B and ∠ C.

Sol: $In\ \Delta ABC$, AB = AC

$\Rightarrow \angle C = \angle B = x$ $(Equal\ sides\ opposite\ angles\ are\ equal)$

$\angle A + \angle B + \angle C = 180^0$ $(Angle\ sum\ property\ of\ a\ triangle)$

$90^0 + x + x = 180^0$

$2x = 90^0$

$x = \frac{90^0}{2} = 45^0$

$\angle B = \angle C = 45^0$

8. Show that the angles of an equilateral triangle are 60° each.

Sol: $Let\ \Delta ABC\ is\ an\ equlateral\ triangle$

$\Rightarrow AB = BC = AC$

$\Rightarrow \angle A = \angle B = \angle C = x$ $(Equal\ sides\ opposite\ angles\ are\ equal)$

$\angle A + \angle B + \angle C = 180^0$

$\Rightarrow x + x + x = 180^0$

$\Rightarrow 3x = 180^0$

$\Rightarrow x = \frac{180^0}{3} = 60^0$

So, each angle of an equilateral triangle is 60^0

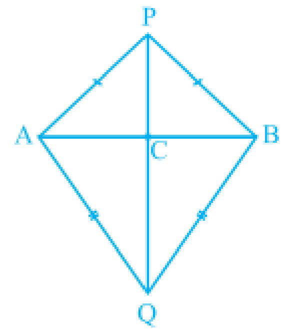
SSS congruence rule:

If three sides of one triangle are equal to the three sides of another triangle, then the two triangles are congruent

RHS congruence rule:

If in two right triangles the hypotenuse and one side of one triangle are equal to the hypotenuse and one side of the other triangle, then the two triangles are congruent.

Example 7 : AB is a line-segment. P and Q are points on opposite sides of AB such that each of them is equidistant from the points A and B (see Fig. 7.37). Show that the line PQ is the perpendicular bisector of AB.



Sol: In Δ PAQ and Δ PBQ.

PA=PB(Given)

AQ=BQ(Given)

PQ=PQ(Common)

Δ PAQ $\cong$ Δ PBQ (SSS rule)

$\angle APQ = \angle BPQ\ (CPCT) \Rightarrow \angle APC = \angle BPC \rightarrow (1)$

In Δ PAC and Δ PBC.

AP = BP (Given)

$\angle$ APC = $\angle$ BPC (From (1))

PC = PC (Common)

Δ PAC $\cong$ Δ PBC (SAS rule)

$AC = BC\ (CPCT) \rightarrow (2)$

$\angle$ ACP = $\angle$ BCP (CPCT)

$\angle$ ACP + $\angle$ BCP = 180° (Linear pair)

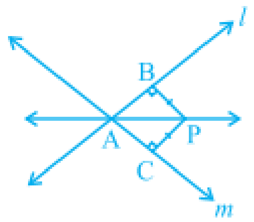
2$\angle$ ACP = 180°

$\angle ACP = 90° \rightarrow (3)$

From (2) and (3)

PQ is perpendicular bisector of AB

Example 8 : P is a point equidistant from two lines l and m intersecting at point A (see Fig. 7.38). Show that the line AP bisects the angle between them.



Sol: Let PB $\perp$ l, PC $\perp$ m. It is given that PB = PC

In Δ PAB and Δ PAC

$\angle$ PBA = $\angle$ PCA = 90° (Given)

PA = PA (Common)

PB = PC (Given)

Δ PAB $\cong$ Δ PAC (RHS rule)

$\angle$ PAB = $\angle$ PAC (CPCT)

AP bisects the angle between l and m

EXERCISE 7.3

1. Δ ABC and Δ DBC are two isosceles triangles on the same base BC and vertices A and D are on the same side of BC (see Fig. 7.39). If AD is extended to intersect BC at P, show that (i) Δ ABD ≅ Δ ACD (ii) Δ ABP≅ Δ ACP (iii) AP bisects ∠ A as well as ∠ D. (iv) AP is the perpendicular bisector of BC.

Sol: $(i) In\ \Delta ABD\ and\ \Delta ACD$

$AB = AC\ (Given)$

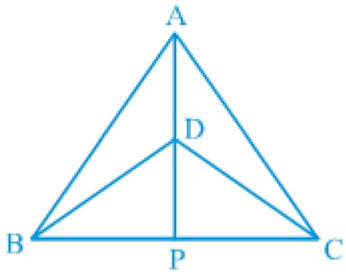
BD = CD(Given)

AD = AD (Common)

ΔABD ≅ ΔACD (SSS rule)

∠BAD = ∠CAD (CPCT)

i.e ∠BAP = ∠CAP → (1)



(ii) Δ ABP≅ Δ ACP

AB = AC (Given)

∠BAP = ∠CAP (From (1))

AP = AP (Common)

Δ ABP ≅ Δ ACP (SAS rule)

∠APB = ∠APC(CPCT) → (2)

i.e ∠DPB = ∠DPC → (3)

Also BP = PC (CPCT) → (4)

(iii) From (1) and (2)

AP bisects ∠ A as well as ∠ D

(iv) $\angle APB + \angle APC = 180^0$ (Linear pair)

From (2) ;∠APB = ∠APC

$\therefore \angle APB = \angle APC = 90^0 \rightarrow (5)$

(iv) From (4) and (5)

AP is the perpendicular bisector of BC.

2. AD is an altitude of an isosceles triangle ABC in which AB = AC. Show that (i) AD bisects BC (ii) AD bisects ∠ A.

Sol: In ΔADB and ΔADC

AB = AC(Given)

$\angle ADB = \angle ADC = 90^0$ (AD ⊥ BC)

AD = AD(Common)

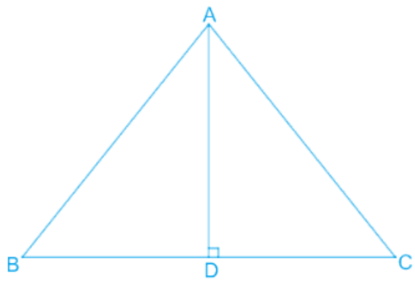
ΔADB ≅ ΔADC (RHS Congruence rule)

BD=CD (CPCT)

Hence , AD bisects BC

∠BAD = ∠CAD (CPCT)

AD bisects ∠A



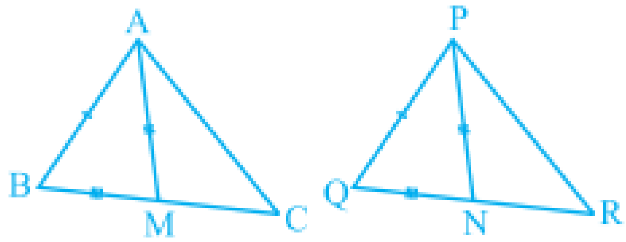
3. Two sides AB and BC and median AM of one triangle ABC are respectively equal to sides PQ and QR and median PN of Δ PQR (see Fig. 7.40). Show that (i) Δ ABM ≅ Δ PQN (ii) Δ ABC ≅ Δ PQR

Sol: In ΔABC , AM is median

$BM = CM = \frac{1}{2}BC$

In ΔPQR , PN is median

$QN = NR = \frac{1}{2}QR$



Given BC=QR

$\frac{1}{2}BC = \frac{1}{2}QR$

$BM = QN \rightarrow (1)$

(i) Δ ABM ≅ Δ PQN

$AB = PQ(Given)$

$AM = PN(Given)$

$BM = QN \ \ (From(1))$

$\Delta\, ABM \ \cong \ \Delta\, PQN\ (SSS\ rule)$

$BM = QN \ \rightarrow (1)$

$\angle ABM = \angle\text{PQN (CPCT)}$

$\angle ABC = \angle\text{PQR} \rightarrow (2)$

(ii) In Δ ABC and Δ PQR

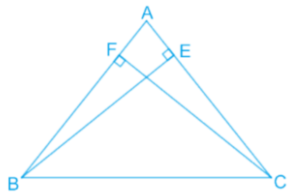
$AB = PQ(Given)$

$\angle ABC = \angle\text{PQR(From(2))}$

$BC = QR\ (Given)$

$\Delta\, ABC \ \cong \ \Delta\, PQR\ (ASA\ rule)$

4. BE and CF are two equal altitudes of a triangle ABC. Using RHS congruence rule, prove that the triangle ABC is isosceles.



Sol: *In* Δ BEC and Δ CFB

∠ BEC = ∠ CFB = 90^0 (BE and CF are two altitudes)

$BC = BC\ (Common)$

$BE = CF\ (Given)$

Δ BEC ≅ Δ CFB (RHS rule)

∠BCE = ∠CBF (CPCT)

i.e, ∠BCA = ∠CBA

AB=AC (Sides opposite to equal angles are equal)

Hence, ΔABC is isosceles triangle.

5. ABC is an isosceles triangle with AB = AC. Draw AP ⊥ BC to show that ∠ B = ∠ C.

Sol: *In* ΔAPB *and* ΔAPC

∠APB = ∠APC = 90^0

AB = AC

AP = AP (Common)

ΔAPB ≅ ΔAPC (RHS rule)

∠B = ∠C (CPCT)